HUNGARIAN ACADEMY OF SCIENCES
WIGNER RESEARCH CENTRE FOR PHYSICS

Topological phases of quantum walks and how they can be detected

Janos Asboth

Dept of Quantum Optics and Quantum Information,
Wigner Research Centre for Physics of the Hungarian Academy of Sciences

HUNQUTECH

[Phys. Rev. B 95, 201407 (2017)]

Banff, Quantum Walks and Information Tasks, 2019 April 23

The plan for today

Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

Two methods to measure topological invariants, with disorder:

- Using scattering matrices
- Using weak measurement & expected displacement

Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

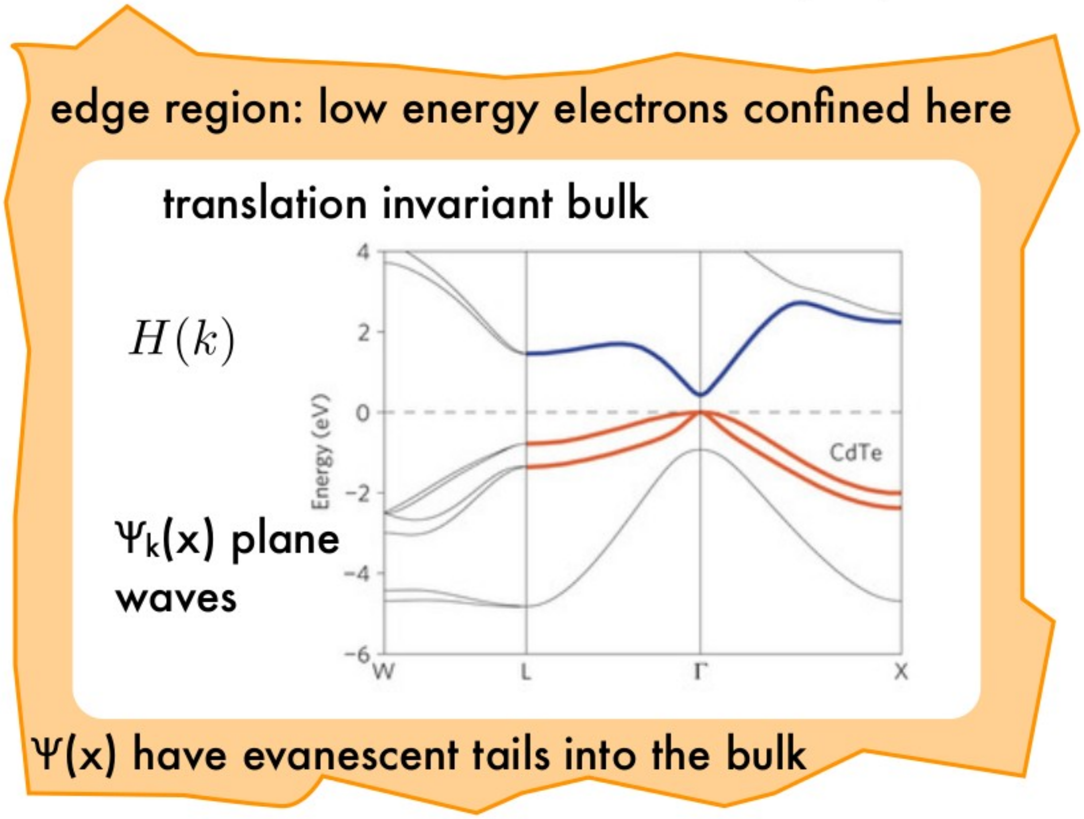
Two methods to measure topological invariants, with disorder:

- Using scattering matrices
- Using weak measurement & expected displacement

Band insulator: has bulk energy gap separating fully occupied bands from fully empty ones

$$\hat{H} = \sum_{\langle xx' \rangle} H_{xx'} \hat{c}^\dagger_{x'} \hat{c}_x$$

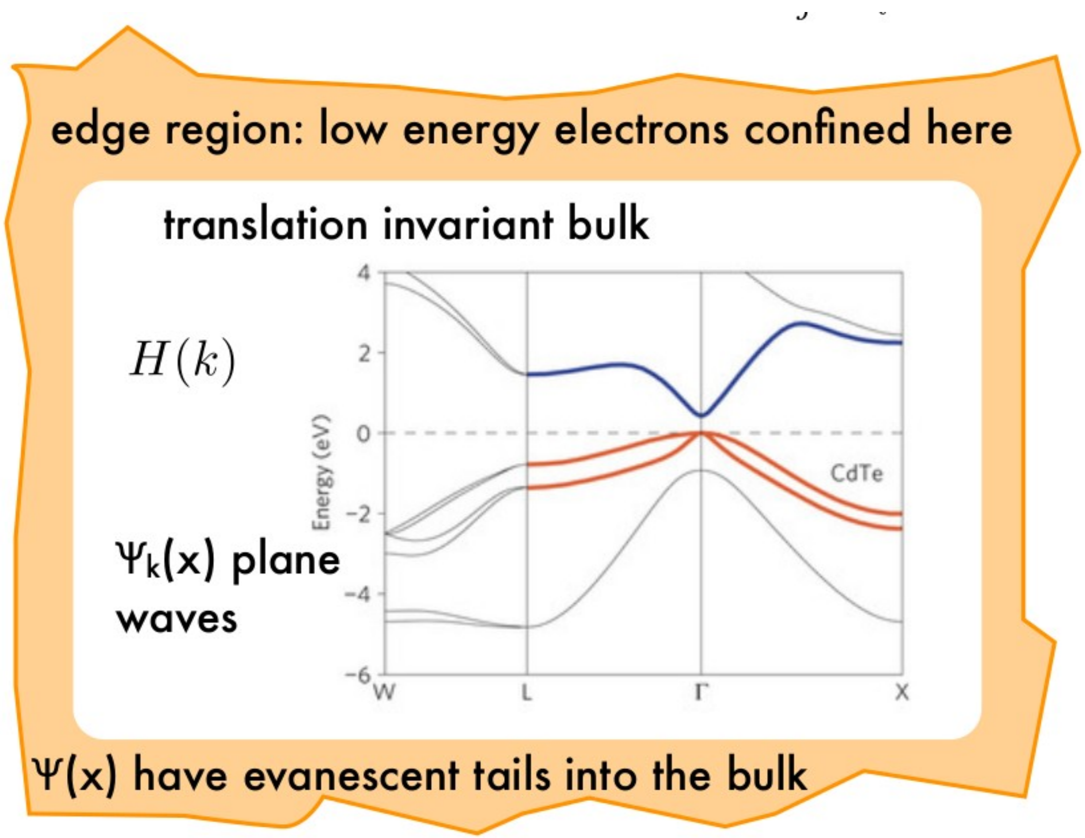
(includes superconductors in mean-field, using Bogoliubov-de Gennes trick)

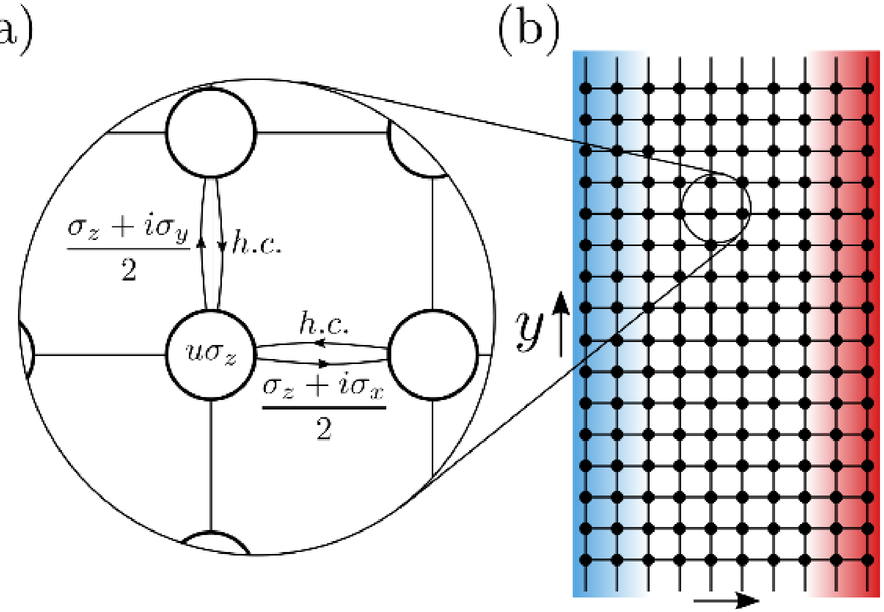


Bulk:
- -simple, can be clean,
- -most of the energy states
- -decides insulator/conductor

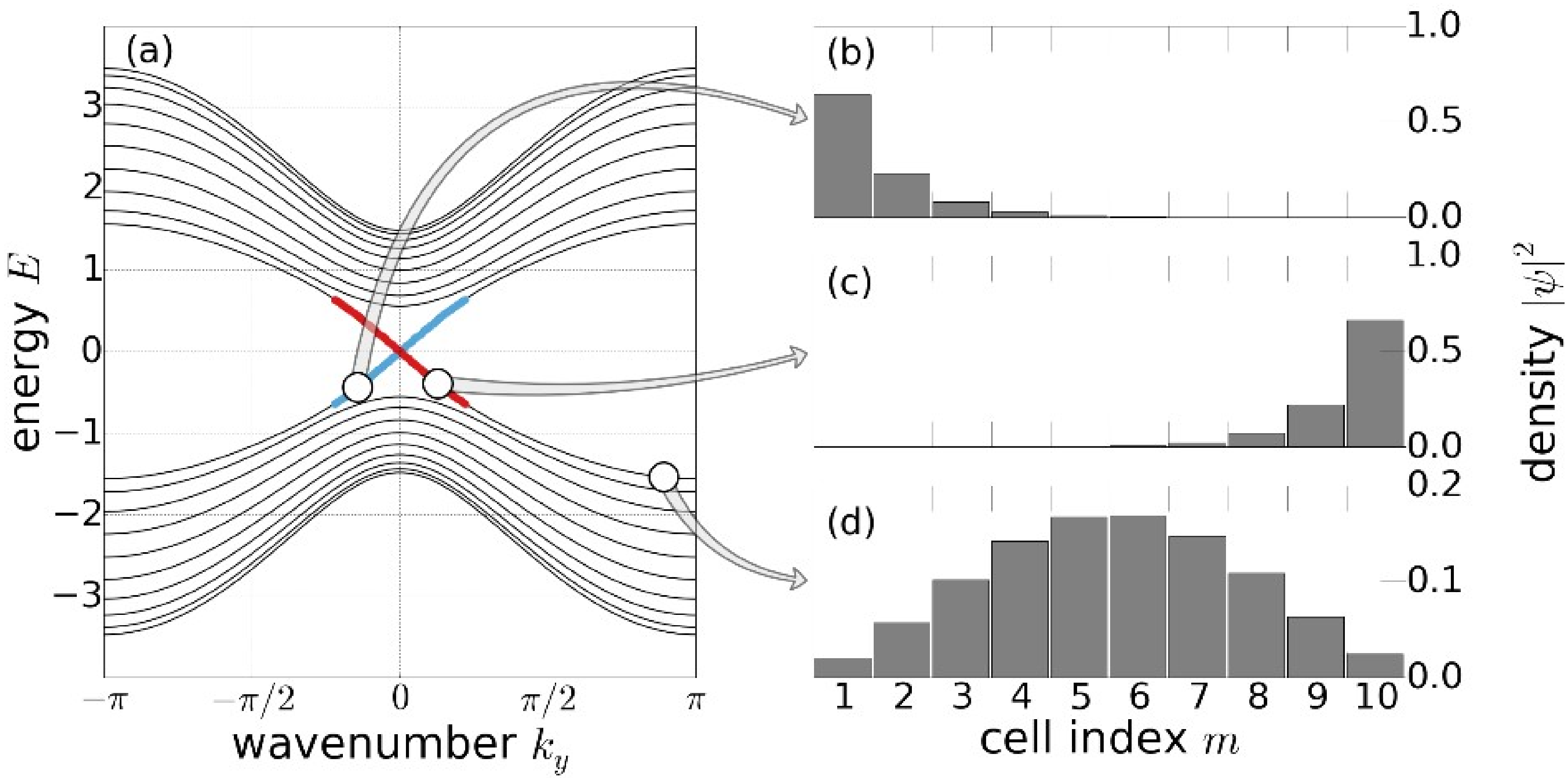
Boundary/edge:
- -disordered
- -few of the energy states
- -can hinder contact

