CAUSALITY OF DIAMAGNETIC SUSCEPTIBILITY AND ITS IMPLI-CATIONS FOR HERGLOTZ THEORY

(as applied to determining non-negative energy expressions)

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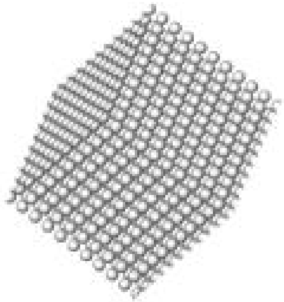
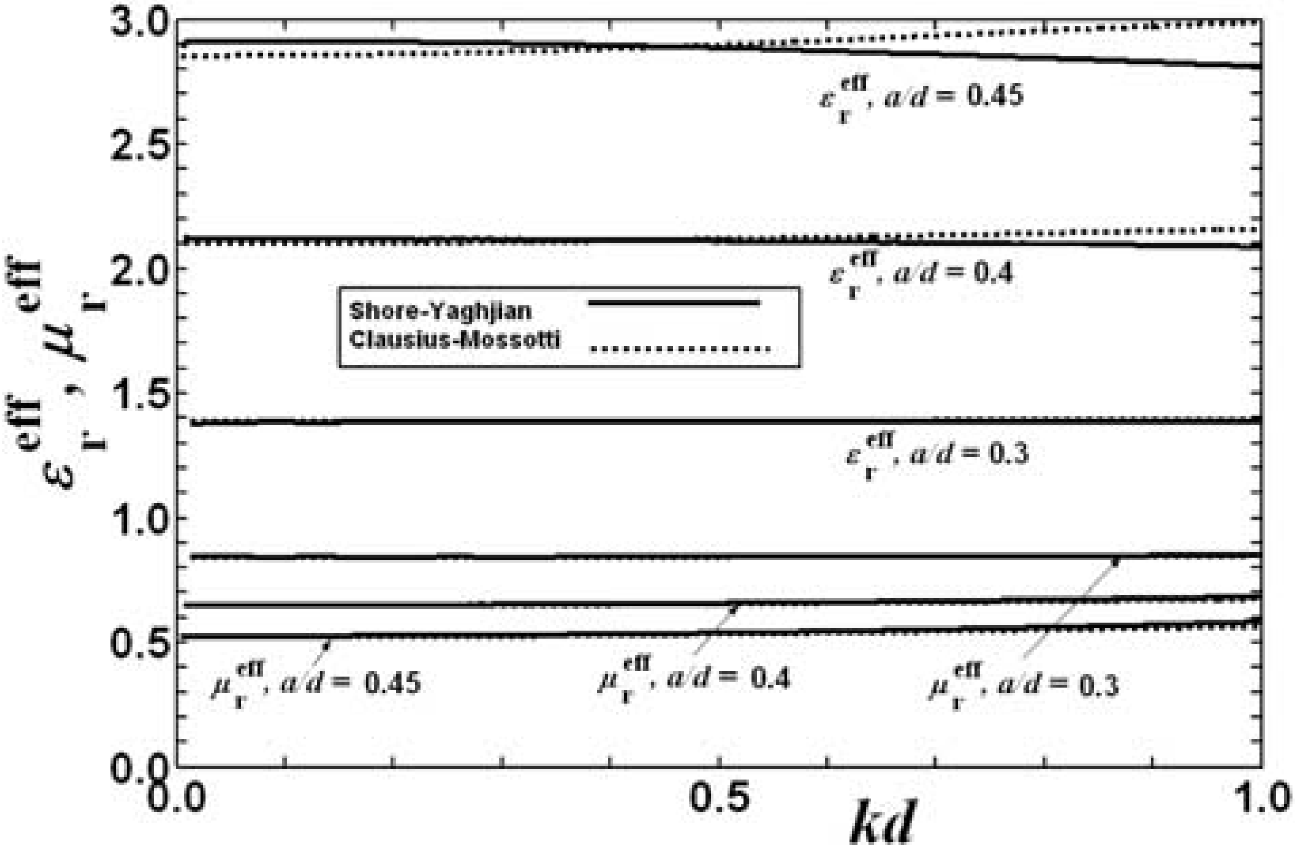
3D ARRAY OF PEC SPHERES





$$\epsilon_{\mathrm{r}}^{\mathrm{eff}} \mu_{\mathrm{r}}^{\mathrm{eff}} \geq 1$$

DIELECTRIC CAUSALITY

$$\mathbf{D}(\omega) = \epsilon(\omega)\mathbf{E}(\omega) \qquad \epsilon(\omega) \overset{|\omega|\to\infty}{\sim} \epsilon_0$$

$$\int_{-\infty}^{+\infty} \epsilon(\omega) e^{-i\omega t} d\omega = 0, \quad t < 0$$

