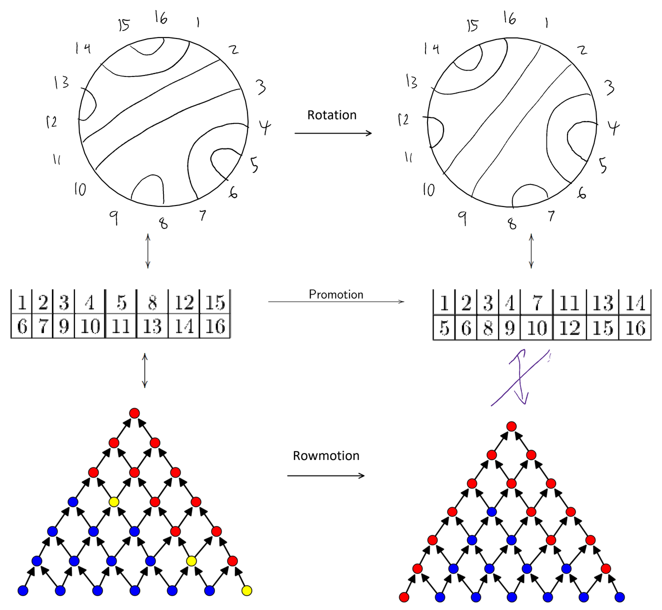
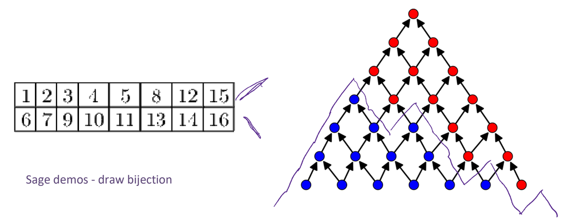
BIRS talk 2020

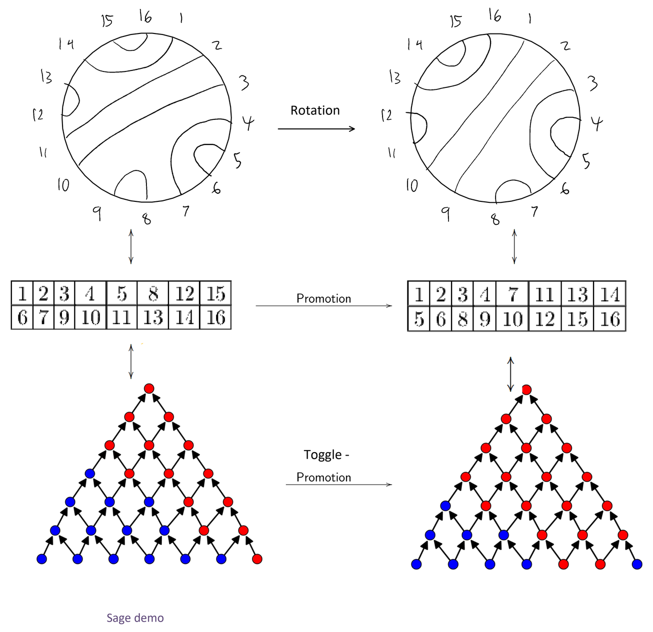


Realm 1a (Type A only - the story is true for other root posets as well)

Promotion		Rowmotion
$2 \times n$ standard Young tableaux	↔	Order ideals of the type A_{n-1} positive root poset

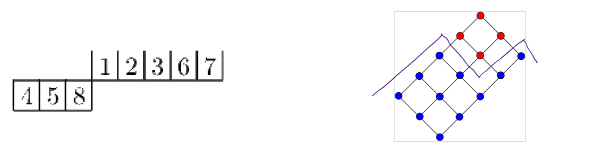


- White showed promotion corresponds to noncrossing matching rotation, so the order is $2n$.
- Panyushev conjectured the order of rowmotion is $2n$.
- Armstrong, Stump, and Thomas' bijection in *A uniform bijection between nonnesting and noncrossing partitions*

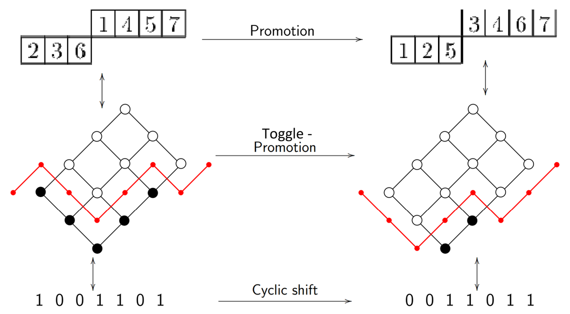


Realm 1b (Type A only - the story is true for other minuscule posets as well)

