BIRS Dynamical Algebraic Combinatorics workshop online
Poster session titles and abstracts

October 23, 2020

- ## Carlos Alejandro Alfaro - The sandpile groups of outerplanar graphs (joint work with Ralihe Raúl Villagrán)

  Abstract: We compute the sandpile groups of families of planar graphs having a common weak dual by evaluating the indeterminates of the critical ideals of the weak dual at the lengths of the cycles bounding the interior faces. This method allows us to determine the algebraic structure of the sandpile groups of outerplanar graphs and can be used to compute the sandpile groups of many other planar graph families.

- ## Joseph Bernstein - P-strict promotion and piecewise-linear rowmotion (joint work with Jessica Striker and Corey Vorland)

  Abstract: We define P-strict labelings, as a generalization of semistandard Young tableaux, and show that promotion on these objects is in equivariant bijection with a piecewise-linear toggle action on an associated poset, that in many nice cases is conjugate to rowmotion. We apply this result to flagged tableaux, Gelfand-Tsetlin patterns, and symplectic tableaux. We also show P-strict promotion can be equivalently defined from Bender-Knuth and jeu de taquin perspectives.

- ## Colin Defant - Promotion Sorting (joint work with Noah Kravitz)

  Abstract: Schützenberger's promotion operator is an extensively-studied bijection that permutes the linear extensions of a finite poset. We introduce a natural extension ∂ of this operator that acts on all labelings of a poset. We prove several properties of ∂; in particular, we show that for every labeling L of an n-element poset P, the labeling $\partial^{n-1}(L)$ is a linear extension of P. Thus, we can view the dynamical system defined by ∂ as a sorting procedure that sorts labelings into linear extensions. For all $0 \leq k \leq n-1$, we characterize the n-element posets P that admit labelings that require at least $n-k-1$ iterations of ∂ in order to become linear extensions. The case in which $k = 0$ concerns labelings that require the maximum possible number of iterations in order to be sorted; we call these labelings tangled. We explicitly enumerate tangled labelings for a large class of posets that we call inflated rooted forest posets. For an arbitrary finite poset, we show how to enumerate the sortable labelings, which are the labelings L such that $\partial(L)$ is a linear extension.

- ## Ben Drucker, Eli Garcia, and Rose Silver - RSK algorithm and the box-ball system (joint work with Aubrey Rumbolt)

  Abstract: A box-ball system (BBS) is a collection of discrete time states each containing elements of a permutation. We can move forward and backward in this system by rearranging the permutation using certain rules. We prove that the soliton decomposition of a permutation is a standard Young tableau if and only if its soliton decomposition and Robinson-Schensted (RS) insertion tableau coincide. Furthermore, we conjecture that, if the BBS soliton partition and the RS partition coincide, then the BBS soliton decomposition and the RS insertion tableau also coincide. Along the way, we show that, for a class of permutations (which include tableau reading words), the soliton decomposition and the

RS insertion tableau coincide. We study the time required for a box-ball system to reach a steady state. We also generalize Fukuda's carrier algorithm to algorithms with multiple carriers and with infinitely many carriers.

This work started as an REU project at the University of Connecticut in Summer 2020. Our mentor is Emily Gunawan.

- Noah Kravitz - Friends and strangers walking on graphs (joint work with Colin Defant and Noga Alon)

  Abstract: Let X and Y be simple graphs on n vertices. Identify the vertices of Y with n people, any two of whom are either friends or strangers (according to the edges and non-edges in Y), and imagine that these people are standing one at each vertex of X. At each point in time, two friends standing at adjacent vertices of X may swap places, but two strangers may not. The friends-and-strangers graph FS(X,Y) has as its vertex set the collection of all configurations of people standing on the vertices of X, where two configurations are adjacent when they are related via a single swap of this form. It is natural to study the connected components of FS(X,Y), which correspond to the equivalence classes of mutually-reachable configurations. This framework provides a common generalization for the famous 15-puzzle, transposition Cayley graphs of symmetric groups, and earlier work of Stanley and Wilson.

  We explicitly describe the connected components of FS(X,Y) in the special cases where X is a path or a cycle; the results for the latter are closely related to toric partial orders. We also obtain bounds on the minimum degrees of X and Y that guarantee FS(X,Y) being connected. In a different direction, we show that if X and Y are n-vertex random graphs with edge probability p, then the threshold probability for the connectedness of FS(X,Y) is $p = n^{-1/2+o(1)}$. Finally, we raise several open problems and avenues for future research.

- Matthew Macauley - Abstract Algebra through Cayley diagrams, actions, and lattices

  Abstract: I am writing a unique undergraduate abstract algebra book that emphasizes Cayley diagrams, group actions, and lattices, and has several hundred colorful pictures. I also plan to (eventually) make it into a series of YouTube lectures. It seems like something that might pique the interest of some people in the DAC community, and I'm interested in getting feedback and ideas, especially involving creative visualizations. So far, I'm 360 pages in, and am happy to share. In this poster, I'll mostly just show off lots of pretty pictures and talk about them.