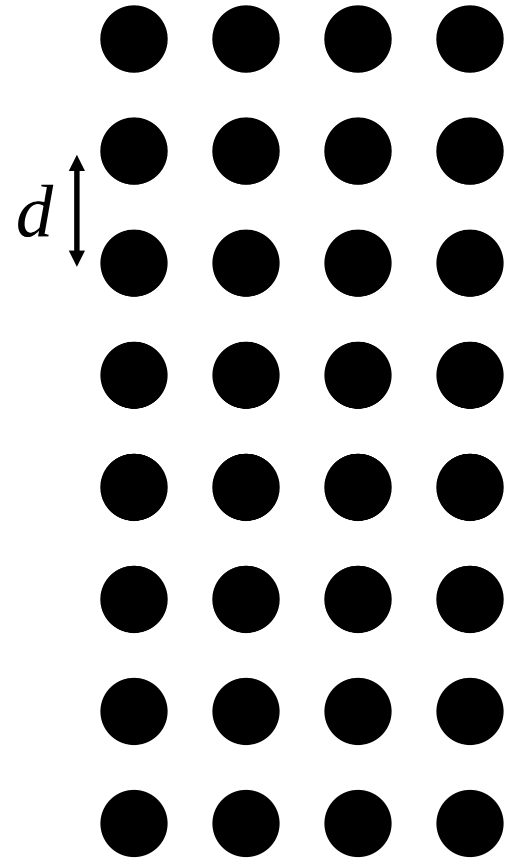
K-K relation $$\epsilon(\omega) - \epsilon_0 = \frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{\epsilon(\nu) - \epsilon_0}{\omega - \nu} d\nu$$

$$\omega \mathrm{Im}[\epsilon(\omega)] \geq 0$$

$$\boxed{\epsilon(\omega \to 0) \geq \epsilon_0}$$

lossless

3D array of PEC spheres



DIAMAGNETIC CAUSALITY ??

$$\mathbf{B}(\omega) = \mu(\omega)\mathbf{H}(\omega) \qquad \mu(\omega) \overset{|\omega|\to\infty}{\sim} \mu_0$$

($\chi_m(\infty) = 0$)

$$\int_{-\infty}^{+\infty} \mu(\omega) e^{-i\omega t} d\omega = 0, \quad t < 0$$

K-K relation $$\mu(\omega) - \mu_0 = \frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{\mu(\nu) - \mu_0}{\omega - \nu} d\nu$$

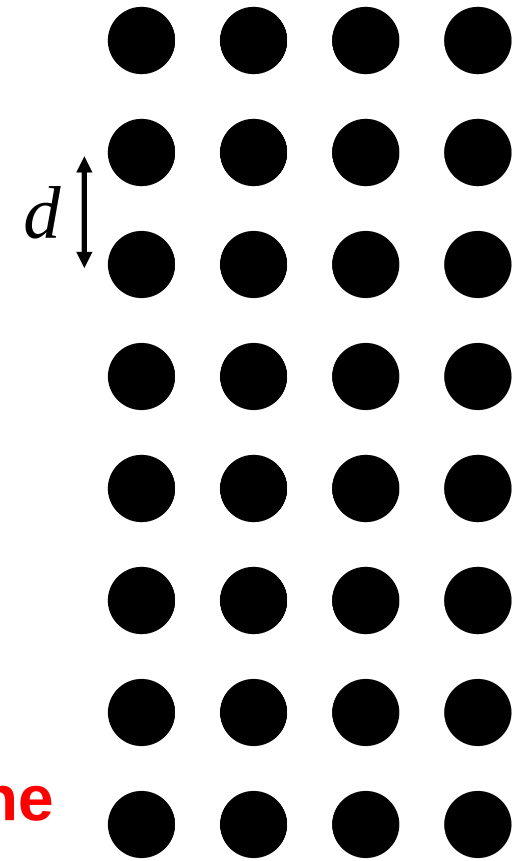
$$\omega \mathrm{Im}[\mu(\omega)] \geq 0$$

?? $\boxed{\mu(\omega \to 0) \geq \mu_0}$??

