The quantum first detection problem

Felix Thiel David A. Kessler Eli Barkai

Bar Ilan University, Ramat Gan, Israel

22.04.2019

The name of the game

Asymptotics of the first detection probability in systems with absolutely continuous energy spectra

The total detection probability in finite dimensional systems

The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\mathsf{in}}\rangle$ first reach $|\psi_{\mathsf{d}}\rangle$?

When is the output of a quantum computer available?

Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\mathsf{in}}\rangle$ first reach $|\psi_{\mathsf{d}}\rangle$?

When is the output of a quantum computer available?

Detection protocol

What one does not measure, thereof one must be silent.

Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

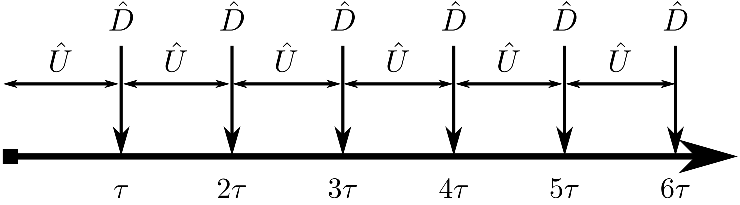
The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\mathsf{in}}\rangle$ first reach $|\psi_{\mathsf{d}}\rangle$?

When is the output of a quantum computer available?

Detection protocol

What one does not measure, thereof one must be silent.



Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

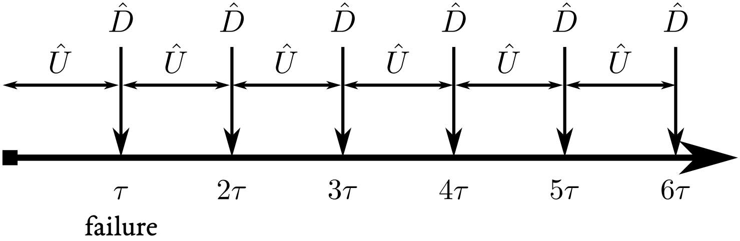
The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\text{in}}\rangle$ first reach $|\psi_{\text{d}}\rangle$?

When is the output of a quantum computer available?

Detection protocol

What one does not measure, thereof one must be silent.



Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

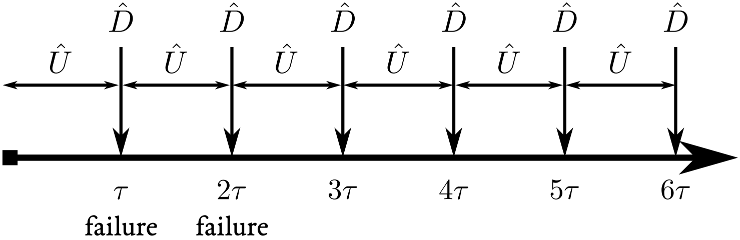
The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\mathsf{in}}\rangle$ first reach $|\psi_{\mathsf{d}}\rangle$?

When is the output of a quantum computer available?

Detection protocol

What one does not measure, thereof one must be silent.



Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

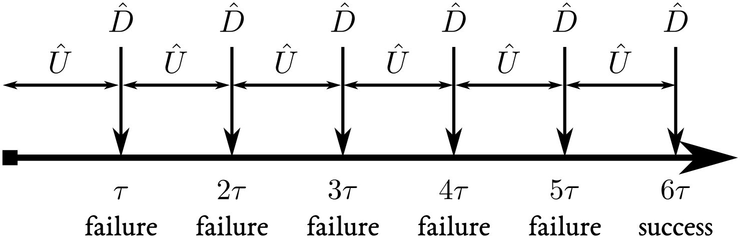
The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\text{in}}\rangle$ first reach $|\psi_{\text{d}}\rangle$?

When is the output of a quantum computer available?

Detection protocol

What one does not measure, thereof one must be silent.



Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

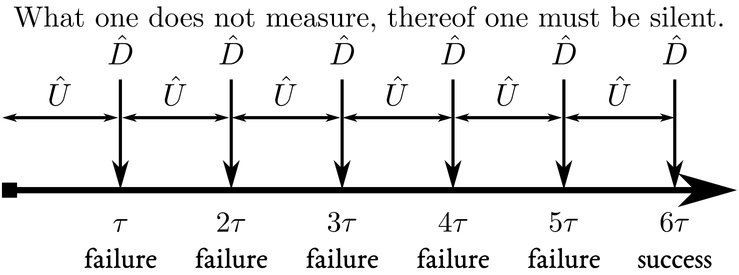
The stroboscopic detection protocol

When does a system/particle starting in $|\psi_{\mathsf{in}}\rangle$ first reach $|\psi_{\mathsf{d}}\rangle$?