Promotion		Rowmotion
Two disjoint standard Young tableaux rows of lengths a and b	↔	Order ideals of the $[a] \times [b]$ poset



- Brouwer and Schrijver showed the order of rowmotion is $a + b$.
- Fon-der-Flaass showed the length of any orbit is $\frac{a+b}{d}$ for some d dividing both a and b.
- Stanley's bijection in *Promotion and Evacuation*



→ Realm 2

Realm 1 unified

Promotion		Rowmotion
2-row (skew-) standard Young tableaux		Order ideals of a poset inside $[a] \times [b]$
↕ easy bijection		↕
Toggle - Promotion		Toggle - Rowmotion
Order ideals	↔	Order ideals

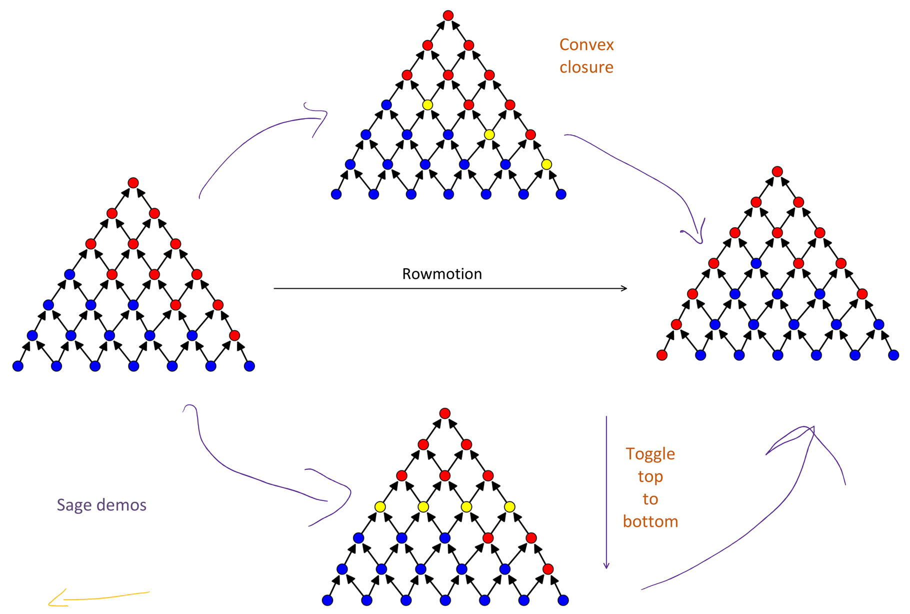
- Emma Sawin's undergraduate thesis generalized Realms 1a and 1b to all 2-row (skew-) standard Young tableaux

- Cameron and Fon-der-Flaass' toggle group interpretation of rowmotion:
- Joint work with Nathan Williams in *Promotion and Rowmotion*:

Toggle - Promotion ↔ Toggle - Rowmotion
Order ideals of P ↑ Order ideals of P

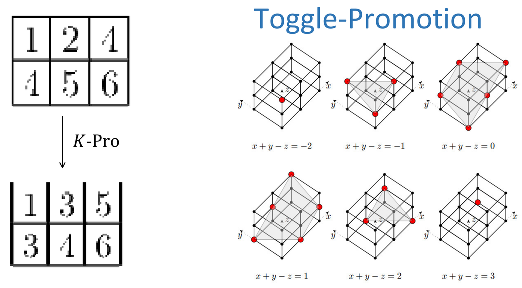
Conjugate elements in the toggle group

In Realm 1, promotion on the tableau is equivalent to toggling left-to-right in the poset (toggle-promotion), which is conjugate to rowmotion. So rowmotion and promotion have the same orbit structure! In Realms 1a and 1b, we also have equivalence to a rotation, so that yields a nice order.



Realm 2 Joint work with Dilks and Pechenik in *Resonance in orbits of plane partitions and increasing tableaux*

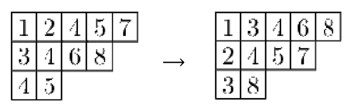
K-Promotion		Rowmotion
$a \times b$ increasing tableaux with entries at most $a+b+c-1$	↔	Order ideals of $[a] \times [b] \times [c]$