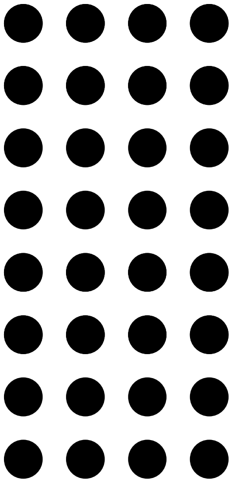
lossless

"$\chi_m(\infty)$ must be less than zero; otherwise one could not have a negative susceptibility at zero frequency." Van Vleck (1957)

3D array of PEC spheres



DIAMAGNETIC CAUSALITY



?? $$\mu(\omega \to 0) \geq \mu_0$$ **??**

The problem is that there is strong spatial dispersion for kd >> 1.

What's needed is a rigorous homogenization theory of spatial dispersion that separates electric and magnetic polarization effects; that is, an exact, (<u>causal)</u> epsilon and mu that depends on the spatial variation of the fields as well as on the frequency.

SPATIALLY DISPERSIVE CAUSALITY RELATIONS (at each fixed β)

$$\mathbf{P}^e_\rho(\boldsymbol{\beta},\omega) = \frac{e^{i\omega t}}{d^3}\int \rho^p(\mathbf{r},t)\mathbf{r}e^{-i\boldsymbol{\beta}\cdot\mathbf{r}}d^3r$$

$$\mathbf{M}^e(\boldsymbol{\beta},\omega) = \frac{e^{i\omega t}}{2d^3}\int \mathbf{r}\times\boldsymbol{\mathcal{J}}^p(\mathbf{r},t)e^{-i\boldsymbol{\beta}\cdot\mathbf{r}}d^3r$$

$$\overline{\mathbf{Q}}^e(\boldsymbol{\beta},\omega) = -\frac{e^{i\omega t}}{i\omega d^3}\int_{\mathrm{V_c}} [\boldsymbol{\mathcal{J}}^p(\mathbf{r},t)\mathbf{r} + \mathbf{r}\boldsymbol{\mathcal{J}}^p(\mathbf{r},t)]e^{-i\boldsymbol{\beta}\cdot\mathbf{r}}d^3r$$

$$\overline{\boldsymbol{\epsilon}}(\boldsymbol{\beta},\omega) - \overline{\boldsymbol{\epsilon}}_\infty(\boldsymbol{\beta}) = \frac{i}{\pi}\,⨍_{-\infty}^{+\infty} \frac{\overline{\boldsymbol{\epsilon}}(\boldsymbol{\beta},\nu) - \overline{\boldsymbol{\epsilon}}_\infty(\boldsymbol{\beta})}{\omega-\nu}d\nu$$

Inverse!

$$\overline{\boldsymbol{\mu}}_{\mathrm{tt}}^{-1}(\boldsymbol{\beta},\omega) - \overline{\boldsymbol{\mu}}_{\mathrm{tt}\infty}^{-1}(\boldsymbol{\beta}) = \frac{i}{\pi}\,⨍_{-\infty}^{+\infty} \frac{\overline{\boldsymbol{\mu}}_{\mathrm{tt}}^{-1}(\boldsymbol{\beta},\nu) - \overline{\boldsymbol{\mu}}_{\mathrm{tt}\infty}^{-1}(\boldsymbol{\beta})}{\omega-\nu}d\nu$$

LOSSY AND LOSSLESS PASSIVITY CONDITIONS

$$\omega \mathrm{Im}\left[\epsilon(\boldsymbol{\beta},\omega) + \frac{|\boldsymbol{\beta}|^2 \mu(\boldsymbol{\beta},\omega)}{\omega^2 |\mu(\boldsymbol{\beta},\omega)|^2}\right] > 0$$

Lossy

$$\mathrm{Im}\left[\epsilon(\boldsymbol{\beta},\omega) + \frac{|\boldsymbol{\beta}|^2 \mu(\boldsymbol{\beta},\omega)}{\omega^2 |\mu(\boldsymbol{\beta},\omega)|^2}\right] = 0$$

Lossless

HOMOGENIZATION THEORY FOR SPATIAL DISPERSON

IT'S MATHEMATICALLY RIGOROUS!

IT'S ELEGANT AND FAIRLY SIMPLE!