When is the output of a quantum computer available?

$$\hat{U} = e^{-i\tau\hat{H}} \qquad \hat{D} = |\psi_{\mathsf{d}}\rangle\langle\psi_{\mathsf{d}}|$$

Detection protocol



Krovi and Brun 2006; Ruschhaupt, Muga, and Hegerfeldt 2009; Grünbaum et al. 2013; Sinkovicz et al. 2015; S. Dhar, Dasgupta, and A. Dhar 2015.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2,$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_{\mathsf{d}} | \hat{U} [(\mathbb{1} - \hat{D}) \hat{U}]^{n-1} | \psi_{\mathsf{in}} \rangle$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_{\mathsf{d}} | \hat{U} [(\mathbb{1} - \hat{D}) \hat{U}]^{n-1} | \psi_{\mathsf{in}} \rangle$$

$$\varphi_n = \langle \psi_{\mathsf{d}} | \hat{U}^n | \psi_{\mathsf{in}} \rangle - \sum_{m=1}^{n-1} \langle \psi_{\mathsf{d}} | \hat{U}^m | \psi_{\mathsf{d}} \rangle \, \varphi_{n-m}$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_{\mathsf{d}} | \hat{U} [(\mathbb{1} - \hat{D}) \hat{U}]^{n-1} | \psi_{\mathsf{in}} \rangle$$

$$\varphi_n = \langle \psi_{\mathsf{d}} | \hat{U}^n | \psi_{\mathsf{in}} \rangle - \sum_{m=1}^{n-1} \langle \psi_{\mathsf{d}} | \hat{U}^m | \psi_{\mathsf{d}} \rangle \, \varphi_{n-m}$$

$$\varphi(z) = \sum_{n=1}^{\infty} \varphi_n z^n$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_{\mathsf{d}} | \hat{U} [(\mathbb{1} - \hat{D}) \hat{U}]^{n-1} | \psi_{\mathsf{in}} \rangle$$

$$\varphi_n = \langle \psi_{\mathsf{d}} | \hat{U}^n | \psi_{\mathsf{in}} \rangle - \sum_{m=1}^{n-1} \langle \psi_{\mathsf{d}} | \hat{U}^m | \psi_{\mathsf{d}} \rangle \, \varphi_{n-m}$$

$$\varphi(z) = \sum_{n=1}^{\infty} \varphi_n z^n = \frac{\langle \psi_{\mathsf{d}} | \frac{z\hat{U}}{\mathbb{1} - z\hat{U}} | \psi_{\mathsf{in}} \rangle}{\langle \psi_{\mathsf{d}} | \frac{\mathbb{1}}{\mathbb{1} - z\hat{U}} | \psi_{\mathsf{d}} \rangle}$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_{\mathsf{d}} | \hat{U} [(\mathbb{1} - \hat{D}) \hat{U}]^{n-1} | \psi_{\mathsf{in}} \rangle$$

$$\varphi_n = \langle \psi_{\mathsf{d}} | \hat{U}^n | \psi_{\mathsf{in}} \rangle - \sum_{m=1}^{n-1} \langle \psi_{\mathsf{d}} | \hat{U}^m | \psi_{\mathsf{d}} \rangle \, \varphi_{n-m}$$

$$\varphi(z) = \sum_{n=1}^{\infty} \varphi_n z^n = \frac{\langle \psi_{\mathsf{d}} | \frac{z\hat{U}}{\mathbb{1} - z\hat{U}} | \psi_{\mathsf{in}} \rangle}{\langle \psi_{\mathsf{d}} | \frac{\mathbb{1}}{\mathbb{1} - z\hat{U}} | \psi_{\mathsf{d}} \rangle}$$

$$\mu(\lambda) := \langle \psi_{\mathsf{d}} | \delta(\hat{U} - e^{-i\lambda}) | \psi_{\mathsf{d}} \rangle \,, \quad \mu(\lambda)\nu(\lambda) := \langle \psi_{\mathsf{d}} | \delta(\hat{U} - e^{-i\lambda}) | \psi_{\mathsf{in}} \rangle$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

First detection amplitudes/probabilities

$$F_n = |\varphi_n|^2, \quad \varphi_n = \langle \psi_\mathsf{d} | \hat{U} [(\mathbb{1} - \hat{D})\hat{U}]^{n-1} | \psi_\mathsf{in} \rangle$$

$$\varphi_n = \langle \psi_\mathsf{d} | \hat{U}^n | \psi_\mathsf{in} \rangle - \sum_{m=1}^{n-1} \langle \psi_\mathsf{d} | \hat{U}^m | \psi_\mathsf{d} \rangle \, \varphi_{n-m}$$

$$\varphi(z) = \sum_{n=1}^{\infty} \varphi_n z^n = \frac{\langle \psi_\mathsf{d} | \frac{z\hat{U}}{\mathbb{1} - z\hat{U}} | \psi_\mathsf{in} \rangle}{\langle \psi_\mathsf{d} | \frac{\mathbb{1}}{\mathbb{1} - z\hat{U}} | \psi_\mathsf{d} \rangle} = z \frac{\int_0^{2\pi} \frac{\mathrm{d}\lambda \mu(\lambda) \nu(\lambda) e^{-i\lambda}}{1 - z e^{-i\lambda}}}{\int_0^{2\pi} \frac{\mathrm{d}\lambda \mu(\lambda)}{1 - z e^{-i\lambda}}}$$

$$\mu(\lambda) := \langle \psi_\mathsf{d} | \delta(\hat{U} - e^{-i\lambda}) | \psi_\mathsf{d} \rangle \,, \quad \mu(\lambda)\nu(\lambda) := \langle \psi_\mathsf{d} | \delta(\hat{U} - e^{-i\lambda}) | \psi_\mathsf{in} \rangle$$

Grünbaum et al. 2013; Friedman, Kessler, and Barkai 2017.