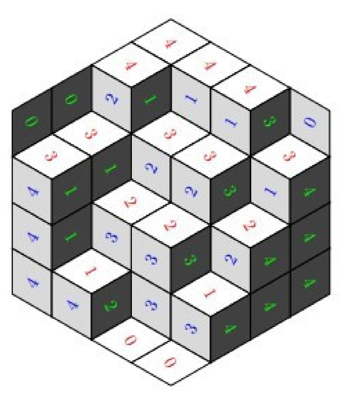
- Cameron and Fon-der-Flaass showed the order of rowmotion for $c = 2$ is $a+b+1$
- In Promotion and Rowmotion, we showed this via a bijection to noncrossing partitions of $a+b+1$ into $b+1$ blocks under rotation.
- Pechenik showed the order of K-promotion for $c = 2$ is $a+b+1$

In Realm 2, K-promotion on the increasing tableau is equivalent to toggling back-to-front in the poset (toggle-promotion), which is conjugate to rowmotion. So rowmotion and K-promotion have the same orbit structure! We can also translate results using the tri-fold symmetry of $[a] \times [b] \times [c]$

K-Promotion on increasing tableaux:

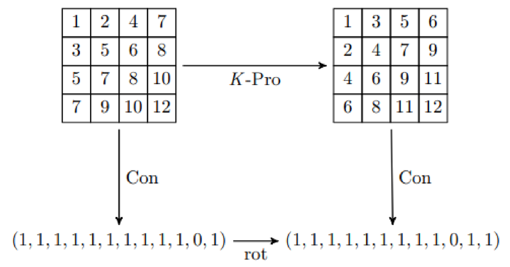


- Thomas and Yong defined K-*jeu de taquin*, which Pechenik used to define K-promotion
- Alternate definition involving *Bender-Knuth involutions* in joint work with Dilks and Pechenik

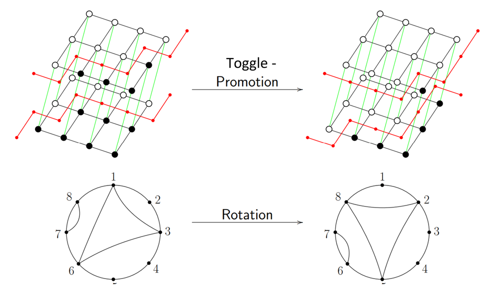


→ Realm 3

Increasing tableaux of shape λ and entries at most q exhibits resonance with frequency q:



- This means there is a projection to something that rotates with that frequency.



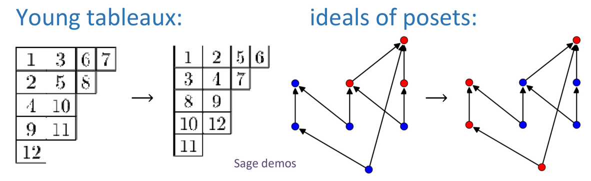
This talk is being recorded

Promotion ↔ Rowmotion

Tableaux Order ideals

- Both actions are bijections on their respective sets, so they each partition their domain into *orbits*
  - *Orbit structure*: the number of orbits of each size
  - *Order:* the lcm of the orbit sizes

Promotion on standard Young tableaux: | Rowmotion on order ideals of posets:

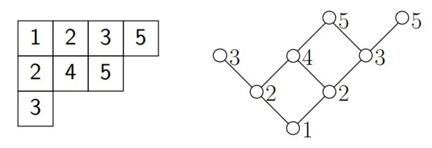


- Schutzenberger defined promotion using *jeu de taquin*
- Alternate definition involving *Bender-Knuth involutions*

- Rowmotion was studied by Duchet, Brouwer and Schrijver, Cameron and Fon-der-Flaass

→ Realm 1

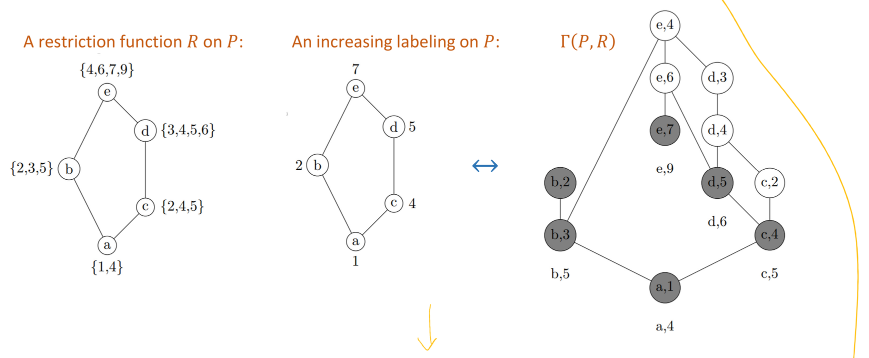
- Stanley studied promotion on linear extensions of any poset, not just partition-shape

