New trends in Mathematics of Dispersive, Integrable and Nonintegrable Models in Fluids, Waves and Quantum Physics (22w5018)

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1 Introduction

This report describes the main scientific and nonscientific outputs obtained in the workshop "New Trends in Mathematics of Dispersive, Integrable and Nonintegrable Models in Fluids, Waves and Quantum Physics", held in Banff on October 09 –14, 2022. The web page of the workshop is `https://www.birs.ca/events/2022/5-day-workshops/22w5018`.

First of all, we acknowledge all the participants of this workshop. We also deeply acknowledge the Banff Research Center for their confidence and approval of our proposal.

Back in 2019, when we submitted this proposal, we had in mind three key elements that we expected would have made it different from other Partial Differential Equations (PDEs) workshops:

- It should have a **strong emphasis on listening to young underrepresented researchers**, specially from Latin American countries.
- A special emphasis was supposed to be given to listening and promoting **women researchers** in our area.
- We also wanted to have an important amount of **free time to allow researchers to interact each day**.

We tried the best we could, taking action in the three previously mentioned directions. We were happy to have with us young researchers from Argentina, Chile, Colombia, Ecuador, Brazil, Venezuela, Mexico, and Cuba, together with European and North-American researchers.

The title of the workshop looks like a mixture of different, unrelated areas. However, if one looks at the people involved here in the workshop, one understands the purpose behind an event with such a title: gathering people in integrable (Benjamin-Ono, KP-II, PCField), nonintegrable (ZK, fKdV) dispersive models, fluids (Navier-Stokes, Euler), waves (Klein Gordon, solitary waves) and quantum physics (Dirac), among other topics. We really hope to continue holding more versions of this workshop!

2 Overview of the Field

This workshop brought together researchers from several areas of PDEs. These areas will be specified below. The final list of participants is given in Table 1.

Table 1: List of participants

Name	Affiliation	Region
Alejo, Miguel Angel	Universidad de Córdoba, Spain	Europe 1
Cassano, Biagio	Universitá degli Studi della Campania, Italy	Europe 2
Cavalcante, Marcio	Universidade Federal de Alagoas, Brazil	Latin America 1
Chen, Gong	Georgia Institute of Technology, USA	USA and Canada 1
Corcho, Adán J.	Universidade Federal do Rio de Janeiro, Brazil	Latin America 2
Cortez, Fernando	Escuela Politécnica Nacional Quito, Ecuador	Latin America 3
Cossetti, Lucrezia	Karlsruhe Institute of Technology, Germany	Europe 3
del Pino, Manuel	University of Bath, UK	Europe 4
Esquivel, Liliana	Universidad del Valle, Colombia	Latin America 4
Fanelli, Luca	Ikerbasque & U. of Basque Country, Spain	Europe 5
Farah Dias, Luiz Gustavo	Universidade Federal de Minas Gerais, Brazil	Latin America 5
Gancedo, Francisco	University of Seville, Spain	Europe 6
García, Claudia	Universidad AutÃ³noma de Madrid, Spain	Europe 7
García-Juárez, Eduardo	Universitat de Barcelona, Spain	Europe 8
Gustafson, Stephen	University of British Columbia, Canada	USA and Canada 2
Ibrahim, Slim	University of Victoria, Canada	USA and Canada 3
Jarrin, Oscar	Universidad de las Americas, Ecuador	Latin America 6
Klaus, Friedrich	Karlsruhe Institute of Technology, Germany	Europe 9
Linares, Felipe	IMPA, Brazil	Latin America 7
Martínez Martini, María E.	Université París Cité, France	Europe 10
Maulen, Christopher	Universitet Bielefeld, Germany	Europe 11
Mizumachi, Tetsu	Hiroshima University, Japan	Japan 1
Muñoz, Claudio	Universidad de Chile, Chile	Latin America 8
Nakanishi, Kenji	Kyoto University, Japan	Japan 2
Ntekoume, Maria	Rice University, USA	USA and Canada 4
Pelinovsky, Dmitry	McMaster University, Canada	USA and Canada 5
Poblete, Felipe	Universidad Austral de Chile, Chile	Latin America 9
Poyato, David	University of Granada, Spain	Europe 12
Riaño, Oscar	Florida International University, USA	USA and Canada 6
Soler, Juan	U. Granada, Spain	Europe 13
Sun, Ruoci	Karlsruhe Institute of Technology, Germany	Europe 14
Trespalacios, Jessica	Universidad de Chile, Chile	Latin America 10
Valet, Frederic	University of Bergen, Norway	Europe 15
Van Den Bosch, Hanne	Universidad de Chile, Chile	Latin America 11
Wittenstein, Alexander	Karlsruhe Institute of Technology, Germany	Europe 16

