

First Principles Model Hamiltonian Ensembles for Light Harvesting: Modeling Dissipative Down Conversion - Signatures of Vibronic Energy Transfer in Nonlinear 2DES Signals - CSDM+PLDM

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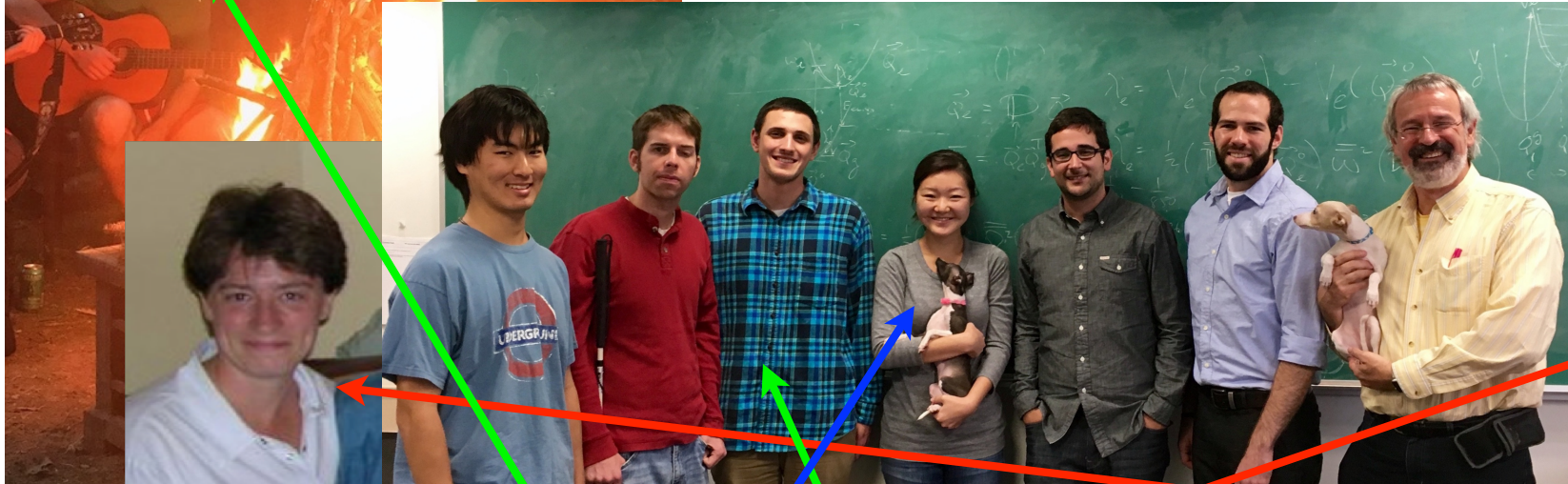
Charge and Energy Transfer Processes: Open Problems in Open Quantum Systems

Banff International Research Center Workshop, Banff, Alberta, Canada

August 18-23, 2019



Ksenia Bravaya



Thanks to: Mi Kyung Lee, Pengfei (Frank) Huo, Sara Bonella
Francesco Segatta, Justin Provazza, Marco Garavelli



Time scale separation?

- (1) Excitation Energy Transfer (EET) dynamics is usually explored using averaged model Hamiltonians.
- (2) However, the EET dynamics can occur faster than the time scale on which these model Hamiltonians change.

So rather than looking at

The dynamics of an average Hamiltonian (fit to average experimental results)

we should be studying

The average dynamics of an ENSEMBLE of model Hamiltonians, e.g. averaging dynamics over site disorder, etc. Fluctuations, perhaps with correlations, around INHERENT STRUCTURES - F. Stillinger J. Chem. Phys. 83, 6413 (1985).

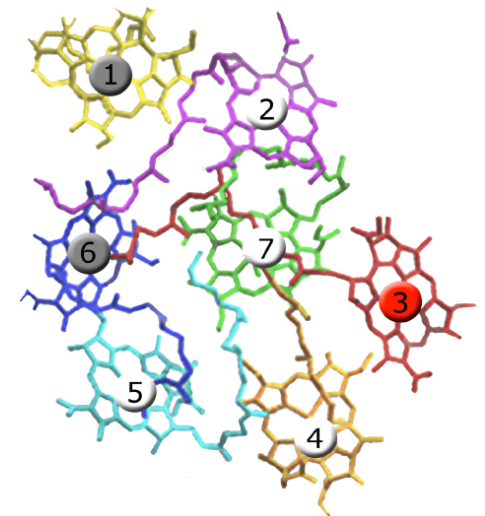
$$\{\epsilon_{\alpha}\} \quad \{\Delta_{\alpha,\beta}\} \quad \{J_{\alpha}(\omega)\}$$

Parameterized Frenkel Exciton Hamiltonian: local Interpolation Model

$$\hat{H} = \sum_{\alpha=1}^{N_{state}} \epsilon_{\alpha} |\alpha\rangle \langle \alpha| + \sum_{\alpha \neq \beta}^{N_{state}} \Delta_{\alpha,\beta} [|\alpha\rangle \langle \beta| + |\beta\rangle \langle \alpha|]$$

$$+ \sum_{\alpha=1}^{N_{state}} \sum_{i=1}^{n^{(\alpha)}} c_i^{(\alpha)} R_i^{(\alpha)} |\alpha\rangle \langle \alpha|$$

$$+ \sum_{\alpha=1}^{N_{state}} \sum_{i=1}^{n^{(\alpha)}} \frac{1}{2} [P_i^{(\alpha)2} + \omega_i^{(\alpha)2} R_i^{(\alpha)2}]$$



7 state FMO test

In principle Site dependent “spectral densities”

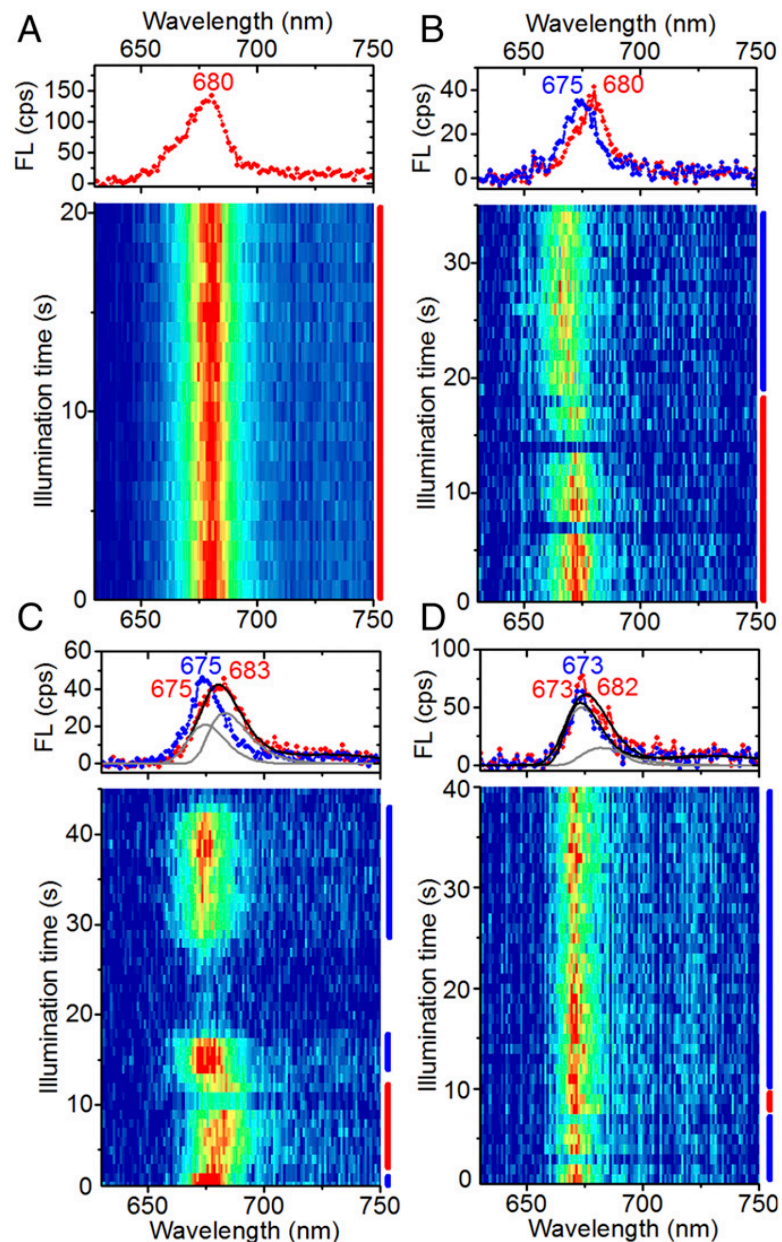
$$J^{(\alpha)}(\omega) = \frac{\pi}{2} \sum_i \frac{c_i^{(\alpha)2}}{\omega_i} \delta(\omega - \omega_i) = 2\lambda\omega\tau_c / (1 + \omega^2\tau_c^2) \quad \tau_c = 50 \text{ fs and } \lambda = 35 \text{ cm}^{-1}$$

Experimental fit assumes all have identical environments!

Fucoxanthin -
Chlorophyll Proteins
(FCP) from light
harvesting antennae
of diatoms
Kruger et al. PNAS
114, E11063 (2017)

Protein fluctuations
drive switching of
exciton delocalization

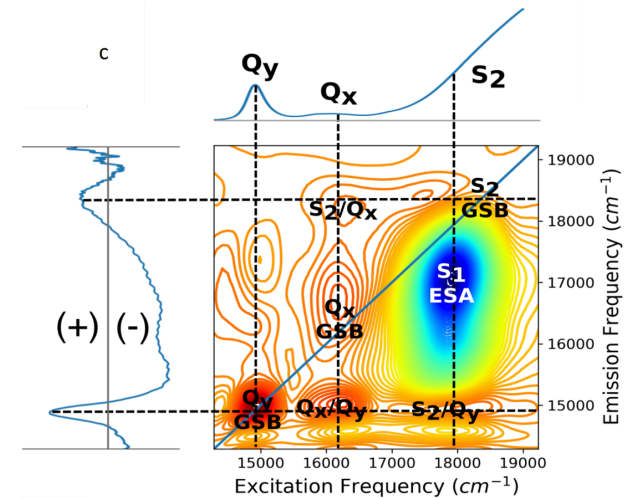
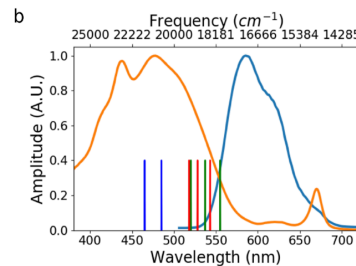
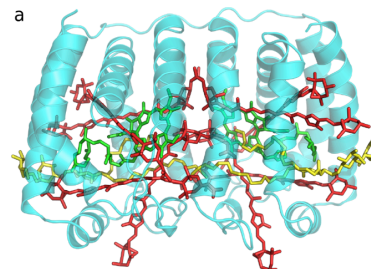
Carotenoid -
Chlorophyll
complexes

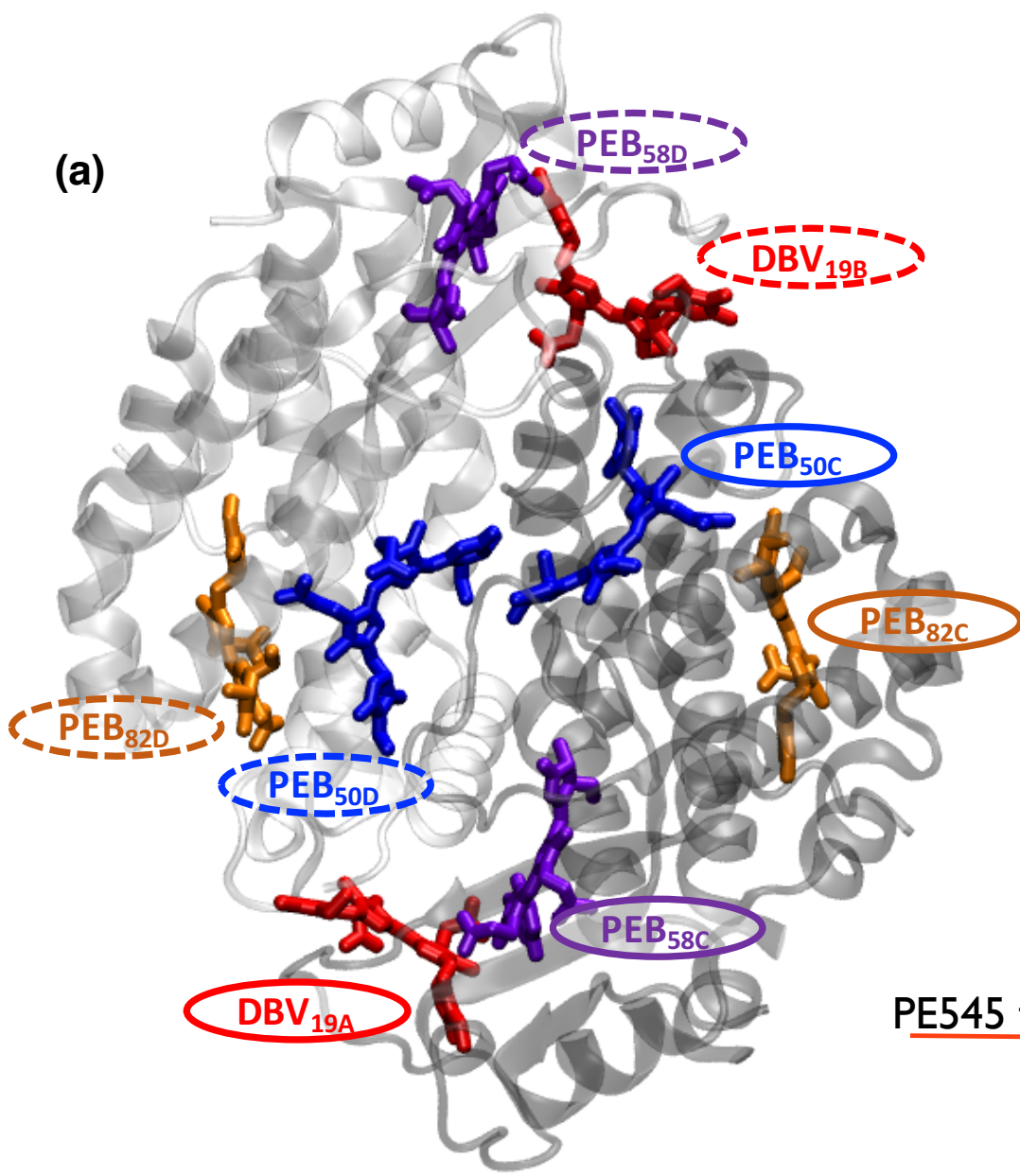


Enhanced Sampling: Replica
Exchange, Parallel Tempering
- Sugita (RIKEN)
Alex Hino

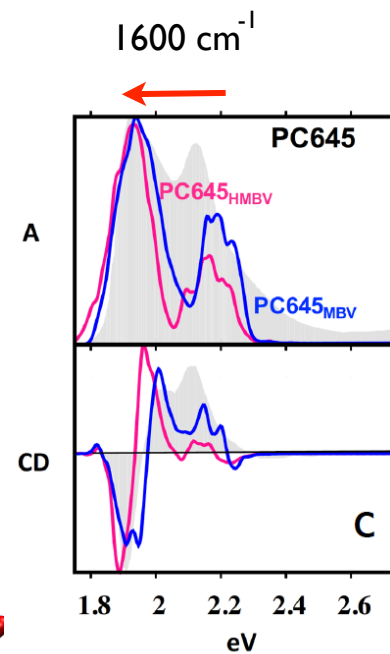
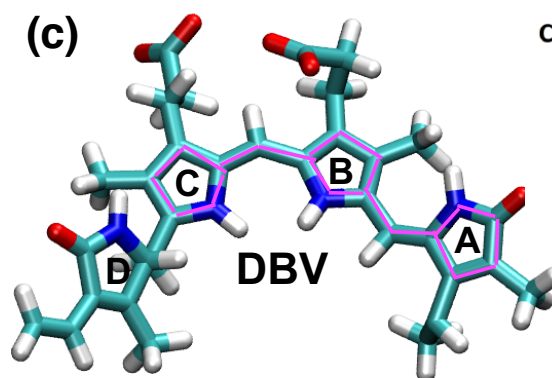
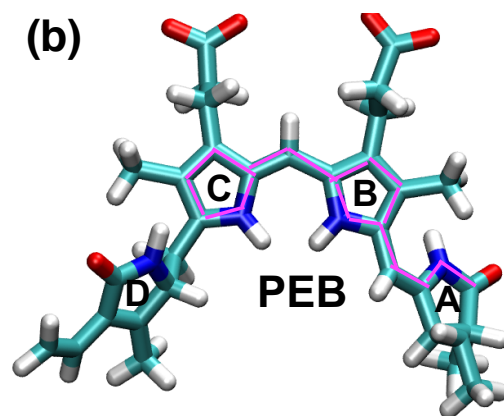
Down Conversion Pigment-Protein Complexes

Down-conversion molecules that enable absorbed blue light to “plug into” red functioning chlorophyll energy transport and transduction networks by using vibronic transitions and engineered environmental dissipation e.g. in Peridinin-Chlorophyll-a Protein (PCP), or phycobiliproteins like PC645





“step down transformer”



PE545 from cryptophyte algae linear tetrapyrrols or “bilins” & phycobili proteins (PC645, PC577, PE555, PC612,...)

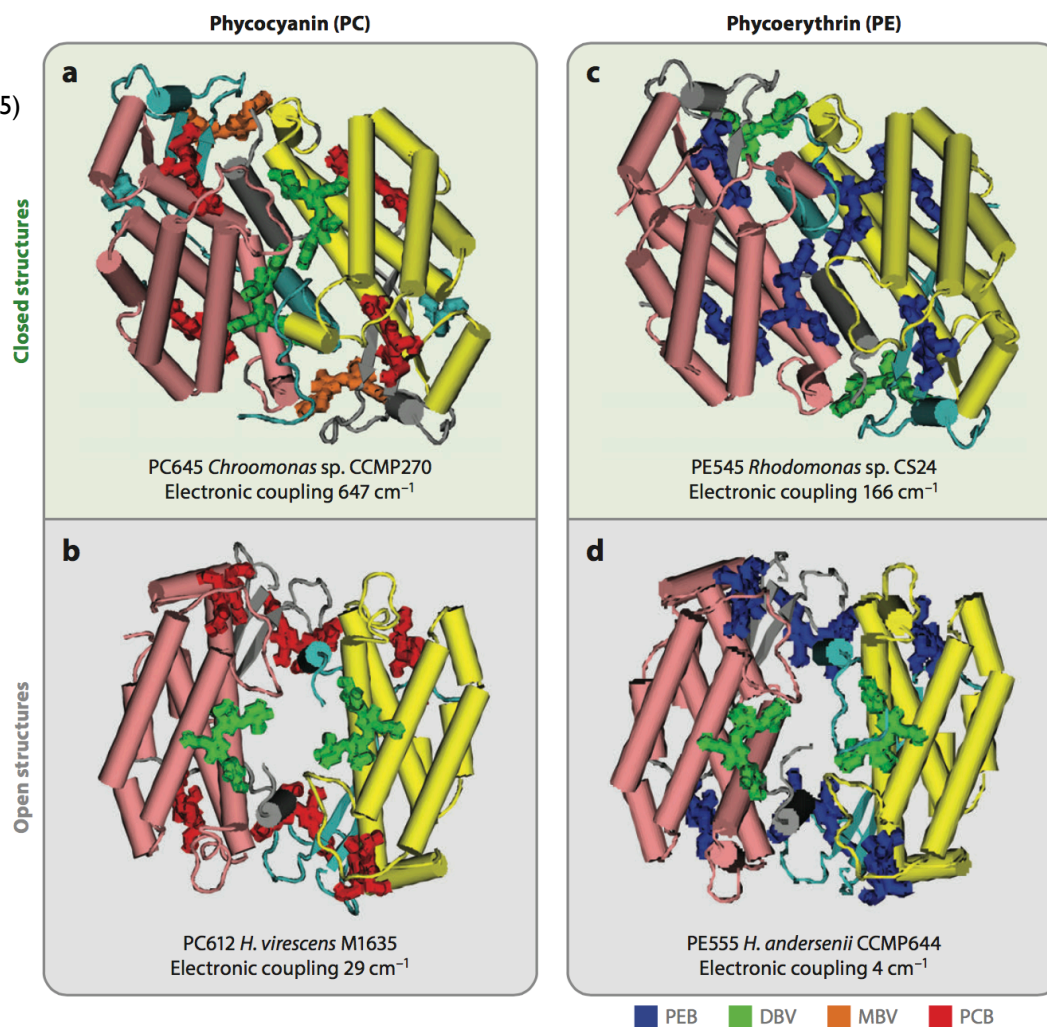


Figure 2

Structures of four representative phycobiliproteins from cryptophyte algae (40, 41): two phycocyanin light-harvesting complexes, (a) PC645 and (b) PC612, and two phycoerythrin light-harvesting complexes, (c) PE545 and (d) PE555. PC645 and PE545 (*top row*) are examples of closed structures, characterized by a large electronic coupling between the chromophores in the indicated central dimer of bilins. PC612 and PE555 (*bottom row*) are examples of open structures, in which the electronic coupling is weak. The large β -subunits are colored pink and yellow, and the small α -subunits are colored cyan and gray. Abbreviations: DBV, 15,16-dihydrobiliverdin; MBV, mesobiliverdin; PCB, phycocyanobilin; PEB, phycoerythrobilin.

Scholes,
Thordarson and
co-workers

Controllable Chimeras with function

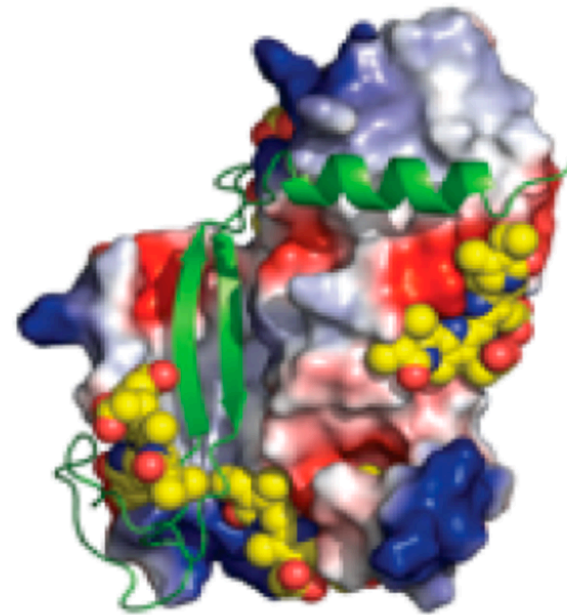
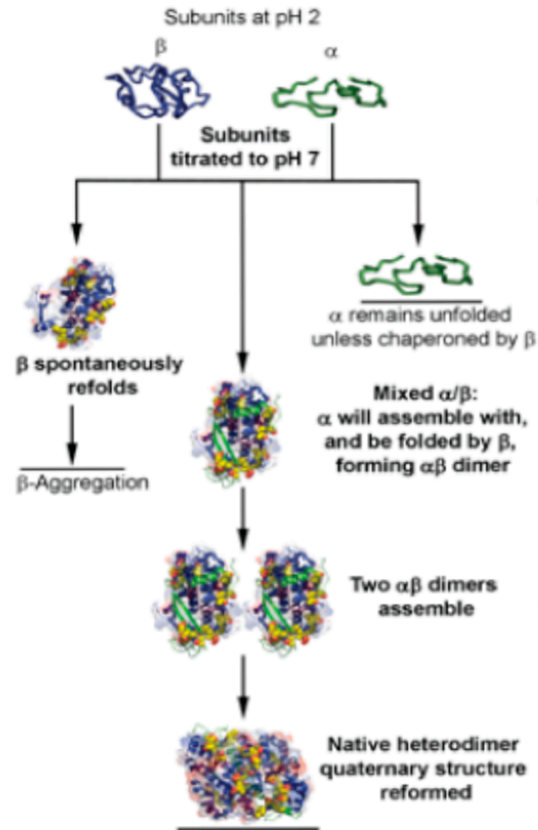
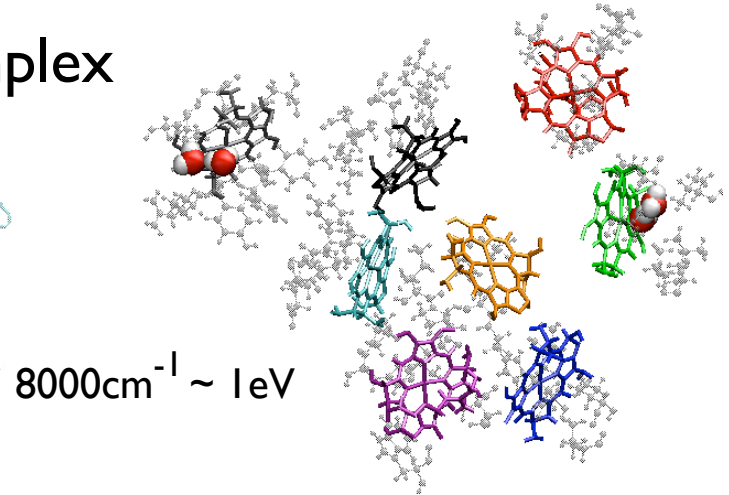
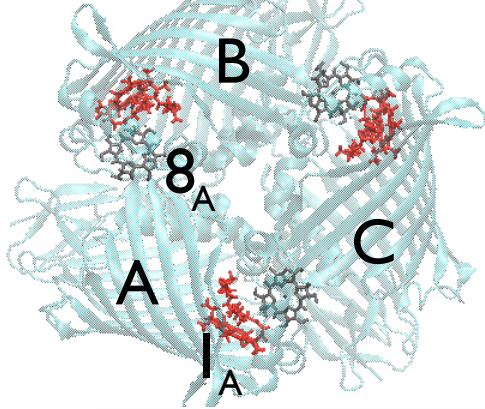
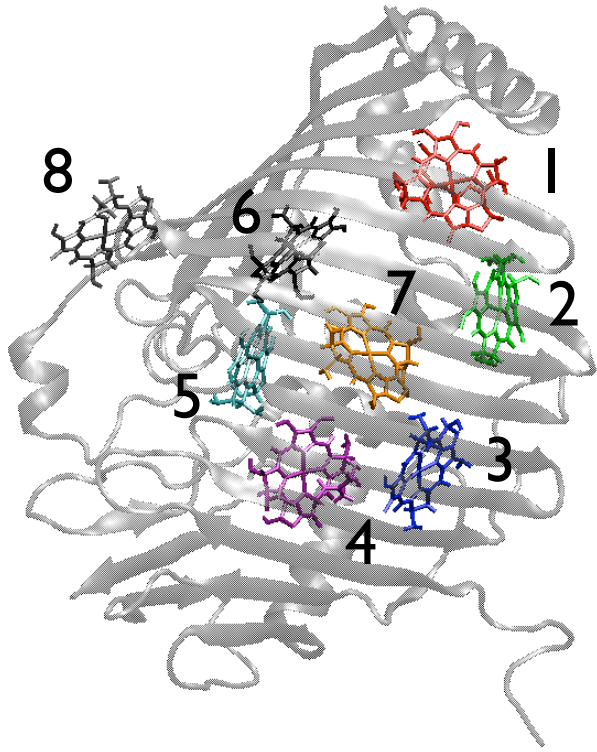


Figure 4: Schematic illustrating how we separate the light harvesting complex α and β subunits, and how we can reassemble them into a functional protein. The right figure shows a single β subunit and, in green, how the α subunit associates with it after assembly of the tetrameric light harvesting complex.