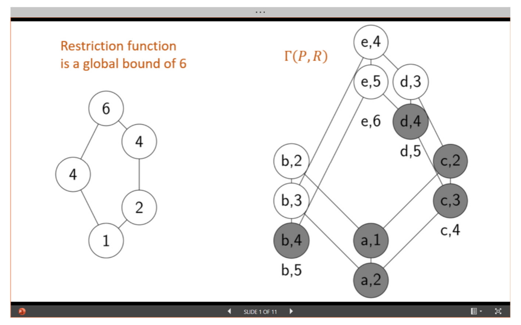


- An increasing tableau is an increasing labeling of a partition-shaped poset

Realm 3: Joint work with Dilks and Vorland in *Rowmotion and increasing labeling promotion*

Inc-Promotion on Increasing labelings of P with entries in a restriction function $R: P \to \mathcal{P}(\mathbb{Z})$	↔	Toggle-Promotion on order ideals of $\Gamma(P,R)$	↔	Rowmotion on Order ideals of $\Gamma(P,R)$

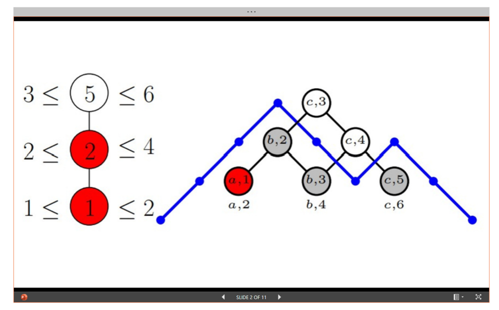




TogPro is toggling in the order of the second coordinate of the element labels

We can get to rowmotion only in the case this order is a *column toggle order*, such as when our restriction function is intervals or a global bound. So in these cases, rowmotion and Inc-promotion have the same orbit structure!

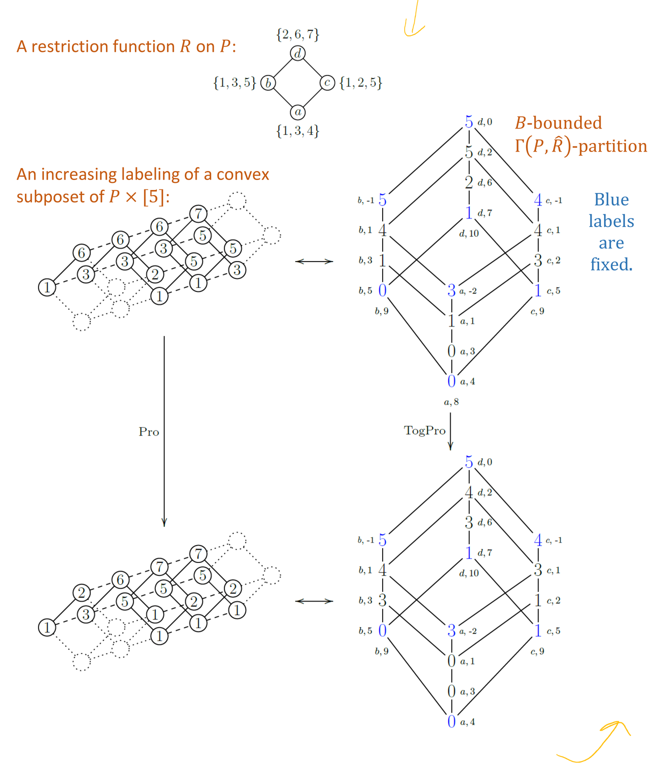
If P is a chain with the restriction function shown, then $\Gamma(P,R)$ is the Type A positive root poset:



→ Realm 4

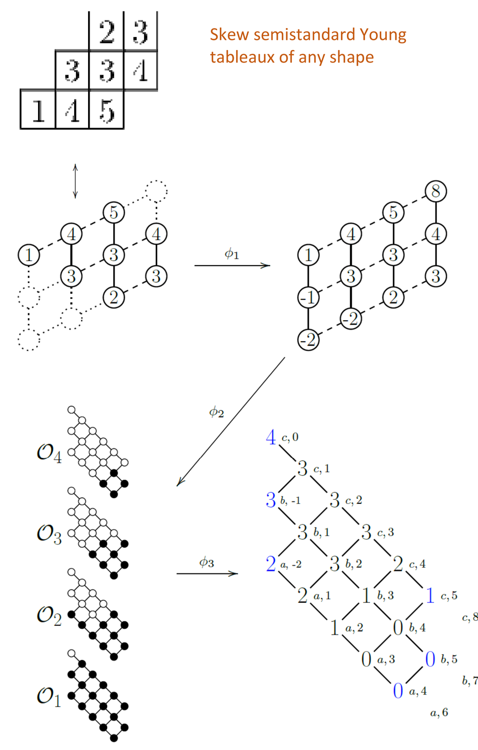
Realm 4: Current joint work with Bernstein and Vorland - will be on the arXiv soon!