Topological Insulator: has protected, extended midgap states on surface, which lead to robust, quantized physics





2D Chern Insulators: 1-way conducting states
→ no backscattering
→ perfect edge conduction



“Why call them *Topological* Insulators?”

a) Robust physics at the edge (e.g., 2D: conductance via edge state channels) quantified by small integers

1D, quantum wire:
# of topologically protected
0-energy states at ends of wire

3D:
# of Dirac cones on surface

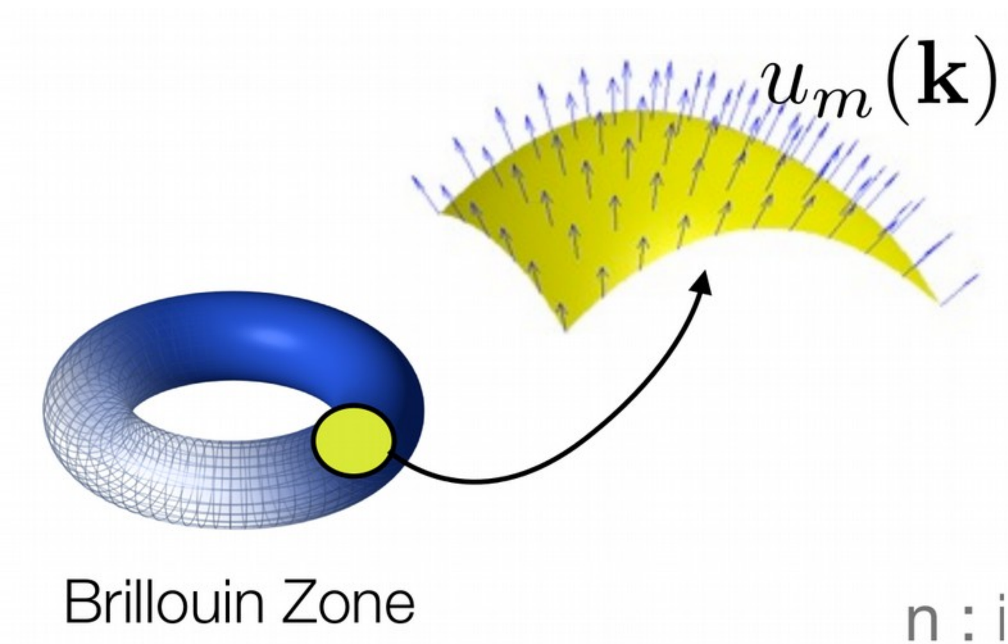
Cannot change by continuous deformation that leaves bulk insulating
→ TOPOLOGICAL INVARIANT

“Why call them *Topological* Insulators?”
b) Bulk description has a topological invariant, generalized “winding” in Brillouin Zone

Example: 2D, two levels:

$$\hat{H}(k) = \vec{h}(k)\hat{\vec{\sigma}}$$

Mapping from d-dimensional torus to Bloch sphere



More general 2D: Chern number of occupied bands

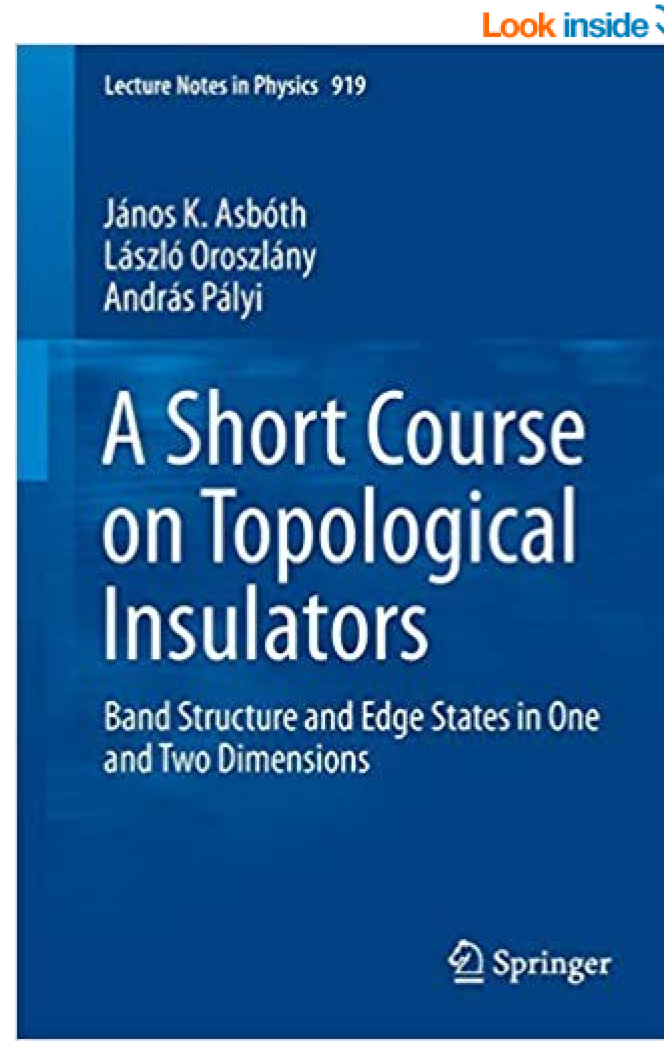
$$A_\mu^{(n)}(k) = -i\langle n(k)|\partial_{k_\mu}|n(k)\rangle$$

$$F_{xy}^{(n)}(k) = \partial_{k_x} A_y^{(n)} - \partial_{k_y} A_x^{(n)}$$

$$Q^{(n)} = \frac{1}{2\pi}\int_{BZ} d^2k F_{xy}^{(n)}(k)$$

Central, beautiful idea of Topological Insulators:
Bulk—boundary correspondence:
“winding number” of bulk = # of edge states

painless introduction: lecture notes



weeks 1-5: gather tools, build intuition, 1D

week 6: Central aim of the course:
prove bulk—boundary correspondence
for the 2-dimensional case

weeks 7-10: generalize/understand

Further accessible sources:
- 3 lectures by Charles Kane (youtube)
- online course by Akhmerov&friends topocondmat.org

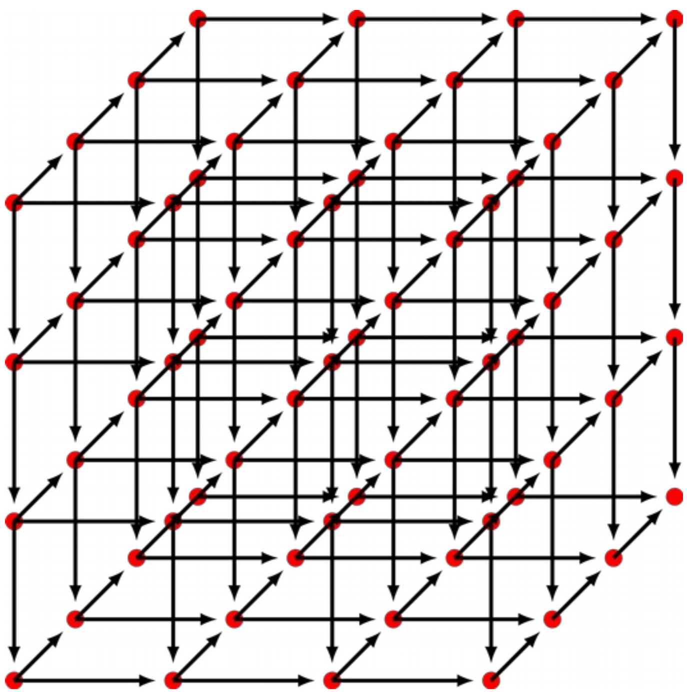
Theory of topological insulators is quite developed. Example: periodic table

Symmetry Θ^2	Ξ^2	Π^2	$\delta = d - D$: 0	1	2	3	4	5	6	7
0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
−1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
−1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
−1	−1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
0	−1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
1	−1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Kitaev (AIP Conf.Proc 2009)
Schnyder et al, NJP (2010)
Teo & Kane, PRB (2010)
Fulga et al, PRB (2012)

Quantum Walks can simulate Topological Insulators. They can be similar to a solid

split-step quantum walk on cubic lattice (3D, 2D, 1D)



Element 1: coin- (spin-) dependent shift,

$$\hat{S}_x = \sum_{\mathbf{r}\in\mathbb{Z}^3} |\mathbf{r}+\mathbf{e}_x,\uparrow\rangle\langle\mathbf{r},\uparrow| + |\mathbf{r}-\mathbf{e}_x,\downarrow\rangle\langle\mathbf{r},\downarrow|$$

Element 2: unitary rotation of coin (spin)

$$\hat{R}(\theta) = \sum_{\mathbf{r}\in\mathbb{Z}^3} |\mathbf{r}\rangle\langle\mathbf{r}| \otimes e^{-i\theta\hat{\sigma}_y} \quad = \hat{1} \otimes \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Timestep operator:

$$\hat{U} = \hat{S}_z\hat{R}_3\hat{S}_y\hat{R}_2\hat{S}_x\hat{R}_1$$

Quantum Walk discrete time evolution:

$$|\Psi(t)\rangle = \hat{U}^t|\Psi(0)\rangle, \quad \text{with} \quad t\in\mathbb{N}$$