From these 35 participants, we count: 11 Latin American based researchers, 16 from Europe, 6 from USA and Canada, and 2 from Japan. We also count 7 women researchers, around 20% of the participants of the workshop. Clearly these numbers are not enough and will be improved in forthcoming realizations of this workshop, and we hope to bring again researchers by 2024 or 2025 in a new edition of this workshop highly concentrated in improving Latin American mathematics.

3 Recent Developments and Open Problems

3.1 Speakers and their current works

Given the multidisciplinary character of this workshop, there is no a solely field represented here, and researchers follow particular paths that are related by the study of physical models coming from Fluids, Quantum Physics, dispersive integrable and non-integrable, and others. In the dispersive part, we can mention recent developments:

In the dispersive community *Cavalcante, Esquivel* and *Corcho* have been advancing dispersive models in the half-line, as well as mixed Schrödinger-KdV type models. *Ntekoume* and *Klaus* have produced remarkable advances in the theory of Nonlinear Schrödinger models. *Gong Chen* and *Nakanishi* have been studying dispersive PDEs from several points of view, in particular via advanced Strichartz estimates applied to modulation models and other dispersive methods, specially applied in the Klein-Gordon case. *Ibrahim* has also studied these models and more recently Skyrmion models. *Farah* has been working in several models of dispersive type, such as Zakharov-Kuznetsov (solitons and blow-up) and nonlinear Schrodinger with a singular potential. *Sun* has considered and advanced during the past years the integrable Benjamin-Ono description of solutions. *Riaño* has worked in the generalized KdV models, as well as *Valet*. *Maulen* has studied the Boussinesq's models in the vicinity of solitons. *Poblete* and *Mizumachi* have been studying KP models in the most demanding case of the energy space. *Linares*, *Sun* and *Esquivel* have been advancing the Benjamin-Ono behavior of solutions in several directions. *Pelinovsky* has been studying solitary waves in non-standard form, such as cusps and peakons. Finally, *Cortez* and *Jarrín* have been studying mixed models of dispersive and fractional diffusion type.

Del Pino has been working in vortex filaments of the Euler's equations, and have given important descriptions of these localized structures in past years. *Gancedo* has advanced the theory of multi-phase flows, specially those of Muskat type. *García* has been working in the construction of vortex patches in Euler models, and *García-Juárez* has recently worked in the Peskin problem. *Gustafson* has been studying Euler flows under particular setting, and *Martínez* has been constructing solitary waves in the water waves model with variable bottom.

Fanelli has described in past years the spectral properties of different type of operators appearing in Physics, in particular those of Dirac and magnetic type. *Cassano* has also described Dirac type models, as well as *Van Den Bosch*. *Trespalacios* has considered the Einstein's field equations under particular symmetries.

Finally, in a fourth line of research, *Soler* has worked on the mean field limits in multi-agent systems, and joint with *Poyato* have advanced the field in an impressive fashion during past years.

3.2 Schedule: main objectives put down on earth

The workshop was scheduled in a fashion different to usual workshops in the area. The idea was to highlight as much as possible young emerging researchers. With this objective in mind, we decided the schedule of our workshop filling the first slots with talks by young promising researchers. While the more senior researchers were supposed to share their expertise and important insights in a second set of talks. Important emphasis was given to highlight as much as possible works by female researchers as well. This new schedule showed interesting outcomes and was very appreciated by the participants. It gave a refreshing style to the workshop, being one able to guess important directions for the future and forthcoming research.

The list of talks on Monday October 10 is given in Table 2.