Promotion on P-strict labelings of a convex subposet of $P \times [\ell]$ with entries in a restriction function $R: P \to \mathcal{P}(\mathbb{Z})$	↔	Piecewise-linear Toggle-promotion on B-bounded $\Gamma(P,\hat{R})$-partitions	↔	Piecewise-linear Rowmotion on B-bounded $\Gamma(P,\hat{R})$-partitions

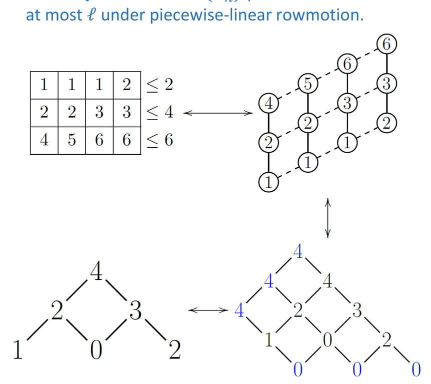


We can get to rowmotion only in the case the poset *is column-adjacent*, such as when our restriction function is intervals or a global bound. So in these cases, piecewise-linear rowmotion and (P-strict) promotion have the same orbit structure! This includes tableau.

Skew semistandard Young tableaux of any shape

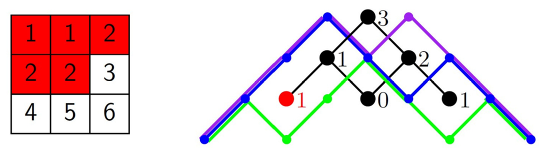


$n \times \ell$ flagged semistandard tableaux with entries at most q and flag $(2,4,6,\ldots)$ under flagged-promotion are in bijection with $\Phi^+(A_n)$-partitions with labels at most ℓ under piecewise-linear rowmotion.

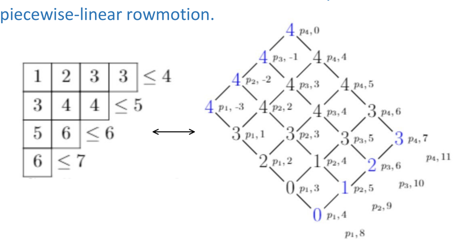


- Grinberg and Roby showed the order of birational rowmotion here is $2(n+1)$, so the order of flagged-promotion here is the same.
- Propp and Hopkins conjectured the CSP for rowmotion, so we have a conjectured CSP for the flagged tableaux too.

Note SSYT-type Bender-Knuth toggles

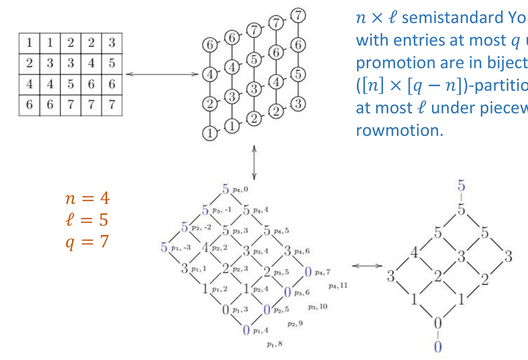


Flagged semistandard tableaux of staircase shape δ_n with entries at most q and flag $(\ell+1, \ell+2, \ldots, \ell+n)$ under flagged-promotion are in bijection with $([n] \times [\ell])$-partitions with labels at most n and bounded below by i under piecewise-linear rowmotion.



- A result of Ceballos, Labbe, and Stump on multi-cluster complexes along with a bijection of Serrano and Stump yields that the order of flagged-promotion here is $n + 1 + 2\ell$
- Serrano and Stump conjectured a CSP here, so we have a conjectured CSP for rowmotion too.

$n \times \ell$ semistandard Young tableaux with entries at most q under promotion are in bijection with $([n] \times [q-n])$-partitions with labels at most ℓ under piecewise-linear rowmotion.



$n = 4$
$\ell = 5$
$q = 7$

- This bijection was noted by Hopkins, Frieden...
- It's the SSYT ↔ Gelfand-Tsetlin pattern bijection, up to a convention.
- The bijection shows:
  - The order of piecewise linear rowmotion here is q. This was proved for *birational rowmotion* by Grinberg and Roby, with more direct proof by Musiker and Roby.
  - This exhibits the *cyclic sieving phenomenon*, as a corollary of Rhoades' cyclic sieving theorem on rectangular SSYT.