IT REQUIRES A LOT OF VARIABLES, E.G., $\epsilon(\beta,\omega)$ and $\mu(\beta,\omega)$ **.**

WE DON'T WANT TO ABANDON

$\epsilon(\omega)$ and $\mu(\omega)$

FOR MANY APPLICATIONS.

ENERGY RELATIONS FOR MACROSCOPIC DIPOLAR CONTINUA

Dipolar continua characterized by extended Herglotz susceptibilities $\omega\Psi_e(\omega)$ and $\omega\Psi_m(\omega)$

P, M

⬍

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t')\right] dt' \geq 0\,, \quad t \geq t_0$$

Glasgow et al., Gustafsson, Welters et al., (Cassier&Milton)

Consider the case: $\mathbf{P} = \epsilon_0 \psi_e \mathbf{E}, \;\; \mathbf{M} = \psi_m \mathbf{H}$

Then: $\psi_e \geq 0, \;\; \psi_m \geq 0$ **??**

Therefore, this is not the positive energy condition for diamagnetism and $\omega\Psi_m(\omega)$ is not Herglotz.

ENERGY RELATIONS FOR MACROSCOPIC DIPOLAR CONTINUA

P, M

Can we find positive semidefinite expressions for the time-domain macroscopic energy density in passive, spatially nondispersive, dipolar continua derived from the microscopic Maxwell equations without requiring that the polarization of the continua satisfy constitutive relations or that the continua are linear?

P, M

Classical Power and Energy Relations for Macroscopic Dipolar Continua Derived from the Microscopic Maxwell Equations
***Progress in Electromagnetics Research B*, 1-37, 2016**

MAXWELL MICROSCOPIC EQUATIONS FOR ELECTRIC CHARGE & CURRENT

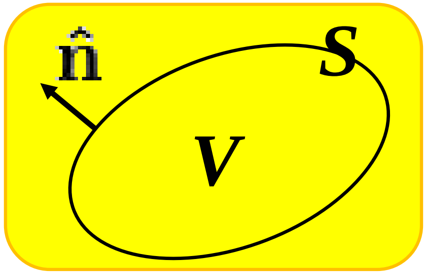
$$\nabla \times \mathbf{e}(\mathbf{r},t) + \frac{\partial \mathbf{b}(\mathbf{r},t)}{\partial t} = 0 \qquad \frac{1}{\mu_0}\nabla \times \mathbf{b}(\mathbf{r},t) - \epsilon_0 \frac{\partial \mathbf{e}(\mathbf{r},t)}{\partial t} = \mathbf{j}(\mathbf{r},t)$$

$$\nabla \cdot \mathbf{b}(\mathbf{r},t) = 0 \qquad \epsilon_0 \nabla \cdot \mathbf{e}(\mathbf{r},t) = \varrho(\mathbf{r},t)$$

Poynting's Theorem for Microscopic Electric Charge and Current

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{e}(\mathbf{r},t) \times \mathbf{b}(\mathbf{r},t)/\mu_0] dS$$

$$= \int_V \mathbf{j}(\mathbf{r},t) \cdot \mathbf{e}(\mathbf{r},t) dV + \frac{1}{2}\frac{d}{dt}\int_V \left[\epsilon_0 |\mathbf{e}(\mathbf{r},t)|^2 + |\mathbf{b}(\mathbf{r},t)|^2/\mu_0\right] dV$$



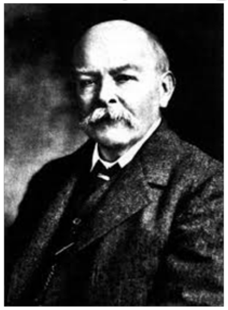
MAXWELL MACROSCOPIC EQUATIONS FOR DIPOLAR CONTINUA

$$\nabla \times \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = 0 \qquad \nabla \times \mathbf{H}(\mathbf{r},t) - \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} = 0$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \qquad \nabla \cdot \mathbf{D}(\mathbf{r},t) = 0$$

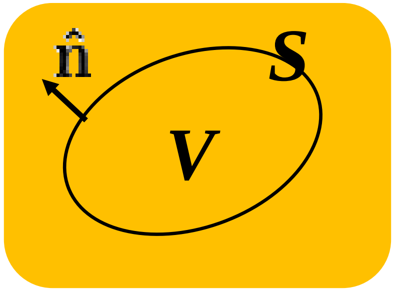
Poynting's Theorem for Macroscopic Dipolar Continua