The detection period

First detection statistics are a spectral property

$$\{\hat{U}, \hat{D}, |\psi_{\mathsf{in}}\rangle\} \Rightarrow \{\mu(\lambda), \nu(\lambda)\} \Rightarrow \varphi(z) \Rightarrow \varphi_n \Rightarrow F_n$$

The detection period

First detection statistics are a spectral property

$$\{\hat{U}, \hat{D}, |\psi_{\mathsf{in}}\rangle\} \Rightarrow \{\mu(\lambda), \nu(\lambda)\} \Rightarrow \varphi(z) \Rightarrow \varphi_n \Rightarrow F_n$$

- Only energy levels with overlap in $|\psi_{\mathsf{d}}\rangle$ contribute.

The detection period

First detection statistics are a spectral property

$$\{\hat{U}, \hat{D}, |\psi_{\mathsf{in}}\rangle\} \Rightarrow \{\mu(\lambda), \nu(\lambda)\} \Rightarrow \varphi(z) \Rightarrow \varphi_n \Rightarrow F_n$$

- Only energy levels with overlap in $|\psi_{\mathsf{d}}\rangle$ contribute.
- Only total overlaps matter $\sum_m \langle\psi_{\mathsf{d}}|E_{l,m}\rangle \langle E_{l,m}|\psi_{\mathsf{in}}\rangle$

The detection period

First detection statistics are a spectral property

$$\{\hat{U}, \hat{D}, |\psi_{\text{in}}\rangle\} \Rightarrow \{\mu(\lambda), \nu(\lambda)\} \Rightarrow \varphi(z) \Rightarrow \varphi_n \Rightarrow F_n$$

$\hat{U} = e^{-i\tau\hat{H}}$ depends on τ!

- Only energy levels with overlap in $|\psi_{\text{d}}\rangle$ contribute.
- Only total overlaps matter $\sum_m \langle\psi_{\text{d}}|E_{l,m}\rangle \langle E_{l,m}|\psi_{\text{in}}\rangle$

The detection period

First detection statistics are a spectral property

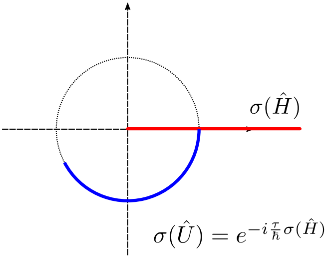
$$\{\hat{U}, \hat{D}, |\psi_{\mathsf{in}}\rangle\} \Rightarrow \{\mu(\lambda), \nu(\lambda)\} \Rightarrow \varphi(z) \Rightarrow \varphi_n \Rightarrow F_n$$

$\hat{U} = e^{-i\tau\hat{H}}$ depends on τ!

- Only energy levels with overlap in $|\psi_{\mathsf{d}}\rangle$ contribute.
- Only total overlaps matter $\sum_m \langle\psi_{\mathsf{d}}|E_{l,m}\rangle \langle E_{l,m}|\psi_{\mathsf{in}}\rangle$
- Equivalent energy levels: $E_l = E_{l'} \mod 2\pi/\tau$

The detection period

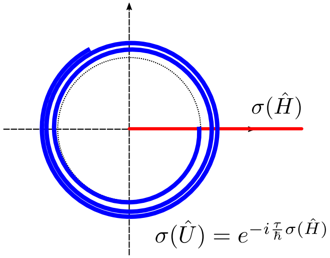
$$\tau_c := \frac{2\pi}{E_{\mathsf{max}} - E_{\mathsf{min}}}$$



$$\tau < \tau_c$$

The detection period

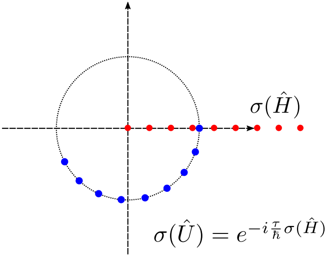
$$\tau_c := \frac{2\pi}{E_{\mathsf{max}} - E_{\mathsf{min}}}$$



$$\tau > \tau_c$$

The detection period

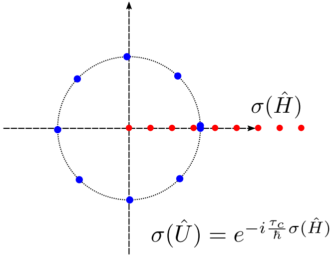
$$\tau_c := \frac{2\pi}{|E_l - E_{l'}|}$$



$$\tau < \tau_c$$

The detection period

$$\tau_c := \frac{2\pi}{|E_l - E_{l'}|}$$



$$\tau = \tau_c$$

The name of the game

Asymptotics of the first detection probability in systems with absolutely continuous energy spectra

The total detection probability in finite dimensional systems

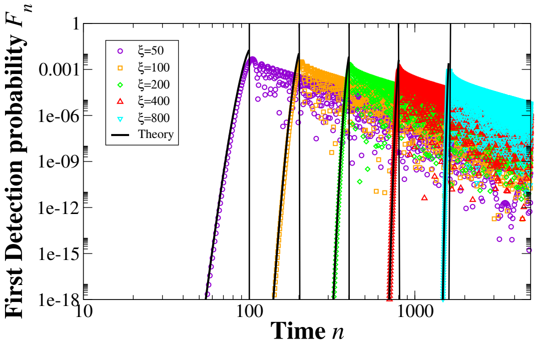
Tight-binding model on the infinite line

$$\hat{H} = \sum_{x \in \mathbb{Z}} \left[2 \left| x \right\rangle\!\left\langle x \right| - \left| x \right\rangle\!\left\langle x+1 \right| - \left| x \right\rangle\!\left\langle x-1 \right| \right]$$

Thiel, Barkai, and Kessler 2018.