As the reader may see, the talks of the first day were supposed to provide the audience with the introductory material for the understanding of all the different topics of the workshop. First of all, professor Gustafson introduced us to inviscid fluids. Then, professor Poyato was in charge of giving the first ideas of the mean field limit results that will be described during the workshop. Then María E. Martínez first introduced us to the collision problem in water wave models. The second part of the morning session was dedicated to new fluid models with the talk by professor García-Juárez.

Table 2: List of Talks given on Monday, October 10

Speaker	Talk
Stephen Gustafson	Growth rates for axisymmetric Euler flows
David Poyato	Mean field limit of non-exchangeable multi-agent systems
María E. Martínez	The soliton problem for the Zakharov water waves system with slowly varying bottom
Eduardo García-Juárez	Recent results for the Peskin problem
Lunch	Break
Oscar Riaño	Well-posedness and dynamics of solutions to the generalized KdV with low power nonlinearity
Christopher Maulén	Asymptotic stability manifolds for solitons in the generalized Good Boussinesq equation
Frederic Valet	Strong interaction of solitary waves for the fmKdV equation

The afternoon session was strongly focused on young researchers working in the area of dispersive PDEs: Professors Riaño, Maulén and Valet described new results in this area, and some of them will be described in further detail in particular sections below.

Tuesday October 11 session was as given in Table 3:

Table 3: List of Talks held on Tuesday, October 11

Speaker	Talk
Maria Ntekoume	Critical well-posedness for the derivative nonlinear Schrödinger equation on the line
Gong Chen	Dynamics of multi-solitons to Klein-Gordon equations
Manuel del Pino	Dynamics of concentrated vorticities in 2D and 3D euler flows
Kenji Nakanishi	Global dynamics around multi-solitons for the nonlinear Klein-Gordon equation
Lunch	Break
Luiz G. Farah	On the intercritical inhomogeneous NLS equation
Liliana Esquivel	On the Benjamin- Ono equation posed in a quarter plane
Felipe Poblete	Long time asymptotics of large data in the Kadomtsev-Petviashvili models
Biagio Cassano	General delta-shell interactions for the two-dimensional Dirac operator
Fernando Cortez	Sharp well-posedness and spatial decaying for a generalized Kuramoto-Velarde-type equation

The morning session started with reporting on the most recent results on dispersive PDEs, in particular on the derivative Schrödinger equation by Maria Ntekoume (Rice U.) and on Nonlinear Klein Gordon models by Gong Chen (Gatech). The second part of the morning session was concentrated on new fluid results by Manuel del Pino. In particular, he presented the most recent advances in the construction of solutions to 2D and 3D Euler equations concentrating vorticity on particular region and geometries in space. Finally, Nakanishi provided results in the same direction to those by Chen, with improved resolution in 3D cases. The afternoon session was again concentrated on listening to young researchers in several areas of research. Here we started with a strong dispersive PDE flavor with results by Farah, Esquivel and Poblete, to then continue with new results in mathematical physics and fluids by Cassano and Cortez.

The Wednesday session October 12 was short and concentrated in morning talks, as show Table 4, given by senior researchers that are the leading experts in the areas topic of our workshop (dispersive, fluids, many particle models, mathematical physics). Gancedo started describing global solutions in the Muskat problem, and then Pelinovsky described solitary waves in models where dispersion is inhomogeneous; Soler presented strong new results on the meand field limit in several key models of many particles, and finally Mizumachi presented important advances towards the understanding of KP-II multi-soliton lines.

Table 4: List of Talks held on Wednesday, October 12

Speaker	Talk
Francisco Gancedo	Global well-posedness for the one-phase Muskat problem
Dmitry Pelinovsky	Solitary waves under intensity-dependent dispersion
Juan Soler	Mean-field limit of Vlasov-Fokker-Planck equations
Tetsu Mizumachi	On linear stability of elastic 2-line solitons for the KP-II equation

Table 5 reviews talks held on Thursday October 13. This session was exclusively centered on online talks. During the morning, we had several talks focused on deep mathematical physics problems; see the talks by Fanelli, Trespalacios and Van Den Bosch. In the afternoon, we had talks by Brazilian and Ecuadorian based researchers dealing with fluids and dispersive models.