- Rene Marczinzik - Distributive lattices and Auslander regular algebras (joint work with Osamu Iyama)

  Abstract: We report on homological properties of distributive lattices. In particular we give a categorification of the rowmotion bijection and discuss at the end some experimental observations that study the interactions of the rowmotion bijection with the Coxeter matrix of a distributive lattice.

- Jaeseong Oh - Cyclic sieving and orbit harmonics (joint work with Brendon Rhoades)

  Abstract: Orbit harmonics is a tool in combinatorial representation theory which promotes the (ungraded) action of a linear group G on a finite set X to a graded action of G on a polynomial ring quotient by viewing X as a G-stable point locus in $\mathbb{C}^n$. The cyclic sieving phenomenon is a notion in enumerative combinatorics which encapsulates the fixed-point structure of the action of a finite cyclic group C on a finite set X in terms of root-of-unity evaluations of an auxiliary polynomial $X(q)$. We apply orbit harmonics to prove cyclic sieving results.

- GaYee Park - Naruse Hook length formula for linear extensions of mobile posets

Abstract: Standard Young tableaux are fundamental objects in combinatorics. The hook length formula by Frame–Robinson–Thrall (1953) counts the number of standard Young tableaux of straight shape and has been intensively studied in numerous context, from combinatorics to probability. There is no known product formula to count standard tableaux of skew shapes like border strips. However in 2014, Naruse gave a formula for a skew tableau as a positive sums over product of hook lengths. I will tell you about Naruse formula and a new extension of this formula to a generalization of border strips called mobile posets.

- Matthew Plante - Periodicity and Homomesy for the $V \times [n]$ poset and center-seeking snakes

Abstract: The poset $V \times [n]$, the Cartesian product of three-element poset with a chain of length $[n]$ has recently emerged as an example of interest in Dynamical Algebraic Combinatorics. Most posets that are known to have "nice" small-order periodicity for rowmotion arise either from representation theory as root or minuscule posets, or are built up inductively in a simple way ("skeletal posets" as defined by Grinberg & Roby). Here we show that the order of rowmotion of $V \times [n]$ is $2(n+2)$, and prove several general homomesies for it.

- Samu Potka - Refined Catalan and Narayana Cyclic Sieving (joint work with Per Alexandersson, Svante Linusson, and Joakim Uhlin)

Abstract: We proved several new instances of the cyclic sieving phenomenon (CSP) on Catalan objects of types A and B. One example is non-crossing (1, 2)-configurations with a q-analog of the Catalan numbers that we have not seen in the context of cyclic sieving before. We also proved a type B version regarding marked non-crossing configurations. Moreover, we refined many known instances of the CSP on Catalan objects. For example, we considered triangulations refined by the number of "ears", non-crossing matchings with a fixed number of short edges, and both unmarked and marked non-crossing configurations with a fixed number of loops and edges.

- James Propp - A Spectral Theory for Combinatorial Dynamics

Abstract: This poster introduces a framework for the study of periodic maps T from a (typically finite) set X to itself when the set X is equipped with one or more real- or complex-valued functions. The main idea, inspired by the time-evolution operator construction from ergodic theory, is the introduction of a time-evolution operator on a vector space. I show that the invariant functions and 0-mesic functions span complementary subspaces associated respectively with the eigenvalue 1 and the other eigenvalues. I also demonstrate computational methods for efficiently discovering homomesies and invariants. The current draft of the associated preprint is available at `http://jamespropp.org/spectrum.pdf`.

- (*) Bruce Sagan - Fences, unimodality, and rowmotion (includes joint work with Thomas McConville and Clifford Smyth)

Abstract: Let $\alpha = (a, b, \ldots)$ be a composition. Consider the associated poset $F(\alpha)$, called a fence, whose covering relations are

$$x_1 \lhd x_2 \lhd \ldots \lhd x_{a+1} \rhd x_{a+2} \rhd \ldots \rhd x_{a+b+1} \lhd x_{a+b+2} \lhd \ldots \ .$$

Let $L(\alpha)$ be the associated distributive lattice consisting of all lower order ideals of $F(\alpha)$. These lattices are important in the theory of cluster algebras and their rank generating functions can be used to define q-analogues of rational numbers. In joint work with Thomas McConville and Clifford Smyth we make progress on a recent conjecture of Morier-Genoud and Ovsienko that $L(\alpha)$ is rank unimodal. I also investigate the orbit decomposition of $F(\alpha)$ under rowmotion.

- (*) Hugh Thomas - Independence posets (joint work with Nathan Williams)

  Abstract: Let G be an acyclic directed graph. For each vertex of G, we define an involution on the independent sets of G. We call these involutions flips, and use them to define the independence poset for G–a new partial order on independent sets of G. We generalization rowmotion to these independence posets in several ways, including computations of rowmotion in slow motion. Poster URL: https://personal.utdallas.edu/~nxw170830/docs/FPSAC2020/independence_posets.html

(*) means the presenter will be at the poster session for the first hour only