Tight-binding model on the infinite line

$$\hat{H} = \sum_{x\in\mathbb{Z}} \left[2\,|x\rangle\langle x| - |x\rangle\langle x+1| - |x\rangle\langle x-1|\right]$$

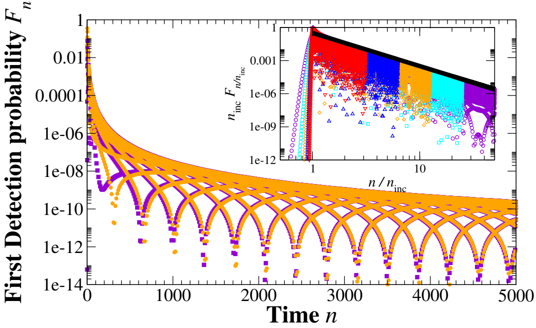


$$n_{\text{inc}} = \frac{\xi}{v_g \tau} = \frac{\xi}{2\tau}, \quad F_n \sim \frac{1}{2\pi\xi}\left(\frac{en}{2n_{\text{inc}}}\right)^{2\xi}, \quad n \to 0$$

Thiel, Barkai, and Kessler 2018.

Tight-binding model on the infinite line

$$\hat{H} = \sum_{x \in \mathbb{Z}} \left[2 \left| x \right\rangle\!\left\langle x \right| - \left| x \right\rangle\!\left\langle x+1 \right| - \left| x \right\rangle\!\left\langle x-1 \right| \right]$$

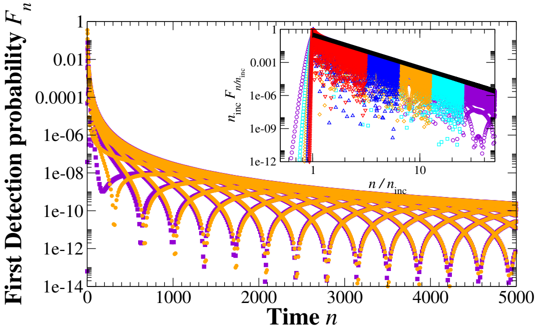


$$F_n \sim \frac{4\tau}{\pi} \frac{r^2(\xi,\tau)}{n^3} \left| \cos\left(2\tau n + \frac{\pi}{4} + \beta(\xi,\tau) \right) \right|^2, \quad n \to \infty$$

Thiel, Barkai, and Kessler 2018.

Tight-binding model on the infinite line

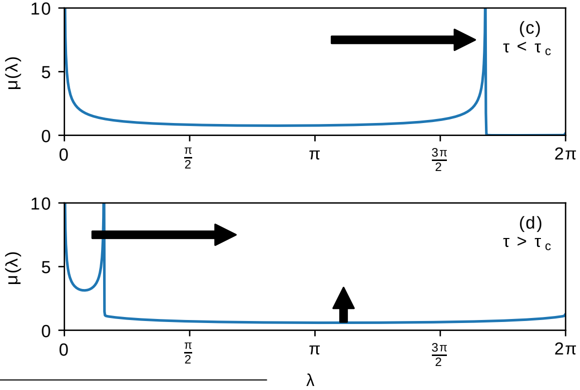
$$\hat{H} = \sum_{x\in\mathbb{Z}} \left[2\left|x\right\rangle\!\left\langle x\right| - \left|x\right\rangle\!\left\langle x+1\right| - \left|x\right\rangle\!\left\langle x-1\right|\right]$$



$$re^{i\beta} \sim -i\frac{\xi}{2\tau}, \quad F_n \sim \frac{1}{\pi}\frac{\xi^2}{\tau}\frac{\left|\cos\left(2\tau n - \frac{\pi}{4}\right)\right|^2}{n^3}, \quad n\to\infty$$

Thiel, Barkai, and Kessler 2018.

The general case, (van Hove) singularities

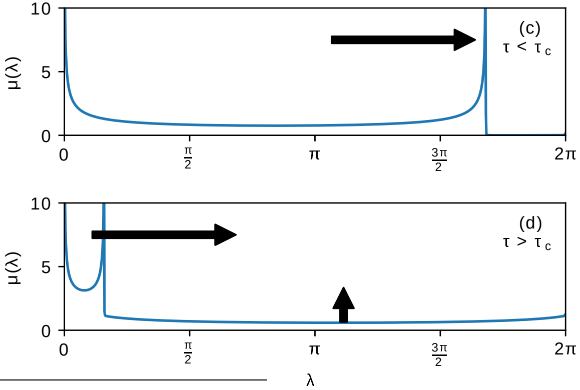


Thiel, Kessler, and Barkai 2018.