Moix, Cao, Huo, DFC JPCL (2011)

FMO bacterial LH complex



Site Dependent Spectral Density Models

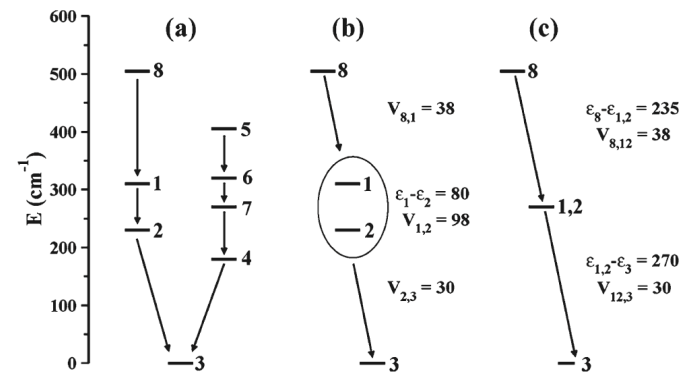
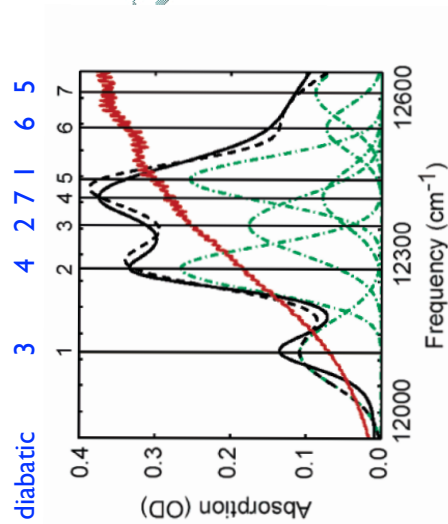


Figure 1. Energy diagrams for the eight-site model (a), the reduced four-site model (b), and the reduced three-site model (c) used in the calculations of Figure 3.



FLN

OUTLINE:

- (1) Spectral Density Calculations
- (2) Excitation Energy Calculations
- (3) Dissipative Quantum Dynamics for General Regimes
- (4) Issues with Linearized Dynamics of Higher Frequency Modes
- (5) Spectra (PC645/HPC645) - Influence of Protonation and “Flickering” Pathways
- (6) Quantum Dynamics and Nonlinear Spectroscopy
- (7) Coherent State Density Matrix Dynamics (+PLDM)

Exciton-Phonon Coupling

$$\hat{H} = \hat{H}_s + \hat{H}_{sb}$$

$$\hat{H}_{sb} = \sum_{\alpha=1}^N \sum_{i_{\alpha}=1}^{n_{\alpha}} \left[\frac{1}{2} (P_{i_{\alpha}}^2 + \omega_{i_{\alpha}}^2 Q_{i_{\alpha}}^2) - c_{i_{\alpha}} Q_{i_{\alpha}} |\alpha\rangle\langle\alpha| \right]$$

$$\hat{H}_s = \sum_{\alpha=1}^N \varepsilon_{\alpha} |\alpha\rangle\langle\alpha| + \sum_{\alpha \neq \gamma}^N \Delta_{\alpha,\gamma} |\alpha\rangle\langle\gamma|$$

$$J_{\alpha}(\omega) \equiv \frac{\pi}{2} \sum_{i_{\alpha}} \frac{c_{i_{\alpha}}^2}{\omega_{i_{\alpha}}} \delta(\omega - \omega_{i_{\alpha}})$$

$$\delta\varepsilon_{\alpha} = \sum_{i_{\alpha}} c_{i_{\alpha}} Q_{i_{\alpha}}$$

Standard Approach: Ground state (MM) MD averaged excitation energy fluctuation correlation function. **Inconsistent MM and electronic structure!**

$$C_{\alpha}(t) = \langle \delta\varepsilon_{\alpha}(0) \delta\varepsilon_{\alpha}(t) \rangle = \sum_{i_{\alpha}} c_{i_{\alpha}}^2 \langle Q_{i_{\alpha}}(0) Q_{i_{\alpha}}(t) \rangle$$

$$J_{\alpha}(\omega) = \beta \omega \int_0^{\infty} dt C_{\alpha}^{cl}(t) \cos \omega t$$

Simple harmonic model of the **standard approach**

Kleinekathoefer & Schulten 2002

$$V_{\alpha}^g(\mathbf{Q}_{\alpha}) = \frac{1}{2} \sum_{i_{\alpha}} \omega_{i_{\alpha}}^2 (Q_{i_{\alpha}} - Q_{i_{\alpha}}^g)^2$$

$$\delta \varepsilon_{\alpha} = \sum_{i_{\alpha}} c_{i_{\alpha}} Q_{i_{\alpha}}$$

$$\delta \varepsilon_{\alpha}(Q_{i_{\alpha}}) = V_{\alpha}^e(Q_{i_{\alpha}}) - V_{\alpha}^g(Q_{i_{\alpha}}) = \text{constant} - \sum_{i_{\alpha}} \omega_{i_{\alpha}}^2 (Q_{i_{\alpha}}^e - Q_{i_{\alpha}}^g) Q_{i_{\alpha}}$$

$$V_{\alpha}^e(\mathbf{Q}_{\alpha}) = \frac{1}{2} \sum_{i_{\alpha}} \omega_{i_{\alpha}}^2 (Q_{i_{\alpha}} - Q_{i_{\alpha}}^e)^2 + E_{\alpha}$$

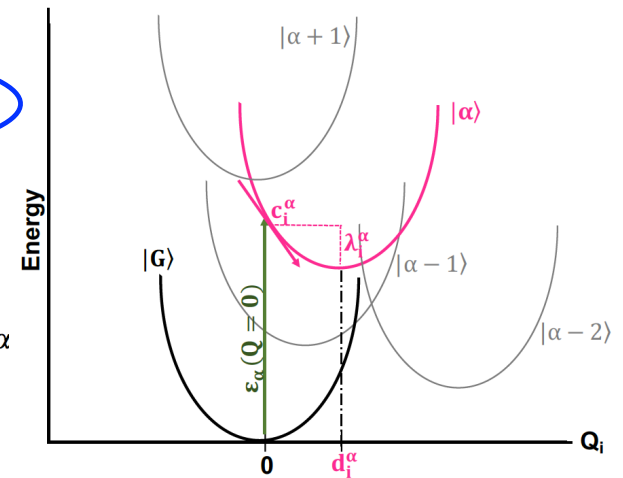
$$V_{\alpha}^{\text{MM}}(\mathbf{q}_{\alpha}) = \frac{1}{2} \sum_{i_{\alpha}} \Omega_{i_{\alpha}}^2 (q_{i_{\alpha}} - q_{i_{\alpha}}^{\text{MM}})^2 + E_{\alpha}^{\text{MM}}$$

$$\lambda_{i_{\alpha}} = \frac{1}{2} \omega_{i_{\alpha}}^2 (Q_{i_{\alpha}}^e - Q_{i_{\alpha}}^g)^2 = S_{i_{\alpha}} \hbar \omega_{i_{\alpha}}$$

Huang-Rhys (HR) Factors

$$c_{i_{\alpha}} = \omega_{i_{\alpha}}^2 (Q_{i_{\alpha}}^e - Q_{i_{\alpha}}^g) = \sqrt{2\lambda_{i_{\alpha}}} \omega_{i_{\alpha}} = \sqrt{2\hbar\omega_{i_{\alpha}} S_{i_{\alpha}}} \omega_{i_{\alpha}}$$

$$\lambda_{i_{\alpha}} = c_{i_{\alpha}}^2 / 2\omega_{i_{\alpha}}^2$$



$$J_{\alpha}(\omega) = \pi \sum_{i_{\alpha}} \omega_{i_{\alpha}} \lambda_{i_{\alpha}} \delta(\omega - \omega_{i_{\alpha}})$$

What about the MD time correlation function with the MM potential??

$$V_{\alpha}^{\text{MM}}(\mathbf{q}_{\alpha}) = \frac{1}{2} \sum_{i_{\alpha}} \overset{\omega_{i_{\alpha}}}{\Omega_{i_{\alpha}}^2} (q_{i_{\alpha}} - q_{i_{\alpha}}^{\text{MM}})^2 + E_{\alpha}^{\text{MM}}$$

Duschinsky-like rotation

$$\mathbf{Q}_{\alpha} = \underline{\underline{D}}_{\alpha} \mathbf{q}_{\alpha} \quad D_{k_{\alpha}i_{\alpha}} = \hat{\mathbf{q}}_{i_{\alpha}}^T \hat{\mathbf{Q}}_{k_{\alpha}}$$

$$H_{\text{MM}}(\mathbf{p}^{\alpha}, \mathbf{q}^{\alpha}) = \frac{1}{2} \sum_{i_{\alpha}} p_{i_{\alpha}}^2 + V_{\text{MM}}^{\alpha}(\mathbf{q}^{\alpha})$$

$$q_{i_{\alpha}}(t) = q_{i_{\alpha}}^{\text{MM}} + (q_{i_{\alpha}}(0) - q_{i_{\alpha}}^{\text{MM}}) \cos \Omega_{i_{\alpha}} t + \frac{p_{i_{\alpha}}(0)}{\Omega_{i_{\alpha}}} \sin \Omega_{i_{\alpha}} t$$

$$C_{\alpha}^{\text{MM}}(t) = \frac{1}{Z^{\text{MM}}} \int d\mathbf{p}_{\alpha}(0) \int d\mathbf{q}_{\alpha}(0) \exp[-\beta H^{\text{MM}}(\mathbf{p}_{\alpha}(0), \mathbf{q}_{\alpha}(0))] \delta \varepsilon_{\alpha}(0) \delta \varepsilon_{\alpha}(t)$$

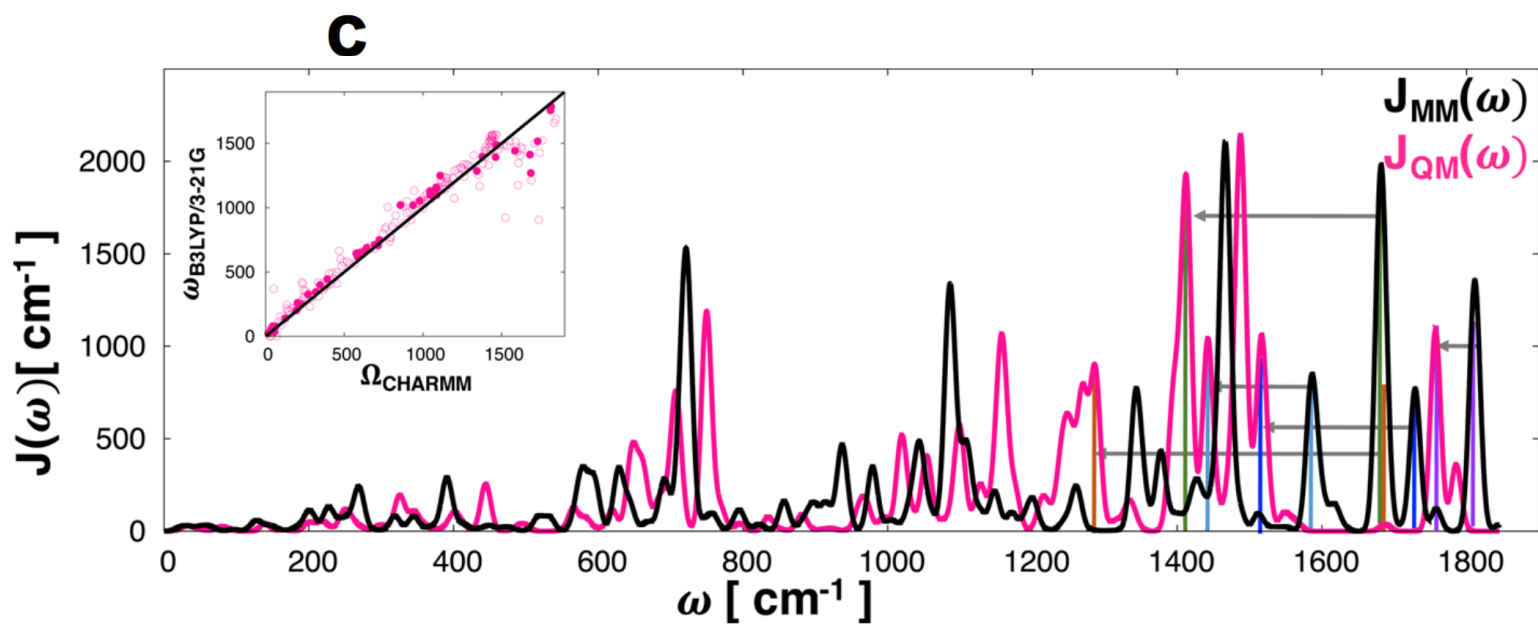
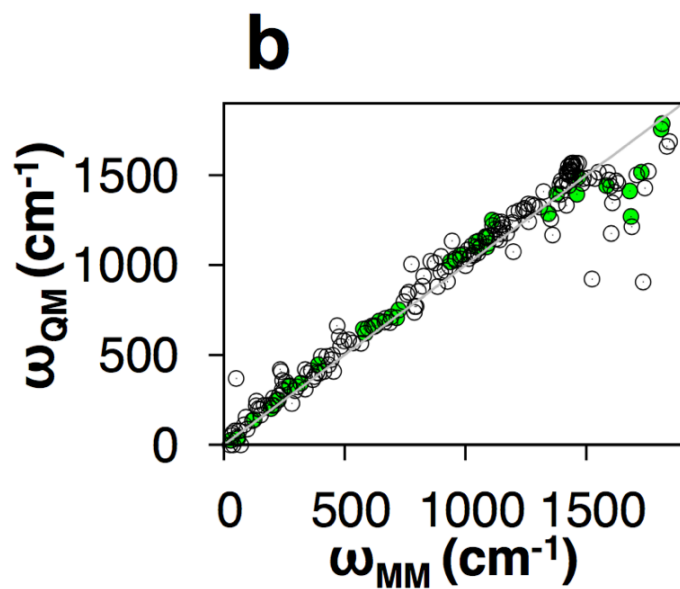
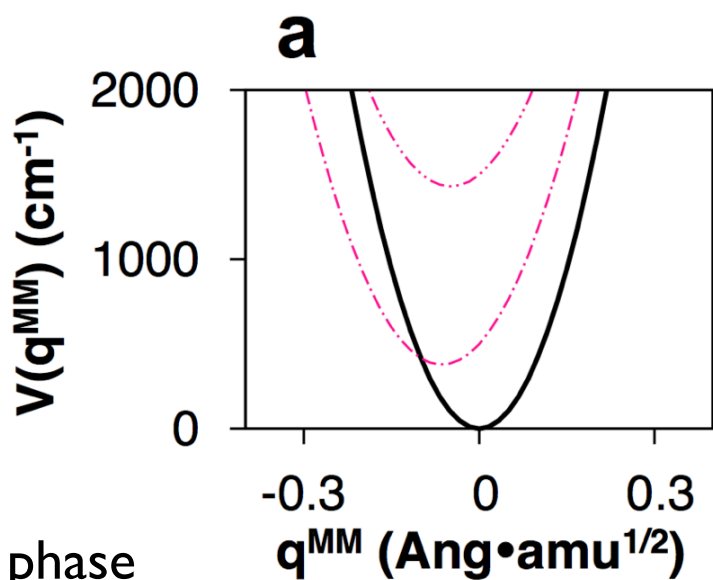
$$Z^{\text{MM}} = \int d\mathbf{p}_{\alpha}(0) \int d\mathbf{q}_{\alpha}(0) \exp[-\beta H^{\text{MM}}(\mathbf{p}_{\alpha}(0), \mathbf{q}_{\alpha}(0))]$$

$$C_{\alpha}^{\text{MM}}(t) = \frac{2}{\beta} \sum_{i_{\alpha}} \sum_{k_{\alpha}} \left(\frac{\omega_{k_{\alpha}} D_{k_{\alpha}i_{\alpha}}}{\Omega_{i_{\alpha}}} \right)^2 \lambda_{k_{\alpha}} \cos \Omega_{i_{\alpha}} t$$

$$J_{\alpha}^{\text{MM}}(\omega) = \pi \omega \sum_{i_{\alpha}} \sum_{k_{\alpha}} \left(\frac{\omega_{k_{\alpha}} D_{k_{\alpha}i_{\alpha}}}{\Omega_{i_{\alpha}}} \right)^2 \lambda_{k_{\alpha}} \delta(\omega - \Omega_{i_{\alpha}})$$

$$J_{\alpha}(\omega) = \pi \sum_{i_{\alpha}} \omega_{i_{\alpha}} \lambda_{i_{\alpha}} \delta(\omega - \omega_{i_{\alpha}})$$

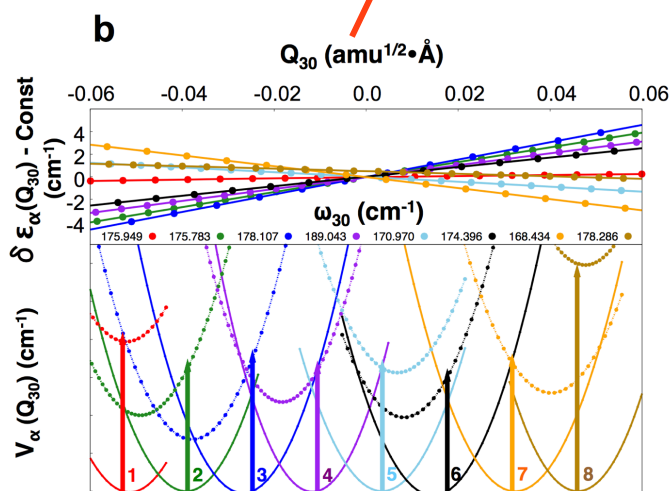
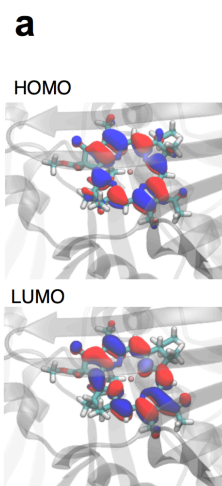
BChl - gas phase



An alternative approach

$$\delta\epsilon_{\alpha}(t) = \delta\epsilon_{\alpha}^{\text{sr}}(t) + \delta\epsilon_{\alpha}^{\text{lr}}(t)$$

$$C_{\alpha}(t) = \langle \delta\epsilon_{\alpha}(0) \delta\epsilon_{\alpha}(t) \rangle$$



$$\delta\epsilon_{\alpha}^{\text{lr}}(t) = \frac{1}{4\pi\epsilon_0} \sum_i \sum_K \frac{\delta z_i^{\alpha} Z_K^{\alpha}}{|r_i^{\alpha}(t) - R_K^{\alpha}(t)|}$$

Charge Density Coupling (CDC)

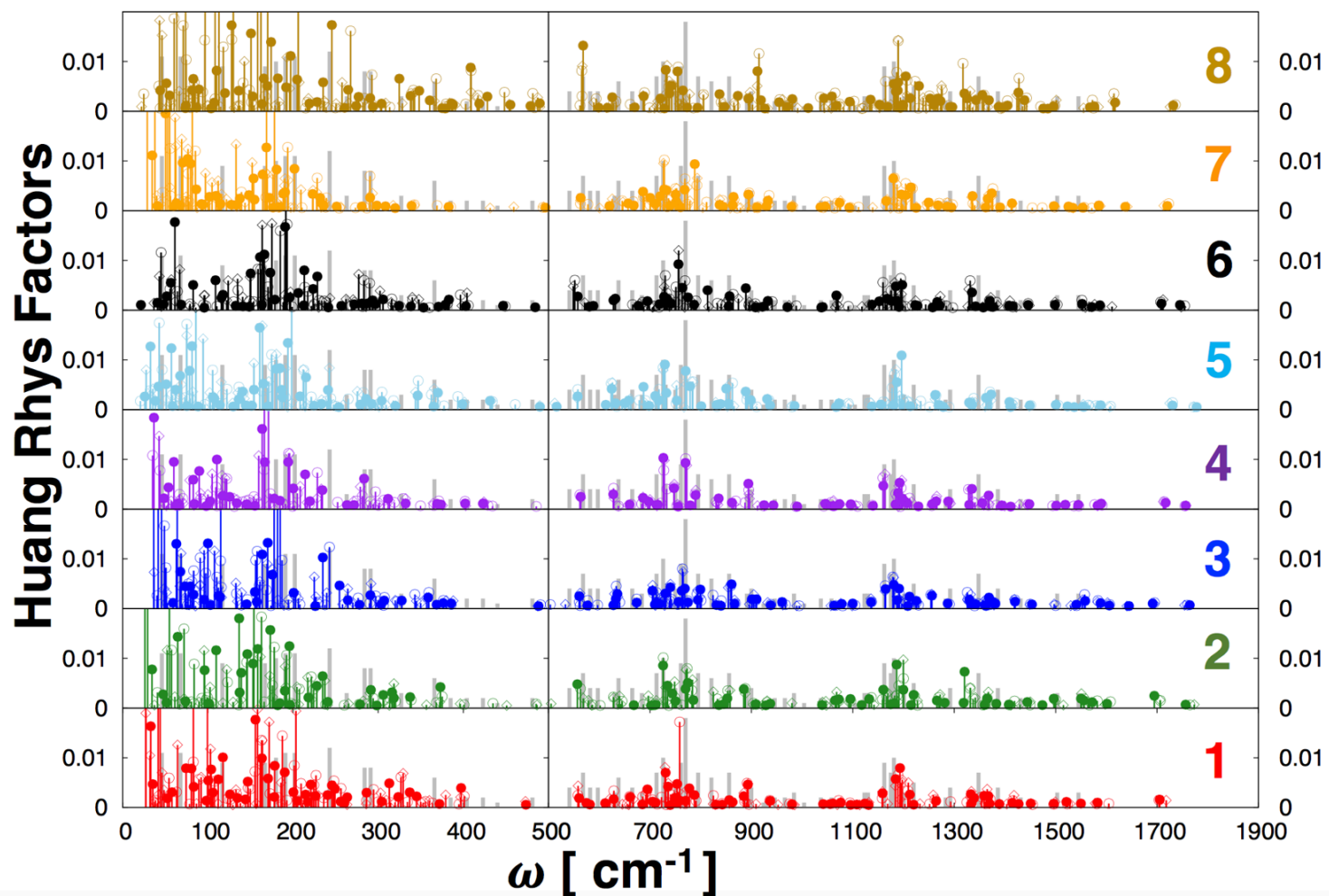
- (1) Adolphs, J.; Muh, F.; Madjet, M. E.-A.; Renger, T. *Photosynth. Res.* 2008, 95, 197–209.
- (2) Renger, T. *Photosynth. Res.* 2009, 102, 471–485.
- (3) Shi and coworkers *J. Phys. Chem. B* 116, 1164 (2012)
- (4) Rivera, Montemayor, Masia, Coker *J. Phys. Chem. B* 117, 5510 (2013)

Mi Kyung Lee, DFC, *J. Phys. Chem. Lett.* 7, 3171 (2016)