Quantum Walk can simulate topological insulators via the (Floquet) Hamiltonian H_{eff} . This gives intuition, e.g. for speedup of spread (ballistic)

Long-time behaviour: eigenstates of timestep operator U
Translation invariant “bulk”: momentum k good quantum number

$$\hat{U} = \hat{S}_y \hat{R}_2 \hat{S}_x \hat{R}_1 = e^{-ik_y \hat{\sigma}_z} e^{-i\theta_2 \hat{\sigma}_y} e^{-ik_x \hat{\sigma}_z} e^{-i\theta_1 \hat{\sigma}_y}$$

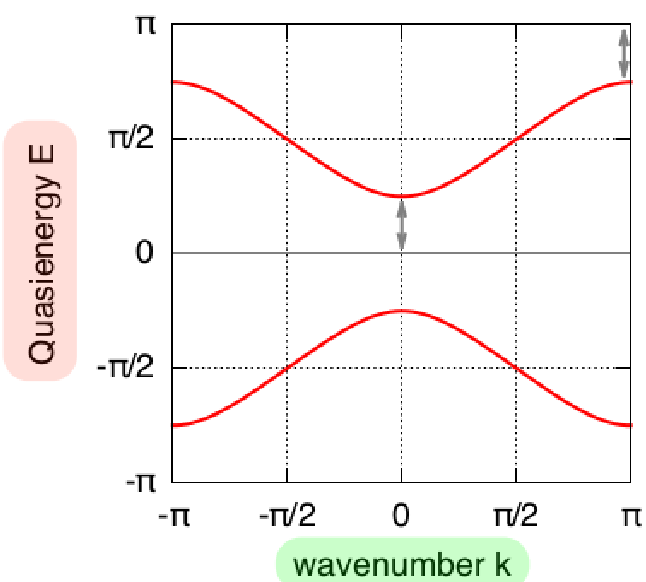
$$\hat{H}_{\text{eff}} = i \log \hat{U}$$

$$|\Psi(t)\rangle = \hat{U}^t |\Psi(0)\rangle = e^{-i\hat{H}_{\text{eff}} t} |\Psi(0)\rangle$$

Stroboscopic simulation of time-independent Heff
(coincide at integer times t)

Eigenstates of the walk are eigenstates of Heff

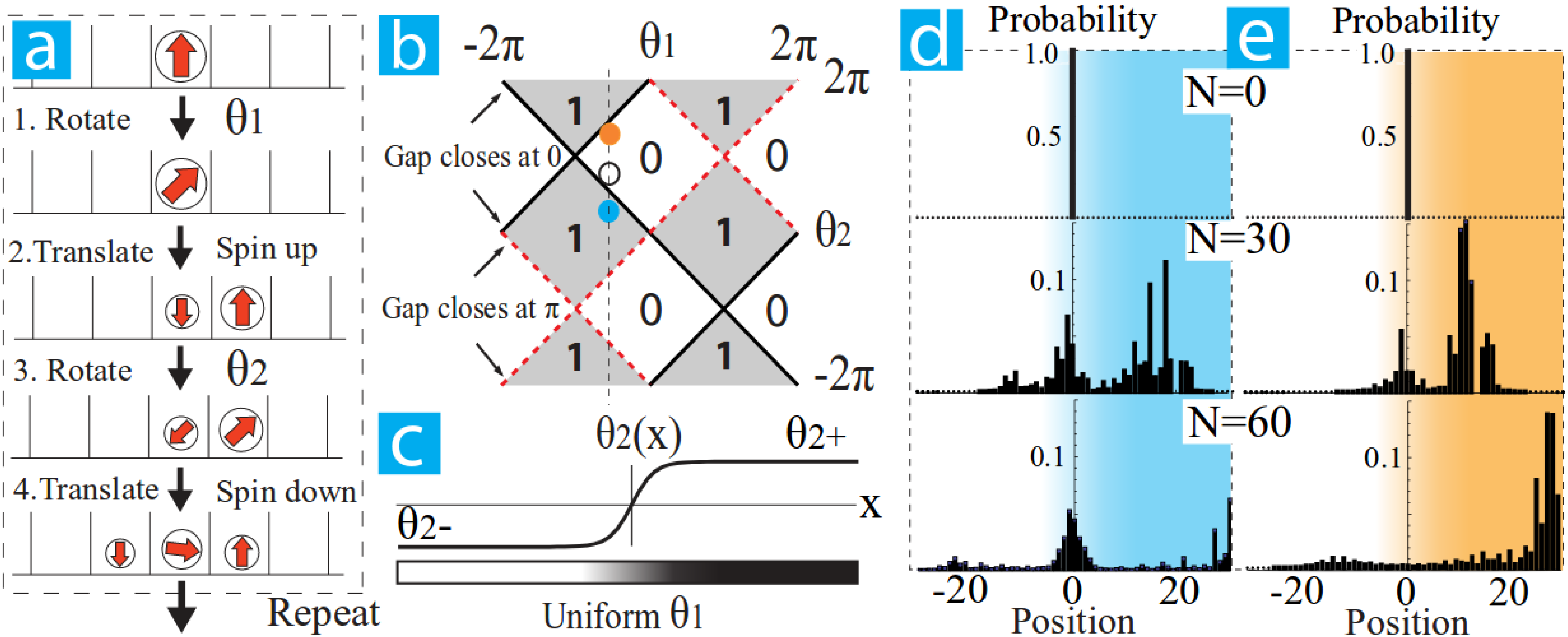
Explains ballistic spread



Discrete time ⇒ *quasi*energy, restricted to energy Brillouin zone: $-\pi < E < \pi$

Discrete positions ⇒ *quasi*momentum, restricted to Brillouin zone: $-\pi < k < \pi$

Kitagawa et al, 2010: recipes for quantum walks to simulate topological insulators via Heff



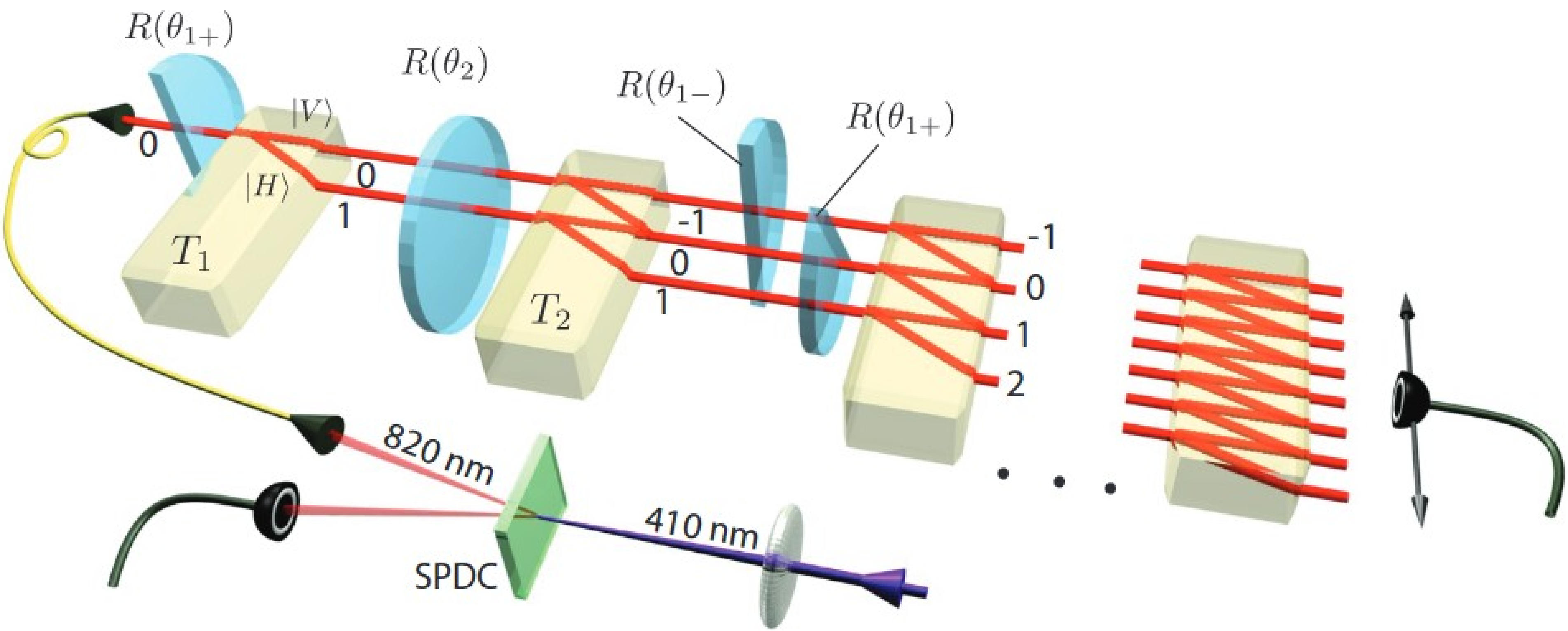
Recipes in 1D, 2D: how to realize all symmetry classes

[Kitagawa, Rudner, Berg, Demler, PRA (2010)] → 233 citations

Experiment, 2011 (White's group):
1-D split-step quantum walk on photons ...

1-D split-step quantum walk, create interface by tuning θ_2

$$\hat{U} = \hat{S}_y \hat{R}_2 \hat{S}_x \hat{R}_1 \qquad \hat{R}_j = e^{-i\theta_j \hat{\sigma}_y/2}$$



[Kitagawa et al, Nat Comm (2012)]

Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

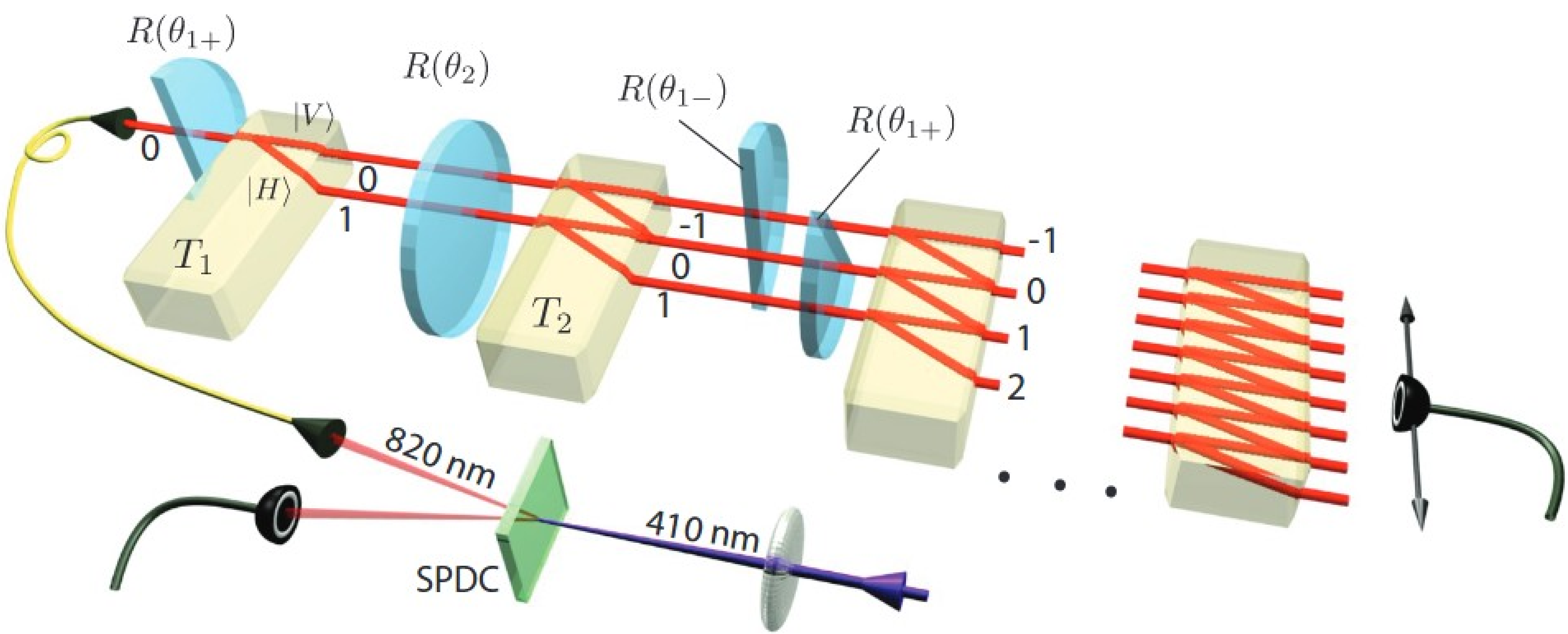
Two methods to measure topological invariants, with disorder:

- Using scattering matrices
- Using weak measurement & expected displacement

Experiment, 2011 (White's group): 1-D split-step quantum walk on photons ...

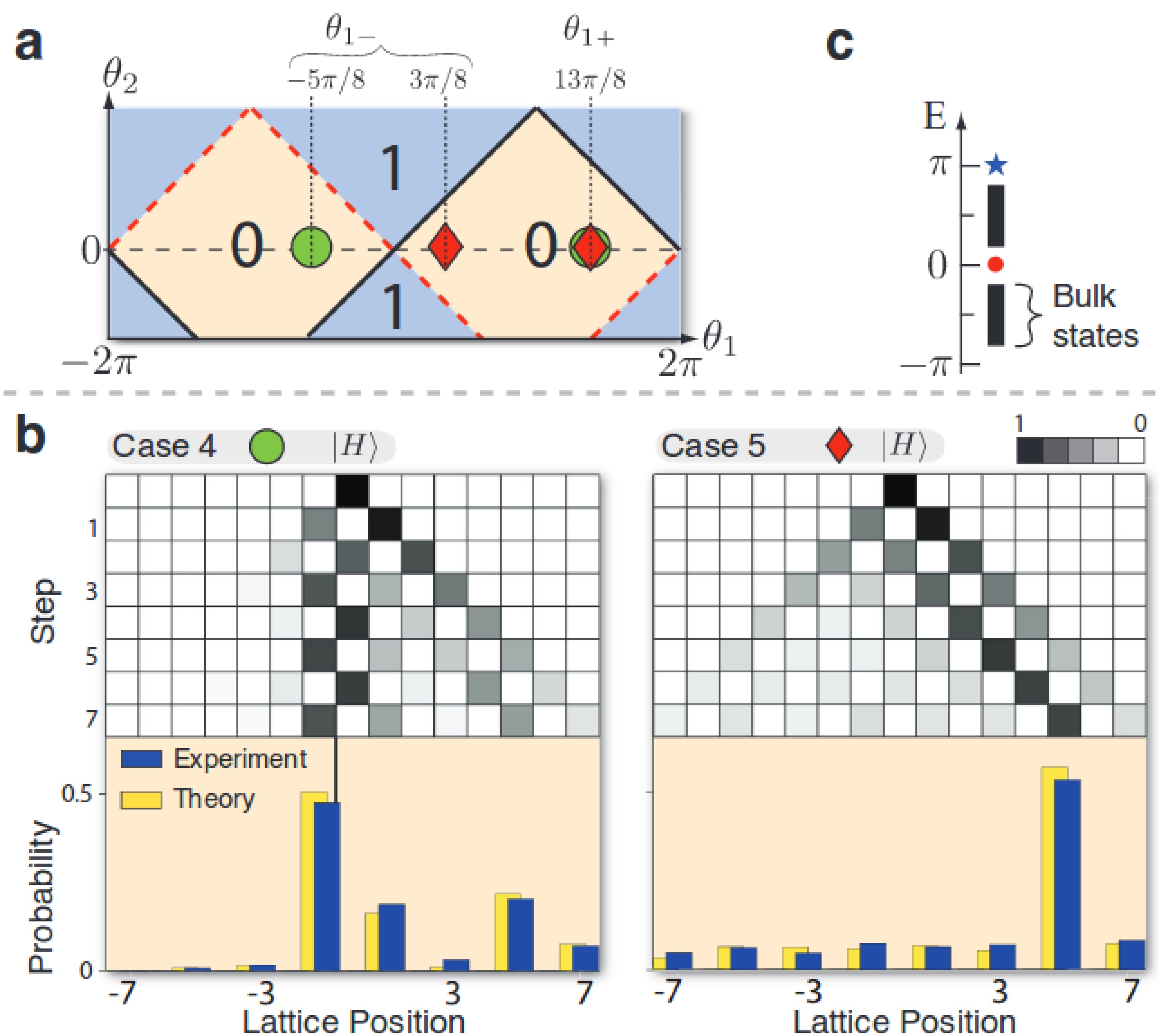
1-D split-step quantum walk, create interface by tuning θ_2

$$\hat{U} = \hat{S}_y \hat{R}_2 \hat{S}_x \hat{R}_1 \qquad \hat{R}_j = e^{-i\theta_j \hat{\sigma}_y/2}$$



[Kitagawa et al, Nat Comm (2012)]

… experiment saw edge states where theory did not predict them



Pair of bound states at quasienergy 0 and π

protected, but not predicted

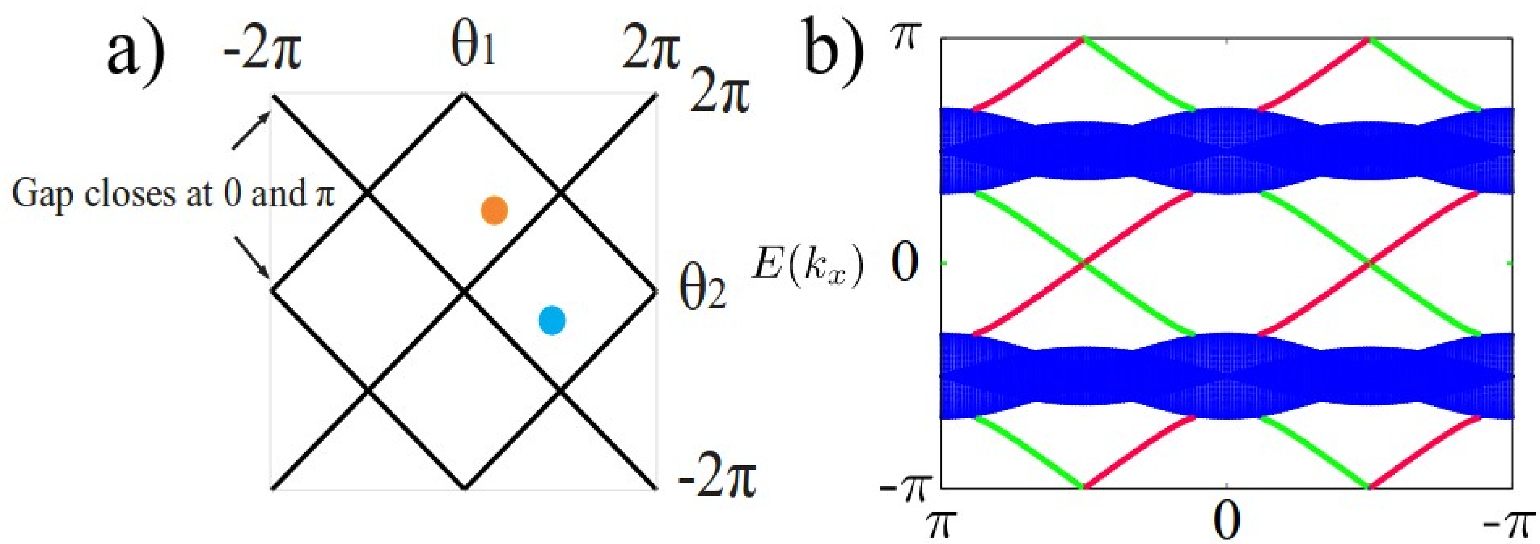
What is the bulk topological invariant?

[Kitagawa et al, Nat Comm (2012)]

Kitagawa, 2011: protected edge state in 2-dimensional quantum walk, no bulk topological invariant

2-D split-step quantum walk has edge states at interface, even though Chern number = 0

$$\hat{U} = \hat{S}_y \hat{R}_2 \hat{S}_x \hat{R}_1 \qquad \hat{R}_j = e^{-i\theta_j \hat{\sigma}_y / 2}$$



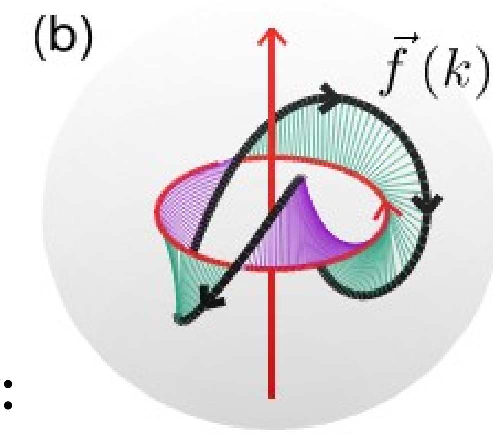
[Kitagawa, Quantum Information Processing (2012)]

k_x

What is the bulk topological invariant?