The general case, (van Hove) singularities

$$\mu(\tau E_l^* \pm \epsilon) \sim M_{\pm} \epsilon^{\frac{d_S}{2}-1}$$

E_l^*... van Hove singularities
d_S... spectral dimension



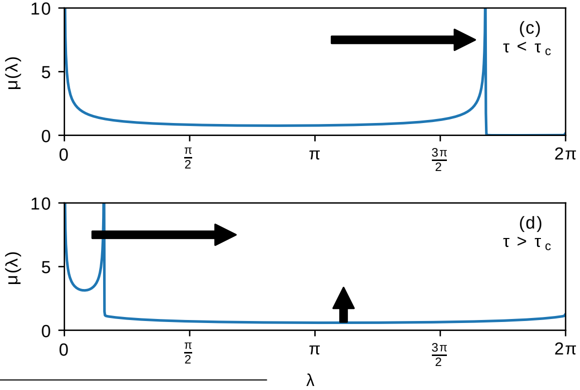
Thiel, Kessler, and Barkai 2018.

The general case, (van Hove) singularities

$$\mu(\tau E_l^* \pm \epsilon) \sim M_{\pm}\epsilon^{\frac{d_S}{2}-1}$$

E_l^*... van Hove singularities
d_S... spectral dimension

$$F_n \sim \frac{1}{n^{\max(4-d_S,d_S)}}\left|\sum_l C_l e^{-in\tau E_l^*}\right|^2$$



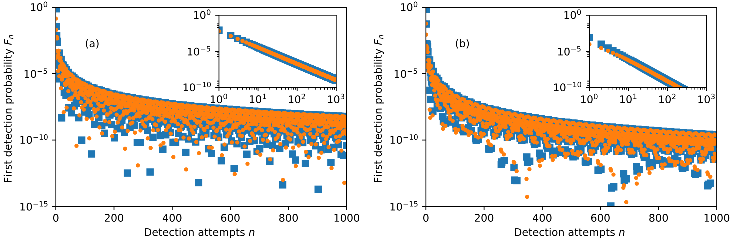
Thiel, Kessler, and Barkai 2018.

The general case, (van Hove) singularities

$$\mu(\tau E_l^* \pm \epsilon) \sim M_\pm \epsilon^{\frac{d_S}{2}-1}$$

E_l^*... van Hove singularities
d_S... spectral dimension

$$F_n \sim \frac{1}{n^{\max(4-d_S, d_S)}} \left| \sum_l C_l e^{-in\tau E_l^*} \right|^2$$



$$\tau_c = \frac{2\pi}{|E_l^* - E_{l'}^*|}$$

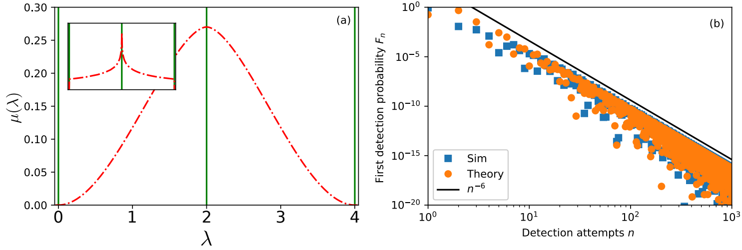
Thiel, Kessler, and Barkai 2018.

The general case, (van Hove) singularities

$$\mu(\tau E_l^* \pm \epsilon) \sim M_\pm \epsilon^{\frac{d_S}{2}-1}$$

E_l^*... van Hove singularities
d_S... spectral dimension

$$F_n \sim \frac{1}{n^{\max(4-d_S,d_S)}} \left| \sum_l C_l e^{-in\tau E_l^*} \right|^2$$



Thiel, Kessler, and Barkai 2018.

The name of the game

Asymptotics of the first detection probability in systems with absolutely continuous energy spectra

The total detection probability in finite dimensional systems

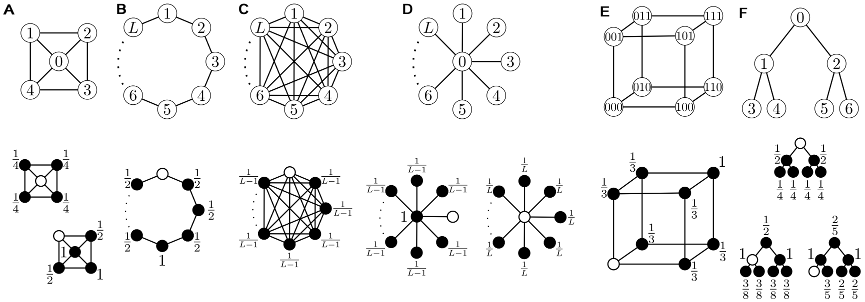
The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}})$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}})$$