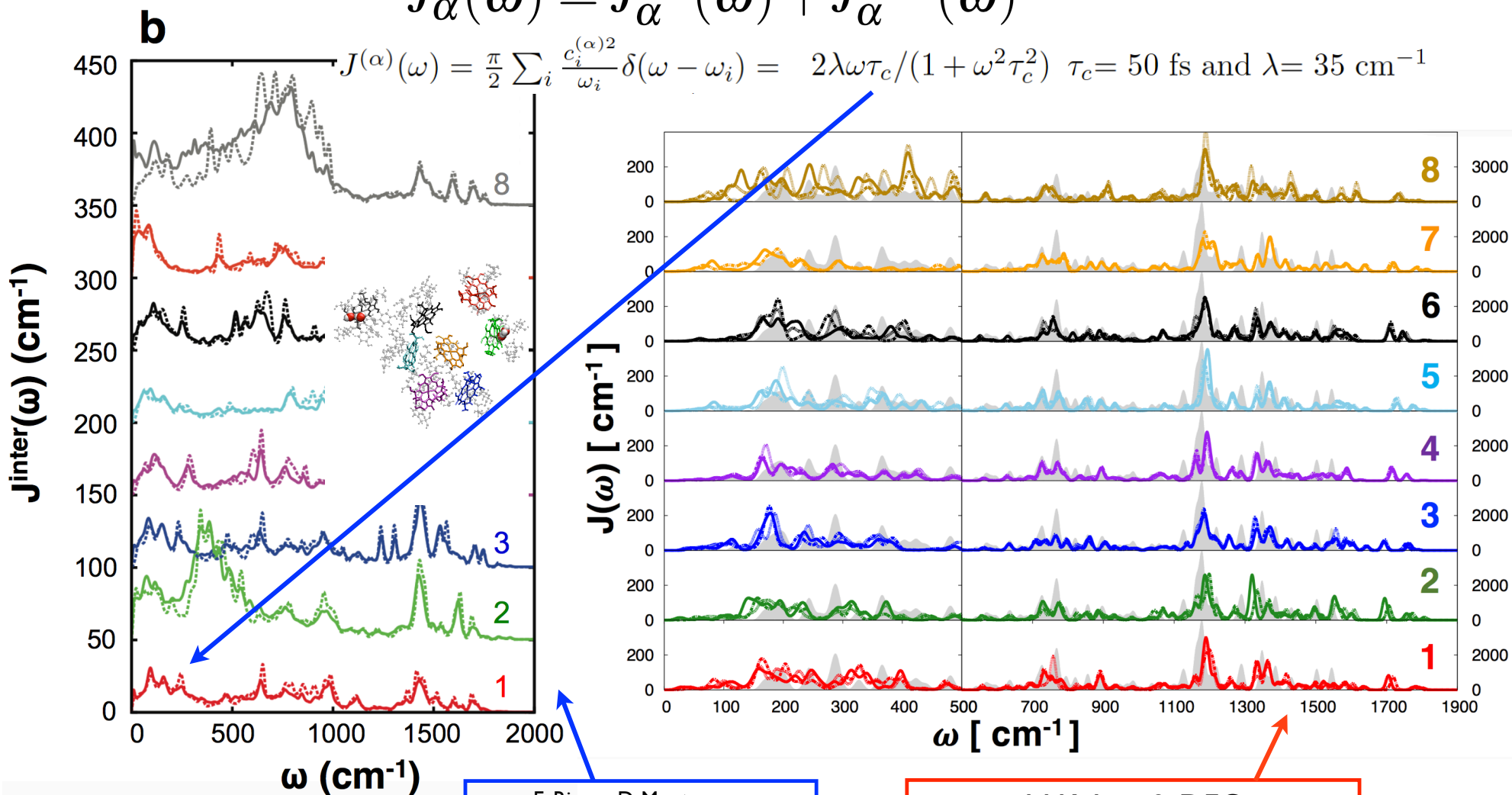
$$C_{\alpha}(t) \sim \langle \delta\epsilon_{\alpha}^{\text{sr}}(0) \delta\epsilon_{\alpha}^{\text{sr}}(t) \rangle + \langle \delta\epsilon_{\alpha}^{\text{lr}}(0) \delta\epsilon_{\alpha}^{\text{lr}}(t) \rangle$$

$$\langle \delta\epsilon_{\alpha}^{\text{sr}}(0) \delta\epsilon_{\alpha}^{\text{lr}}(t) \rangle \sim 0$$

FMO HR factors - gray bars (expt. Freiburg and co-workers)

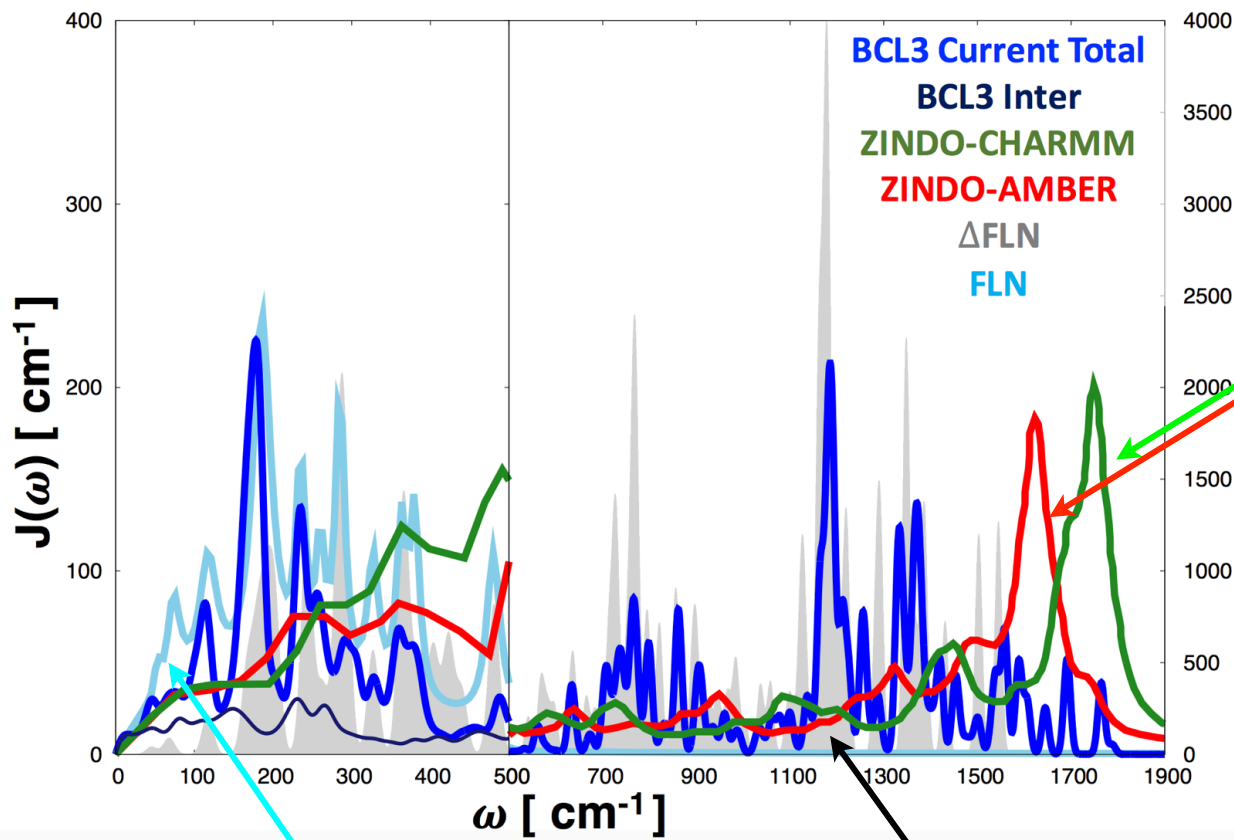


$$J_{\alpha}(\omega) = J_{\alpha}^{\text{vib}}(\omega) + J_{\alpha}^{\text{inter}}(\omega)$$



E. Rivera, D. Montemayor,
M. Masia and D.F. Coker,
J. Phys. Chem. B, 117, 55105521 (2013)

M.K. Lee & DFC,
J. Phys. Chem. Lett. 7, 3171 (2016)



Kleinekathoefer, et al.
JPCB (2015)

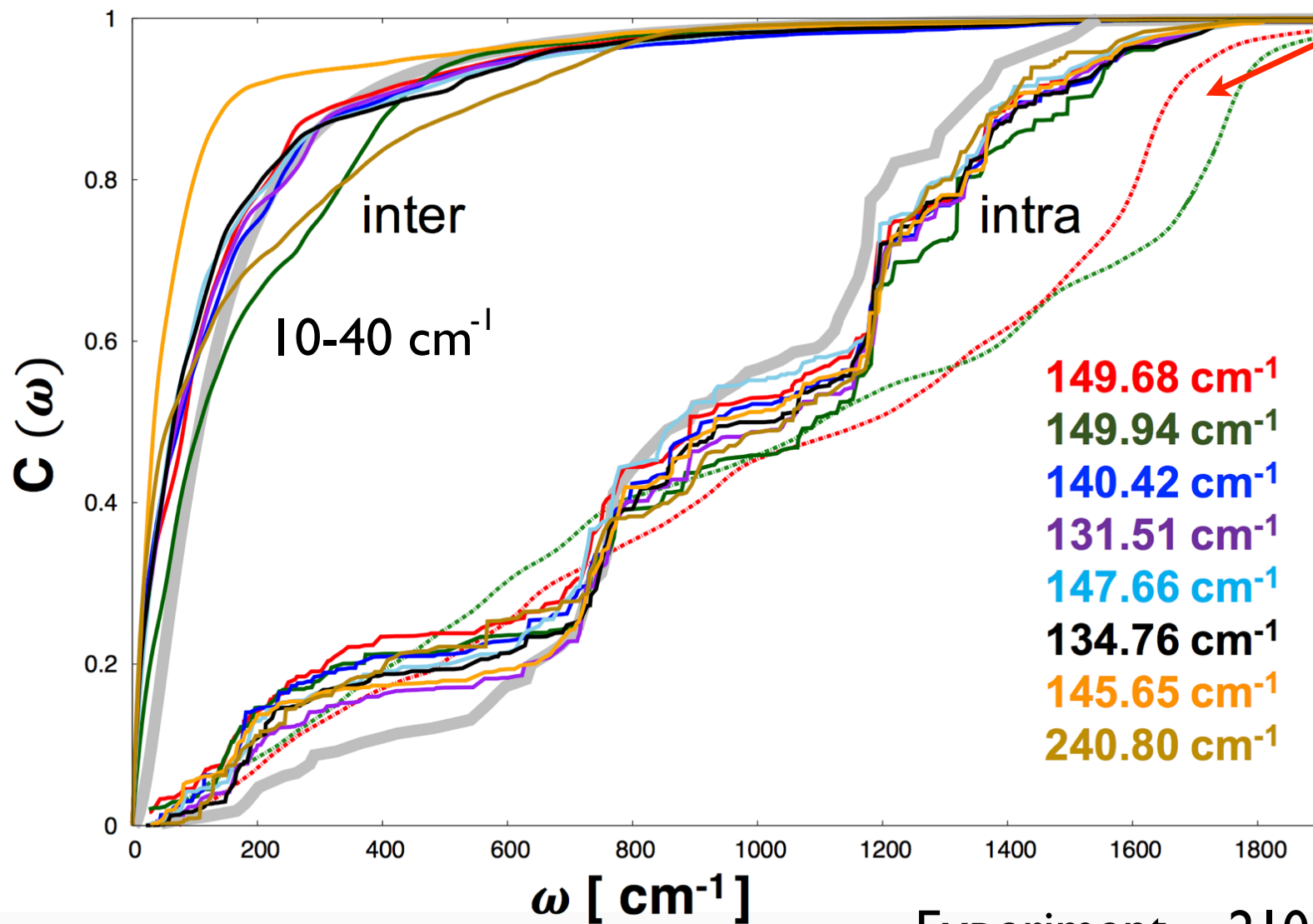
$$J_{\alpha}^{MM}(\omega) = \pi\omega \sum_{i_{\alpha}} \sum_{k_{\alpha}} \left(\frac{\omega_{k_{\alpha}} D_{k_{\alpha}i_{\alpha}}}{\Omega_{i_{\alpha}}} \right)^2 \lambda_{k_{\alpha}} \delta(\omega - \Omega_{i_{\alpha}})$$

$$J_{\alpha}(\omega) = \pi \sum_{i_{\alpha}} \omega_{i_{\alpha}} \lambda_{i_{\alpha}} \delta(\omega - \omega_{i_{\alpha}})$$

M. Ratsep & A. Freiberg J. Lumin. 127 (2007) 251.

M. Wendling et al. J. Phys. Chem. B 104 (2000) 5825.

$$\int_0^\omega d\omega' J_\alpha(\omega')/\omega'/(\pi\lambda_\alpha)$$



MM 700-1000 cm^{-1}

Experiment $\sim 210 \text{ cm}^{-1}$

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Parameterized Model Site Hamiltonian

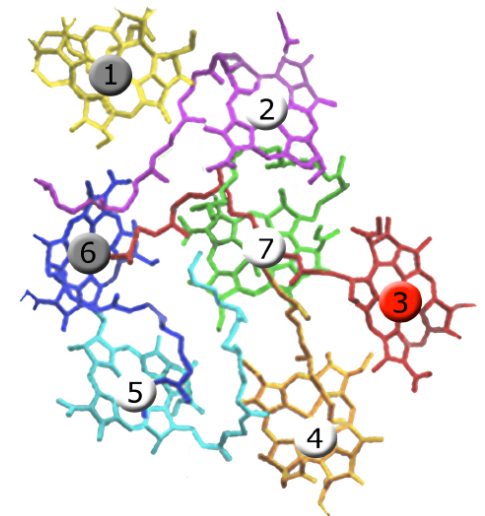
estimate from transition dipoles
or densities - small fluctuations

$$\hat{H} = \sum_{\alpha=1}^{N_{state}} \underline{\epsilon_{\alpha}} |\alpha\rangle\langle\alpha| + \sum_{\alpha \neq \beta}^{N_{state}} \underline{\Delta_{\alpha,\beta}} [|\alpha\rangle\langle\beta| + |\beta\rangle\langle\alpha|]$$

Fluctuate like crazy!

$$+ \sum_{\alpha=1}^{N_{state}} \sum_{i=1}^{n^{(\alpha)}} c_i^{(\alpha)} Q_i^{(\alpha)} |\alpha\rangle\langle\alpha|$$

$$+ \sum_{\alpha=1}^{N_{state}} \sum_{i=1}^{n^{(\alpha)}} \frac{1}{2} [P_i^{(\alpha)2} + \omega_i^{(\alpha)2} Q_i^{(\alpha)2}]$$



7 state FMO test

In principle Site dependent “spectral densities”

$$J^{(\alpha)}(\omega) = \frac{\pi}{2} \sum_i \frac{c_i^{(\alpha)2}}{\omega_i} \delta(\omega - \omega_i) = 2\lambda\omega\tau_c / (1 + \omega^2\tau_c^2) \quad \tau_c = 50 \text{ fs and } \lambda = 35 \text{ cm}^{-1}$$

Experimental fit assumes all have identical environments
continuum of intermolecular modes

Scaled Second-Order Perturbation Corrections to Configuration Interaction Singles: Efficient and Reliable Excitation Energy Methods

Young Min Rhee and Martin Head-Gordon*

Department of Chemistry, University of California and Chemical Sciences Division,
Lawrence Berkeley National Laboratory, Berkeley, California 94720

Received: December 7, 2006; In Final Form: March 9, 2007

$$E^{\text{CIS(D)}} = \langle \Phi_{\text{CIS}} | V | U_2 \Phi_0 \rangle + \langle \Phi_{\text{CIS}} | V | T_2 U_1 \Phi_0 \rangle$$

Double excitation from reference HF wavefunction. "Direct term" that describes excited state correlation pairs.

Double excitation of the "CIS inactive" electrons. This second "indirect" term describes correlation with electrons not involved in the excitation.

Single excitation of the "CIS active" electrons

$$\omega^{\text{SOS-CIS(D)}} = \langle \Phi_{\text{CIS}} | V | c_U U_2^{\text{OS}} \Phi_0 \rangle + \langle \Phi_{\text{CIS}} | V | c_T T_2^{\text{OS}} U_1 \Phi_0 \rangle$$

Two modifications of the perturbative doubles correction to configuration interaction with single substitutions (CIS(D)) are suggested, which are excited state analogues of ground state scaled second-order Møller–Plesset (MP2) methods. The first approach employs two parameters to scale the two spin components of the direct term of CIS(D), starting from the two-parameter spin-component scaled (SCS) MP2 ground state, and is termed SCS–CIS(D). An efficient resolution-of-the-identity (RI) implementation of this approach is described. The second approach employs a single parameter to scale only the opposite-spin direct term of CIS(D), starting from the one-parameter scaled opposite-spin (SOS) MP2 ground state, and is called SOS–CIS(D). By utilizing auxiliary basis expansions and a Laplace transform, a fourth-order algorithm for SOS–CIS(D) is described and implemented. The parameters that describe SCS–CIS(D) and SOS–CIS(D) are optimized based on a training set that includes valence excitations of various organic molecules and Rydberg transitions of water and ammonia, and they significantly improve upon CIS(D) itself. The accuracy of the two methods is found to be comparable. This arises from a strong correlation between the same-spin and the opposite-spin portions of the excitation energy terms. The methods are successfully applied to the zincbacteriochlorin–bacteriochlorin charge-transfer transition, for which time-dependent density functional theory, with presently available exchange-correlation functionals, is known to fail. The methods are also successfully applied to describe various electronic transitions outside of the training set. The efficiency of the SOS–CIS(D) and the auxiliary basis implementation of CIS(D) and SCS–CIS(D) are confirmed with a series of timing tests.

Standard MP2 correlation energy

$$E_C = E_C^{SS} + E_C^{OS},$$

Correlation for same spin

$$E_C^{SS} = \frac{1}{2} \sum_{ij} e_{ij} + \frac{1}{2} \sum_{\bar{i}\bar{j}} e_{\bar{i}\bar{j}},$$

Correlation for opposite spin

$$E_C^{OS} = \sum_{i\bar{j}} e_{i\bar{j}}.$$

But in SOS-MP2:

$$E_C[SOS - MP2] = c_{OS} E_C^{OS}[MP2]$$

Neglect E^{SS} because much smaller than E^{OS} .

c_{OS} = empirical parameter

$$e_{ij} = \sum_{ab} (T_{ij}^{ab} - T_{ij}^{ba})(ia|jb),$$

$$e_{\bar{i}\bar{j}} = \sum_{\bar{a}\bar{b}} (T_{\bar{i}\bar{j}}^{\bar{a}\bar{b}} - T_{\bar{i}\bar{j}}^{\bar{b}\bar{a}})(\bar{i}\bar{a}|\bar{j}\bar{b}),$$

$$e_{i\bar{j}} = \sum_{a\bar{b}} T_{i\bar{j}}^{a\bar{b}}(ia|\bar{j}\bar{b}).$$

$$T_{ij}^{ab} = \frac{(ia|jb)}{\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b}, \quad (ia|jb) = (i(1)a(1)|j(2)b(2))$$

Advanced Review

Spin-component-scaled electron correlation methods

Stefan Grimme,^{1*} Lars Goerigk² and Reinhold F. Fink³



Spin-component-scaled (SCS) electron correlation methods for electronic structure theory are reviewed. The methods can be derived theoretically by applying special conditions to the underlying wave functions in perturbation theory. They are based on the insight that low-order wave function expansions treat the correlation effects of electron pairs with opposite spin (OS) and same spin (SS) differently because of their different treatment at the underlying Hartree-Fock level. Physically, this is related to the different average inter-electronic distances in the SS and OS electron pairs. The overview starts with the original SCS-MP2 method and discusses its strengths and weaknesses and various ways to parameterize the scaling factors. Extensions to coupled-cluster and excited state methods as well the connection to virtual-orbital dependent density functional approaches are highlighted. The performance of various SCS methods in large thermochemical benchmarks and for excitation energies is discussed in comparison with other common electronic structure methods. © 2012 John Wiley & Sons, Ltd.

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WIREs Comput Mol Sci 2012, 2: 886–906 doi: 10.1002/wcms.1110

TABLE III. Computed (deviations between method and reference are shown, results were obtained with the def2-TZVPP basis set) vertical singlet excitation energies ΔE (in eV) for the complete dye bench-mark set.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Reference ^a	2.95	2.02	2.12	2.72	2.33	2.66	3.66	2.52	3.37	3.15	2.60	3.60
B-LYP	-0.72	-0.06	-0.42	-0.35	-0.61	-0.26	-0.43	0.15	-0.53	-0.65	-0.55	-1.43
B3-LYP	-0.32	0.13	-0.09	-0.43	-0.38	-0.02	-0.20	0.38	-0.14	-0.41	-0.30	-0.86
PBE38	-0.01	0.27	0.19	0.00	-0.19	0.18	0.01	0.55	0.15	-0.22	-0.11	-0.39
BMK	0.00	0.30	0.25	0.06	-0.15	0.20	0.04	0.55	0.17	-0.17	-0.06	-0.36
CAM-B3LYP	0.06	0.26	0.30	0.07	-0.11	0.23	0.04	0.55	0.24	-0.15	-0.02	-0.13
B2-PLYP	-0.29	0.06	0.20	-0.13	-0.27	-0.01	-0.02	0.24	-0.07	-0.28	-0.16	-0.63
B2GP-PLYP	-0.17	0.08	0.35	-0.03	-0.18	0.08	0.08	0.28	0.06	-0.18	-0.06	-0.38
CIS	0.90	0.79	1.02	0.69	0.45	0.92	0.51	1.27	1.03	0.42	0.53	0.74
CIS(D)	0.05	0.02	0.77	0.19	0.09	0.28	0.45	0.16	0.55	0.21	0.23	0.05
SCS-CIS(D)'	0.13	0.02	0.96	0.26	0.20	0.41	0.49	0.19	0.60	0.28	0.33	0.12
SCS-CIS(D) ^{$\lambda=0$}	-0.04	-0.30	0.74	0.03	-0.02	0.18	0.31	-0.06	0.46	0.12	0.12	-0.02
SCS-CIS(D) ^{$\lambda=1$}	-0.17	-0.30	0.67	-0.05	-0.11	0.09	0.24	-0.13	0.33	0.00	0.02	-0.18
SOS-CIS(D)	-0.08	-0.36	0.70	-0.03	-0.10	0.13	0.27	-0.10	0.40	0.04	0.03	-0.09
CC2	-0.26	0.04	0.51	-0.06	-0.13	0.01	0.24	0.20	0.00	-0.10	0.01	-0.48
SCS-CC2	-0.06	0.05	0.77	0.08	0.06	0.20	0.34	0.25	0.12	0.04	0.17	-0.23

^aExperimental vertical excitation energies from Table II.

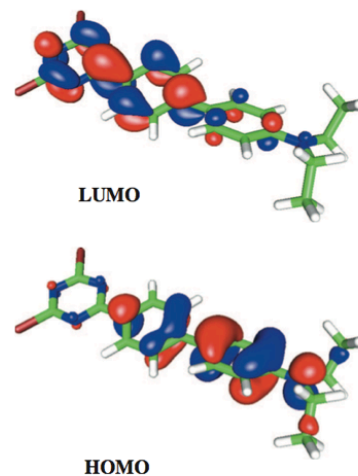


FIG. 2. Lowest unoccupied (LUMO; top) and highest occupied molecular orbitals (HOMO; bottom) for DBQ (12) obtained at the HF/def2-TZVPP level of theory.

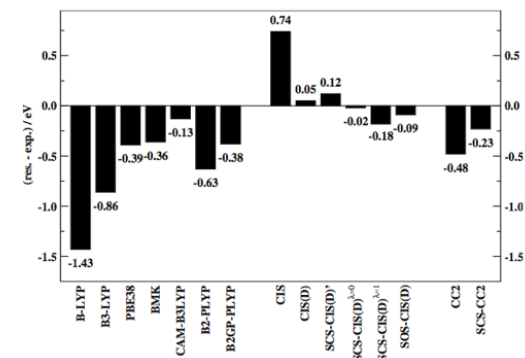


FIG. 3. Deviations for DBQ (12) in eV. All results were obtained with the def2-TZVPP basis.

OUTLINE:

- (1) Spectral Density Calculations
- (2) Excitation Energy Calculations
- (3) Dissipative Quantum Dynamics for General Regimes
- (4) Issues with Linearized Dynamics of Higher Frequency Modes
- (5) Spectra (PC645/HPC645) - Influence of Protonation and “Flickering” Pathways
- (6) Quantum Dynamics and Nonlinear Spectroscopy
- (7) Coherent State Density Matrix Dynamics (+PLDM)

Partial Linearized Density Matrix (PLDM) dynamics:

Ehrenfest - like partially linearized propagator
useful for longer time propagation

Partial Linearized Density Matrix Propagation

PLDM: P. Huo and D.F. Coker, J. Chem. Phys. 135 201101 (2011).

ILDLM: Dunkel, Bonella, Coker, J. Chem. Phys. 129, 114106 (2008).