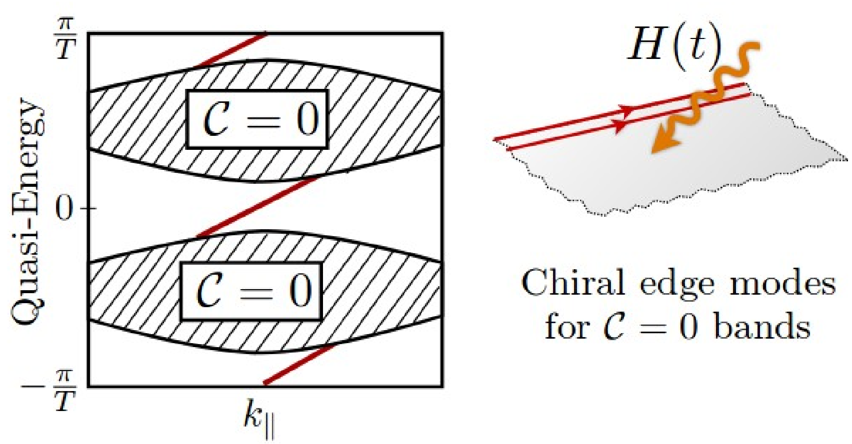
We found the bulk topological invariant for both mysterious types of edge states

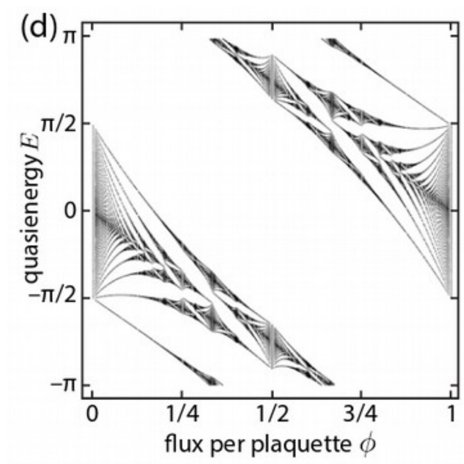
1-dimensional chiral symmetric quantum walks:
2 topological invariants
[Asboth & Obuse Phys Rev B (2013)]
[Asboth, Tarasinski, Delplace, Phys Rev B (2014)]



2-dimensional quantum walks without symmetry:
[Asboth & Edge, Phys Rev A (2015)]
by mapping to model of Rudner et al, Phys. Rev. X (2013)

- affects localization in 2D quantum walks [Edge & Asboth, Phys Rev B (2015)]
- can be measured by pseudomagnetic field [Asboth & Alberti, Phys Rev Lett (2017)]





Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

Two methods to measure topological invariants, with disorder:

- Using scattering matrices
- Using weak measurement & expected displacement

Scattering theory of topological phases in discrete-time quantum walks
B Tarasinski, JK Asbóth, JP Dahlhaus, Phys Rev A (2014)

Detecting topological invariants in chiral symmetric insulators via losses
T Rakovszky, JK Asbóth, A Alberti, Phys Rev B (2017)

Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

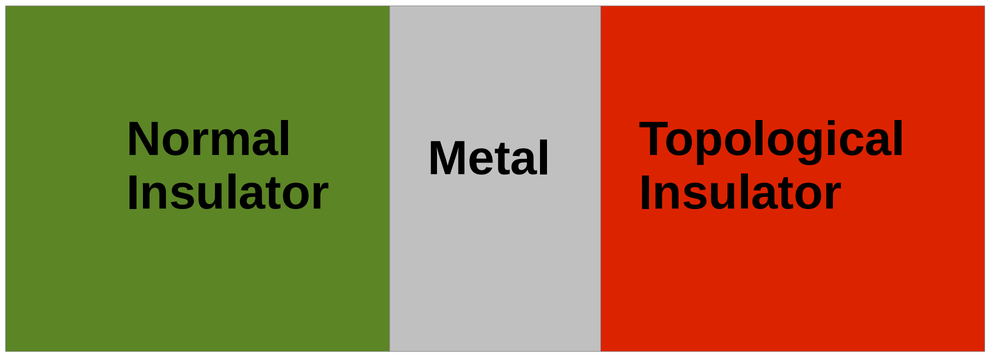
Two methods to measure topological invariants, with disorder:

- **Using scattering matrices**
- Using weak measurement & expected displacement

Scattering theory of topological phases in discrete-time quantum walks
B Tarasinski, JK Asbóth, JP Dahlhaus, Phys Rev A (2014)

Detecting topological invariants in chiral symmetric insulators via losses
T Rakovszky, JK Asbóth, A Alberti, Phys Rev B (2017)

First method, borrowed from Hamiltonians: measure the scattering matrix



$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

Are there bound states at zero energy between the two insulators?

Does an electron interfere constructively with itself? Bohr-Sommerfeld quantization

$$\det(1 - r_N r_{TI}) = 0$$

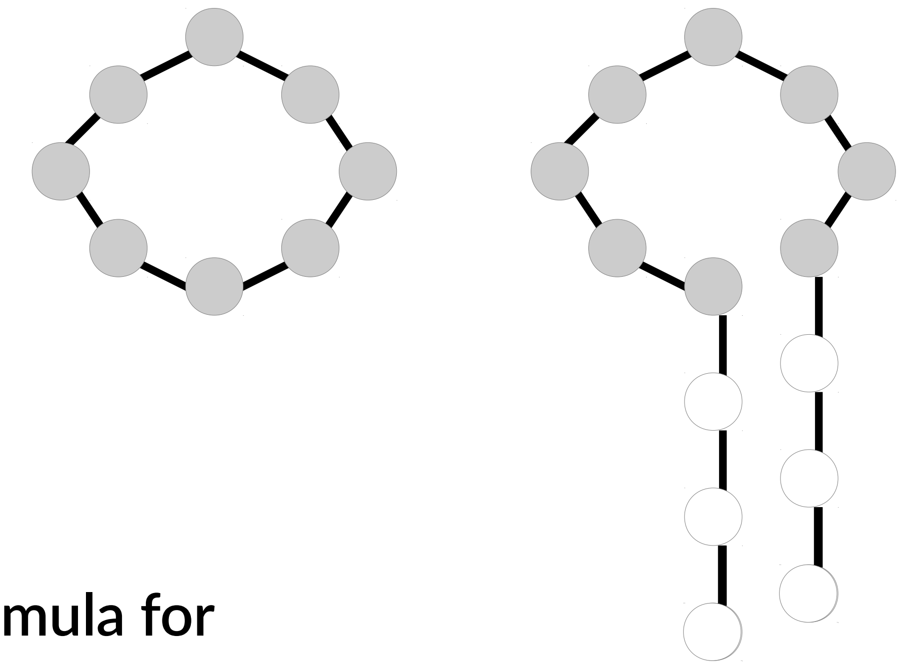
Simple formulas for all symmetry classes in 1D

[Fulga, Hassler, Akhmerov, Beenakker, Phys. Rev. B (2011)]

Generalizes via dimensional reduction to all dimensions, symmetry classes

[Fulga, Hassler, Akhmerov, Phys. Rev. B (2012)]

To define the scattering matrix, the system needs to be "opened up"



1) Open up the system
2) Attach leads
3) Define scattering matrix S

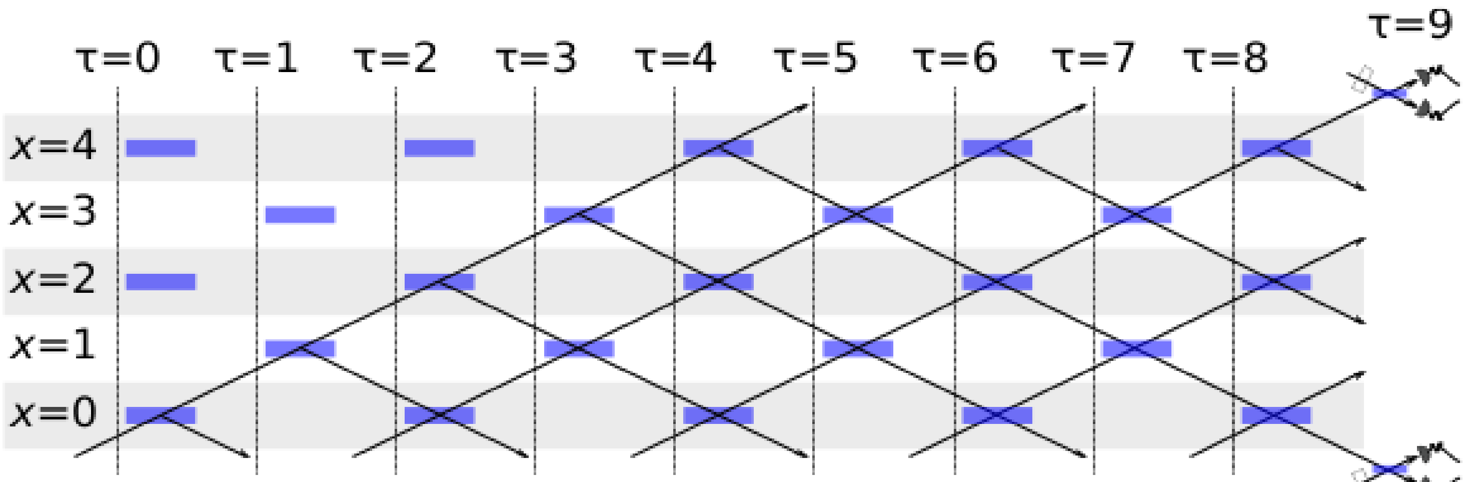
Mahaux-Weidemüller formula for continuous-time sytems:

$$S = 1 + 2\pi i W^{\dagger} (\tilde{H} - i\pi W W^{\dagger})^{-1} W.$$

Rewritten for discrete-time systems by Fyodorov&Sommers:

$$S(\epsilon) = \sigma_x e^{i\epsilon} \left[w_2 \frac{1}{e^{-i\epsilon} - A} w_1 + S_0 \right]$$

Can be transcribed to quantum walk on beam splitter array

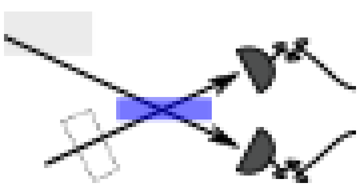


Measure transmission

Measure reflection

Introduce light from one edge at every timestep

→ Measure reflection after transients

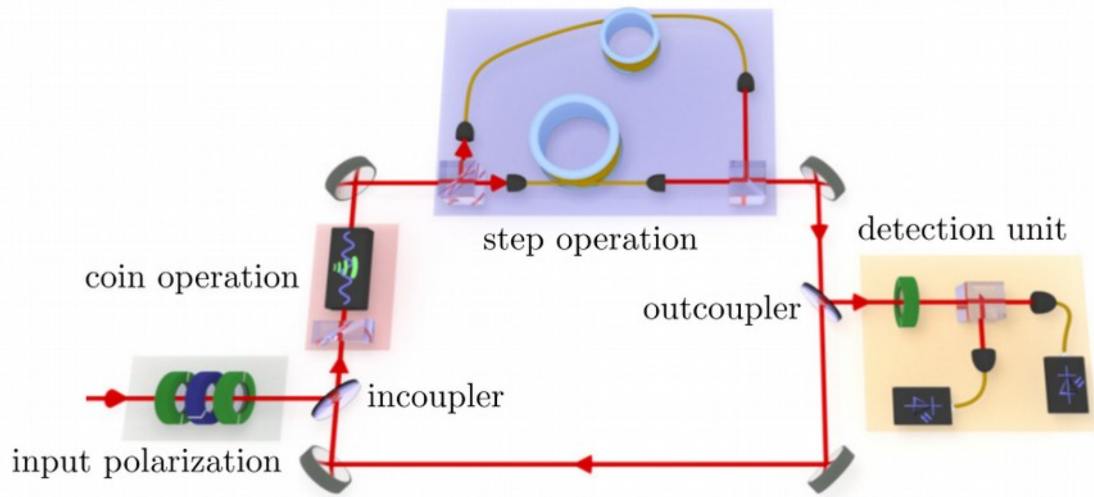


Introduce light only at t=0,
→ Measure reflection at every t

$$r(\varepsilon) = \sum_{\tau=0}^{\infty} e^{i\varepsilon\tau} r(\tau)$$

[B Tarasinski, JK Asbóth, JP Dahlhaus, Phys Rev A (2014)]