Table 5: List of Talks held on Thursday, October 13

Speaker	Talk
Luca Fanelli	On the eigenvalues of the Heisenberg Sublaplacian with a potential
Jessica Trespalacios	Global Existence and Long Time Behavior in the 1+1 dimensional Principal Chiral Model with Applications to Solitons
Hanne Van Den Bosch	Spectral stability in the nonlinear Dirac equation with Soler type nonlinearity
Ruoci Sun	Explicit formula of multi-solitary waves of the Benjamin-Ono equation
Lunch	Break
Felipe Linares	Local energy decay for solutions of the Benjamin-Ono equation
Adán Corcho	On the Cauchy Problem associated to a Nonequilibrium Bose-Einstein Condensate
Marcio Cavalcante	Stability of mKdV Breathers the half-line
Oscar Jarrin	From anomalous to classical diffusion in a non-linear heat equation

Finally, Table 6 describes the short Friday session October 14, where we ended the workshop with three talks describing the last advances in fluids, high energy physics and dispersive models.

Table 6: List of Talks held on Friday, October 14

Speaker	Talk
Claudia García	Self-similar spirals for the generalized surface quasi-geostrophic equations
Slim Ibrahim	Phase transition threshold and stability of magnetic skyrmions
Friedrich Klaus	NLS with slowly decaying and non-decaying initial data

As one can notice, we decided to have always mixed sessions were all the topics of the workshop were covered. This allowed people to immerse themselves in every proposed topic of the workshop. From the point of view of the organizers, this method of work was excellent and allowed an important comprehension of the converging fields.

In the next sections, we will describe in deeper detail the main contributions made by the speakers and researchers during this workshop.

4 Scientific Progress Made

Many highly relevant results were presented in this workshop. We do not plan to highlight all of them by space reasons, but only some particular ones.

4.1 Fluid dynamics

This section of the workshop presented impressive new advances. Starting with the upper and lower bounds on the growth of Euler flows obtained by **Gustafson**, Miller and Tsai, we have had the opportunity to listen a series of notable results in an area of huge advances today. More precisely, Gustafson presented upper and lower bounds on the growth of Euler flows, in three and higher dimensions, of vortex-tube-pair-type (axisymmetric, swirl-free, vorticity satisfying certain sign, oddness, and decay properties), generalizing and improving bounds of Choi-Jeong in three dimensions.

Presentation Highlight. In another impressive result, **García** constructed a large class of non-trivial (non radial) self-similar solutions of the generalized surface quasi-geostrophic equation [18]. Being probably the first rigorous construction of any self-similar solution for these equations. Solutions of this form are of spiral type, locally integrable and may have a change of sign.

4.1.1 Advances in the Peskin problem

Following the schedule described in previous sections, **García-Juárez** showed important advances in the Peskin problem, that models the dynamics of a closed elastic membrane immersed in an incompressible Stokes fluid [19]. This set of equations was proposed as a simplified model for the motion of red blood cells, and it serves as a canonical test problem for numerical methods. Mathematically, the problem can be seen as a generalization of the Stokes two-phase interface with surface tension, and it shares the linear structure with the Muskat problem. García-Juárez reviewed the latest well-posedness results, and proved the global regularity for the 2D Peskin with viscosity jump and the local well-posedness for 3D membranes.

4.1.2 Concentrated vorticities in 2D and 3D Euler models

Presentation Highlight 1. In an impressive talk, **Manuel del Pino** considered the classical problem of the dynamics of solutions to the Euler equations of an inviscid incompressible fluid when the vorticity of the solution is initially concentrated near isolated points in 2d or vortex lines in 3d [13]. This problem as a huge historical development that traces back to Helmholtz and Kirchhoff. del Pino discussed recent results on these solutions, including their existence and asymptotic behavior. Indeed, in collaboration with J. Dávila, A. Fernández, M. Musso and J. Wei, he described precise asymptotics of interacting vortices and traveling helices, and even more, their extension to the 2d generalized SQG.

Presentation Highlight 2. Another very much highlighted talk was given by **Francisco Gancedo**, who described new results of global well-posedness for the one-phase Muskat problem [20]. Indeed, the Muskat problem models the evolution of an incompressible fluid filtered in porous media driven by gravity. In his talk, he described the construction of global-in-time critical Muskat solutions, in the sense that initial Lipschitz graphs of arbitrary size provide global-in-time well-posedness for the stable scenario.