Miller-Meyer
Stock-Thoss

$$\hat{h}_{el} = \sum_{\beta, \lambda} |\beta\rangle \langle \beta | \hat{h}_{el}(\hat{R}) | \lambda \rangle \langle \lambda | \quad |\beta\rangle \langle \lambda| \rightarrow \hat{a}_{\beta}^{\dagger} \hat{a}_{\lambda}$$

$$|n\rangle \rightarrow |m_n\rangle = |0_1, \dots, 1_n, \dots, 0_{N_s}\rangle$$

$$\hat{a}_{\lambda} = \frac{1}{\sqrt{2\hbar}} (\hat{q}_{\lambda} - i\hat{p}_{\lambda}) \quad \hat{a}_{\lambda}^{\dagger} = \frac{1}{\sqrt{2\hbar}} (\hat{q}_{\lambda} + i\hat{p}_{\lambda})$$

Total and “mapping” hamiltonian

$$\begin{aligned} \hat{H} = \hat{P}^2/2M + h_m(\hat{R}, \hat{p}, \hat{q}) \quad \hat{h}_m(\hat{R}) = & \frac{1}{2\hbar} \sum_{\beta} h_{\beta, \beta}(\hat{R}) (\hat{q}_{\beta}^2 + \hat{p}_{\beta}^2 - \hbar) \\ & + \frac{1}{2\hbar} \sum_{\lambda \neq \beta} h_{\beta \lambda}(\hat{R}) (\hat{q}_{\beta} \hat{q}_{\lambda} + \hat{p}_{\beta} \hat{p}_{\lambda}) \end{aligned}$$

Density matrix propagation

$$\begin{aligned} \langle R_t n_t | \hat{\rho}(t) | R'_t n'_t \rangle = & \sum_{n_0, n'_0} \int dR_0 dR'_0 \langle R_t n_t | e^{-\frac{i}{\hbar} \hat{H} t} | R_0 n_0 \rangle \\ & \times \langle R_0 n_0 | \hat{\rho}(0) | R'_0 n'_0 \rangle \langle R'_0 n'_0 | e^{\frac{i}{\hbar} \hat{H} t} | R'_t n'_t \rangle \end{aligned}$$



Pengfei (Frank)
Huo

Path integral mapping hamiltonian propagator

$$\langle R_N m_{n_t} | e^{-\frac{i}{\hbar} \hat{H} t} | R_0 m_{n_0} \rangle = \int \prod_{k=1}^{N-1} dR_k \frac{dP_k}{2\pi\hbar} \frac{dP_N}{2\pi\hbar} e^{\frac{i}{\hbar} S_0} \quad \text{Pechukas JCP 1969}$$

$$\times \langle n_t | e^{-\frac{i}{\hbar} \epsilon \hat{h}_m(R_{N-1})} \dots e^{-\frac{i}{\hbar} \epsilon \hat{h}_m(R_0)} | n_0 \rangle$$

$$S_0 = \epsilon \sum_{k=1}^N \left[P_k \frac{(R_k - R_{k-1})}{\epsilon} - \frac{P_k^2}{2M} \right] \quad T_{[n_t, n_0]} = \langle n_t | e^{-\frac{i}{\hbar} \epsilon \hat{h}_m(R_{N-1})} \dots e^{-\frac{i}{\hbar} \epsilon \hat{h}_m(R_0)} | n_0 \rangle$$

$$T_{[n_t, n_0]} = \int dq_0 dp_0 \frac{1}{4} (q_{n_t} + ip_{n_t})(q_{n_0} - ip_{n_0}) c_t e^{iS_1(t)}$$

$$\times e^{-\frac{i}{2} \sum_{\beta} (q_{\beta t} p_{\beta t} - q_{\beta 0} p_{\beta 0})} e^{-\frac{1}{2} \sum_{\beta} (q_{\beta 0}^2 + p_{\beta 0}^2)}$$

Coherent State
description

$$c_t = e^{-\frac{i}{2\hbar} \int_0^t d\tau \sum_{\beta} \hat{h}_{\beta\beta}(R)}$$

$$S_1(t) = \int_0^t L_1(\tau) d\tau$$

$$L_1 = \sum_{\beta} p_{\beta} \dot{q}_{\beta} - h_m^{cl}(R) + \frac{1}{2} \sum_{\beta} h_{\beta\beta}(R)$$

$$h_m^{cl}(R, p, q) = \frac{1}{2} \sum_{\beta} h_{\beta\beta}(R) (p_{\beta}^2 + q_{\beta}^2)$$

$$+ \frac{1}{2} \sum_{\lambda \neq \beta} h_{\lambda\beta}(R) (p_{\lambda} p_{\beta} + q_{\lambda} q_{\beta})$$

Bonella, Coker, J. Chem. Phys. 118,4370 (2003)
Dunkel, Bonella, Coker, J. Chem. Phys. 129,114106 (2008)

$$\langle R_t n_t | \hat{\rho}(t) | R'_t n'_t \rangle = \sum_{n_0, n'_0} \int dR_0 dR'_0 \langle R_t n_t | e^{-\frac{i}{\hbar} \hat{H} t} | R_0 n_0 \rangle$$

$$\times \langle R_0 n_0 | \hat{\rho}(0) | R'_0 n'_0 \rangle \langle R'_0 n'_0 | e^{\frac{i}{\hbar} \hat{H} t} | R'_t n'_t \rangle$$

Partial Linearization in
Forward-Backward Environment Path
Difference

Transform to mean and difference paths

$$\bar{R} = (R + R')/2 \quad Z = (R - R')$$

$$\bar{P} = (P + P')/2 \quad Y = (P - P')$$

$$(S_0 - S'_0) = \bar{P}_N Z_N - \bar{P}_1 Z_0 - \sum_{k=1}^{N-1} (\bar{P}_{k+1} - \bar{P}_k) Z_k$$

$$- \sum_{k=1}^N \left[\frac{\epsilon}{m} \bar{P}_k - (\bar{R}_k - \bar{R}_{k-1}) \right] Y_k$$

Truncate to linear order in difference paths

$$\sum_{\beta} p_{\beta} \dot{q}_{\beta} - h_m^{cl}(\bar{R}, p, q) \sim \sum_{\beta} p_{\beta} \dot{q}_{\beta} - h_m^{cl}(R, p, q) = \frac{1}{2} \frac{d}{d\tau} \left(\sum_{\beta} p_{\beta} q_{\beta} \right)$$

$$(S_1 - S'_1)[R(t), R'(t)]$$

$$= \int_0^t \left[\frac{1}{2} \frac{d}{d\tau} \sum_{\beta} (p_{\beta\tau} q_{\beta\tau} - p'_{\beta\tau} q'_{\beta\tau}) + \mathcal{O}(Z_{\tau}^2) \right.$$

$$\left. + \frac{1}{2} \left(\nabla_{\bar{R}} h_m^{cl}(\bar{R}_{\tau}, p_{\tau}, q_{\tau}) + \nabla_{\bar{R}} h_m^{cl}(\bar{R}_{\tau}, p'_{\tau}, q'_{\tau}) \right) Z_{\tau} \right] d\tau$$

Q. Shi & E. Geva J. Phys. Chem. A
108, 6109 (2004)

R. Lambert & N. Makri J. Chem. Phys.
137, 22A552-3 (2010)

$$\begin{aligned}
\langle \bar{R}_N + \frac{Z_N}{2} n_t | \hat{\rho}(t) | \bar{R}_N - \frac{Z_N}{2} n'_t \rangle &= \sum_{n_0, n'_0} \int d\bar{R}_0 dq_0 dp_0 dq'_0 dp'_0 G_0 G'_0 \\
&\times \frac{1}{4} (q_{n_t} + ip_{n_t})(q_{n_0} - ip_{n_0}) \frac{1}{4} (q'_{n'_t} - ip'_{n'_t})(q'_{n'_0} + ip'_{n'_0}) \\
&\times \int \prod_{k=1}^{N-1} d\bar{R}_k \frac{d\bar{P}_k}{2\pi\hbar} \frac{d\bar{P}_N}{2\pi\hbar} (\hat{\rho})_W^{n_0, n'_0} (\bar{R}_0, \bar{P}_1) e^{\frac{i}{\hbar} \bar{P}_N Z_N} \\
&\times \prod_{k=1}^{N-1} \delta \left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k \right) \prod_{k=1}^N \delta \left(\frac{\bar{P}_k}{M} - \frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon} \right)
\end{aligned}$$

initial mapping distributions

$$G_0 = e^{-\frac{1}{2} \sum_{\beta} (q_{\beta 0}^2 + p_{\beta 0}^2)}$$

$$G'_0 = e^{-\frac{1}{2} \sum_{\beta'} (q'_{\beta' 0} + p'_{\beta' 0})^2}$$

mapping equations of motion

$$\dot{q}_{n_t} = \partial h_m^{cl}(\bar{R}_t) / \partial p_{n_t}$$

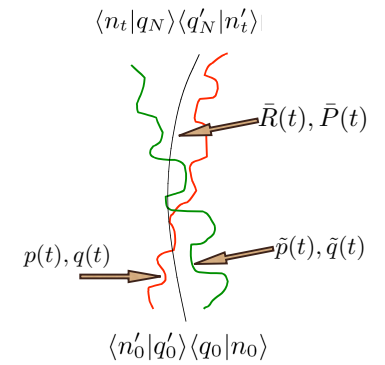
$$\dot{p}_{n_t} = -\partial h_m^{cl}(\bar{R}_t) / \partial q_{n_t}$$

Effective back-reaction force

$$\begin{aligned}
F_k &= -\frac{1}{2} \left[\nabla_{\bar{R}_k} h_m^{cl}(\bar{R}_k, p_k, q_k) + \nabla_{\bar{R}_k} h_m^{cl}(\bar{R}_k, p'_k, q'_k) \right] \\
&= -\frac{1}{4} \sum_{\beta} \nabla_{\bar{R}_k} h_{\beta\beta}(\bar{R}_k) (p_{\beta k}^2 + q_{\beta k}^2 + p'_{\beta k} + q'_{\beta k}) \\
&\quad -\frac{1}{4} \sum_{\lambda \neq \beta} \nabla_{\bar{R}_k} h_{\lambda\beta}(\bar{R}_k) (p_{\lambda k} p_{\beta k} + q_{\lambda k} q_{\beta k}) \\
&\quad -\frac{1}{4} \sum_{\lambda \neq \beta} \nabla_{\bar{R}_k} h_{\lambda\beta}(\bar{R}_k) (p'_{\lambda k} p'_{\beta k} + q'_{\lambda k} q'_{\beta k})
\end{aligned}$$

Partial Linearized Density Matrix (PLDM) Dynamics: Approximate Open System Quantum Dynamics

P. Huo and D.F. Coker, J. Chem. Phys. 135 201101 (2011).



Canonical transformation:

$$\tilde{p}_\alpha = p_\alpha / \sqrt{2} \quad \tilde{q}_\alpha = q_\alpha / \sqrt{2}$$

$$\tilde{p}'_\alpha = p'_\alpha / \sqrt{2} \quad \tilde{q}'_\alpha = q'_\alpha / \sqrt{2}$$

Full system hamiltonian dynamics:

$$H = \frac{\bar{P}^2}{2M} + h_m^{can}(\bar{R}, \tilde{p}, \tilde{q}, \tilde{p}', \tilde{q}')$$

$$\begin{aligned} h_m^{can}(\bar{R}, \tilde{p}, \tilde{q}, \tilde{p}', \tilde{q}') &= \frac{1}{2} \left[\sum_{\alpha} h_{\alpha,\alpha}(\bar{R}) (\tilde{p}_\alpha^2 + \tilde{q}_\alpha^2) + \sum_{\gamma \neq \alpha} h_{\gamma,\alpha}(\bar{R}) (\tilde{p}_\gamma \tilde{p}_\alpha + \tilde{q}_\gamma \tilde{q}_\alpha) \right. \\ &\quad \left. + \sum_{\alpha} h_{\alpha,\alpha}(\bar{R}) (\tilde{p}'_\alpha{}^2 + \tilde{q}'_\alpha{}^2) + \sum_{\gamma \neq \alpha} h_{\gamma,\alpha}(\bar{R}) (\tilde{p}'_\gamma \tilde{p}'_\alpha + \tilde{q}'_\gamma \tilde{q}'_\alpha) \right] \end{aligned}$$

Iterative Implementation (IPLDM):

Second linearized density matrix propagation segment

Dunkel, Bonella, Coker, J. Chem. Phys. 129, 114106 (2008)

P. Huo & D.F. Coker, J. Chem. Phys. 137 22A535 (2012)

$$\begin{aligned}
 \langle \bar{R}_{2N} + \frac{Z_{2N}}{2} n_{2t} | \hat{\rho}(2t) | \bar{R}_{2N} - \frac{Z_{2N}}{2} n'_{2t} \rangle &= \int \prod_{k=N+1}^{2N-1} d\bar{R}_k \frac{d\bar{P}_k}{2\pi} \frac{d\bar{P}_{2N}}{2\pi} e^{i\bar{P}_{2N} Z_{2N}} \\
 &\quad \times r_{2t, n_{2t}}(\{\bar{R}_k\}) r'_{2t, n'_{2t}}(\{\bar{R}_k\}) e^{-i\epsilon \sum_{k=N+1}^{2N} (\theta_{n_{2t}}(\bar{R}_k) - \theta_{n'_{2t}}(\bar{R}_k))} \\
 &\quad \times \prod_{k=N+1}^{2N-1} \delta\left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k^{\text{can}}\right) \prod_{k=N+1}^{2N} \delta\left(\frac{\bar{P}_k}{M} - \frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon}\right) \\
 &\quad \sum_{n_t, n'_t} \int d\bar{R}_N \frac{d\bar{P}_N}{2\pi} dq_N dp_N dq'_N dp'_N r'_{t, n'_t} e^{-i\Theta'_{t, n'_t}} G'_t r_{t, n_t} e^{i\Theta_{t, n_t}} G_t \\
 &\quad \quad \quad \times \delta(\bar{P}_N - \bar{P}_{N+1}) \delta\left(\frac{\bar{P}_N}{M} - \frac{\bar{R}_N - \bar{R}_{N-1}}{\epsilon}\right) \\
 &\quad \times \int \prod_{k=1}^{N-1} d\bar{R}_k \frac{d\bar{P}_k}{2\pi} r_{t, n_t}(\{\bar{R}_k\}) r'_{t, n'_t}(\{\bar{R}_k\}) e^{-i\epsilon \sum_{k=1}^N (\theta_{n_t}(\bar{R}_k) - \theta_{n'_t}(\bar{R}_k))} \\
 &\quad \quad \quad \times \prod_{k=1}^{N-1} \delta\left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k^{\text{can}}\right) \prod_{k=1}^N \delta\left(\frac{\bar{P}_k}{M} - \frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon}\right) \\
 &\quad \times \sum_{n_0, n'_0} \int d\bar{R}_0 dq_0 dp_0 dq'_0 dp'_0 r'_{0, n'_0} e^{-i\Theta'_{0, n'_0}} G'_0 r_{0, n_0} e^{i\Theta_{0, n_0}} G_0 [\hat{\rho}]_W^{n_0, n'_0}(\bar{R}_0, \bar{P}_1)
 \end{aligned}$$

An Algorithm:

$$\left(\sum_{n_t, n'_t} \int dq_N dp_N dq'_N dp'_N \right)_{n_{2t}, n'_{2t}}$$

Intermediate mapping integrals performed by steepest descent

$$(p_{n_0}^o, q_{n_0}^o) = (1/\sqrt{2}, 1/\sqrt{2}) \quad (p_{n_0}^u, q_{n_0}^u) = (0, 0)$$

Intermediate state sums performed by importance sampled MC

Focusing

Bonella, Coker, J. Chem. Phys. 118,4370 (2003)

$$r_{\tau, n_t} r'_{\tau, n'_t} \exp\left[-i \int_0^\tau d\tau' (\theta_{n_t}(\tau') - \theta_{n'_t}(\tau'))\right]$$

$$M_{n_t, n'_t} = r_{t, n_t} r'_{t, n'_t} / \mathcal{N}(t)$$

$$\mathcal{N}(t) = \sum_{n_t, n'_t} r_{t, n_t} r'_{t, n'_t}$$

Trajectory weights

$$\Omega_K = \left\{ \prod_{k=1}^{K-1} \mathcal{N}(kt) \exp\left[-i \int_{(k-1)t}^{kt} d\tau' (\theta_{n_{kt}}(\tau') - \theta_{n'_{kt}}(\tau'))\right] \right\}$$

$$\times r_{Kt, n_{Kt}} r'_{Kt, n'_{Kt}} \exp\left[-i \int_{(K-1)t}^{Kt} d\tau' (\theta_{n_{Kt}}(\tau') - \theta_{n'_{Kt}}(\tau'))\right]$$

Algorithm: Iterative Scheme LAND-Map

$$M_{n_t, n'_t} = r_{t, n_t} r'_{t, n'_t} / \sum_{n_t, n'_t} r_{t, n_t} r'_{t, n'_t}$$

$$C_{m_t, m'_t} = \sum_{n_t=1}^{m_t} \sum_{n'_t=1}^{m'_t} M_{n_t, n'_t}$$

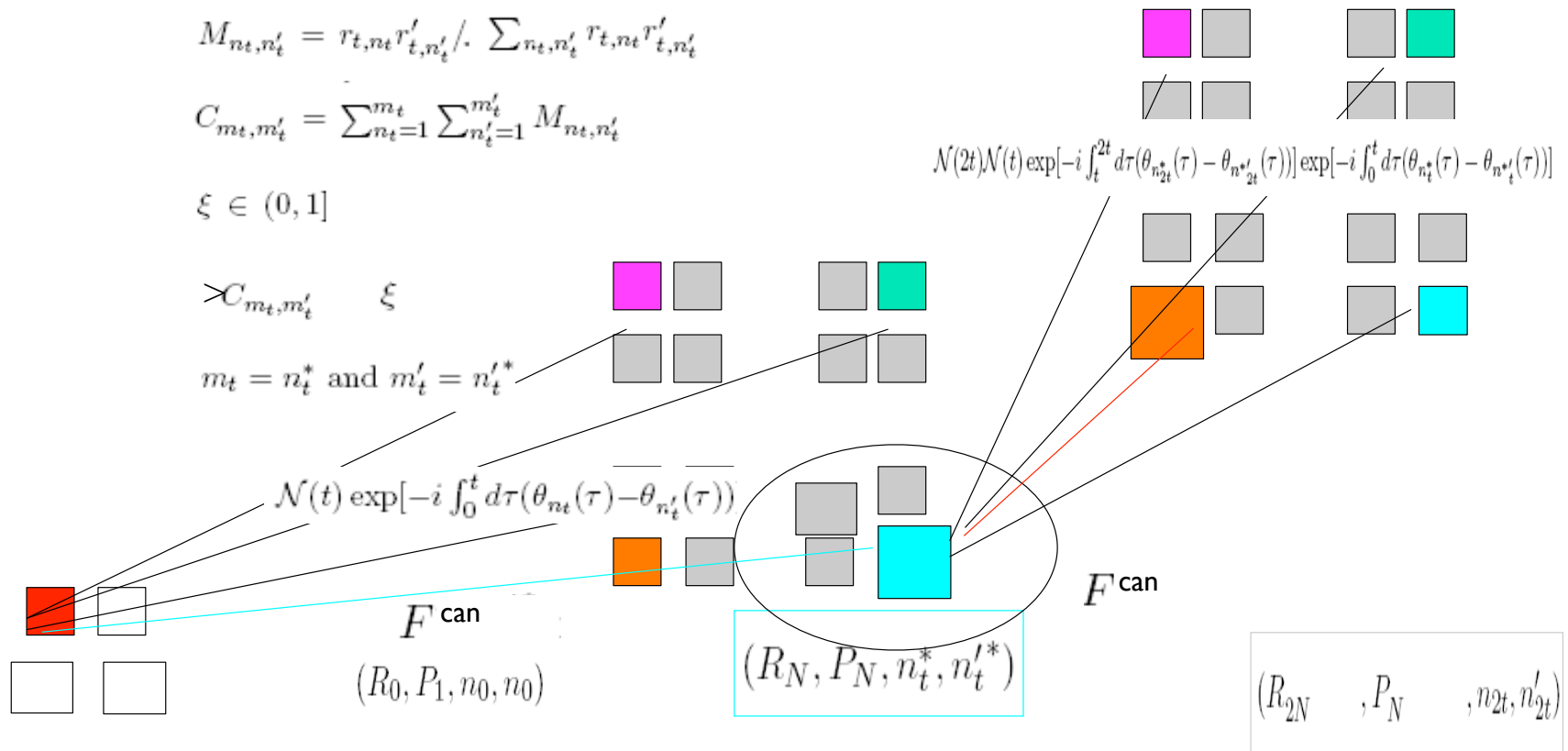
$$\xi \in (0, 1]$$

$$\xi C_{m_t, m'_t}$$

$$m_t = n_t^* \text{ and } m'_t = n'_t^*$$

$$\mathcal{N}(t) \exp[-i \int_0^t d\tau (\theta_{n_t}(\tau) - \theta_{n'_t}(\tau))]$$

$$\mathcal{N}(2t) \mathcal{N}(t) \exp[-i \int_t^{2t} d\tau (\theta_{n_{2t}}(\tau) - \theta_{n'_{2t}}(\tau))] \exp[-i \int_0^t d\tau (\theta_{n_t}(\tau) - \theta_{n'_t}(\tau))]$$



$$\left(\boxed{\rho_{11}(0)} \quad \rho_{12}(0) \right) \xrightarrow{\quad} \left(\rho_{11}(t) \quad \rho_{12}(t) \right) \xrightarrow{\quad} \left(\rho_{11}(2t) \quad \rho_{12}(2t) \right) \xrightarrow{\quad} \dots$$

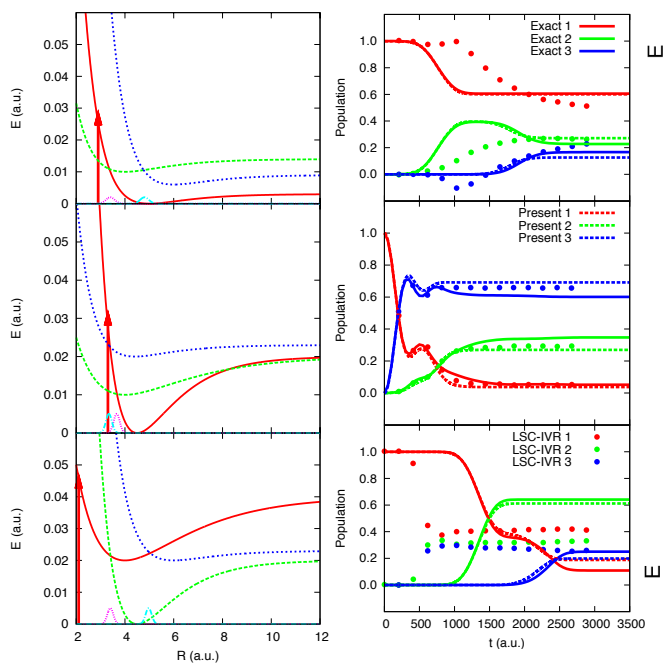
$\left(\begin{array}{cc} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cc} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \rho_{22}(t) \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{cc} \rho_{11}(2t) & \rho_{12}(2t) \\ \rho_{21}(2t) & \rho_{22}(2t) \end{array} \right) \xrightarrow{\quad} \dots$

$$\begin{pmatrix} \boxed{\rho_{11}(0)} & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \boxed{\rho_{22}(t)} \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} \rho_{11}(2t) & \rho_{12}(2t) \\ \rho_{21}(2t) & \rho_{22}(2t) \end{pmatrix} \xrightarrow{\quad} \dots$$

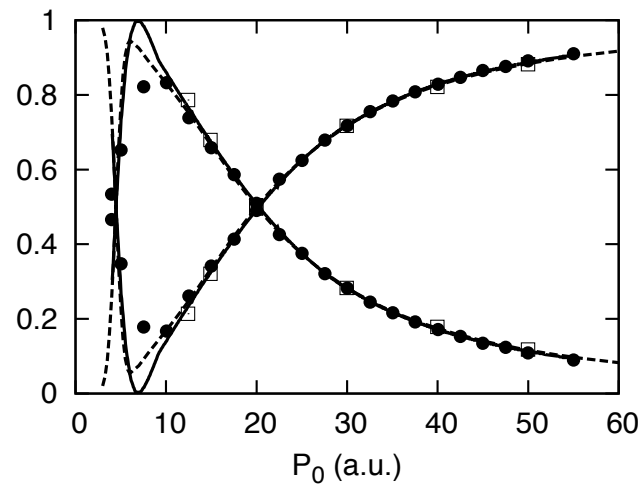
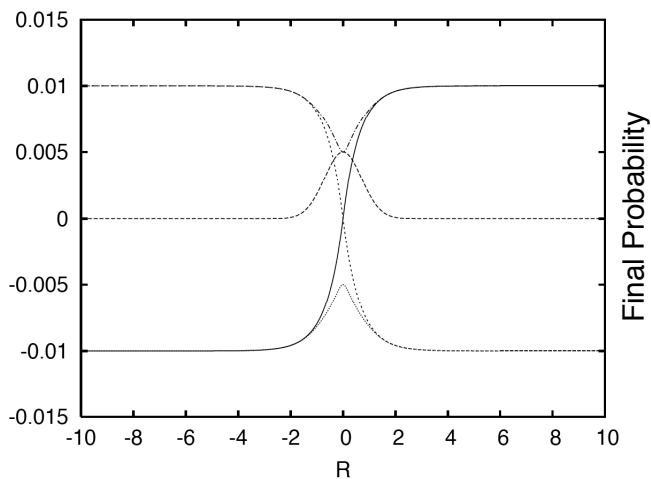
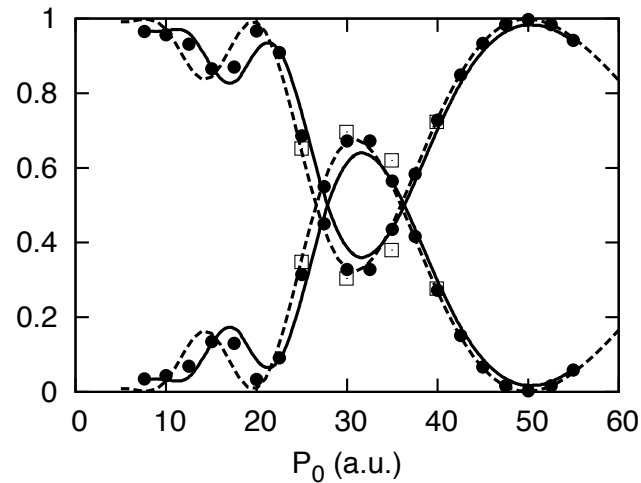
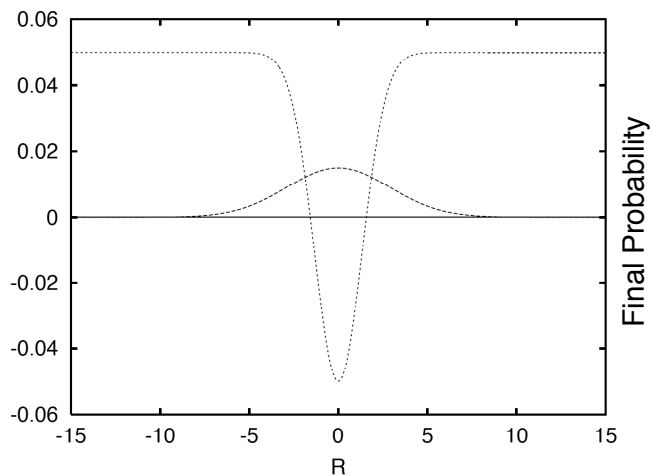
$$\begin{pmatrix} \boxed{\rho_{11}(0)} & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \boxed{\rho_{22}(t)} \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} \rho_{11}(2t) & \rho_{12}(2t) \\ \boxed{\rho_{21}(2t)} & \rho_{22}(2t) \end{pmatrix} \xrightarrow{\quad} \dots$$

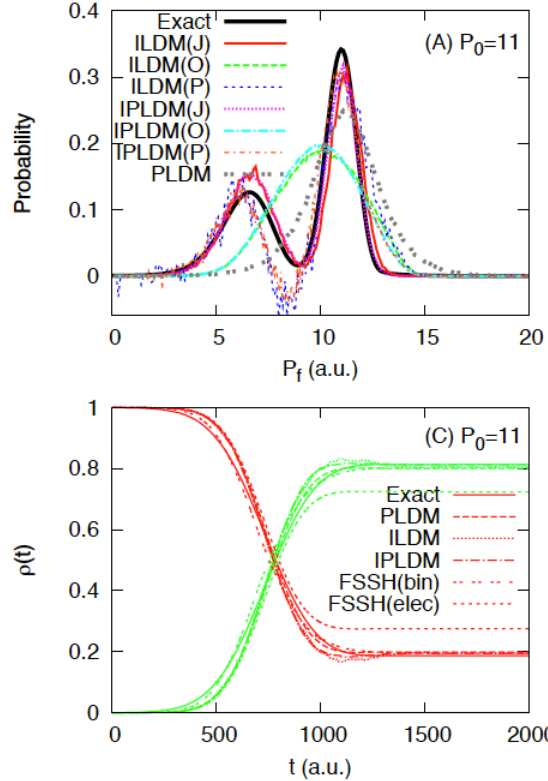
Accumulating phase weights for
different branching trajectories
gives a representation of
interference effects