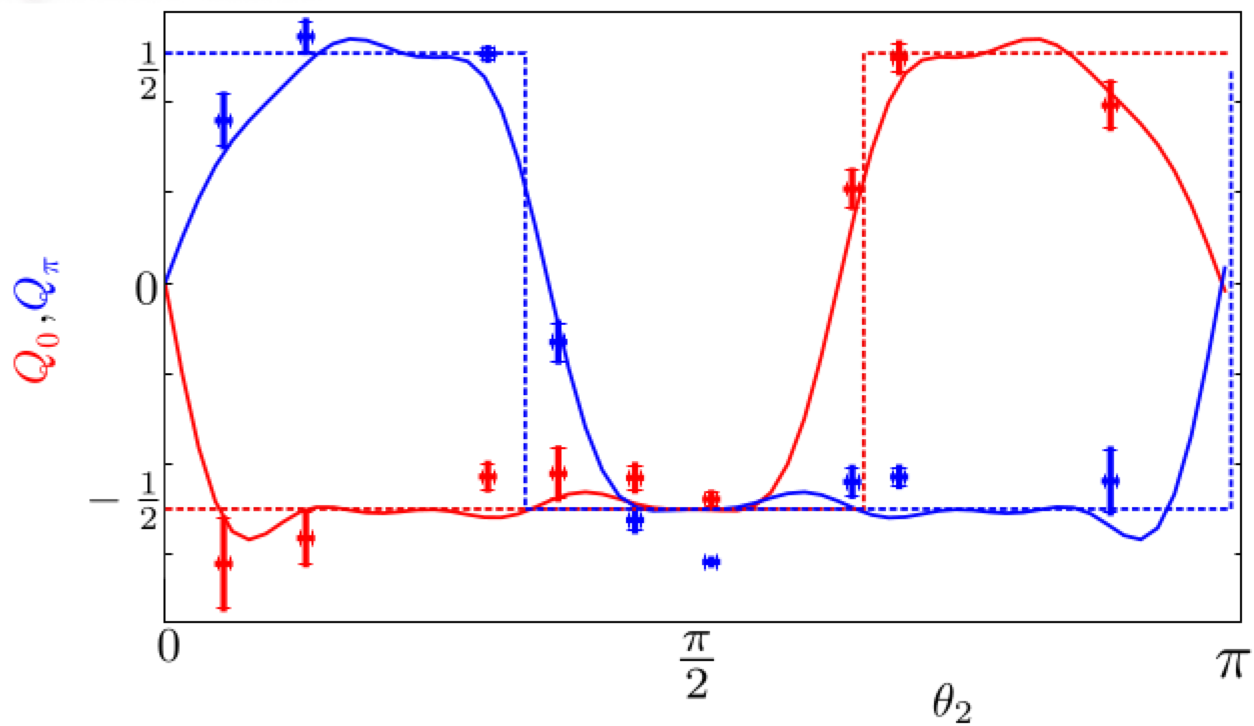
Experiment using our proposal: 2017, Silberhorn group



Previously demostrated:
fluctuating disorder
→ diffusion
time-independent disorder
→ Anderson localization
[Schreiber et al, PRL (2011)]

- Implemented scattering setup
- Quantized reflection amplitudes
- Also with time-independent disorder (localized)
- Transition smoothened by finite sampling time

[Barkhofen et al, Phys. Rev. A (2017)



Quantum Walks as simulators for solid state

Topological insulators: interesting Hamiltonians to simulate

Extra topological invariants of quantum walks

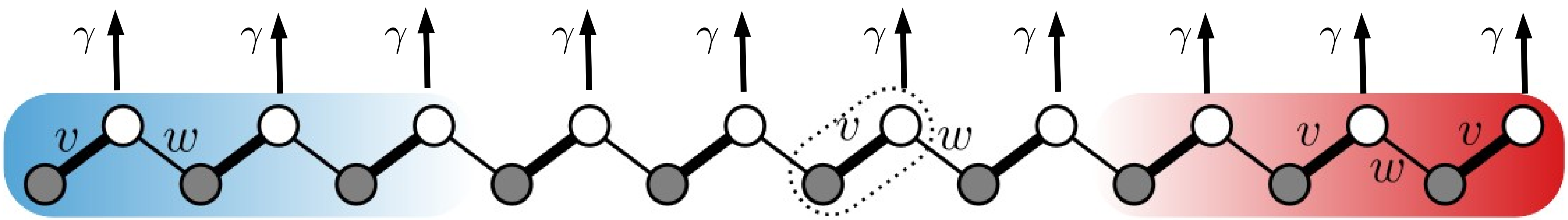
Two methods to measure topological invariants, with disorder:

- Using scattering matrices
- Using weak measurement & expected displacement

Scattering theory of topological phases in discrete-time quantum walks
B Tarasinski, JK Asbóth, JP Dahlhaus, Phys Rev A (2014)

Detecting topological invariants in chiral symmetric insulators via losses
T Rakovszky, JK Asbóth, A Alberti, Phys Rev B (2017)

Second method, generalizing results of Rudner & Levitov about non-Hermitian SSH model

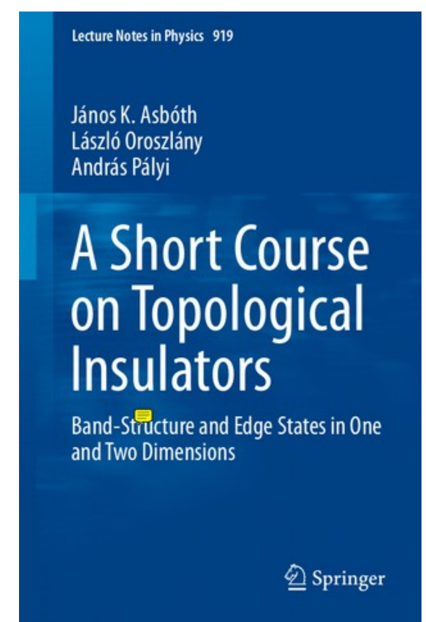


$$\hat{H} = v \sum_{m=1}^{L} \left(|m,B\rangle\langle m,A| + h.c. \right) + w \sum_{m=1}^{L-1} |m,B\rangle\langle m+1,A| + h.c.) - i\gamma \sum_{m=1}^{L} |m,B\rangle\langle m,B|$$

γ=0 : Su-Schrieffer-Heeger (SSH) model for polyacetylene (1979)
mother of all topological insulators

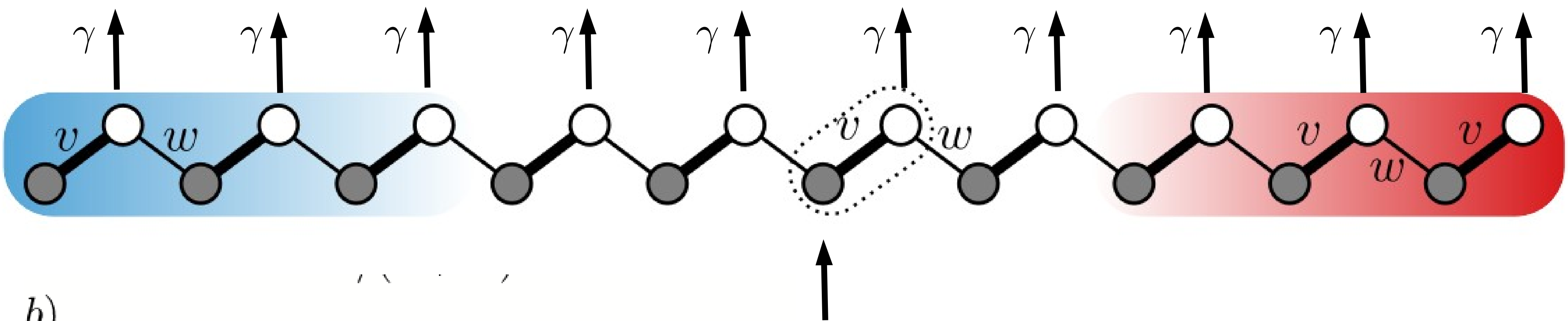
γ>0 : added by Rudner & Levitov to represent losses
→ Nonhermitian Hamiltonian for conditional time evolution.
Condition: no decay events.
Norm of wavefunction = prob(condition holds)

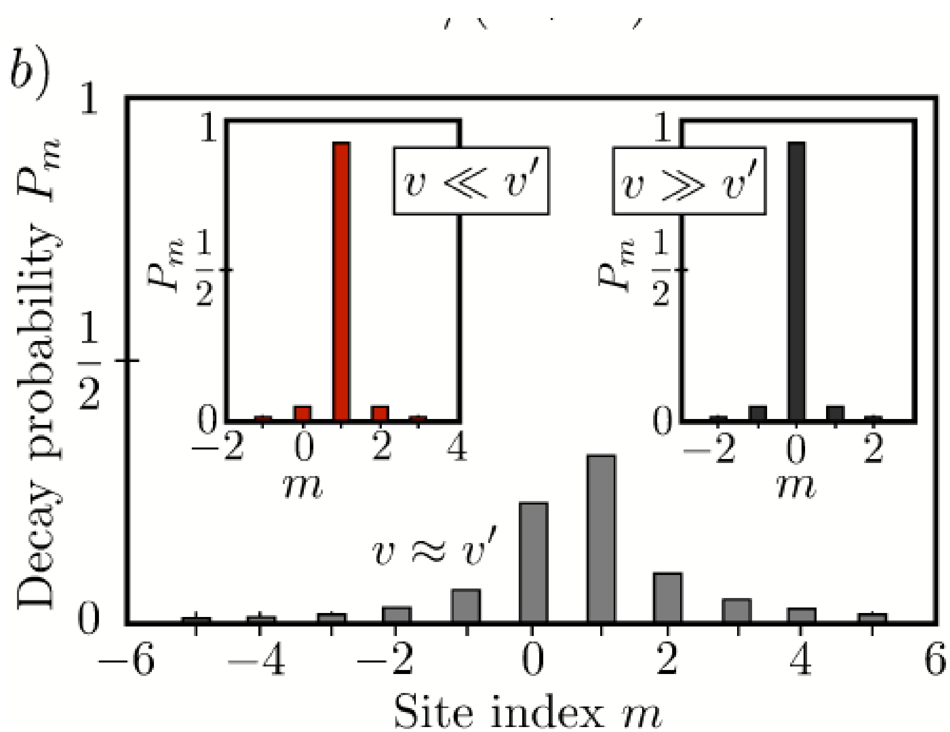
[Rudner and Levitov, Phys. Rev. Lett. (2009)]



Rudner and Levitov (2009): Nonhermitian SSH, expected displacement until decay = top. inv.