4.2 Dispersive models

4.2.1 Water wave models

This represented one of the most relevant topic of the workshop. As a matter of fact, the talks on this subject held during the workshop reported on striking and impressive advances in the field.

Presentation Highlight. The young researcher **Maria E. Martínez** contributed with probably the most exciting development, namely the construction of solitary waves and the study of collisions in the Zakharov water waves system with variable bottom [28]. This system arises as a free surface model for an irrotational and incompressible fluid under the influence of gravity. Such fluid is considered in a domain with rigid bottom and a free surface. When considering the pressure over the surface, Amick-Kirchgässner proved the existence of solitary waves (solutions that maintain its shape as they travel in time) of constant speed for the flat-bottom case. In her talk, Martínez described the behavior of the solitary wave solution of the flat-bottom problem when the bottom presents a (slight) change at some point. She presented the construction of solitary waves for the Zakharov water waves system with non-flat bottom, thus generalizing the Amick-Kirchgässner soliton.

4.2.2 The derivative and the inhomogeneous nonlinear Schrödinger models

Presentation Highlight. Additionally, **Maria Ntekoume** presented to us the most advanced results in the global well-posedness of the derivative nonlinear Schrödinger equation on the line [29]. This model is known to be completely integrable and L^2-critical with respect to scaling. However, until recently not much was known regarding the well-posendess of the equation below $H^{1,2}$, the Sobolev space with suitable weights. In her talk she presented joint work with Benjamin Harrop-Griffiths, Rowan Killip, and Monica Visan, and proved that the problem is well-posed in the critical space L^2 on the line, and she highlighted several recent results that led to this resolution.

The NLS model has been also mentioned during this workshop, with interesting new results in the case of Hardy type potentials. **Farah** considered the inhomogeneous nonlinear Schrödinger (INLS) equation [3]-[5], a generalization of the classical nonlinear Schrödinger equation. Focusing on the intercritical case, he discussed well-posedness, scattering and blow-up results in the radial and non-radial settings.

A third result of high interest was provided by **Corcho**, who presented a non-equilibrium Gross-Pitaevskii type system recently proposed to model exciton-polariton condensates [10]. This model presents numerous mathematical challenges, and the known previous methods do not seem to apply in a standard way to study the global dynamics and singularity formation. He considered initial data in Sobolev spaces defined on euclidean and periodic domains and proved global in-time existence results for small data (in all dimensions).

A final talk on the NLS model was presented by **Klaus**, who described NLS equations with slowly decaying and non-decaying initial data [25]. He presented global well-posedness for the tooth problem (that is initial data in $H^{s_1}(\mathbb{R})+H^{s_2}(\mathbb{T})$). He also mentioned local and global wellposedness results in modulation spaces, which include slowly decaying functions with a finite number of slowly decaying derivatives.

4.2.3 The soliton resolution in the Klein-Gordon and Boussinesq models

Presentation Highlight. An important third focus was given by the talks by **Gong Chen and Kenji Nakanishi**. Both presented the most recent new results on the dynamics of multi-solitons to Klein-Gordon equations in 3D, including their asymptotic stability and classification. In particular, Nakanishi considered the Klein-Gordon equation with the focusing cubic power on $\mathbb{R}^3$ [22]. From Côte and Muñoz (2014) [12] there are solutions asymptotic to sum of solitons generated by the Lorentz transform from the ground state. In view of their instability and the soliton resolution conjecture, it is natural to study global behavior of solutions initially close to those multi-solitons. Assuming that the solitons are far enough and getting away from each other, he classified the initial data in a small neighborhood into open sets of scattering and blow-up, and a connected union of manifolds consisting of multi-solitons of various numbers. To treat globally the remainder, a key ingredient is a local energy estimate for the Klein-Gordon equation along multi-soliton trajectories, together with uniform decay for their mutual distance. It may be regarded as an extension of the classical estimate by Morawetz, including the proof, but it becomes tricky for the uniform decay, since we may not simply localize the Morawetz multiplier, which requires a space-time vector field with non-negative Jacobian matrix.