J.H. Poynting



1852-1914

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)] dS$$

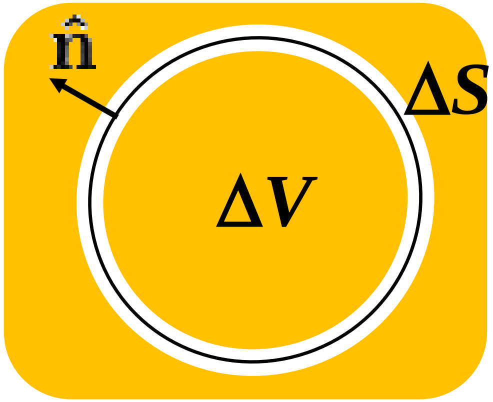


$$= \int_V \left[\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \cdot \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \cdot \mathbf{H}(\mathbf{r},t)\right] dV$$

ENERGY RELATIONS FOR MACROSCOPIC DIPOLAR CONTINUA

In the free-space shell

$$\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H}) = \hat{\mathbf{n}} \cdot (\mathbf{e} \times \mathbf{h})$$



which implies

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t') \right] dt'$$

Spatially nondispersive, passive continua with bound charge carriers (time-independent media)

$$\approx \frac{1}{\Delta V} \int_{t_0}^{t} \int_{\Delta V} \mathbf{j} \cdot \mathbf{e} dV dt' + \frac{1}{2\Delta V} \int_{\Delta V} \left[\epsilon_0 |\mathbf{e}^{\rm ins} - \mathbf{E}^{\rm ins}|^2 + \frac{1}{\mu_0} |\mathbf{b}^{\rm ins} - \mathbf{B}^{\rm ins}|^2 \right]_{t_0}^{t} dV$$

≥ 0 for diamagnetic continua.

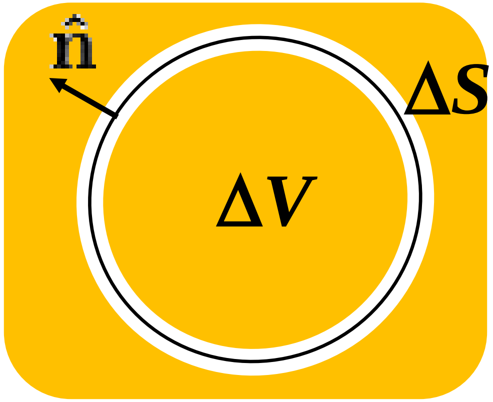
(Diamagnetism: conductors (e.g., wire loops) or molecules with no primary current)

ENERGY RELATIONS FOR MACROSCOPIC DIPOLAR CONTINUA

In the free-space shell

$$\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H}) = \hat{\mathbf{n}} \cdot (\mathbf{e} \times \mathbf{h})$$

which implies



Spatially nondispersive continua with bound charge carriers (time-independent media)

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t') \right] dt'$$

$$\approx \frac{1}{\Delta V} \int_{t_0}^{t} \int_{\Delta V} \mathbf{j} \cdot \mathbf{e} dV dt' + \frac{1}{2\Delta V} \int_{\Delta V} \left[\epsilon_0 |\mathbf{e}^{\text{ins}} - \mathbf{E}^{\text{ins}}|^2 + \frac{1}{\mu_0} |\mathbf{b}^{\text{ins}} - \mathbf{B}^{\text{ins}}|^2 \right]_{t_0}^{t} dV$$

$\not\geq 0$ for nondiamagnetic continua.

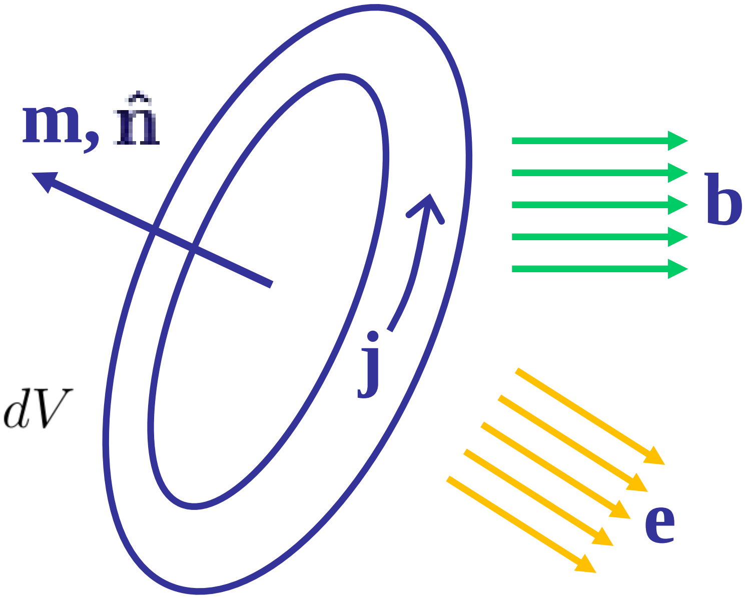
(Nondiamagnetism: PEC wire loops or molecules with large primary current (e.g., paramagnetism or ferro(i)magnetism)