ID nonadiabatic test models: PLDM, LSC-IVR, PBME



Tully models

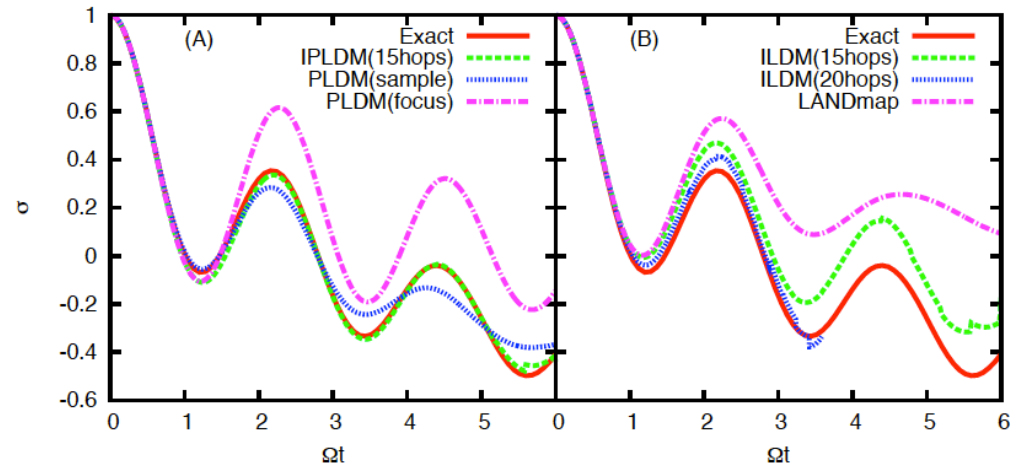




(2) Asymmetric Spin-Boson Relaxation-Thermalization

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{s-b}$$

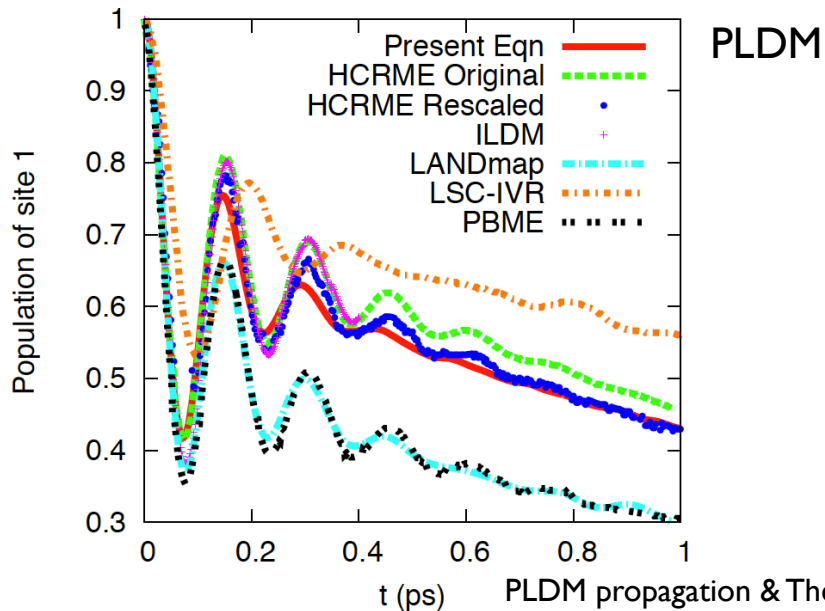
$$= \epsilon \hat{\sigma}_z - \hbar \Omega \hat{\sigma}_x + \sum_j \left\{ \left[\frac{P_j^2}{2M_j} + \frac{1}{2} \omega_j^2 R_j^2 \right] \hat{1} - \hbar c_j R_j \hat{\sigma}_z \right\}$$



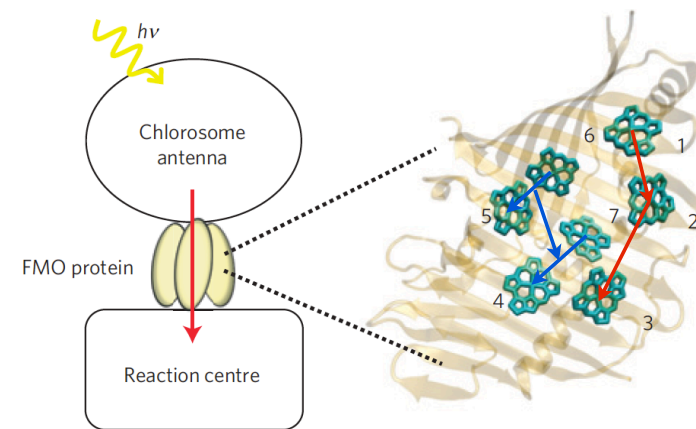
(1) Tully I momentum distribution bifurcation

P. Huo & D.F. Coker, J. Chem. Phys. 137 22A535 (2012)

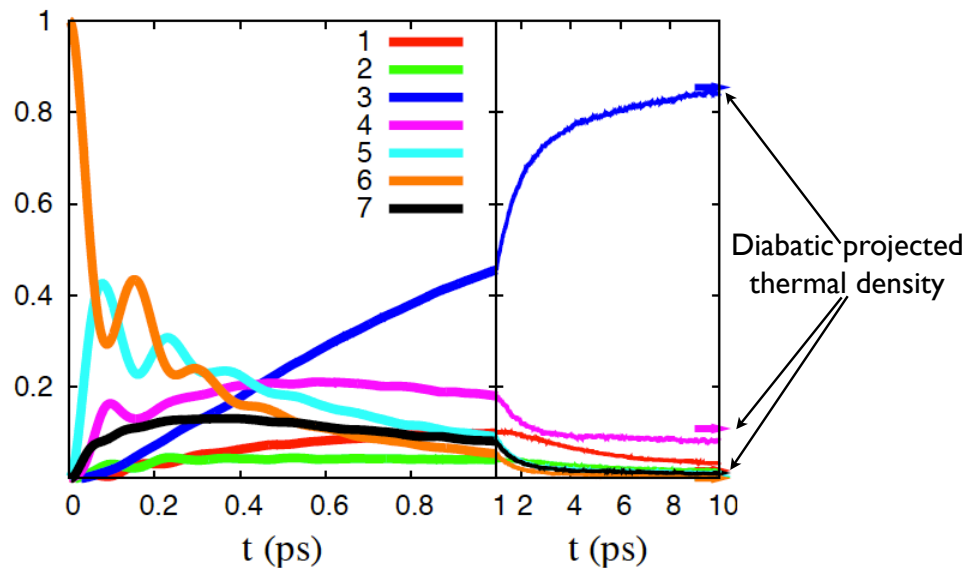
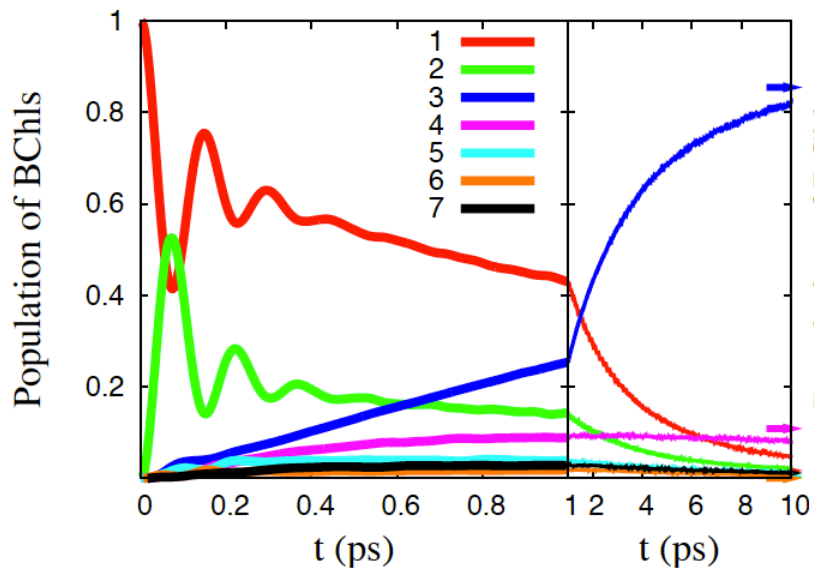
FIG. 7: $\langle \sigma_z(t) \rangle$ versus Ωt for spin-boson model, with $\Omega/\omega_c = 0.4$, $\zeta = 0.13$, $\epsilon = 0.4$, $\beta \hbar \omega_c = 12.5$. (A) Left panel gives results obtained with linearized and iterative algorithms based on the PLDM approximate short time propagator. (B) Right panel shows results obtained using LANDmap approximate short time propagator as the basis of iteration. In each panel results are explored as a function on the number of iterations.



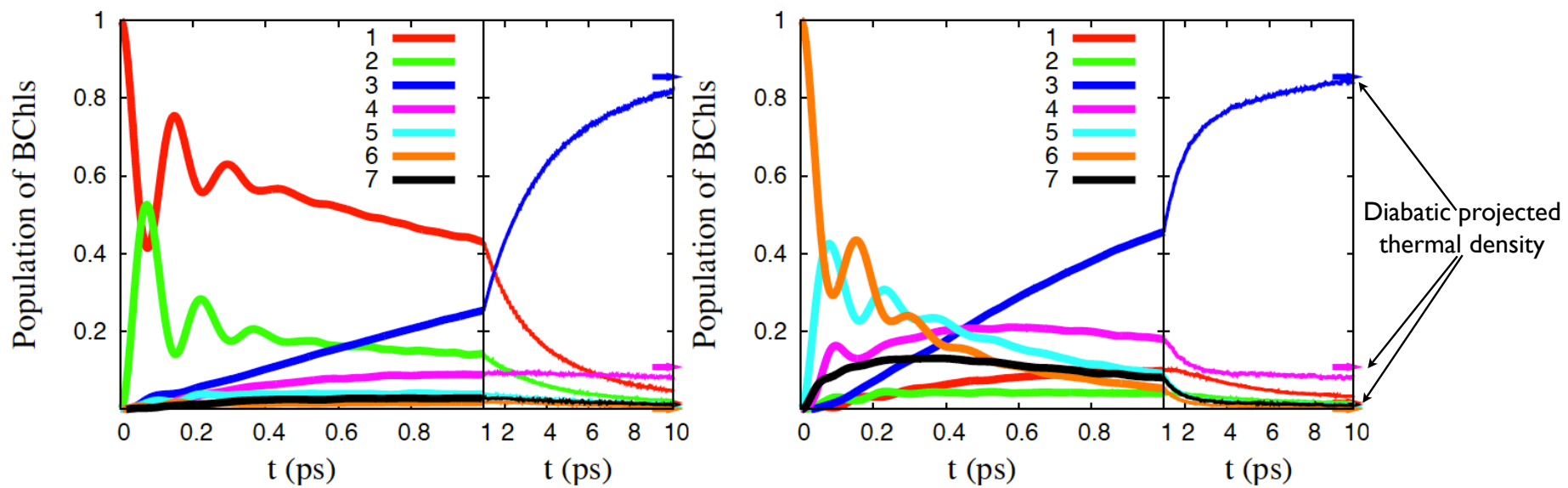
Seven state FMO spin-boson



Experimentally fit spectral density



PLDM propagation & Thermal Equilibration



C.-Y. Hsieh and R. Kapral, JCP 138, 134110 (2013)

IPLDM = “Jump FB solution”

Equivalent to exact solution of
Mixed Quantum Classical Liouville (MQCL) equation

Important properties:

(1) Exact for the spin-boson:

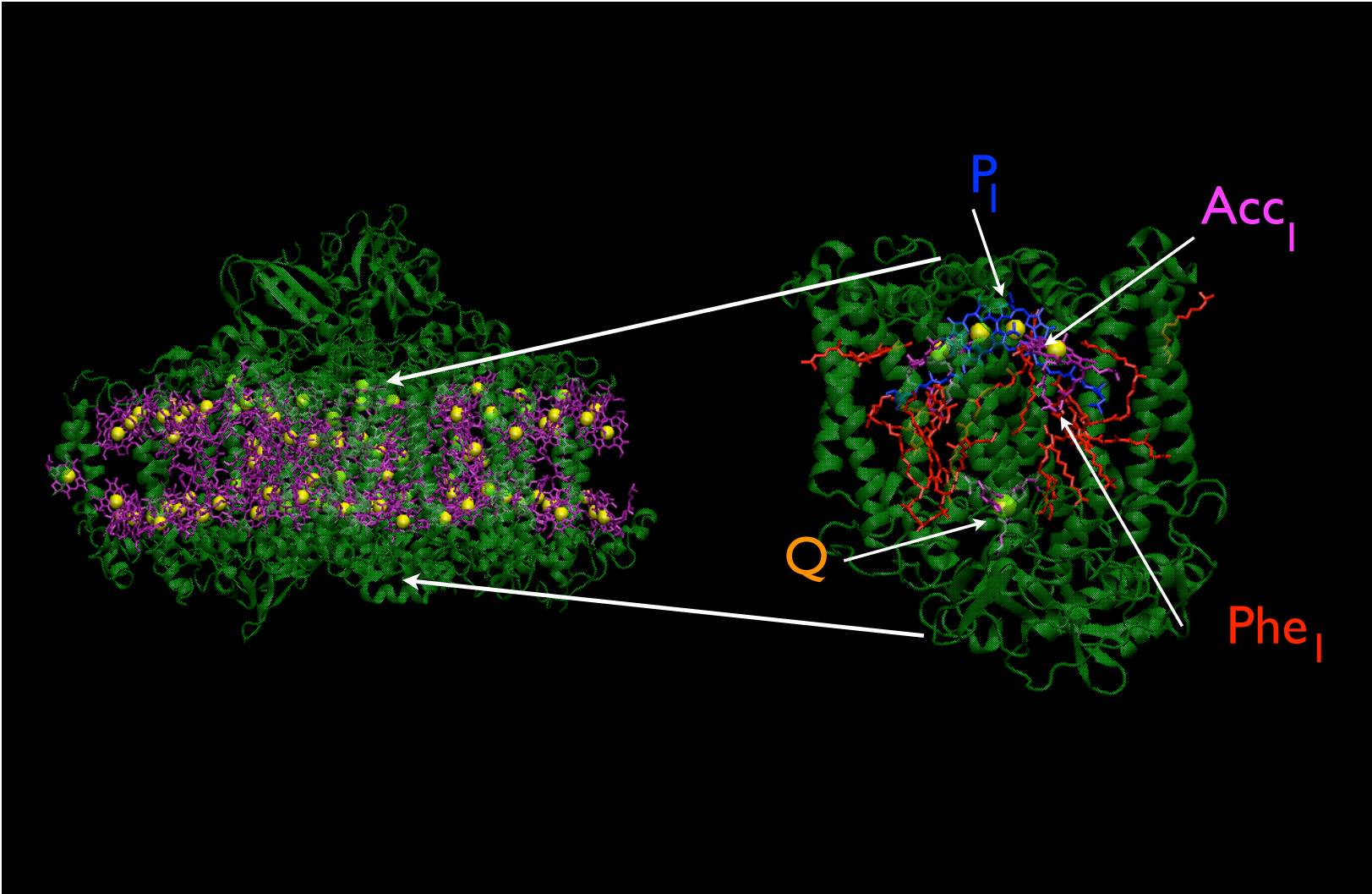
D. MacKernan, G. Ciccotti, R. Kapral, JCP 116, 2346 (2002)

(2) Stationary solution of MQCL equation, $\hat{\rho}_{We}(R, P)$, and the partial Wigner transform of the quantum equilibrium canonical density matrix, $\hat{\rho}_{We}^Q(R, P)$, agree to $\mathcal{O}(\hbar)$:

S. Neilsen, G. Ciccotti, R. Kapral, JCP 115, 5805 (2001)

OUTLINE:

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- (2) Excitation Energy Calculations
- (3) Dissipative Quantum Dynamics for General Regimes
- (4) Issues with Linearized Dynamics of Higher Frequency Modes
- (5) Spectra (PC645/HPC645) - Influence of Protonation and “Flickering” Pathways
- (6) Quantum Dynamics and Nonlinear Spectroscopy
- (7) Coherent State Density Matrix Dynamics (+PLDM)



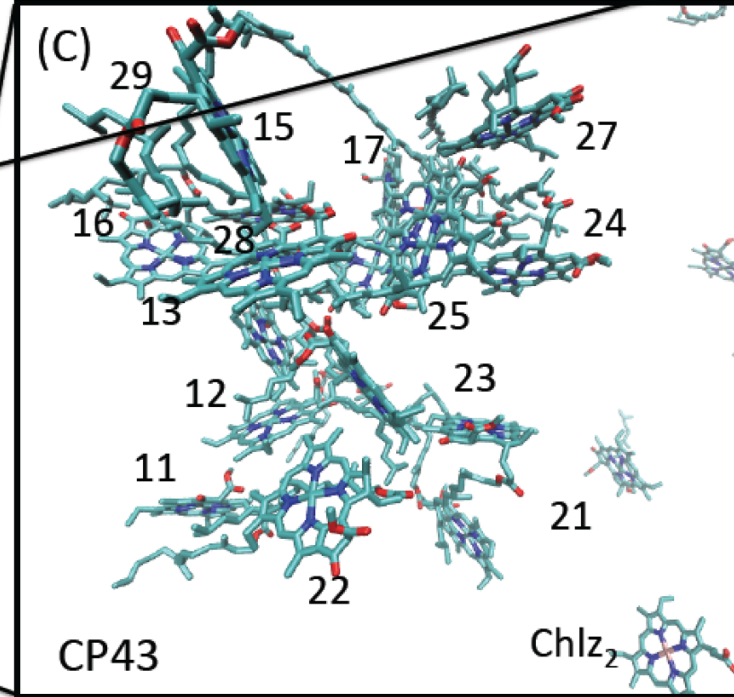
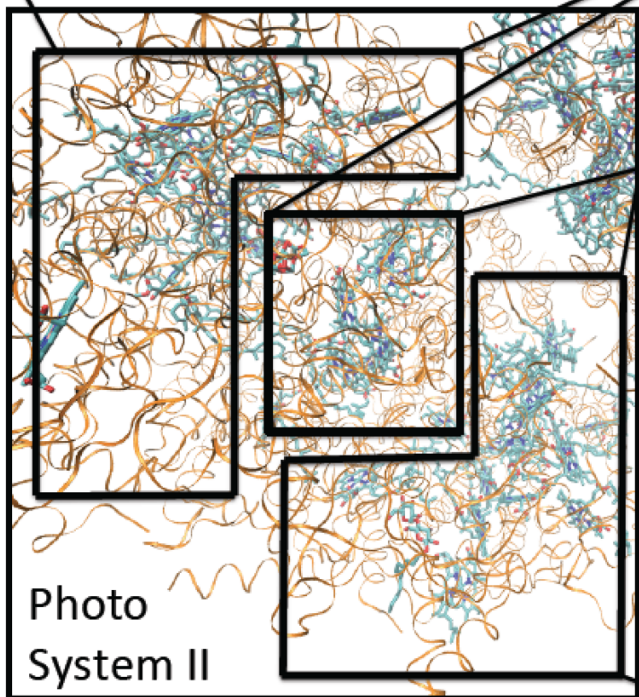
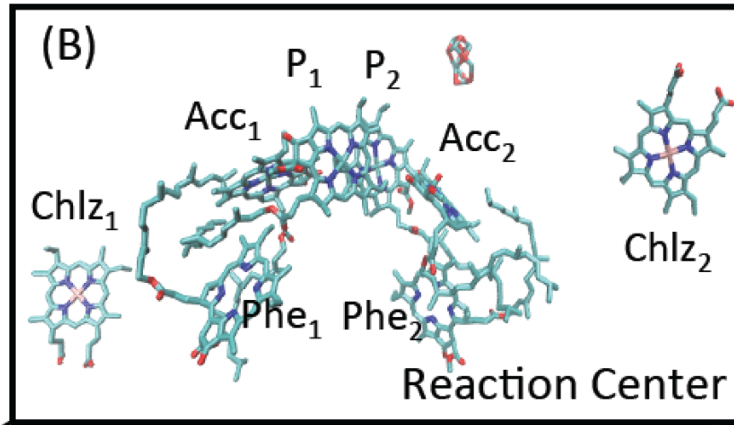
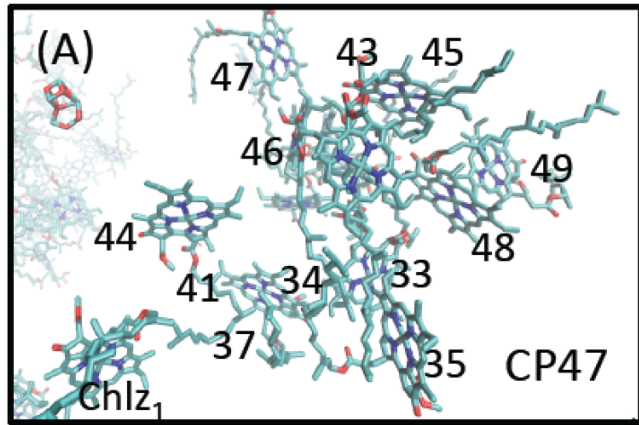


Photo System II Harvesting, Photo Protection and Charge Separation

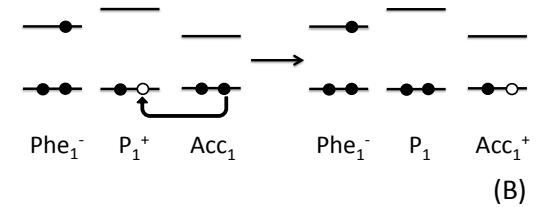
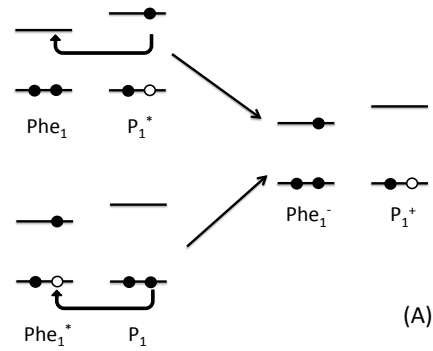
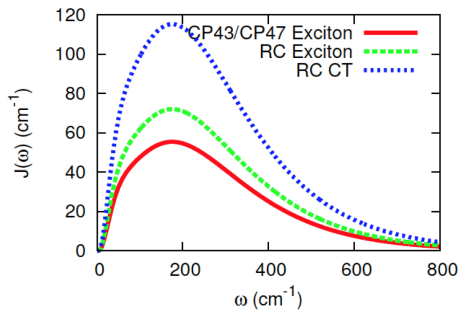
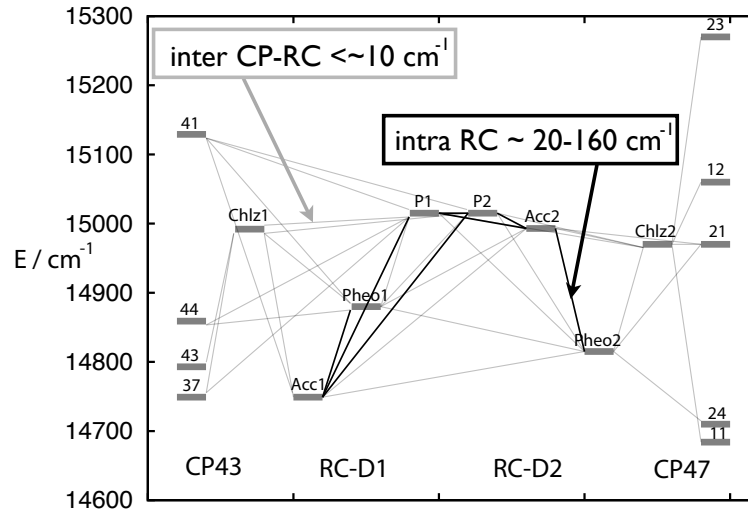
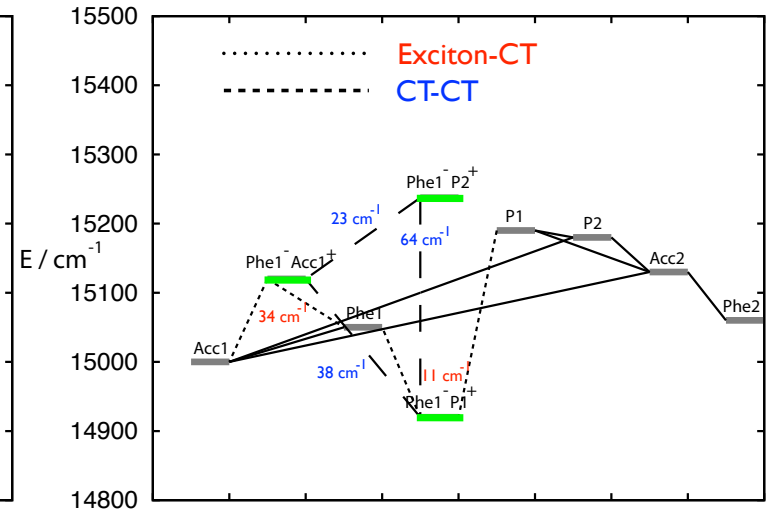


FIG. 2: Spectral density used in the model hamiltonian of PSII complex[14]. The exciton state spectral density in the harvesting complex CP43, CP47 is represented by red solid line. The spectral density for exciton (EX) and charge transfer (CT) state in reaction center are represented by green dash and blue dotted line respectively.