A closely related model is the Good-Boussinesq model in 1D. **Maulén** considered in his talk the so-called generalized Good-Boussinesq model in one dimension, with power nonlinearity and data in the energy space [30]. He presented the long time behavior of zero speed solitary waves, or standing waves. By using virial identities, in the spirit of Kowalczyk, Martel and Muñoz [26]-[27], he constructed and characterized a manifold of even-odd initial data around the standing wave for which there is asymptotic stability in the energy space.

4.2.4 The dynamics in the Benjamin-Ono model

Another model that was deeply studied in this workshop was the Benjamin-Ono equation. **Linares** presented recent results regarding the asymptotic behavior of solutions to the associated initial value problem, and solutions decay to zero in the energy space in an appropriate domain [17]. The result is independent of the integrability of the equation involved and it does not require any size assumption. **Sun** presented an explicit formula of multi-solitary waves. Indeed, every multi-soliton manifold of the Benjamin-Ono equation on the line is invariant under the flow [35]. Its generalized action-angle coordinates allowed him to solve this equation by quadrature, obtaining an explicit expression of every multi-solitary wave.

Finally, **Esquivel** considered the inhomogeneous Dirichlet initial boundary value problem for the Benjamin-Ono equation formulated on the half line. First of all, she studied the global in time existence of solutions to this equation. The novelty of her work is that it combines two different approaches between the real and the complex analysis.

4.2.5 The Kadomtsev-Petviashvili models

Presentation Highlight. An important model to be considered in forthcoming years is the Kadomtsev-Petviashvili (KP) equations posed on $\mathbb{R}^2$ and other geometries. **Poblete** provided sequential in time asymptotic descriptions of solutions obtained from arbitrarily large initial data, inside regions of the plane not containing lumps or line solitons, and under minimal regularity assumptions [31]. These results are consequence of two new virial identities adapted to the KP dynamics and do not require the use of the integrability of KP. In a second talk, **Mizumachi** considered the KP-II equation, a generalization of the KdV equation which takes slow variations in the transversal direction into account [32]. The KP-II equation has explicit multi-line soliton solutions. In his talk, he proved the linear stability of the 2-line soliton solutions of P-type, whose line solitons interact elastically.

4.2.6 The mKdV model: Fractional and breather dynamics

Valet presented important advances for the fractional modified Korteweg-de Vries equation, which enjoys the existence of solitary waves [15]; those solutions keep their form along the time and move with a constant velocity in one direction. He was interested in constructing solutions behaving at large time as a sum of two solitary waves with the same velocity. He first introduced the equation and the asymptotic behavior of solitary waves, and then stated the existence of solutions whose asymptotic behavior is a sum of two strongly interacting solitary waves with almost the same velocity.

A second important result was given by **Cavalcante**, who discussed the stability problem for mKdV breathers on the left half-line [1]. Indeed, he showed that leftwards moving breathers, initially located far away from the origin, are strongly stable for the problem posed on the left half-line, when assuming homogeneous boundary conditions. The proof involves a Lyapunov functional which is almost conserved by the mKdV flow once we control some boundary terms which naturally arise. Also, recent results about orbital and asymptotic stability of solitons on the positive half-line were discussed [7, 8].

4.2.7 Other models

There were other models treated in our workshop, not fitting directly into the previous ones. **Jarrín** described anomalous and classical diffusion in a non-linear heat equation with the natural polynomial non-linear term, and with two different cases in the diffusion term [24]. The first case (anomalous diffusion) concerns with the fractional Laplacian operator with parameter $1 < \alpha < 2$, while, the second case (classical diffusion) involves the classical Laplacian operator. When α approaches 2, he proved the uniform convergence of the solutions of the anomalous diffusion case to a solution of the classical diffusion case. Moreover, he rigorously derived a convergence rate, which was experimentally exhibited in previous related works.

Additionally, **Cortez** presented a generalization of the well-known Kuramoto-Velarde equation that also, at the same time, and under certain particularities, represents others interesting dispersive-dissipative equations in the field of fluid mechanics [11]. He showed that the initial value problem is locally well-posed (and globally in some cases) in certain Sobolev spaces.