ENERGY RELATIONS FOR NONDIAMAGNETIC DIPOLAR CONTINUA

$$\mathbf{e}(\mathbf{r},t) + \mathbf{v}(\mathbf{r},t) \times \mathbf{b}(\mathbf{r},t) = 0$$

$$\int_V \mathbf{j}(\mathbf{r},t) \cdot \mathbf{e}(\mathbf{r},t) dV$$

$$= \int_V \mathbf{j}(\mathbf{r},t) \cdot \mathbf{e}_{\text{ext}}(\mathbf{r},t) dV + \int_V \mathbf{j}(\mathbf{r},t) \cdot \mathbf{e}_{\text{ind}}(\mathbf{r},t) dV$$



PEC wire loop with large primary current

$$\int_V \mathbf{j}(\mathbf{r},t) \cdot \mathbf{e}(\mathbf{r},t) dV = \mathbf{b}_{\text{ext}}(\mathbf{r}_0,t) \cdot \left(\frac{d\mathbf{m}}{dt} - \hat{\mathbf{n}} \frac{dm}{dt} \right) \approx \mathbf{b}_{\text{ext}}(\mathbf{r}_0,t) \cdot \frac{d\mathbf{m}(t)}{dt}$$

This is the same result one would get for magnetic-charge dipoles!

$$\int_V \mathbf{j}_m(\mathbf{r},t) \cdot \mathbf{h}(\mathbf{r},t) dV = \mathbf{b}_{\text{ext}}(\mathbf{r}_0,t) \cdot \frac{d\mathbf{m}(t)}{dt}$$

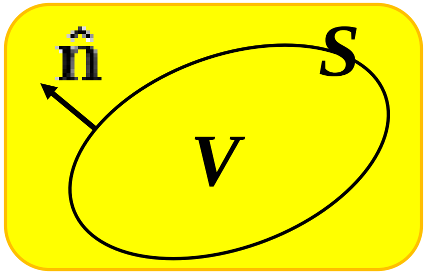
MAXWELL MICROSCOPIC EQUATIONS WITH MAGNETIC CHARGE & CURRENT

$$\nabla \times \mathbf{e}(\mathbf{r},t) + \mu_0 \frac{\partial \mathbf{h}(\mathbf{r},t)}{\partial t} = -\mathbf{j}_m(\mathbf{r},t) \qquad \nabla \times \mathbf{h}(\mathbf{r},t) - \epsilon_0 \frac{\partial \mathbf{e}(\mathbf{r},t)}{\partial t} = \mathbf{j}(\mathbf{r},t)$$

$$\nabla \cdot \mathbf{h}(\mathbf{r},t) = \varrho_m(\mathbf{r},t)/\mu_0 \qquad \epsilon_0 \nabla \cdot \mathbf{e}(\mathbf{r},t) = \varrho(\mathbf{r},t)$$

Poynting's Theorem with Microscopic Magnetic Charge and Current

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{e}(\mathbf{r},t) \times \mathbf{h}(\mathbf{r},t)]dS$$



$$= \int_V [\mathbf{j}(\mathbf{r},t)\cdot\mathbf{e}(\mathbf{r},t) + \mathbf{j}_m(\mathbf{r},t)\cdot\mathbf{h}(\mathbf{r},t)]dV + \frac{1}{2}\frac{d}{dt}\int_V \left[\epsilon_0|\mathbf{e}(\mathbf{r},t)|^2 + \mu_0|\mathbf{h}(\mathbf{r},t)|^2\right] dV.$$