Exciton Couplings (Tom Renger & Coworkers) BioPhys.J. 986 (2005), JACS 130 4431 (2008)



Exciton-CT & CT-CT Couplings Mukamel & Coworkers, JCP 133, 184501 (2010)



Exciton-Charge Transfer
"wiring" diagram
Light harvesting &
Reaction Center
Sub-Units

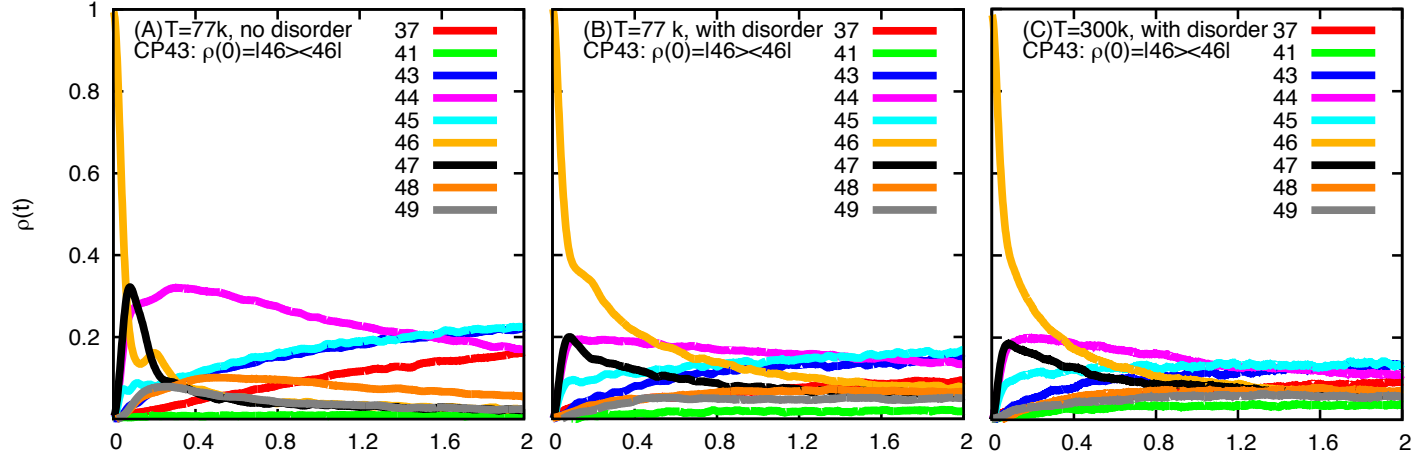
Harvesting Network Dynamics

Site disorder:

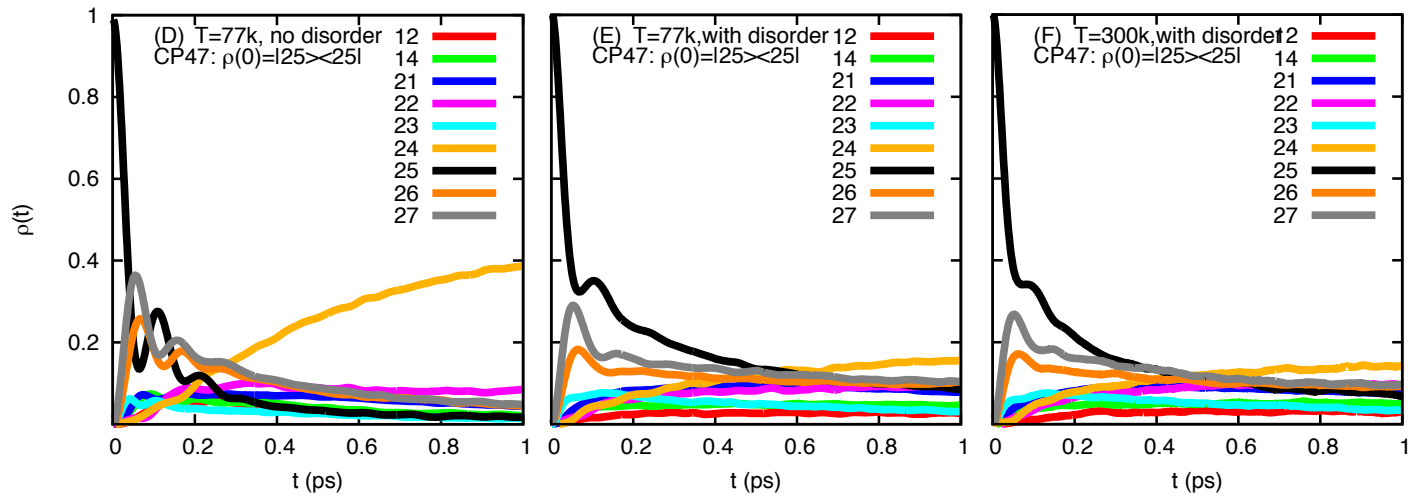
$\sigma = 0 \text{ cm}^{-1}$

$\sigma = 180 \text{ cm}^{-1}$

CP47

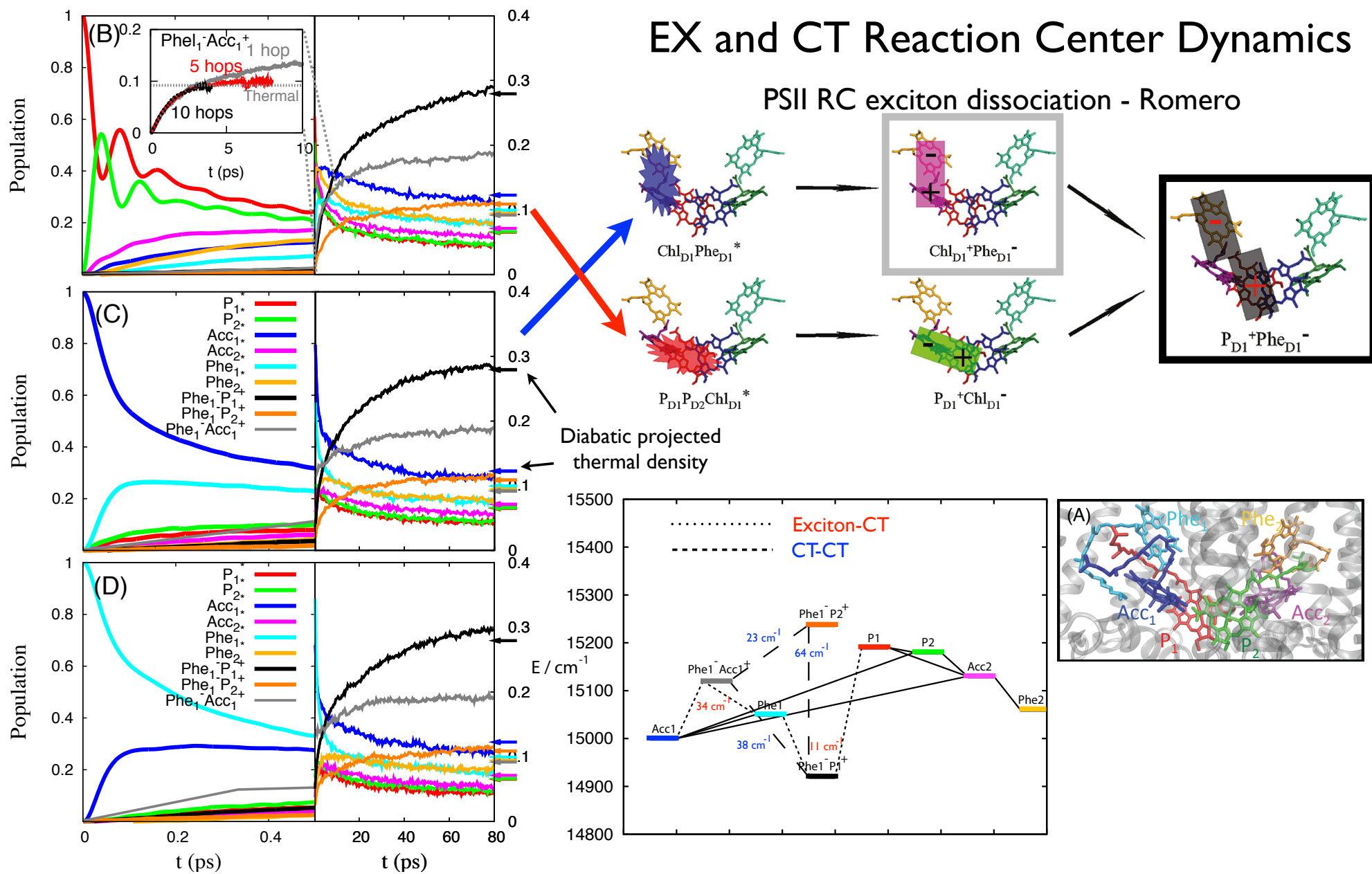


CP43



EX and CT Reaction Center Dynamics

PSII RC exciton dissociation - Romero



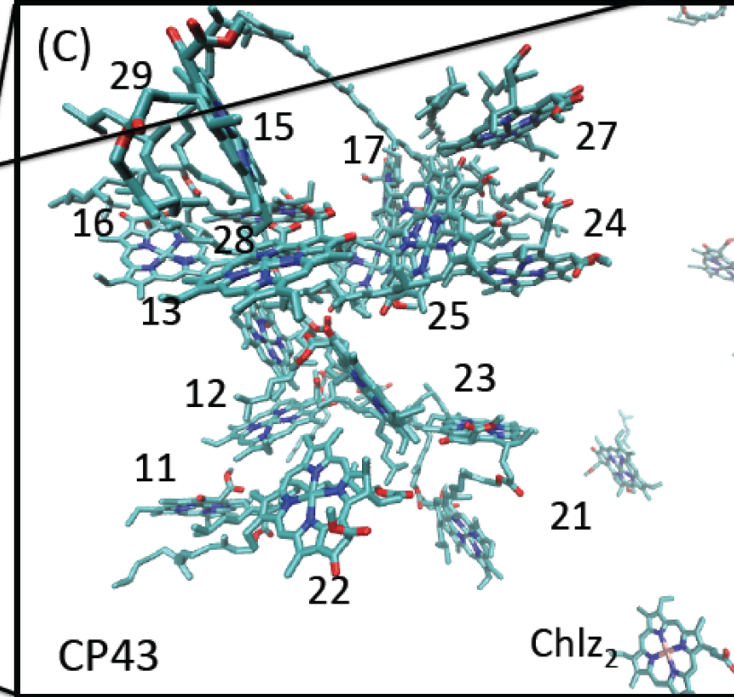
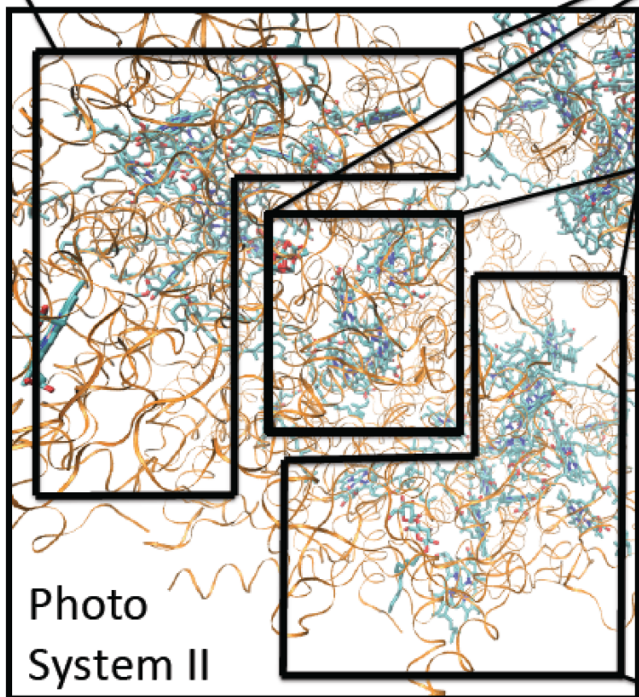
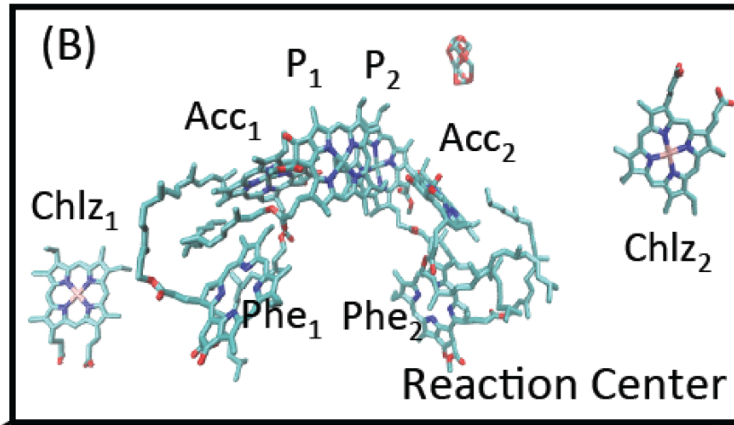
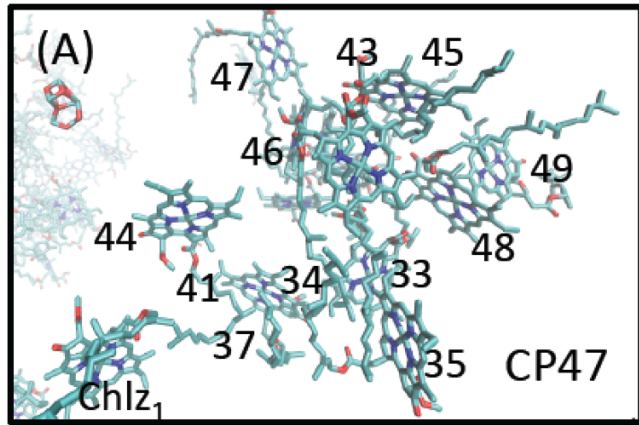
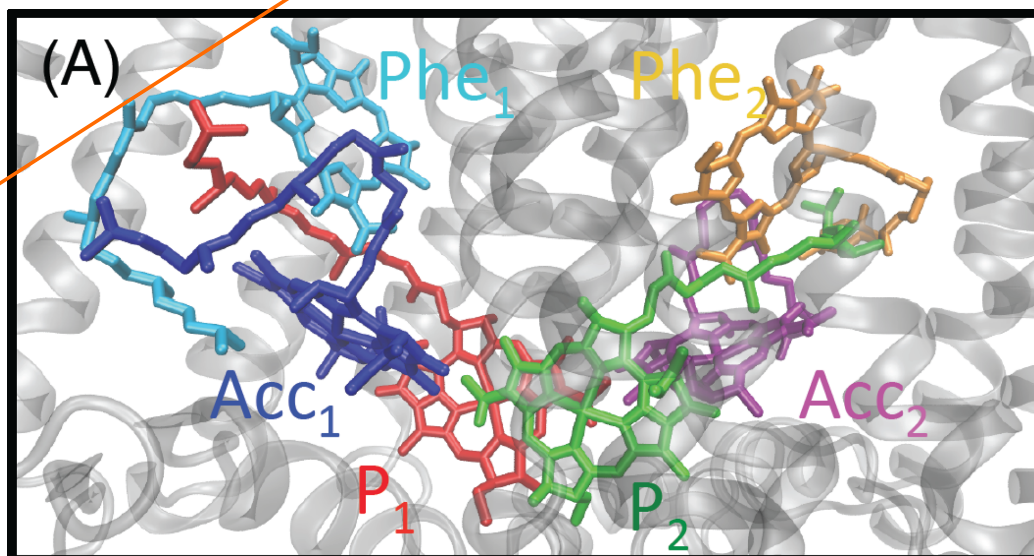
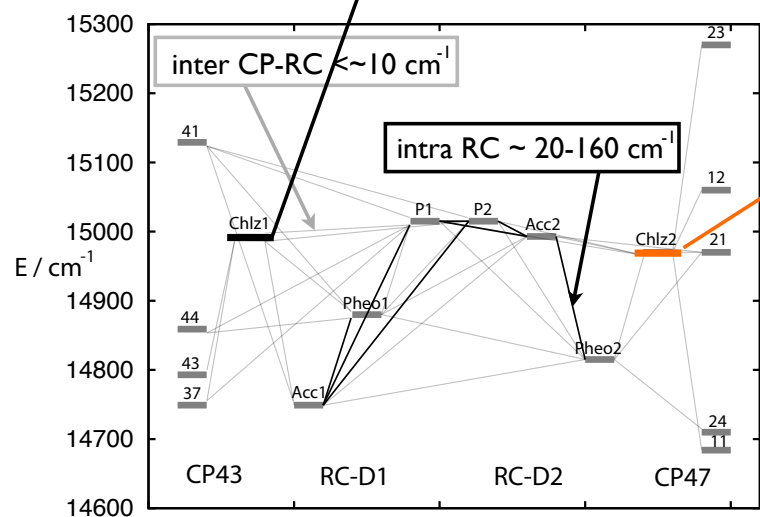
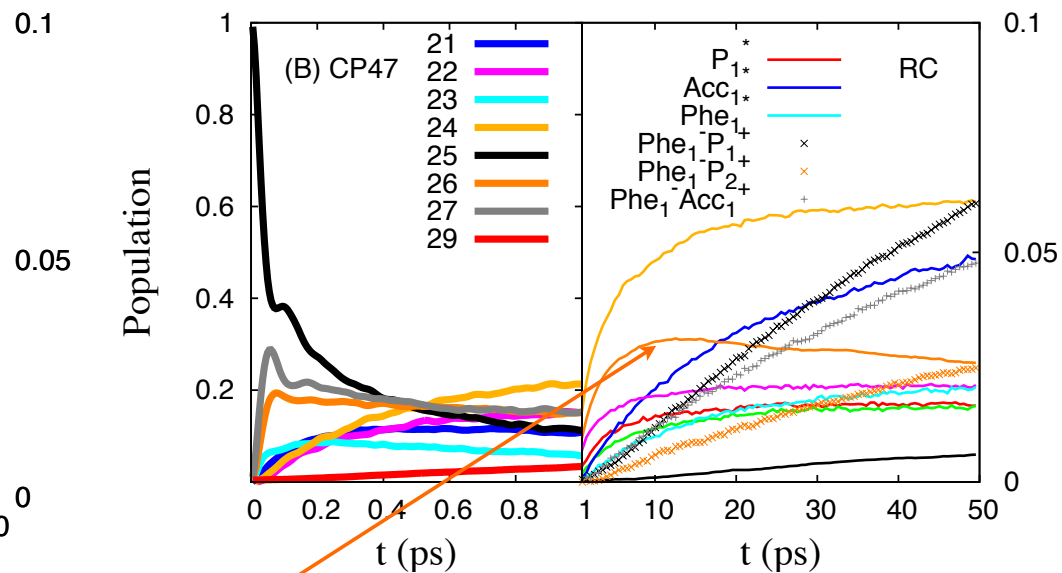
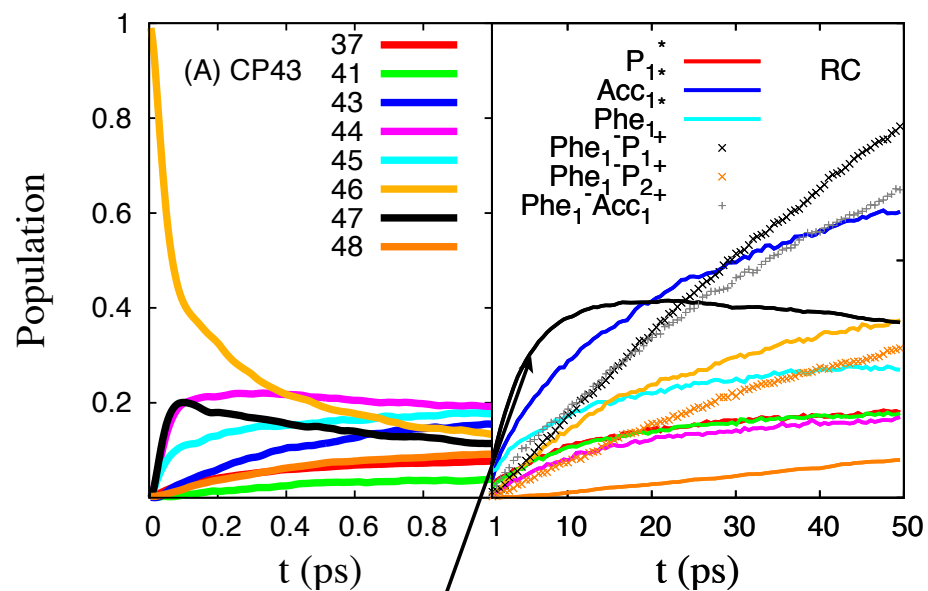
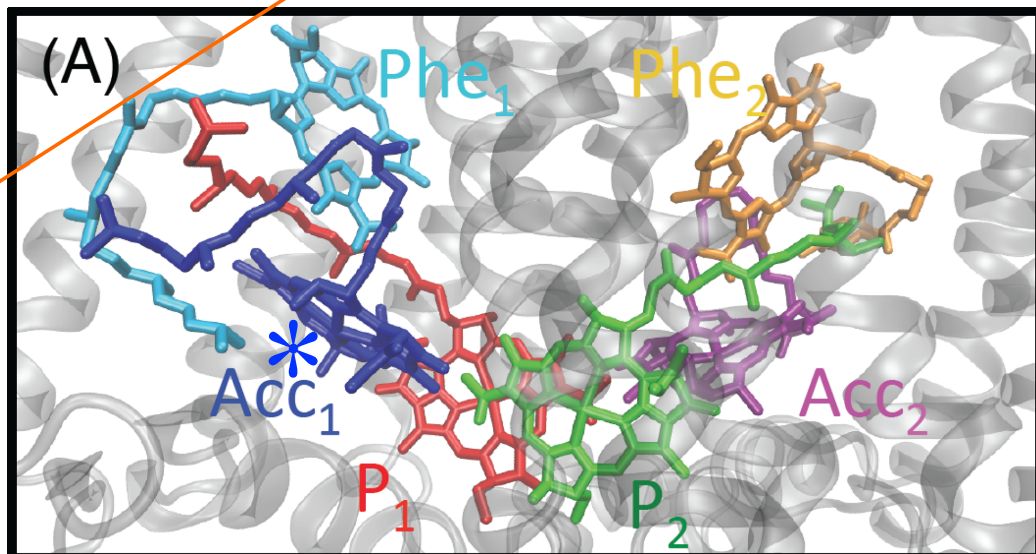
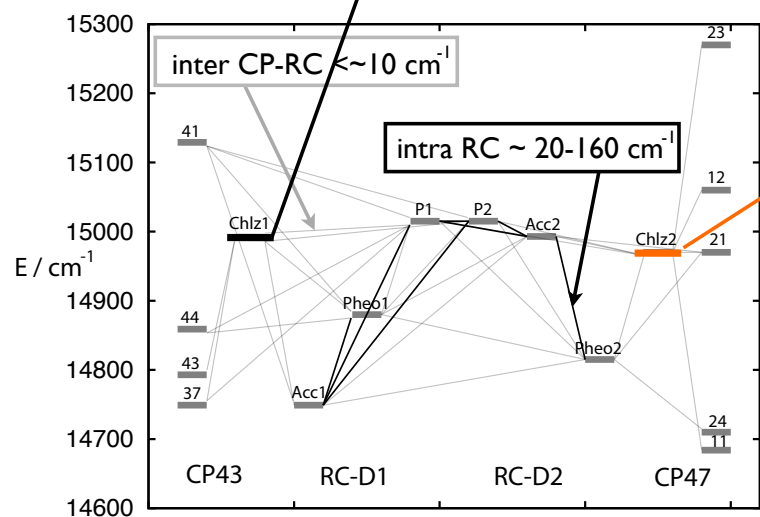
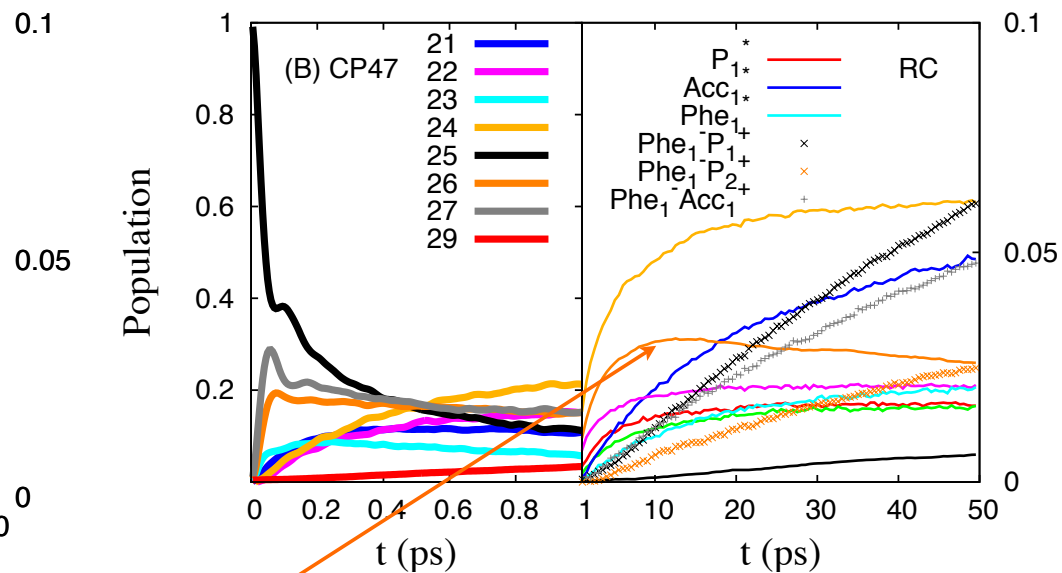
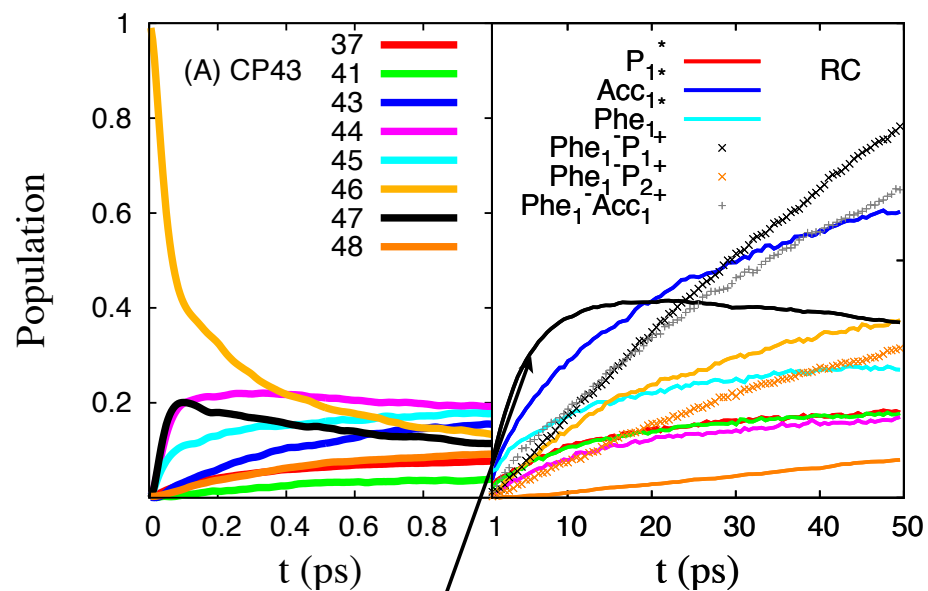
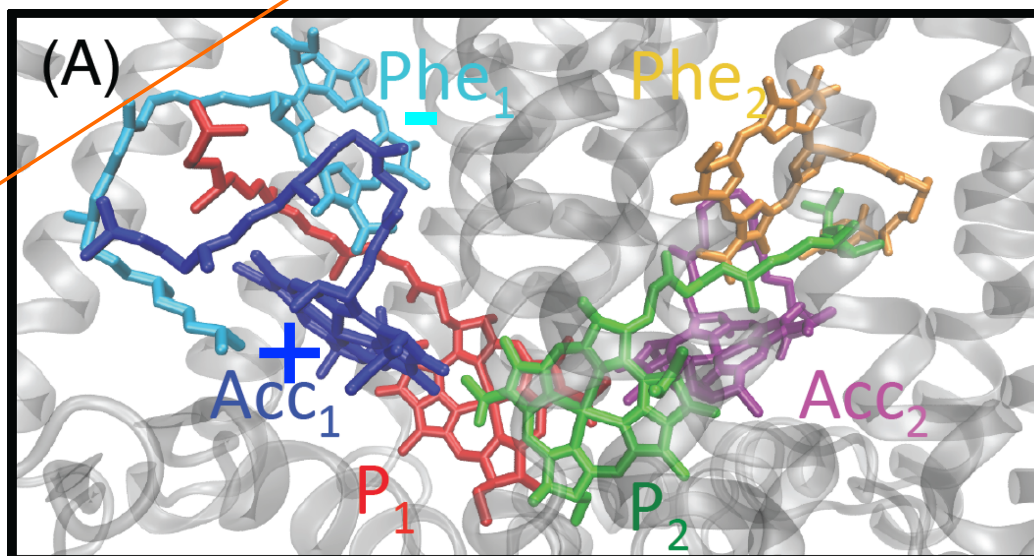
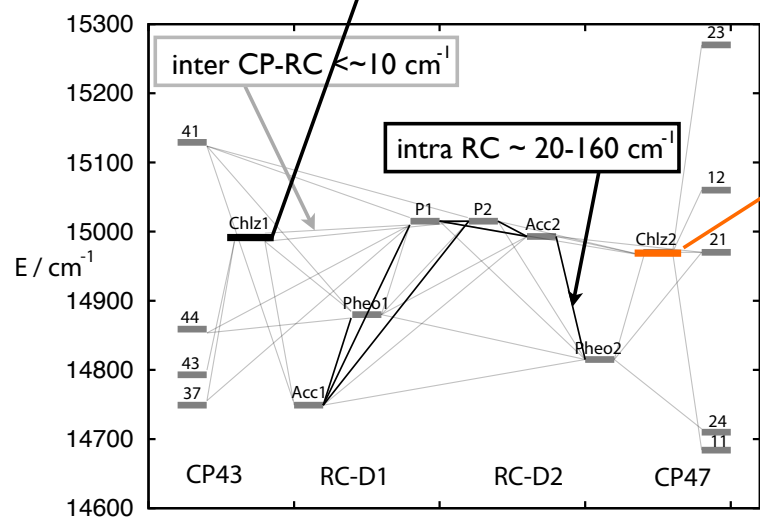
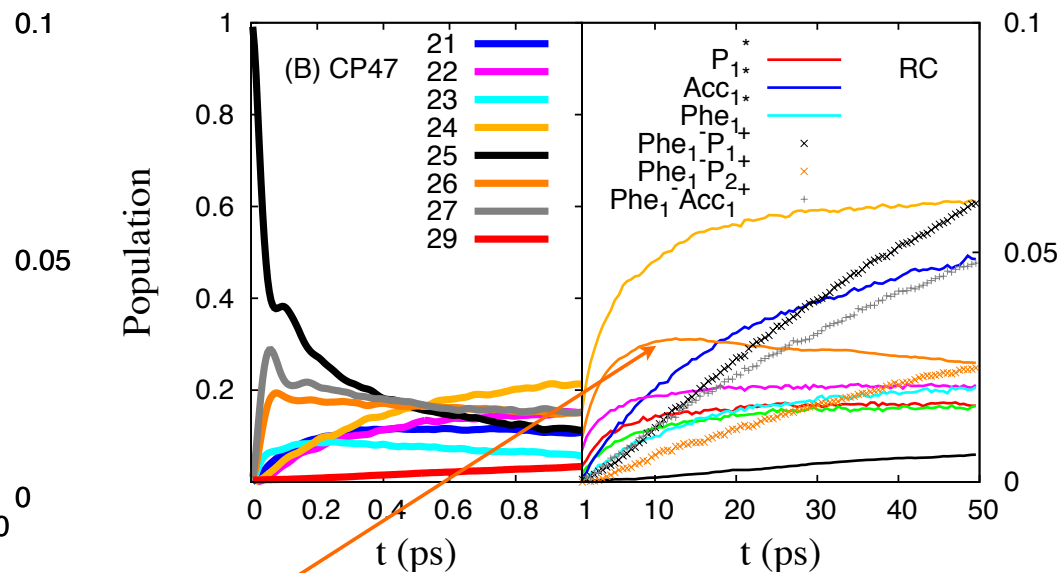
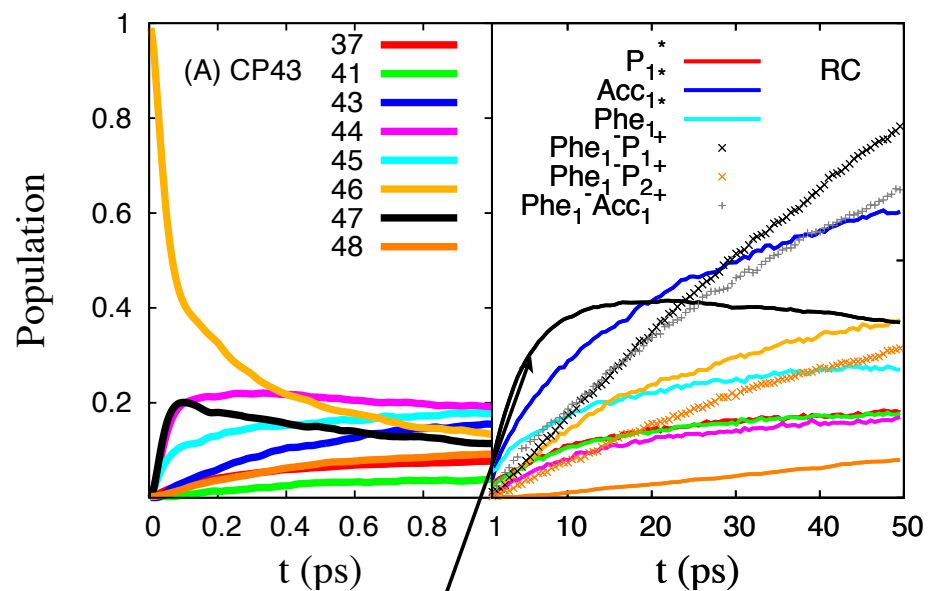
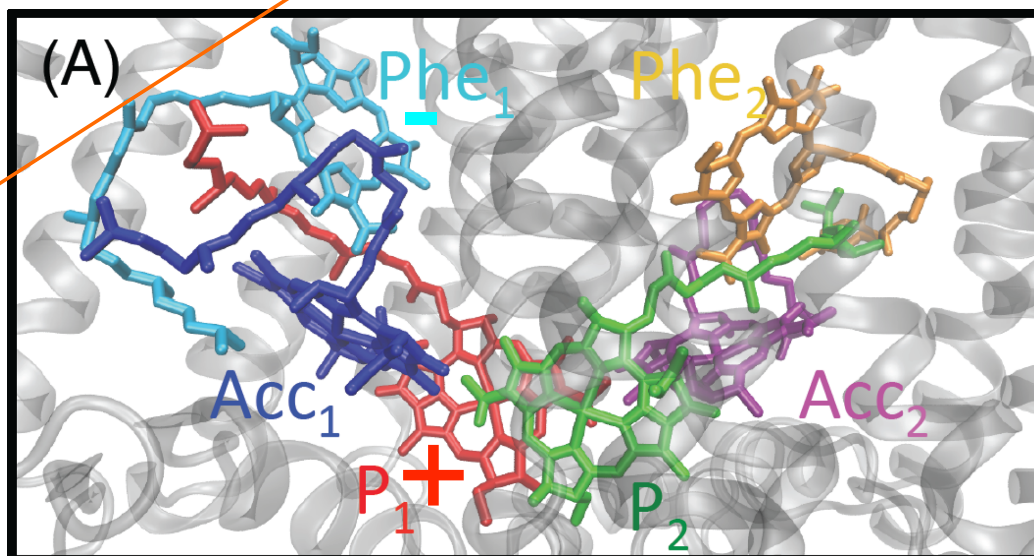
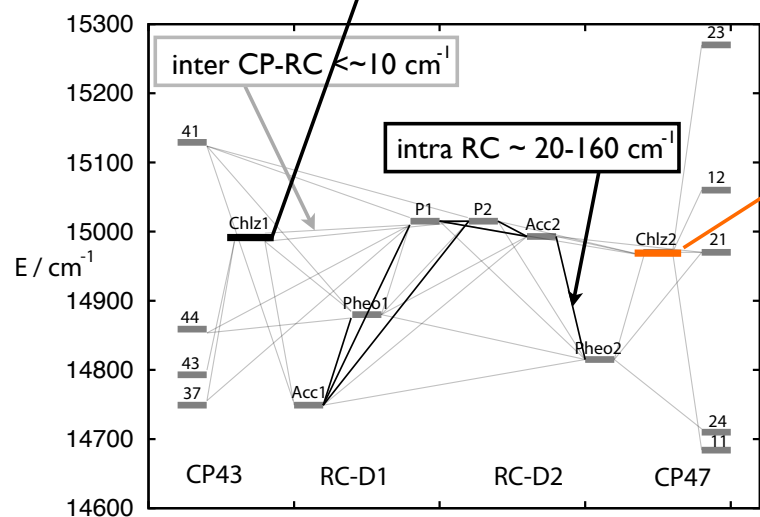
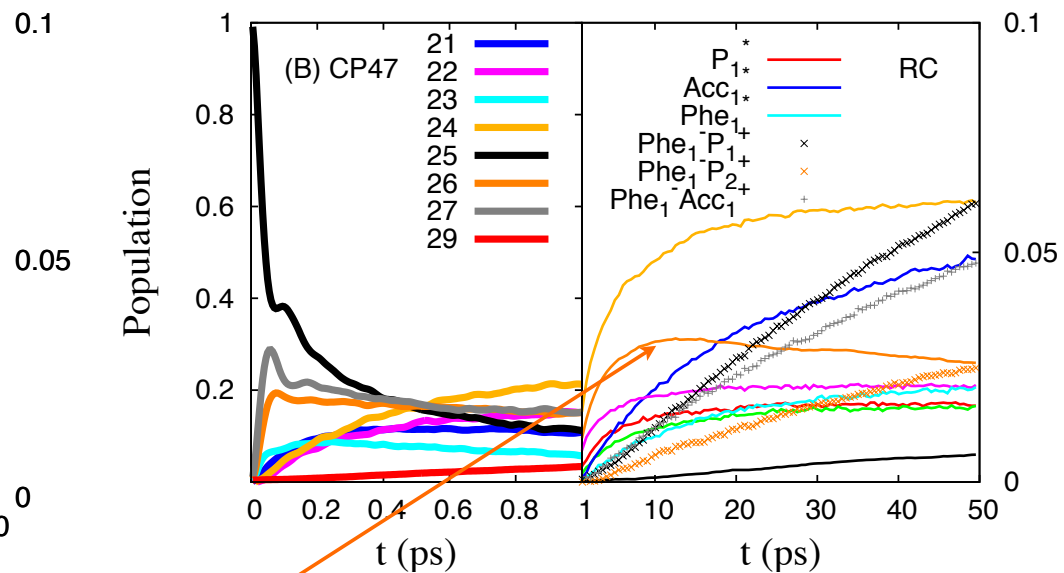
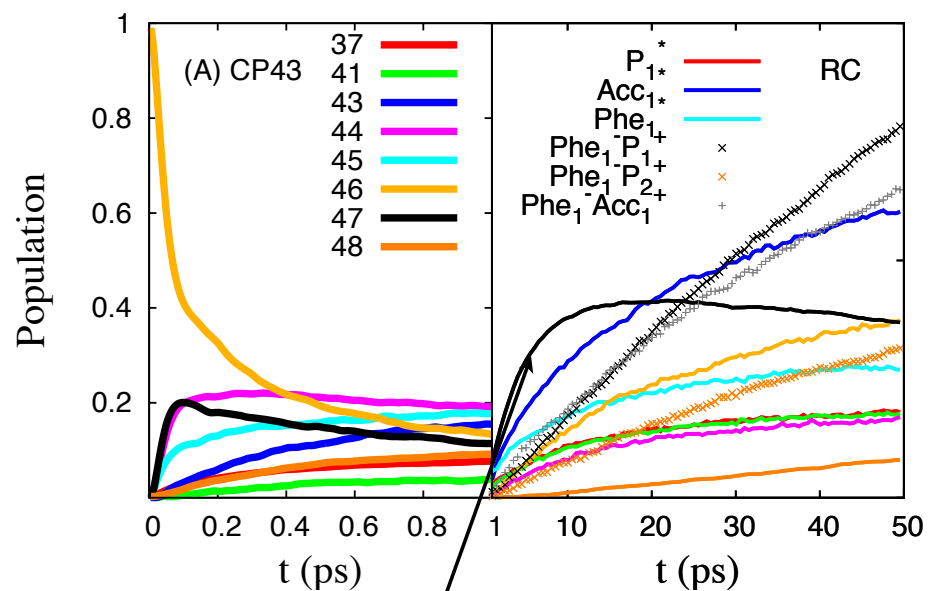