Finally, **Riaño** showed well-posedness and the dynamics of solutions to the generalized KdV with low power nonlinearity. He studied two types of the generalized Korteweg-De Vries equation, where the nonlinearity is given with or without absolute value [34]. An example of this is the Schamel equation. He studied the local well-posedness of both equations in polynomial weighted Sobolev spaces, to then investigate the large time behavior of solutions numerically. He also ”confirmed” the soliton resolution for the positive and negative data.

4.3 Many agents systems

Presentation Highlight. This was another highlight of the workshop. **David Poyato** presented the mean field limit of non-exchangeable multi-agent systems [23]. More precisely, he showed the recent derivation of the mean-field limit for multi-agent systems on a large class of sparse graphs. The case of non-exchangeable multi-agent systems is consisting of non-identical agents, where the heterogeneous distribution of connectivities in the network is known to have critical effects on the collective dynamics. An important new result in this direction is that the method of proof does not only involve PDEs and stochastic analysis, but also graph theory through a novel concept of limits of sparse graphs (extended graphons) for the structure of the network, which can be regarded as a new non-trivial extension of the seminal works by L. Lovasz and B. Szegedy for dense graph limits. The proof is so stable that allows removing some of the main restrictive hypotheses in the previous literature on the connectivities between agents (dense graphs) and the cooperation between them (symmetric interactions).

Presentation Highlight. In a second, not less important talk, **Soler** introduced a novel approach to the mean-field limit of stochastic systems of interacting particles [2], leading to the first ever derivation of the mean-field limit to the Vlasov-Poisson-Fokker-Planck system for plasmas in dimension 2 together with a partial result in dimension 3. The method is broadly compatible with second order systems that lead to kinetic equations and it relies on novel estimates on the BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy. By taking advantage of the diffusion in velocity, those estimates bound weighted L^p norms of the marginals or observables of the system uniformly in the number of particles. This allows to treat very singular interaction kernels between the particles, including repulsive Poisson interactions.

4.4 Mathematical Physics

4.4.1 Massive nonlinear Dirac models

This topic was strongly described in this workshop, with several high level talks. **Hanne Van Den Bosch** described to the audience the spectral stability in the massive nonlinear Dirac equation with Soler type nonlinearity [14]. This nonlinearity takes the form of a space-dependent mass. Van Den Bosch described the standing wave solutions for frequencies below the mass in the Dirac operator. These standing waves are generally expected to be stable (i.e., small perturbations in the initial conditions stay small) based on numerical simulations, but there are very few mathematically rigorous results in this direction. During her talk, she discussed the simpler question of spectral stability (and instability), i.e., the absence (or presence) of exponentially growing solutions to the linearized equation around a solitary wave. As in the case of the nonlinear Schrödinger equation, this is equivalent to the presence or absence of "unstable eigenvalues" of a non-self-adjoint operator with a particular block structure. She presented some partial results for the one-dimensional case, highlighted the differences and similarities with the Schrödinger case, and discussed (a lot of) open problems.

In a second talk, **Cassano** considered the two-dimensional Dirac operator with general local singular interactions supported on a closed curve [6]. A systematic study of the interaction was performed by decomposing it into a linear combination of four elementary interactions: electrostatic, Lorentz scalar, magnetic, and a fourth one which can be absorbed by using unitary transformations. He addressed the self-adjointness and the spectral description of the underlying Dirac operator, describing its approximation by Dirac operators with regular potentials.

4.4.2 Solitons in General Relativity

An important new area of research is the soliton dynamics in General Relativity. Here **Jessica Trespalacios** showed the global existence and long time behavior in the 1+1 dimensional Principal Chiral Model (PCF), and mentioned its applications to solitons [36]. Indeed, this model is obtained as a simplification of the Vacuum Einstein Field equations under the Belinski-Zakharov symmetry. PCF is an integrable model, but a rigorous description of its evolution is far from complete. She provided existence of local solutions in a suitable chosen energy space, as well as small global smooth solutions under a certain non degeneracy condition. She also constructed virial functionals which provide a clear description of decay of smooth global solutions inside the light cone. Finally, she presented some applications in the case of PCF solitons, a first step towards the study of its nonlinear stability.