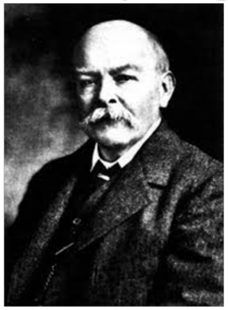
MAXWELL MACROSCOPIC EQUATIONS FOR DIPOLAR CONTINUA

$$\nabla \times \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = 0 \qquad \nabla \times \mathbf{H}(\mathbf{r},t) - \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} = 0$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \qquad \nabla \cdot \mathbf{D}(\mathbf{r},t) = 0$$

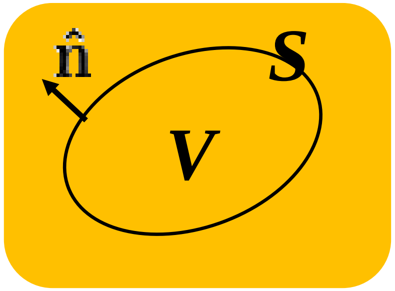
Poynting's Theorem for Macroscopic Dipolar Continua

J.H. Poynting



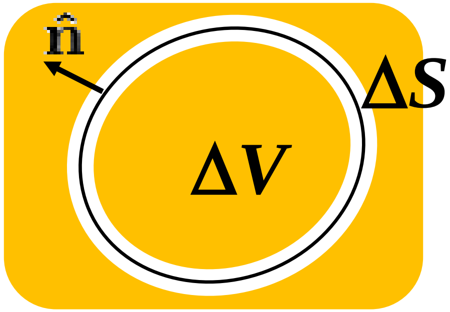
1852-1914

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)] dS$$



$$= \int_V \left[\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \cdot \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \cdot \mathbf{H}(\mathbf{r},t) \right] dV$$

ENERGY RELATIONS FOR MACROSCOPIC DIPOLAR CONTINUA



Spatially nondispersive continua with bound charge carriers (time-independent media)

In the free-space shell

$$\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H}) = \hat{\mathbf{n}} \cdot (\mathbf{e} \times \mathbf{h})$$

which implies

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r}, t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r}, t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r}, t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r}, t') \right] dt' \geq 0 \, , \quad t \geq t_0$$

for nondiamagnetic continua.

(Nondiamagnetism: PEC wire loops or molecules with large primary current (e.g., paramagnetism or ferro(i)magnetism)

RECAPITULATION OF ENERGY RELATIONS FOR DIPOLAR CONTINUA

$$\int_V \int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} + \nabla' \times \mathbf{M}(\mathbf{r},t')\right] \cdot \mathbf{E}(\mathbf{r},t') dt' dV \;=$$

Diamagnetic
$$\int_V \int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t')\right] dt' dV \geq 0$$

Nondiamagnetic
$$\int_V \int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t')\right] dt' dV \geq 0$$

"Hidden Power" = $\dfrac{\partial}{\partial t}\left[\mathbf{M} \cdot \left(\mathbf{B} - \dfrac{\mu_0}{2}\mathbf{M}\right)\right]$

The microscopic derivation has revealed that this "hidden power" originates from the reservoir of energy in the pre-existing Amperian primary magnetic-dipole current.

APPLICATION OF ENERGY RELATIONS TO LOSSLESS BIANISOTROPIC CONTINUA

$$\mathbf{D}_\omega(\mathbf{r}) = \overline{\boldsymbol{\epsilon}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r}) + \overline{\boldsymbol{\tau}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r})$$

$$\mathbf{B}_\omega(\mathbf{r}) = \overline{\boldsymbol{\mu}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r}) + \overline{\boldsymbol{\nu}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r})$$

Nondiamagnetic $\int\limits_V \int\limits_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t') \right] dt' dV \geq 0$

$$\mathrm{Re}\left\{ \mathbf{E}_\omega^* \cdot (\omega \overline{\boldsymbol{\epsilon}})' \cdot \mathbf{E}_\omega + \mathbf{H}_\omega^* \cdot (\omega \overline{\boldsymbol{\mu}})' \cdot \mathbf{H}_\omega + \mathbf{E}_\omega \cdot \left[(\omega(\overline{\boldsymbol{\nu}}^{\mathrm{T}} + \overline{\boldsymbol{\tau}}^*) \right]' \cdot \mathbf{H}_\omega^* \right\} \geq \left[\epsilon_0 |\mathbf{E}_\omega|^2 + \mu_0 |\mathbf{H}_\omega|^2 \right]$$

$$[(\omega \epsilon_{ll})' - \epsilon_0] \geq \omega \epsilon_{ll}'/2 \geq 0$$

$$[(\omega \mu_{ll})' - \mu_0] \geq \omega \mu_{ll}'/2 \geq 0$$

$$\epsilon_{ll} - \epsilon_0 \geq 0, \qquad \omega \to 0$$

$$\mu_{ll} - \mu_0 \geq 0, \qquad \omega \to 0$$

These are obtained without using the K-K relations.