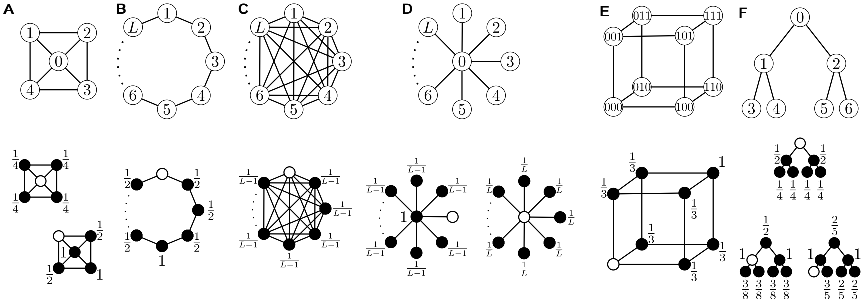


Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathrm{det}}(\psi_{\mathrm{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathrm{in}})$$

is (almost) independent of τ for finite-dimensional systems



Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}})$$

is (almost) independent of τ for finite-dimensional systems

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_D$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}})$$

is (almost) independent of τ for finite-dimensional systems

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_D$$

$$|\beta\rangle \in \mathcal{H}_B \Rightarrow P_{\mathsf{det}}(\beta) = 1, \quad |\delta\rangle \in \mathcal{H}_D \Rightarrow P_{\mathsf{det}}(\delta) = 0$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\text{det}}(\psi_{\text{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\text{in}}) = \langle \psi_{\text{in}} | \hat{P}_{\mathcal{H}_B} | \psi_{\text{in}} \rangle$$

is (almost) independent of τ for finite-dimensional systems

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_D$$

$$|\beta\rangle \in \mathcal{H}_B \Rightarrow P_{\text{det}}(\beta) = 1, \quad |\delta\rangle \in \mathcal{H}_D \Rightarrow P_{\text{det}}(\delta) = 0$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}}) = \langle \psi_{\mathsf{in}} | \hat{P}_{\mathcal{H}_B} | \psi_{\mathsf{in}} \rangle$$

is (almost) independent of τ for finite-dimensional systems

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_D$$

$$|\beta\rangle \in \mathcal{H}_B \Rightarrow P_{\mathsf{det}}(\beta) = 1, \quad |\delta\rangle \in \mathcal{H}_D \Rightarrow P_{\mathsf{det}}(\delta) = 0$$

$$\hat{U} = \sum_l \hat{P}_l e^{-iE_l\tau}, \quad \mathcal{H}_B = \mathrm{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\}$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

The total detection probability

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_{n=1}^{\infty} F_n(\psi_{\mathsf{in}}) = \langle \psi_{\mathsf{in}} | \hat{P}_{\mathcal{H}_B} | \psi_{\mathsf{in}} \rangle = \sum_l{}' \frac{\left| \langle \psi_{\mathsf{d}} | \hat{P}_l | \psi_{\mathsf{in}} \rangle \right|^2}{\langle \psi_{\mathsf{d}} | \hat{P}_l | \psi_{\mathsf{d}} \rangle}$$

is (almost) independent of τ for finite-dimensional systems

$$\mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_D$$

$$|\beta\rangle \in \mathcal{H}_B \Rightarrow P_{\mathsf{det}}(\beta) = 1, \quad |\delta\rangle \in \mathcal{H}_D \Rightarrow P_{\mathsf{det}}(\delta) = 0$$

$$\hat{U} = \sum_l \hat{P}_l e^{-iE_l\tau}, \quad \mathcal{H}_B = \operatorname{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\}$$

Facchi and Pascazio 2003; Krovi and Brun 2006; Caruso et al. 2009; Grünbaum et al. 2013; Novo et al. 2015; Thiel, Mualem, et al. 2019.

Some bounds on P_{det}

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left| \langle \psi_{\mathsf{d}} | \hat{P}_l | \psi_{\mathsf{in}} \rangle \right|^2}{\langle \psi_{\mathsf{d}} | \hat{P}_l | \psi_{\mathsf{d}} \rangle}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \operatorname{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\}$$

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left| \langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle \right|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \mathrm{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left| \left\langle \psi_{\mathsf{d}} \middle| \hat{P}_l \middle| \psi_{\mathsf{in}} \right\rangle \right|^2}{\left\langle \psi_{\mathsf{d}} \middle| \hat{P}_l \middle| \psi_{\mathsf{d}} \right\rangle}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\} = \mathrm{span}\{\hat{H}^k \, |\psi_{\mathsf{d}}\rangle\}$$

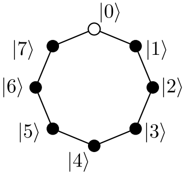
$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \text{span}\{\hat{P}_l \left|\psi_{\text{d}}\right\rangle\} = \text{span}\{\hat{H}^k \left|\psi_{\text{d}}\right\rangle\}$$

$$\left|\left\langle\psi_{\text{d}}\middle|\psi_{\text{in}}\right\rangle\right|^2 + \frac{\left|\langle\psi_{\text{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\text{d}}\rangle\right|^2}{\text{Var}[\hat{H}]_{\psi_{\text{d}}}} \leq P_{\text{det}}(\psi_{\text{in}}) = \sum_l{}' \frac{\left|\langle\psi_{\text{d}}|\hat{P}_l|\psi_{\text{in}}\rangle\right|^2}{\langle\psi_{\text{d}}|\hat{P}_l|\psi_{\text{d}}\rangle}$$