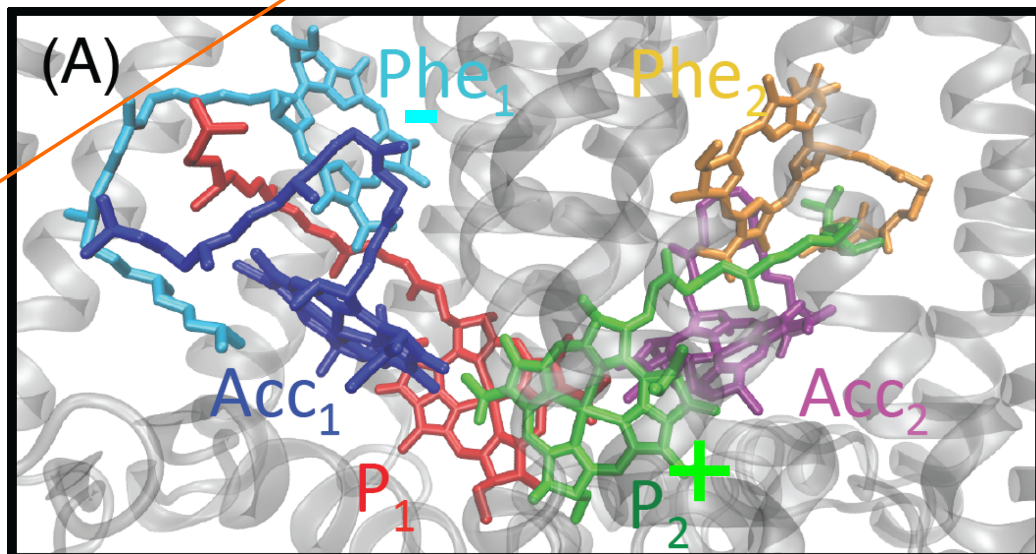
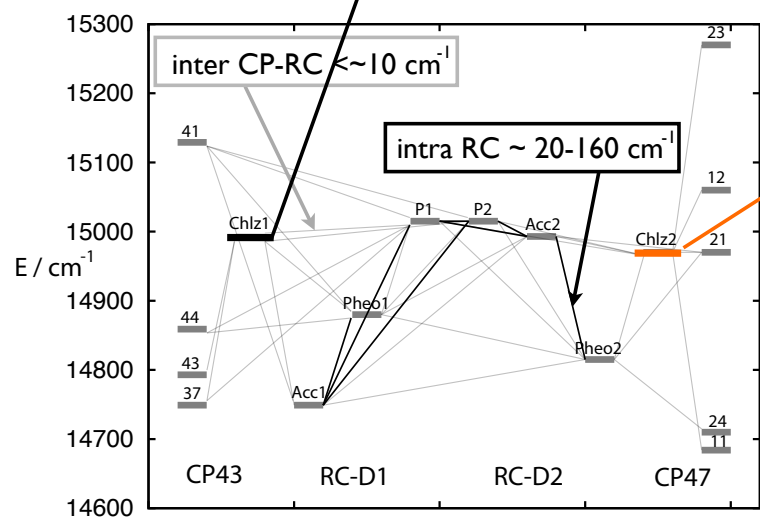
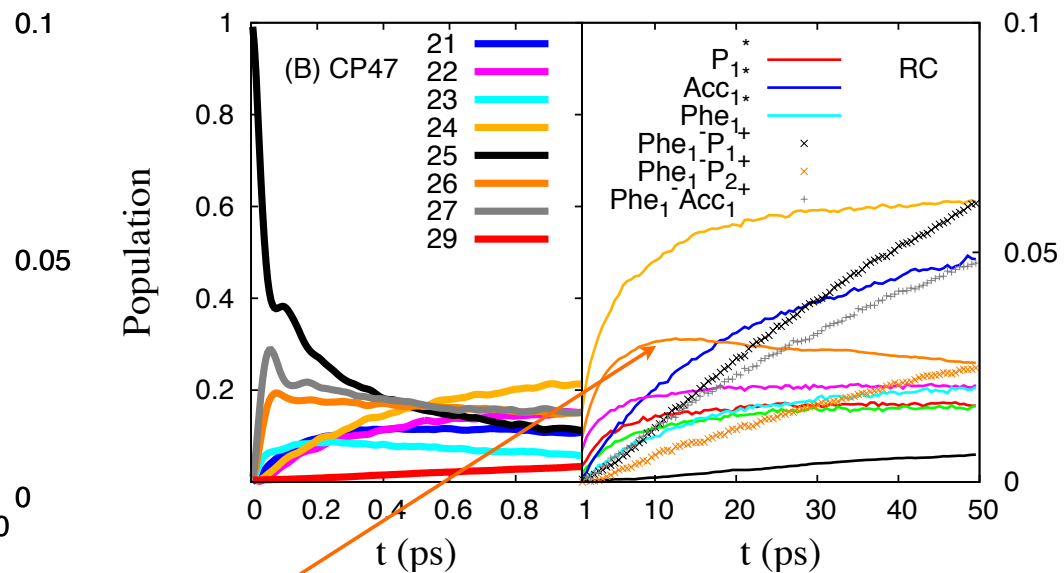
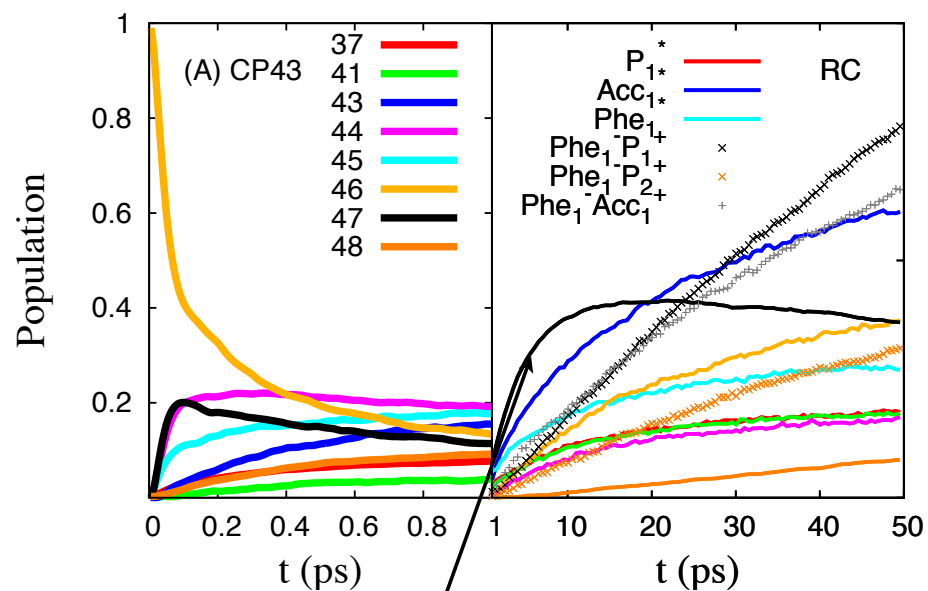
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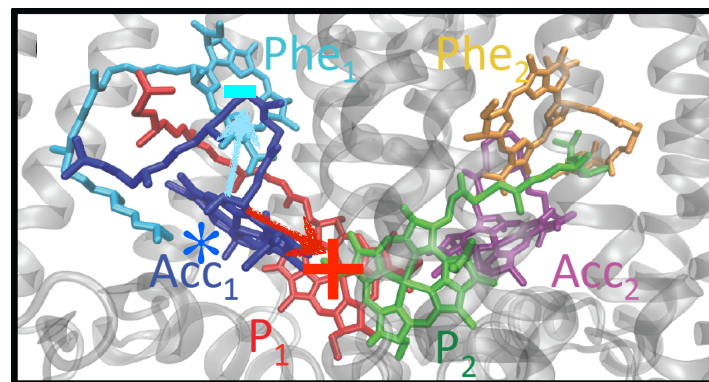
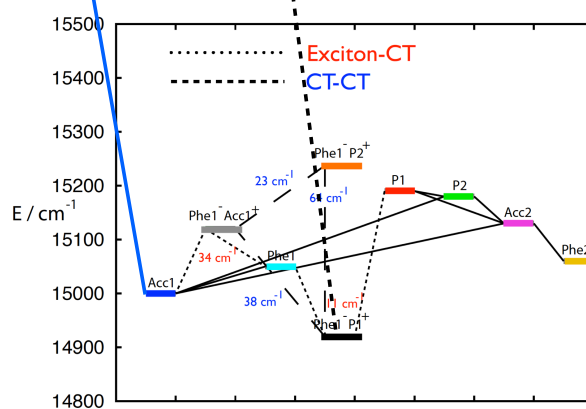
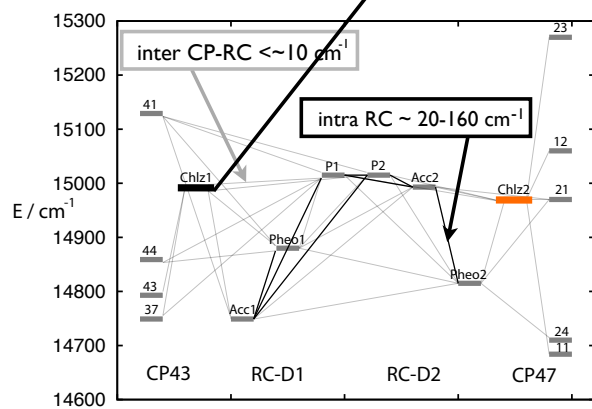
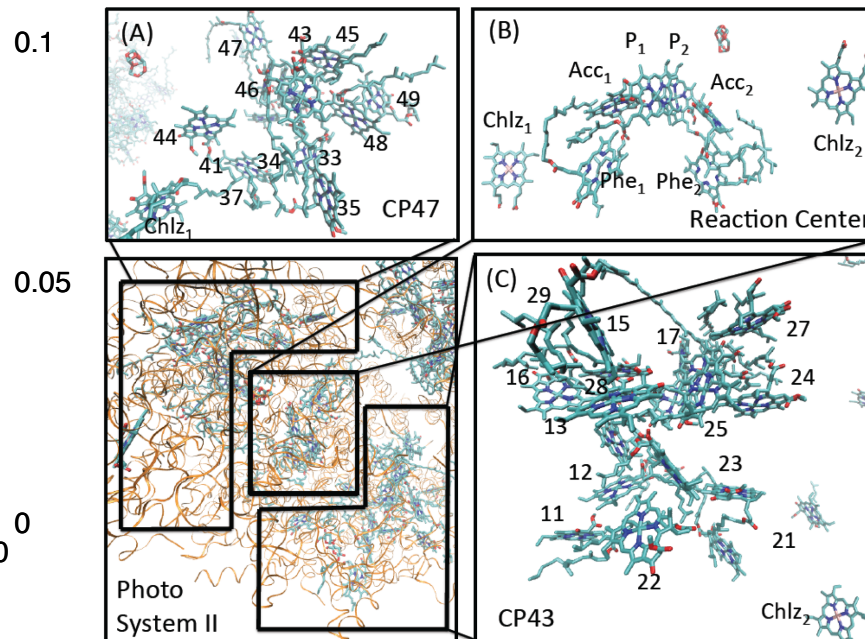
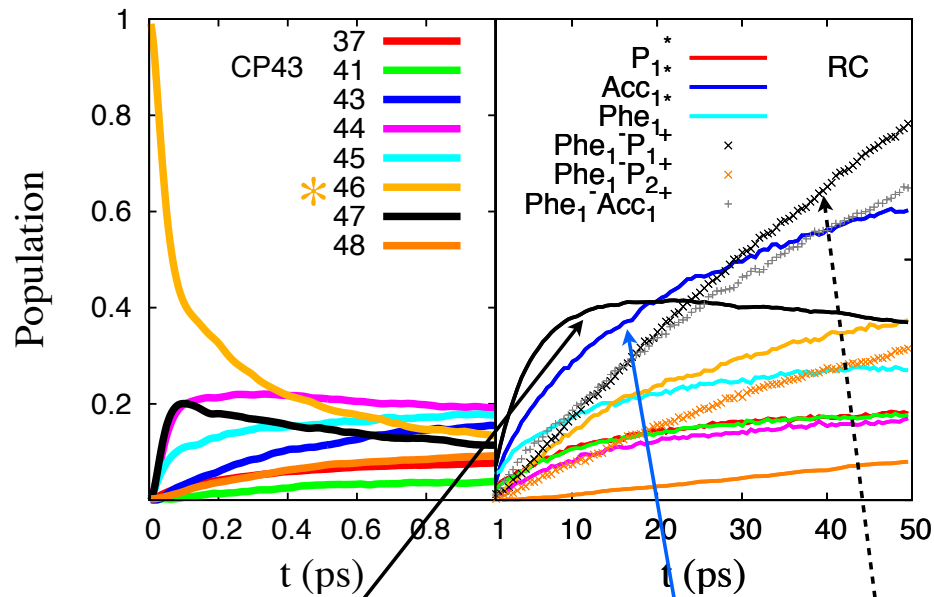