APPLICATION OF ENERGY RELATIONS TO LOSSLESS BIANISOTROPIC CONTINUA

$$\mathbf{D}_\omega(\mathbf{r}) = \overline{\boldsymbol{\epsilon}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r}) + \overline{\boldsymbol{\tau}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r})$$

$$\mathbf{B}_\omega(\mathbf{r}) = \overline{\boldsymbol{\mu}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r}) + \overline{\boldsymbol{\nu}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r})$$

Diamagnetic

$$\int_V \int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t')\right] dt' dV \geq 0$$

$$\mathrm{Re}\left\{\mathbf{E}_\omega^* \cdot (\omega\overline{\boldsymbol{\epsilon}})' \cdot \mathbf{E}_\omega + \mathbf{H}_\omega^* \cdot (\omega\overline{\boldsymbol{\mu}})' \cdot \mathbf{H}_\omega + \mathbf{E}_\omega \cdot \left[(\omega(\overline{\boldsymbol{\nu}}^{\mathrm{T}} + \overline{\boldsymbol{\tau}}^*)\right]' \cdot \mathbf{H}_\omega^*\right\} \geq \left[\epsilon_0|\mathbf{E}_\omega|^2 + \frac{1}{\mu_0}|\mathbf{B}_\omega|^2\right]$$

$$[(\omega\epsilon_{ll})' - \epsilon_0] \geq \omega\epsilon_{ll}'/2 \geq 0$$

$$[(\omega\mu_{ll})' - \mu_{ll}^2/\mu_0] \geq \omega\mu_{ll}'/2 \geq 0$$

$$\epsilon_{ll} - \epsilon_0 \geq 0, \quad \omega \to 0$$

$$0 \leq \mu_{ll} \leq \mu_0, \quad \omega \to 0$$

These are obtained without using the K-K relations.

SUMMARY

- Microscopic and macroscopic Poynting theorems have been combined with electric- and magnetic-field boundary conditions to find non-negative macroscopic energy relations for diamagnetic and nondiamagnetic (paramagnetic or ferro(i)magnetic) dipolar continua.
- The key to deriving the nondiamagnetic energy relation is to prove that changes in energy (nonpassive) in the alignment of pre-existing Amperian magnetic dipoles can be modeled by energy changes in the alignment of passive magnetic-charge magnetic dipoles.
- Remarkably, the microscopic derivation reveals that a “hidden energy” for nondiamagnetic Amperian magnetic dipoles is drawn from the reservoir of inductive energy in the pre-existing Amperian magnetic dipole moments.
- The two energy relations predict consistent results for the permittivity and permeability of both diamagnetic and nondiamagnetic dipolar continua satisfying bianisotropic constitutive relations.

As I understand it, a Herglotz function f(w) is analytic in the upper half plane and has Im[f(w)] non-negative in the upper half plane. It can then be proven from analytic function theory that wf(w) approaches 0 as |w| approaches infinity in the upper half plane. On the real axis, a Herglotz function may not be continuous and may even be singular. Also, Im[f(w)] may be negative on the real axis. Therefore, it is often assumed that the Herglotz function is continuous in the upper half plane that includes the real axis. Such continuous Herglotz w(susceptibilities) can be proven to be a necessary and sufficient condition for the non-negative EM energy expression

$$\boxed{\int_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') + \mu_0 \frac{\partial \mathbf{M}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r},t')\right] dt' \geq 0\,, \quad t \geq t_0}$$

provided one assumes the susceptibilities themselves, not w(susceptibilities), approach positive real values as |w| approaches infinity in the upper half plane that includes the real axis. Diamagnetism does not satisfy this non-negative energy expression. If, as Cassier&Milton do, we work with the total Poynting energy and we work with w(mu) and w(eps) with mu and eps approaching positive real values as w approaches infinity, we then get constant eps and mu are equal to or greater than 0. However, these eps and mu obey the K-K relations and thus their w=0 values equal their values at infinity.