When decay happens, collect particle. Position of decay=displacement until decay





Insert single particle
at m=0, A

$$\bar{m} = \langle \Delta x \rangle = \nu$$

topological proof: mapping to a winding number

Our questions

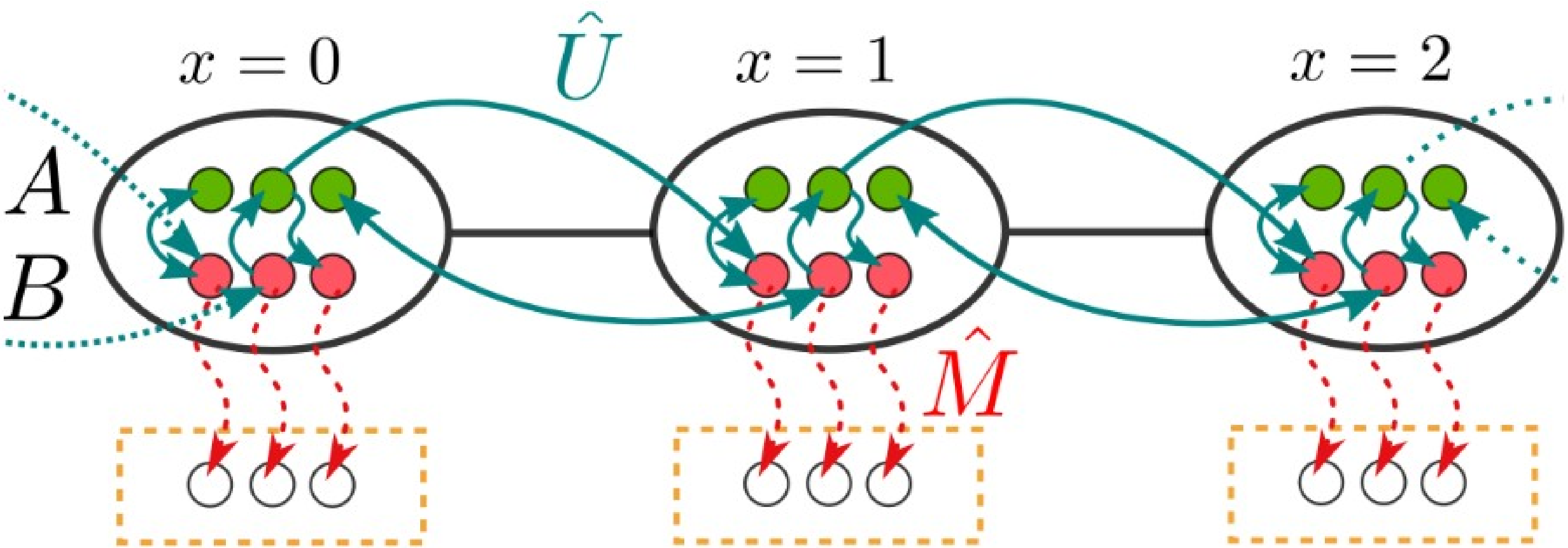
- Is Rudner & Levitov result general, or only specific to two-band model? (Their proof only works for two-band model)
- Is it valid for disordered systems?
- How to translate this to periodically driven systems?

$$\hat{H}(t) = \hat{H}(t+1) \qquad \hat{U} = \mathcal{T} e^{-i\int_0^1 \hat{H}(t)dt} = e^{-i\hat{H}_{\text{eff}}}$$

energy → quasienergy E

pair of winding numbers at E=0, E=π [Asboth & Obuse, PRB (2013)]

The way to realize losses is by weak measurement on sublattice B at the end of each driving cycle



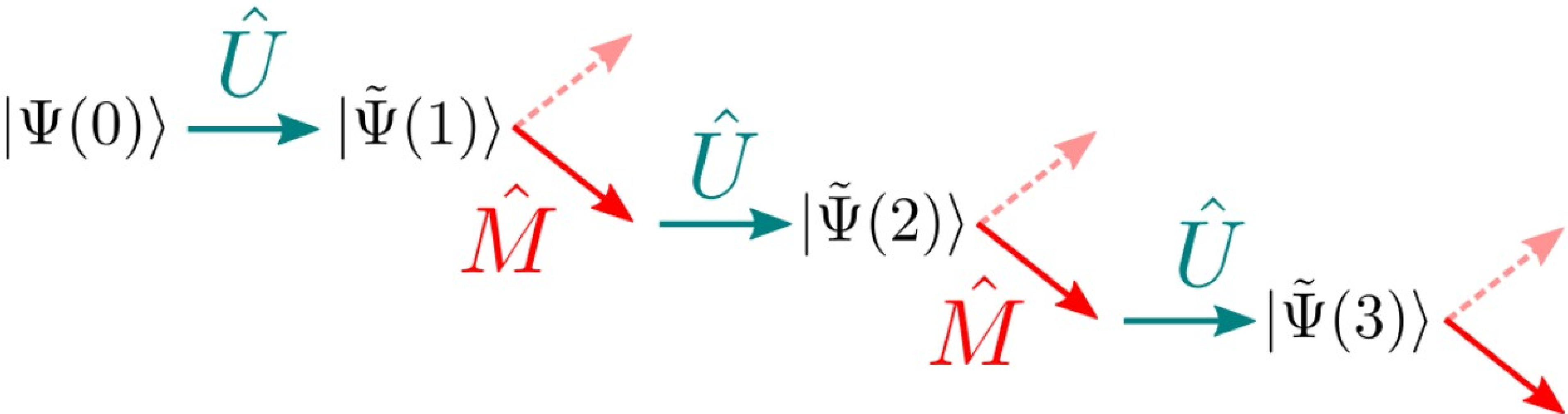
Effect of negative measurement:

(particle not detected)

$$\hat{M} = \hat{P}_A + \sqrt{1 - p_M}\, \hat{P}_B$$

Measurement efficiency

Continue time evolution until particle is detected



Conditional wavefunction: $$|\tilde{\Psi}(t)\rangle = \hat{U}\left[\hat{M}\hat{U}\right]^{t-1}|\Psi(0)\rangle$$

$$\hat{M} = \hat{P}_A + \sqrt{1-p_M}\,\hat{P}_B$$

Static case: $\text{period time} \to 0,\ p_M \to 0$

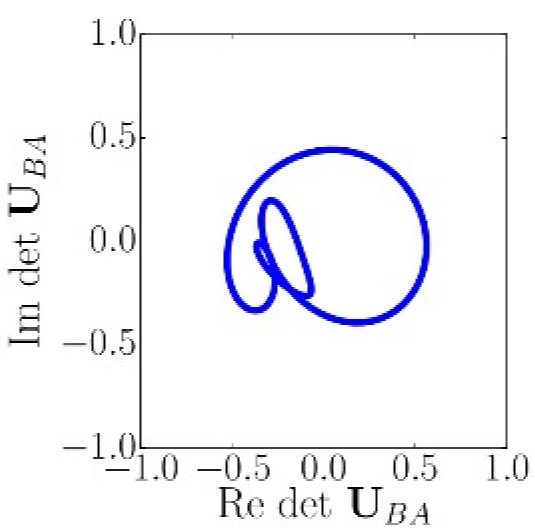
Expected displacement ⟨Δx⟩= topological invariant υ/N

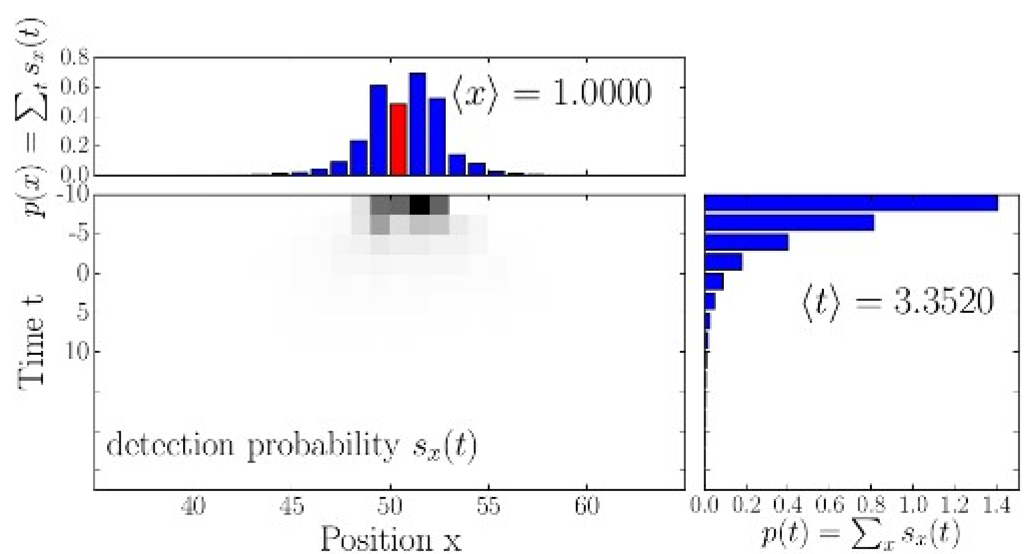
Expectation value of measured position:

$$\langle x \rangle \equiv \frac{p_M}{N} \sum_{t\in\mathbb{Z}^+} \sum_{x\in\mathbb{Z}} x \sum_{b=N+1}^{2N} \sum_{a=1}^{N} \left| \langle x, b | \hat{U} [\hat{M}\hat{U}]^{t-1} | x_0, a \rangle \right|^2$$

Translation invariance ➡

$$\boxed{\langle \Delta x \rangle \equiv \langle x \rangle - x_0 = \nu/N}$$



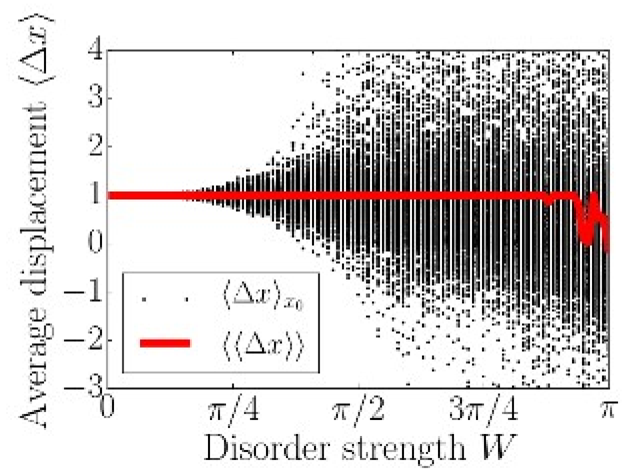


In the disordered case, averaging over initial position is needed: $\langle\langle \Delta x \rangle\rangle = \nu / N$