$$\langle 0|\hat{U}^n|1\rangle = \langle 0|\hat{U}^n|7\rangle$$

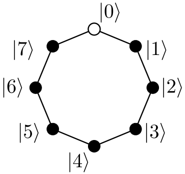


Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \mathrm{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$\left|\left\langle\psi_{\mathsf{d}}\middle|\psi_{\mathsf{in}}\right\rangle\right|^2 + \frac{\left|\left\langle\psi_{\mathsf{in}}\right|(\mathbb{1}-\hat{D})\hat{H}\left|\psi_{\mathsf{d}}\right\rangle\right|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left|\left\langle\psi_{\mathsf{d}}\right|\hat{P}_l\left|\psi_{\mathsf{in}}\right\rangle\right|^2}{\left\langle\psi_{\mathsf{d}}\right|\hat{P}_l\left|\psi_{\mathsf{d}}\right\rangle}$$

$$\varphi_n(1) = \varphi_n(7)$$

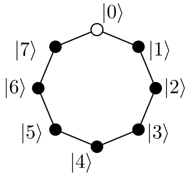


Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \,|\psi_{\mathsf{d}}\rangle\} = \mathrm{span}\{\hat{H}^k \,|\psi_{\mathsf{d}}\rangle\}$$

$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = {\sum_l}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle}$$

$$\varphi_n(1) = \varphi_n(7)$$



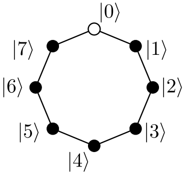
$$|\psi_\pm\rangle := \frac{1}{\sqrt{2}}(|1\rangle \pm |7\rangle)$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\} = \mathrm{span}\{\hat{H}^k \, |\psi_{\mathsf{d}}\rangle\}$$

$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \le P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle}$$

$$\varphi_n(1) = \varphi_n(7)$$



$$|\psi_\pm\rangle := \frac{1}{\sqrt{2}}(|1\rangle \pm |7\rangle)$$

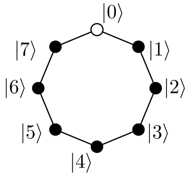
$$|\psi_-\rangle \in \mathcal{H}_D$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \mathrm{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$\left|\left\langle\psi_{\mathsf{d}}\middle|\psi_{\mathsf{in}}\right\rangle\right|^2 + \frac{\left|\left\langle\psi_{\mathsf{in}}\right|(\mathbb{1}-\hat{D})\hat{H}\left|\psi_{\mathsf{d}}\right\rangle\right|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \le P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left|\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{in}}\right\rangle\right|^2}{\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{d}}\right\rangle}$$

$$\varphi_n(1) = \varphi_n(7)$$



$$\left|\psi_{\pm}\right\rangle := \frac{1}{\sqrt{2}}(\left|1\right\rangle \pm \left|7\right\rangle)$$

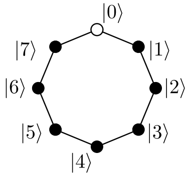
$$P_{\mathsf{det}}(1) = \frac{1}{2}P_{\mathsf{det}}(\psi_+)$$

Some bounds on P_{det}

$$\mathcal{H}_B = \operatorname{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \operatorname{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$\left|\left\langle\psi_{\mathsf{d}}\middle|\psi_{\mathsf{in}}\right\rangle\right|^2 + \frac{\left|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle\right|^2}{\operatorname{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{\left|\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{in}}\right\rangle\right|^2}{\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{d}}\right\rangle}$$

$$\varphi_n(1) = \varphi_n(7)$$



$$\left|\psi_\pm\right\rangle := \frac{1}{\sqrt{2}}(\left|1\right\rangle \pm \left|7\right\rangle)$$

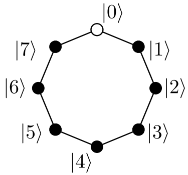
$$P_{\mathsf{det}}(1) = \frac{1}{2}P_{\mathsf{det}}(\psi_+) \leq \frac{1}{2}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \mathrm{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle} \leq \frac{1}{\nu}$$

$$\varphi_n(1) = \varphi_n(7)$$



$$|\psi_\pm\rangle := \frac{1}{\sqrt{2}}(|1\rangle \pm |7\rangle)$$

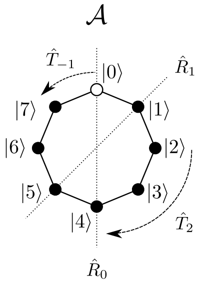
$$P_{\mathsf{det}}(1) = \frac{1}{2}P_{\mathsf{det}}(\psi_+) \leq \frac{1}{2}$$

ν...number of physically equivalent states

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \, |\psi_{\mathsf{d}}\rangle\} = \mathrm{span}\{\hat{H}^k \, |\psi_{\mathsf{d}}\rangle\}$$

