Quantum Damour Equation

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1 Overview of the Field

Physics on null hypersurfaces is rich of unusual features. Gravity induces constraints on a null hypersurface, which are the projection of the equations of motion. These are the celebrated Raychaudhuri [1] and Damour [2] equations. They describe how the gravitational data on a null hypersurface are constrained as time evolves. In [3], we focused on the Raychaudhuri constraint and proved 3 main results. First, we showed that it can be recast fully intrinsically as the conservation law for a Carrollian stress tensor, reviewing and ameliorating the membrane paradigm [4]. Inspired by [5, 6], we secondly constructed the phase space and charges, proving in particular how the various symplectic sectors of the theory intertwine together in the symplectic structure and Poisson brackets. In particular, the gravitational data induced on a null hypersurface can be conveniently split into a spin-0, spin-1, and spin-2 symplectic pairs, where ”spin” here stands for the number of indices of a field on a cut. Lastly, we introduced the notion of dressing time, which is a particular clock with which we dressed the symplectic variables. This time is conjugated to the Raychaudhuri constraint. In a forthcoming paper [7], we will further report on the Raychaudhuri equation, and how quantization of gravity on null hypersurfaces is dramatically different from the quantization on a time- or space-like hypersurface. We regard this as a promising avenue of investigation, where the unexplored and surprising features of gravity on null hypersurfaces could help us unveiling a controlled quantization of the gravitational phase space.

The dynamics induced by the Raychaudhuri and Damour equations inherently follows the split into spin-0, 1, and 2 data. The spin-2 data encode the radiative degrees of freedom sourcing the geometric spin-0 and spin-1 fields. The dynamical equations display a similar pattern: the Raychaudhuri equation dictates how the spin-0 data evolve due to radiation and external matter, while the Damour constraint expresses how the spin-1 data evolve due to these effects. In our analysis prior to this research group meeting, we have consistently restricted our attention to the spin-0 evolution only, that is, to the Raychaudhuri constraint. Clearly, a step further in this direction requires to include the Damour constraint, and thus the spin-1 sector, in our analysis. This presents two challenges: the first one is to study the classical phase space, and dressing apparatus, while the second one is the quantization of the phase space including the Damour symplectic data. It was the primary purpose of our BIRS research group to address these two challenges.

2 Scientific Progress Made

We made progress in understanding how to incorporate the spin-1 data in the phase space.

We first showed how to parameterize the phase space, such that the three sectors (spin-0, 1, and 2) appear in the canonical symplectic potential on the hypersurface. We then focused on the freedom to shift the (Ehresmann) connection used in the geometric description of null hypersurfaces [8]. We demonstrated that this is indeed a freedom: the stress tensor and equations of motion are independent of this shift. Importantly, we then proved that such a freedom is pure gauge: the charge associated to this transformation identically vanishes. This allows us to use this gauge transformation to couple with the hypersurface diffeomorphisms, such that the Ehresmann connection does not change on phase space. This is useful to introduce a universal notion of time, such that we can construct the Poisson brackets of all the gravitational data. We then discussed how to introduce not only a dressing time but also a dressing shift, such that both the Raychaudhuri and Damour constraints appear in the symplectic structure.

We secondly addressed the challenge of quantization. We briefly touched upon this more far-reaching goal, studying how the brackets can be canonically quantized, and the consequences on the transverse charges, which are intimately related to the Damour constraint. Setting up a perturbative semi-classical scheme brings various choices and a certain level of arbitrariness that we carefully studied. In particular, we focussed on understanding under which conditions the various sectors of the theory can be disentangled and studied separately.

3 Outcome of the Meeting

We plan to report in two separate manuscripts the results and ideas discussed in this research group.

The first manuscript will collect the classical results: the introduction of the shift symmetry, the analysis of the complete phase space, and the construction of the Poisson brackets including the spin-1 data. It will then display the dressing apparatus, such that the Raychaudhuri constraint is conjugated to the dressing time, while the Damour constraint is conjugated to the dressing shift. The outcome will be a complete description of the classical phase space of gravity induced to a arbitrary null hypersurface. It will in particular encompass horizons, but also more general null hypersurfaces, characterized by non-trivial expansion, shear, and angular momentum.

The second manuscript is more in an embryonal phase. It will discuss aspects of the quantization of the phase space on null hypersurfaces. An important part of it will be devoted to the semi-classical approach. It will then present a more systematic discussion of what we intend to call ”mesoscopic quantum gravity”, which is a scale between semi-classical gravity and more top/down approaches to quantum gravity, such as string theory or loop quantum gravity. At this mesoscopic scale we maintain contact with classical aspects and control over symmetries, but avoid quantum field theoretical divergences arising without imposing the gravitational constraints.

References

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