Disorder → Displacement depends on starting position

So let's average over them!

$$\langle\langle \Delta x \rangle\rangle = \frac{1}{L} \sum_{x_0} \langle \Delta x \rangle_{x_0}$$



Most general statement:

$$\langle\langle \Delta x \rangle\rangle = \frac{-2}{LN} \mathrm{Tr} \left\{ \hat{X} \hat{G} \hat{P}_{(E>0)} \right\} = \frac{\nu}{N}$$

$$\hat{G} = \hat{P}_A - \hat{P}_B$$

We proved ⟨⟨Δx⟩⟩=υ using non-commutative geometry formulation of winding number

Noncommutative geometry for topological insulators: Lori & Hastings, Prodan
for chiral symmetric (AIII): Mondragon-Shem et al, PRL (2014)

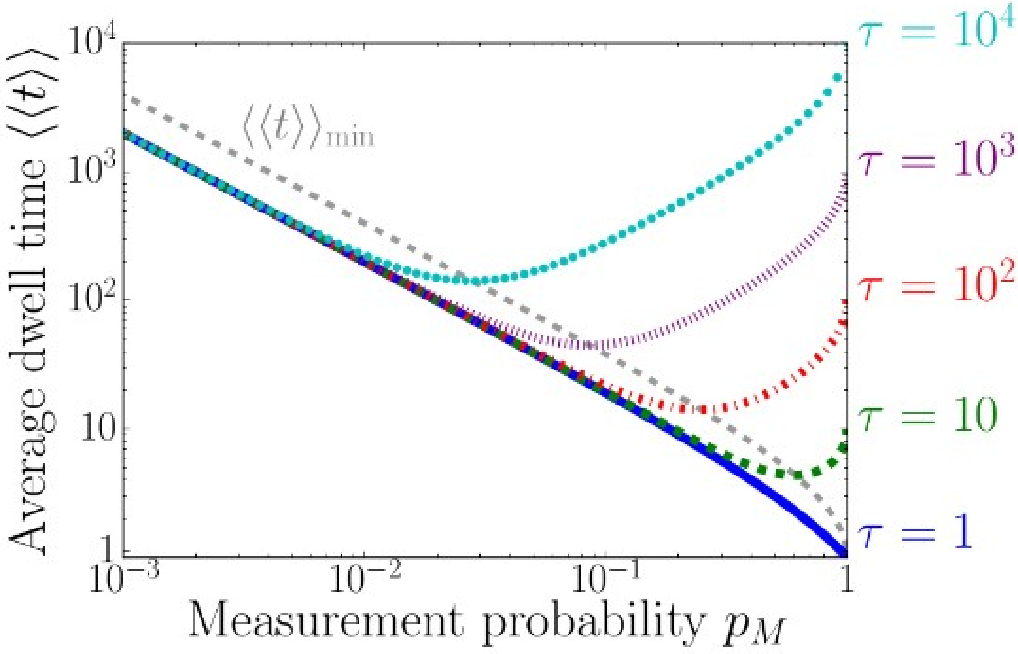
$$\nu = \frac{-(\pi i)^n}{(2n+1)!!} \sum_{\rho} (-1)^{\rho} \mathcal{T} \left\{ \prod_{i=1}^{2n+1} Q_{-+} [X_{\rho_i}, Q_{+-}] \right\}$$

Used this before on quantum walk, compared to scattering formulation of topological invariant [Rakovszky & Asboth, PRA (2015)]

Fast readout can require weak measurement, if almost-dark states are present

Average dwell time:

$$\langle\langle t \rangle\rangle = \frac{p_M}{(1+\sqrt{1-p_M})^2} \underbrace{\int_{E=0}^{\pi} \frac{\rho(E)}{\sin^2 E} \mathrm{d}E}_{\tau} + \frac{2\sqrt{1-p_M}}{p_M}$$



$$\langle\langle t \rangle\rangle|_{p_M=1} = \tau$$

$$\langle\langle t \rangle\rangle_{\min} \approx \sqrt{2\tau} \quad \text{for } \tau \gg 1$$

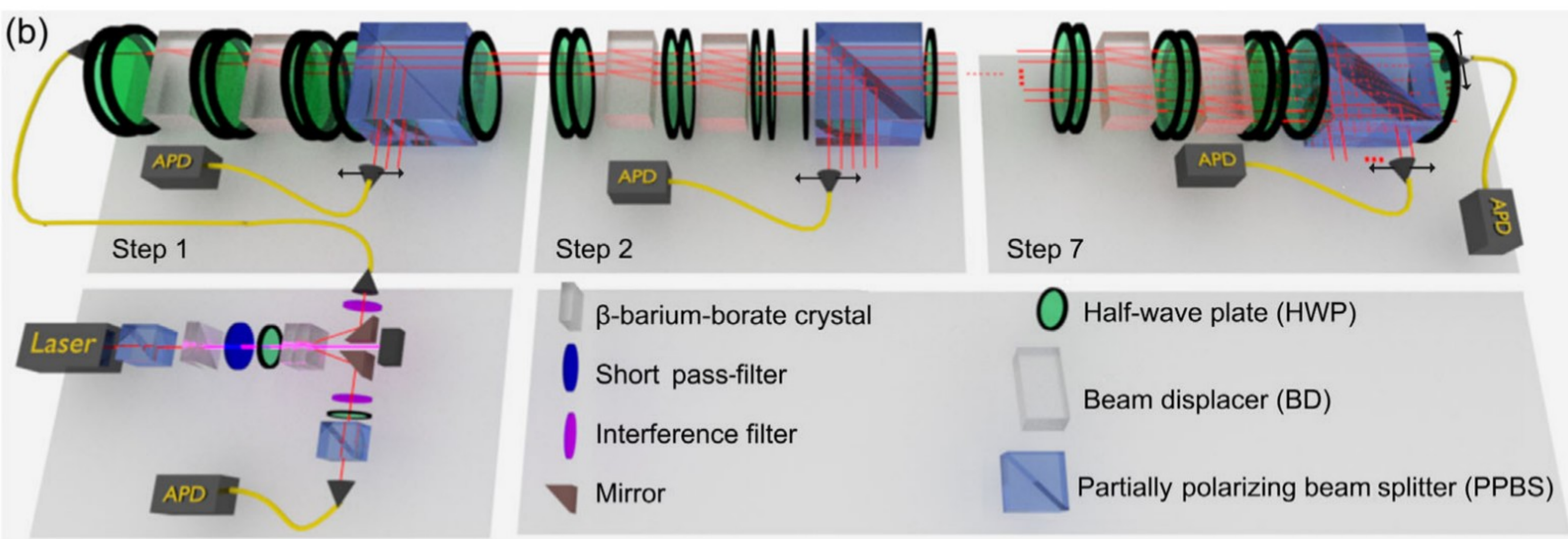
$$p_M^* \approx \sqrt{8/\tau}$$

Experiment using our proposal: 2017, Peng Xue's group

PRL **119**, 130501 (2017) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2017

Detecting Topological Invariants in Nonunitary Discrete-Time Quantum Walks

Xiang Zhan,[1] Lei Xiao,[1] Zhihao Bian,[1] Kunkun Wang,[1] Xingze Qiu,[2,3] Barry C. Sanders,[3,4,5,6] Wei Yi,[2,3,*] and Peng Xue[1,7,†]



Topological invariants using displacement: Open questions, related work

- Does something like this work in 3 dimensions?
- Massignan & collaborators have since found similar results for ⟨Δx⟩ defined for Hermitian Hamiltonians, in long-time limit. Precise equivalence?

1. **arXiv:1802.02109 [pdf, other]**

   Observation of the topological Anderson insulator in disordered atomic wires
   Eric J. Meier, Fangzhao Alex An, Alexandre Dauphin, Maria Maffei, Pietro Massignan, Taylor L. Hughes, Bryce Gadway
   Comments: 6 pages, 3 figures; 9 pages of supplementary materials
   Subjects: **Quantum Gases (cond-mat.quant-gas)**; Disordered Systems and Neural Networks (cond-mat.dis-nn); Quantum Physics (quant-ph)

2. **arXiv:1708.02778 [pdf, other]**

   Topological characterization of chiral models through their long time dynamics
   Maria Maffei, Alexandre Dauphin, Filippo Cardano, Maciej Lewenstein, Pietro Massignan
   Journal-ref: New J. Phys. 20, 013023 (2018)
   Subjects: **Other Condensed Matter (cond-mat.other)**; Mesoscale and Nanoscale Physics (cond-mat.mes-hall); Quantum Gases (cond-mat.quant-gas); Quantum Physics (quant-ph)

3. **arXiv:1610.06322 [pdf, other]**

   Detection of Zak phases and topological invariants in a chiral quantum walk of twisted photons
   F. Cardano, A. D'Errico, A. Dauphin, M. Maffei, B. Piccirillo, C. de Lisio, G. De Filippis, V. Cataudella, E. Santamato, L. Marrucci, M. Lewenstein, P. Massignan
   Comments: 10 pages, 7 color figures (incl. appendices) Close to the published version
   Journal-ref: Nature Commun. 8, 15516 (2017)
   Subjects: **Mesoscale and Nanoscale Physics (cond-mat.mes-hall)**; Quantum Gases (cond-mat.quant-gas); Optics (physics.optics); Quantum Physics (quant-ph)

Summary of this talk

- Quantum Walks as simulators for solid state

  Topological insulators: interesting Hamiltonians to simulate
- Extra topological invariants of quantum walks
- Two methods to measure topological invariants, with disorder:
  - Using scattering matrices
  - Using weak measurement & expected displacement

Scattering theory of topological phases in discrete-time quantum walks
B Tarasinski, JK Asbóth, JP Dahlhaus, Phys Rev A (2014)

Detecting topological invariants in chiral symmetric insulators via losses
T Rakovszky, JK Asbóth, A Alberti, Phys Rev B (2017)

My collaborators on these projects

Andrea Alberti,
Uni Bonn

Tibor Rakovszky,
TU München

Brian Tarasinski,
QuTech Delft

Jan Dahlhaus,
project manager,
Munchen

Currently funded by: National Research, Development and Innovation Office of Hungary,

→ FK 124723: *From Topologically Protected States to Topological Quantum Computation*

→ National Quantum Technology Program, 2017-1.2.1-NKP-2017-00001 *Preparation and distribution of quantum bits, and development of quantum networks*

HUNQUTECH

NATIONAL RESEARCH, DEVELOPMENT AND INNOVATION OFFICE
HUNGARY

PROJECT FINANCED FROM THE NRDI FUND

MOMENTUM OF INNOVATION