$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle} \leq \frac{1}{\nu}$$

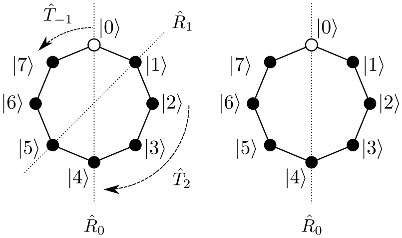


Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \,|\psi_{\mathsf{d}}\rangle\} = \mathrm{span}\{\hat{H}^k \,|\psi_{\mathsf{d}}\rangle\}$$

$$|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle} \leq \frac{1}{\nu}$$

$\mathcal{A}$ $\qquad\qquad$ $\mathcal{S}_{|0\rangle}$

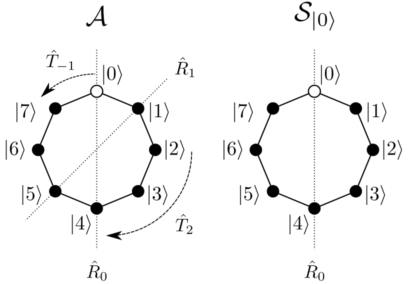


$$\mathcal{S}_{|\psi_{\mathsf{d}}\rangle} := \{\hat{S} \in \mathcal{A} | 0 = [\hat{S}, \hat{D}]\}$$

Some bounds on P_{det}

$$\mathcal{H}_B = \mathrm{span}\{\hat{P}_l \left|\psi_{\mathsf{d}}\right\rangle\} = \mathrm{span}\{\hat{H}^k \left|\psi_{\mathsf{d}}\right\rangle\}$$

$$\left|\left\langle\psi_{\mathsf{d}}\middle|\psi_{\mathsf{in}}\right\rangle\right|^2 + \frac{\left|\left\langle\psi_{\mathsf{in}}\right|(\mathbb{1}-\hat{D})\hat{H}\left|\psi_{\mathsf{d}}\right\rangle\right|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = {\sum_l}' \frac{\left|\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{in}}\right\rangle\right|^2}{\left\langle\psi_{\mathsf{d}}\middle|\hat{P}_l\middle|\psi_{\mathsf{d}}\right\rangle} \leq \frac{1}{\nu}$$

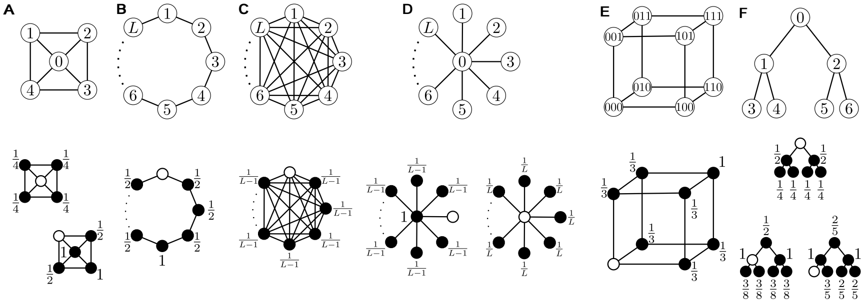


$$\mathcal{S}_{|\psi_{\mathsf{d}}\rangle} := \{\hat{S} \in \mathcal{A} | 0 = [\hat{S}, \hat{D}]\}$$

$$\nu = \dim\{\mathcal{S}_{|\psi_{\mathsf{d}}\rangle} \left|\psi_{\mathsf{in}}\right\rangle\}$$

Some bounds on P_{det}

$$\underbrace{|\langle\psi_{\mathsf{d}}|\psi_{\mathsf{in}}\rangle|^2 + \frac{|\langle\psi_{\mathsf{in}}|(\mathbb{1}-\hat{D})\hat{H}|\psi_{\mathsf{d}}\rangle|^2}{\mathrm{Var}[\hat{H}]_{\psi_{\mathsf{d}}}}}_{=1/d_{\mathrm{x}_d}} \leq P_{\mathsf{det}}(\psi_{\mathsf{in}}) = \sum_l{}' \frac{|\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{in}}\rangle|^2}{\langle\psi_{\mathsf{d}}|\hat{P}_l|\psi_{\mathsf{d}}\rangle} \leq \frac{1}{\nu}$$



Summary

- A detection protocol defines the first "arrival" time
- (Pseudo-)energies and overlaps determine the first detection statistics
- Continuous energy spectra: Power law decay and oscillations from van-Hove singularities
- Finite dimensional systems: Dark and bright space, P_{det}, Krylov and symmetry bound

You've made it!
Thank you for the attention!

References

- F Thiel et al: in preparation (2019)
- F Thiel, E Barkai, and D A Kessler: Phys Rev Lett **120**, 040502 (2018)
- F Thiel, D A Kessler, and E Barkai: Phys Rev A **97**, 062105 (2018)
- H Friedman, D A Kessler, and E Barkai: Phys Rev E **96**, 032141 (2017)
- S Dhar et al: Phys Rev A **91**, 062115 (2015)
- L Novo et al: nature scient rep **5**, 13304 (2015)
- P Sinkovicz et al: Phys Rev A **91**, 042108 (2015)
- F A Grünbaum et al: Comm Math Phys **320**, 543–569 (2013)
- F Caruso et al: J Chem Phys **131**, 105106 (2009)
- A Ruschhaupt, J G Muga and G C Hegerfeldt: Lect Notes Phys **789**, 65–99 (2009)
- H Krovi and T A Brun: Phys Rev A **73**, 032341 (2006)
- P Facchi and S Pascazio: J Phys Soc Japan **72**, Supp C, 30–33 (2003)