4.4.3 Solitary waves with inhomogeneous dispersion

Dmitry Pelinovsky discussed solitary waves under intensity-dependent dispersion. These are NLS models where bright and dark solitons may exist [33]. For bright solitons, a continuous family of singular solitons exists with a cusped soliton as the limiting lowest energy state. He showed in his talk that this family can be obtained variationally by minimization of mass at fixed energy and fixed length between two singularities. For the case of dark solitons, he showed that the spectral stability problem possesses only isolated eigenvalues on the imaginary axis and the energetic stability argument holds in Sobolev spaces with exponential weights.

4.4.4 The Sublaplacian in Heisenberg groups

An important advance in the knowledge of the Laplacian in Carnot groups was presented by **Fanelli**. Joint with L. Roncal and N. Schiavone, he mentioned spectral properties of the Sublaplacian on the Heisenberg Group with a potential [16]. Then he showed a priori estimates similar to the so called Kato-Yajima estimate for the Euclidean setting. Their results contain a new method that allows them to prove their results in this subriemannian geometry by using horizontal weights and direct methods. Finally, a connection with local smoothing properties of the associated Schrödinger equation was also given.

4.4.5 Skyrmions

An exciting new area of research is the one describing the behavior of Skyrmions. **Ibrahim** presented the stability of vortex-like configurations of magnetization in magnetic materials, the so-called magnetic Skyrmion [21]. This corresponds to a critical point of the Landau-Lifshitz energy with the Dzyaloshinskii-Moriya (DM) interactions. From an earlier work of Doring and Melcher, it is known that the Skyrmion is a ground state when the coefficient of the DM term is small. Ibrahim proved that there is an explicit critical value of the coefficient above which the Skyrmion is unstable, while stable below this threshold. In the unstable regime the infimum of energy is not bounded from below, by giving an explicit counterexample with a sort of helical configuration. This mathematically explains the occurrence of phase transition observed in some experiments.

5 Outcome of the Meeting

After more than four years of preparation, our event "New Trends in Mathematics of Dispersive, Integrable and Nonintegrable Models in Fluids, Waves and Quantum Physics" took place in Fall 2022 and we, as organizers, could not have much higher expectations for the success of it. Back in 2019, when we planned to apply for this BIRS workshop, we spent a lot of energies for the choice of a workshop topic that could cover our main fields of expertise and the related ones, we hardly worked on the selection of the speakers, trying to balance among valuable underrepresented communities and more exposed ones and we planned a schedule that, due to the hybrid form of the event, had to appropriately include both in person and online talks in a way that the audience could smoothly pass from one format to the other and therefore still enjoying the overall schedule. Despite the effort we put in the organization, after hearing the extremely positive feedback from our participants (both on-site and online), experiencing the excellence quality of the talks, living the vibrant atmosphere that was created during the workshop, we should say that we could not be more satisfied with the outcomes of our meeting. We synthetically highlight the main achievements in the meeting as follows: we believe that a good balance between talks and free time for interaction among the participant was extremely beneficial. Many discussions and many potential new projects and lines of research were activated after these intentional slots in the program of the meeting. Due to the success of this first experience, we will definitely try to keep this balance in future applications, reinforcing and encouraging free interactions. A second main outcome of the meeting was the number of talks of promising female researchers that, in total, represented the 20% of the participants. Not less important, one should also notice that we tried to keep a balance also in the structure of the organizing committee (1 female and 2 male organizers). Of course we are aware that we have not reached the balance yet, but we plan to increase these numbers in our next application. Even more, we believe that scheduling again their talks into first hours of the program, will highlight them and will increase their visibility. As a third main outcome of the meeting was the promotion of underrepresented Latin America researchers, that usually do not have the opportunity to present their works in such events. Although the final number of participants with this origin was good, we would like to increase it in our next application. Another very important aspect was the presence of an extremely variegated community, both from a thematic point of view and from different stage career point of view. The presence of both young and expert people in a good balance was a crucial point in the success of the event. Indeed we believe that the synergy between early stage career researchers and more experienced ones represents an extremely important value for future and successful development of any scientific sector, for ensuring high-quality research and innovation and for keeping high the motivation of the young researchers. For all these reasons we believe that our communities would strongly benefit from a follow-up event or, even better, from a consolidation of the event in the academic calendar.

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