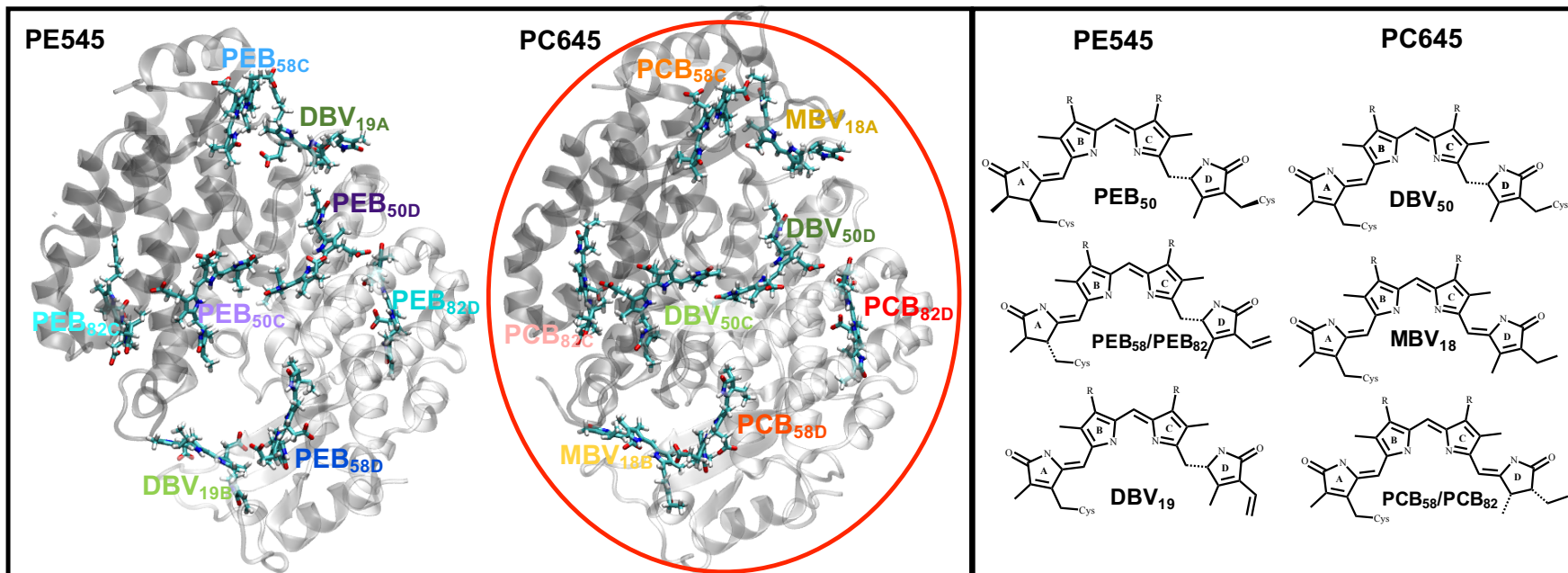




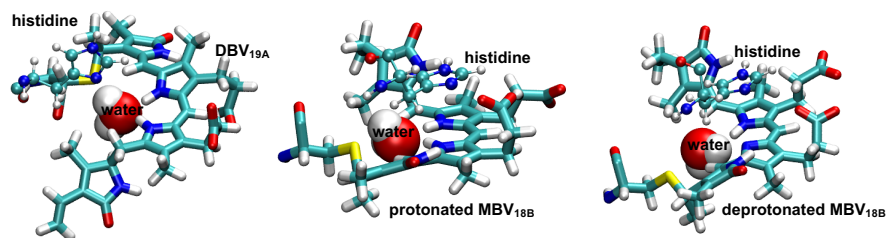


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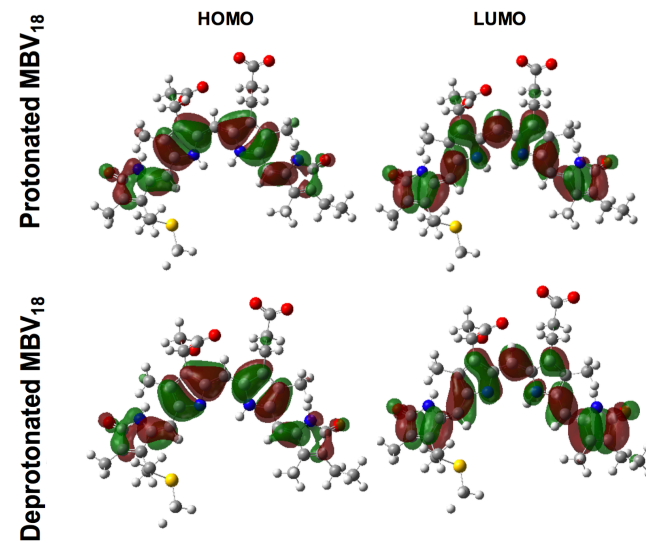
MBV - no
aspartic acid
coordination



PE545

PC645

M.K. Lee, K. Bravaya, DFC, J. Am. Chem. Soc., 139, 7803-7814 (2017)



$$\hat{H}_{\text{ex-vib}} = \sum_{\alpha} \sum_i^{n(\alpha)} c_i^{(\alpha)} \hat{Q}_i^{(\alpha)} |\alpha\rangle \langle \alpha|$$

Approximate Exciton - Vibronic Lineshapes
Renger & Marcus JCP 116, 9997 (2002)

$$\hat{H} = \hat{H}_{\text{ex}} + \hat{H}_{\text{ex-vib}} + \hat{H}_{\text{vib}} \quad \hat{H}_{\text{vib}} = \sum_{\alpha} \sum_i^{n(\alpha)} \frac{1}{2} [\hat{P}_i^{(\alpha)2} + \omega_i^{(\alpha)2} \hat{Q}_i^{(\alpha)2}]$$

$$\hat{H}_{\text{ex}} = \sum_{\alpha} \epsilon_{\alpha} |\alpha\rangle \langle \alpha| + \sum_{\alpha \neq \beta} \Delta_{\alpha\beta} |\alpha\rangle \langle \beta| \quad (H_{\text{ex}} \text{ eigen states}) \quad |M\rangle = \sum_{\alpha} a_{\alpha}^{(M)} |\alpha\rangle \quad \hbar\omega_{LK} = E_L - E_K$$

Linear Absorption $A(\omega) \propto \omega \sum_M |\mu_M|^2 D_M(\omega)$ $D_M(\omega) = \text{Re} \int_0^{\infty} dt e^{i\omega t} \rho_{M0}(t)$ $\mu_M = \langle 0|\mu|M\rangle = \sum_{\alpha} a_{\alpha}^{(M)} \mu_{\alpha}$

Non-Markovian, 2nd order cumulant Master Equation for reduced density matrix in secular approximation

$$\dot{\rho}_{M0}(t) = -i\omega_{M0}\rho_{M0}(t) - \frac{1}{\hbar^2} \sum_{KL} \int_0^t d\tau C_{MKKL}(\tau) e^{i\omega_{LK}\tau} \rho_{L0}(t) \quad C_{MNKL}(t) = \sum_{\alpha} \sum_{\beta} a_{\alpha}^{(M)} a_{\alpha}^{(N)} a_{\beta}^{(K)} a_{\beta}^{(L)} \langle \delta\epsilon^{\alpha}(t) \delta\epsilon^{\beta}(0) \rangle$$

$$\langle \delta\epsilon^{\alpha}(t) \delta\epsilon^{\alpha}(0) \rangle = \langle e^{iH_{\text{vib}}^{(\alpha)}t/\hbar} \delta\epsilon^{\alpha} e^{-iH_{\text{vib}}^{(\alpha)}t/\hbar} \delta\epsilon^{\alpha} \rangle = \frac{\hbar}{\pi} \int_0^{\infty} d\omega J^{(\alpha)}(\omega) [(n(\omega) + 1)e^{-i\omega t} + n(\omega)e^{i\omega t}]$$

$$\rho_{M0}(t) = \exp[-i\omega_{M0}t - \frac{1}{\hbar^2} \sum_K \int_0^t d\tau (t - \tau) C_{MKKM}(\tau) e^{i\omega_{MK}\tau}] \quad n(\omega) = 1/(e^{\beta\hbar\omega} - 1)$$

$$D_M(\omega) = \text{Re} \int_0^{\infty} dt e^{i(\omega - \tilde{\omega}_{M0})t} e^{[G_M(t) - G_M(0)]} e^{-t/\tau_M}$$

$$\tilde{C}_{MK}(\omega_{MK}) = \int_0^{\infty} d\tau C_{MKKM}(\tau) e^{i\omega_{MK}\tau}$$

$$\tilde{\omega}_{M0} = \omega_{M0} - \frac{E_{\lambda}^M}{\hbar} + \sum_{K \neq M} \text{Im} \tilde{C}_{MK}(\omega_{MK})$$

$$E_{\lambda}^M = \frac{1}{\pi} \int_0^{\infty} d\omega J^{MM}(\omega) / \omega$$

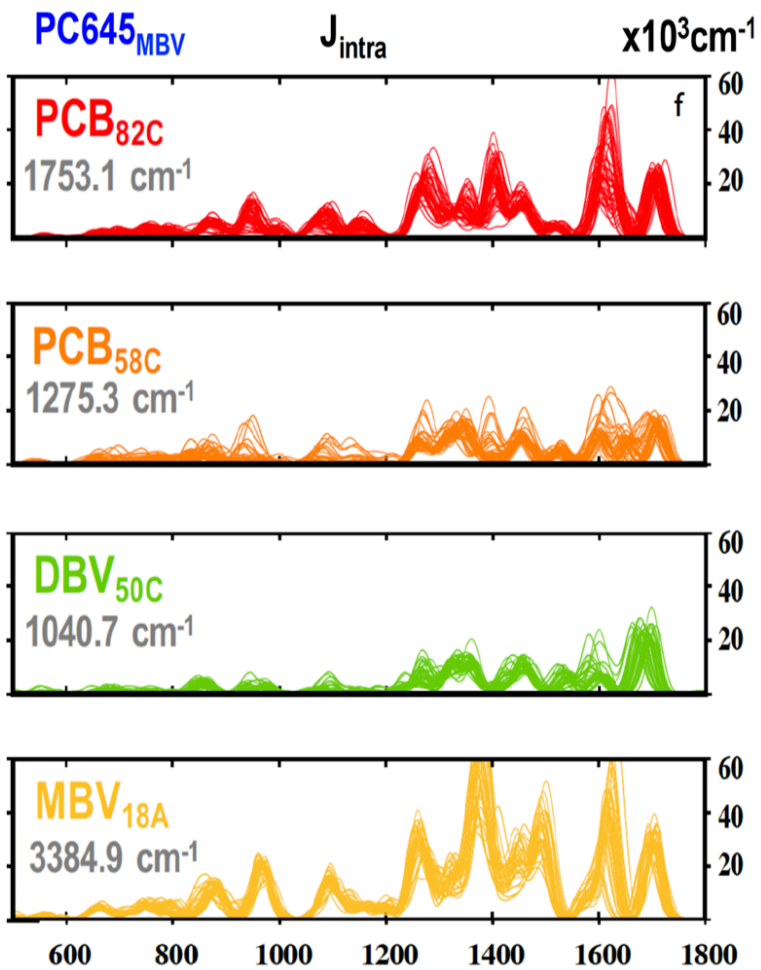
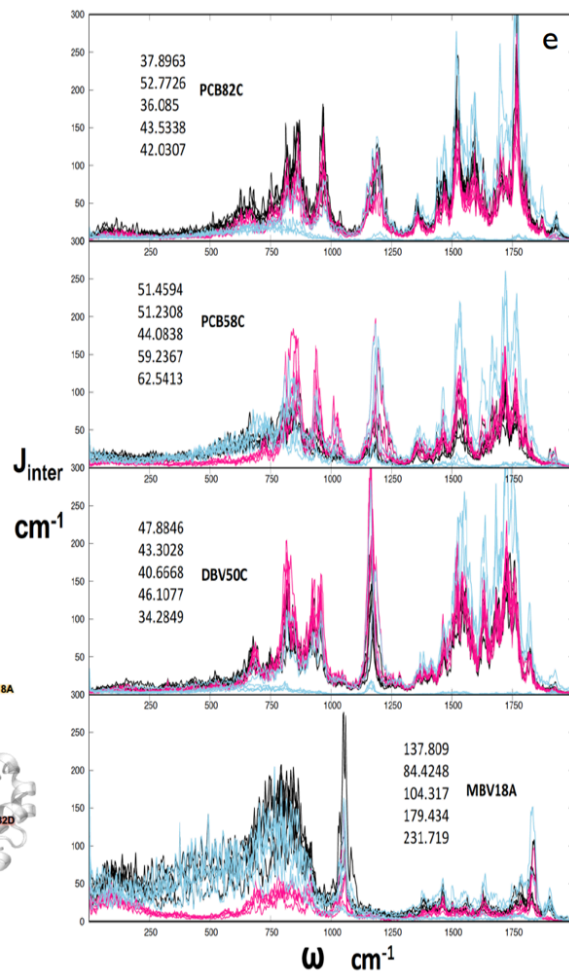
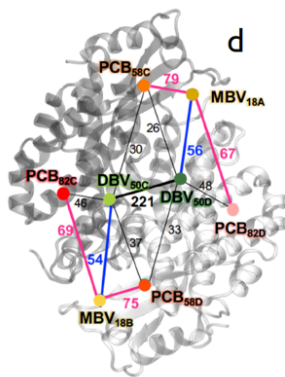
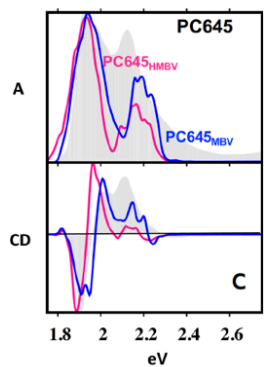
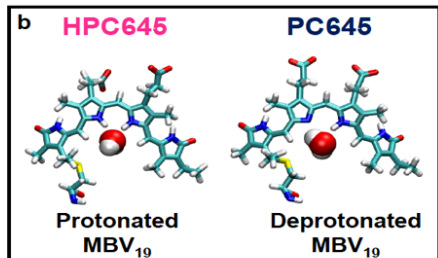
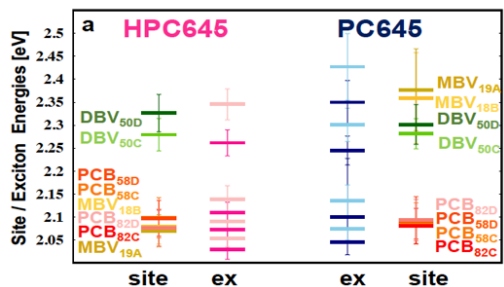
$$\text{Re} \tilde{C}_{MK}(\omega) = \frac{1}{\hbar} [(1+n(\omega)) J^{MK}(\omega) + n(-\omega) J^{MK}(-\omega)]$$

$$J^{MK}(\omega) = \sum_{\alpha} |a_{\alpha}^{(M)}|^2 |a_{\alpha}^{(K)}|^2 J^{(\alpha)}(\omega)$$

$$\tau_M^{-1} = \frac{1}{2} \sum_K k_{M \rightarrow K}$$

$$k_{M \rightarrow K} = 2 \text{Re} \tilde{C}_{MK}(\omega_{MK})$$

$$G_M(t) = \frac{\hbar}{\pi} \int_0^{\infty} d\omega (J^{MM}(\omega) / \omega^2) [(n(\omega) + 1)e^{-i\omega t} + n(\omega)e^{i\omega t}]$$



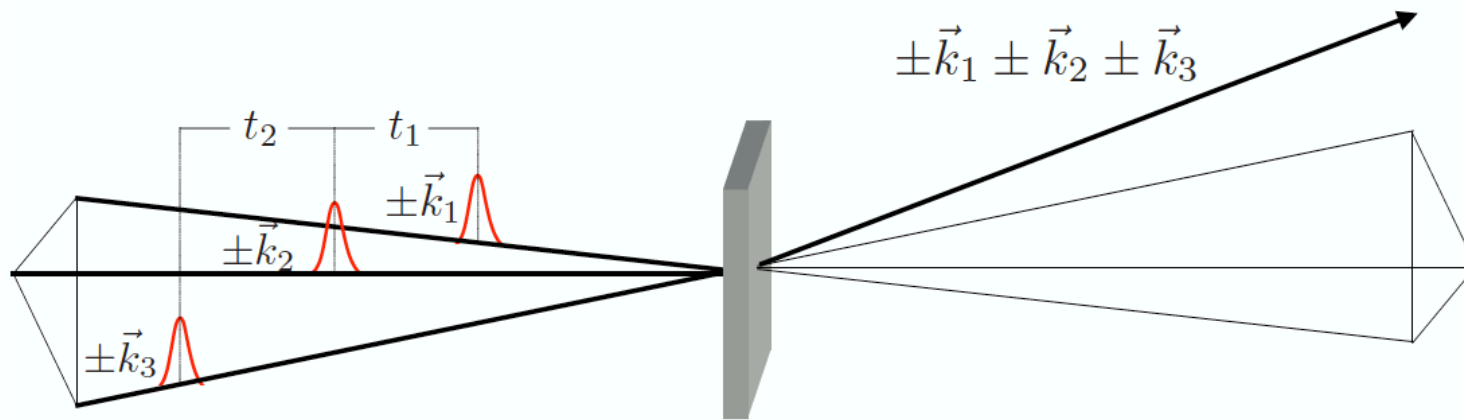
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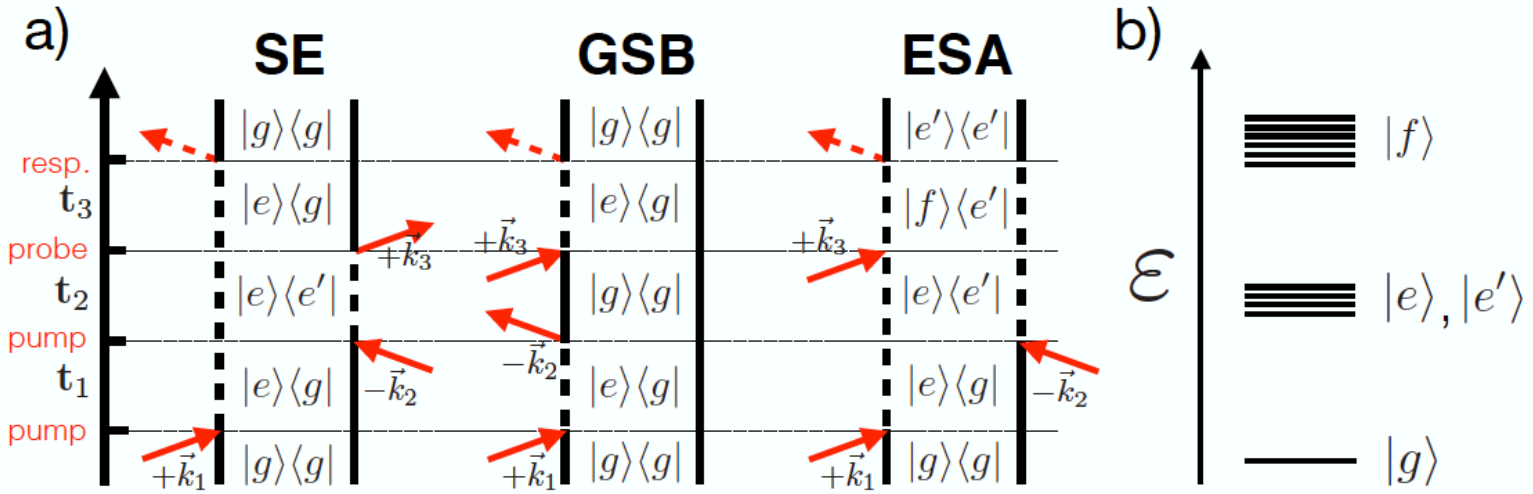
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Ultrafast Nonlinear Spectroscopy

$$\hat{H}_{tot}(t) = \hat{H} + \hat{H}'(t) \quad \hat{H}'(t) = -\hat{\boldsymbol{\mu}} \cdot \mathbf{E}(\mathbf{r}, t)$$

$$P^{(n)} = \text{Tr}[\hat{\boldsymbol{\mu}} \hat{\rho}^{(n)}] \quad \hat{\rho}^{(n)} = [\hat{\boldsymbol{\mu}}, \hat{\rho}^{(n-1)}]$$





$$P^{(1)}(t) = \int_0^\infty dt_1 R^{(1)}(t_1) E(t - t_1) \quad R^{(1)}(t_1) = i\text{Tr}[\hat{\mu}(t_1)\hat{\mu}^\times(0)\hat{\rho}(0)]$$

$$\hat{\mu}(t) = e^{+i\hat{H}t}\hat{\mu}e^{-i\hat{H}t}$$

$$P^{(3)}(t) = \int_0^\infty dt_3 \int_0^\infty dt_2 \int_0^\infty dt_1 R^{(3)}(t_3, t_2, t_1) \\ \times E(t - t_3)E(t - t_3 - t_2)E(t - t_3 - t_2 - t_1)$$

$$\hat{\mu}^\times(t) \bullet = [\hat{\mu}(t), \bullet]$$

$$R^{(3)}(t_3, t_2, t_1) = (i)^3 \langle \hat{\mu}(t_3 + t_2 + t_1) \hat{\mu}^\times(t_2 + t_1) \hat{\mu}^\times(t_1) \hat{\mu}^\times(0) \rangle \\ = (i)^3 \text{Tr}[\hat{\mu}(t_3 + t_2 + t_1) \hat{\mu}^\times(t_2 + t_1) \hat{\mu}^\times(t_1) \hat{\mu}^\times(0)\hat{\rho}(0)]$$

$$\begin{aligned}
R^{(3)}(t_3, t_2, t_1) &= (i)^3 \sum_{n_{t_3}} \left(\prod_{k=N_2+N_1+1}^{N_3+N_2+N_1} \int d\bar{R}_k \frac{d\bar{P}_k}{2\pi} \right) \sum_{n_{t_2}, n'_{t_2}} \int d\bar{R}_{N_2+N_1} \frac{d\bar{P}_{N_2+N_1}}{2\pi} dx_{t_2} dp_{t_2} dx'_{t_2} dp'_{t_2} G_{t_2} G'_{t_2} \\
&\times (\mu \rho^{(3)})_{n_{t_3} n_{t_3}} \prod_{k=N_2+N_1+1}^{N_3+N_2+N_1-1} \delta \left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k \right) \prod_{k=N_2+N_1+1}^{N_3+N_2+N_1} \delta \left(\frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon} - \frac{\bar{P}_k}{M} \right) \delta (\bar{P}_{N_2+N_1+1} - \bar{P}_{N_2+N_1}) \\
&\times \delta \left(\frac{\bar{R}_{N_2+N_1} - \bar{R}_{N_2+N_1-1}}{\epsilon} - \frac{\bar{P}_{N_2+N_1}}{M} \right) \left(\prod_{k=N_1+1}^{N_2+N_1-1} \int d\bar{R}_k \frac{d\bar{P}_k}{2\pi} \right) \sum_{n_{t_1}, n'_{t_1}} \int d\bar{R}_{N_1} \frac{d\bar{P}_{N_1}}{2\pi} dx_{t_1} dp_{t_1} dx'_{t_1} dp'_{t_1} G_{t_1} G'_{t_1} \\
&\times (\mu^\times \rho^{(2)})_{n_{t_2} n'_{t_2}} \prod_{k=N_1+1}^{N_2+N_1-1} \delta \left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k \right) \prod_{k=N_1+1}^{N_2+N_1} \delta \left(\frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon} - \frac{\bar{P}_k}{M} \right) \delta (\bar{P}_{N_1+1} - \bar{P}_{N_1}) \\
&\times \delta \left(\frac{\bar{R}_{N_1} - \bar{R}_{N_1-1}}{\epsilon} - \frac{\bar{P}_{N_1}}{M} \right) \left(\prod_{k=1}^{N_1-1} \int d\bar{R}_k \frac{d\bar{P}_k}{2\pi} \right) \sum_{n_0, n'_0} \int d\bar{R}_0 dx_0 dp_0 dx'_0 dp'_0 G_{t_0} G'_{t_0} \\
&\times (\mu^\times \rho^{(1)})_{n_{t_1} n'_{t_1}} \prod_{k=1}^{N_1-1} \delta \left(\frac{\bar{P}_{k+1} - \bar{P}_k}{\epsilon} - F_k \right) \prod_{k=1}^{N_1} \delta \left(\frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon} - \frac{\bar{P}_k}{M} \right) (\mu^\times \rho^{(0)})_{n_0 n'_0}^W (\bar{R}_0, \bar{P}_1)
\end{aligned}$$

$$\text{Tr } \hat{\mu} \left(\begin{array}{|c|c|c|} \hline & \blacksquare & \\ \hline \blacksquare & & \blacksquare \\ \hline & \blacksquare & \\ \hline \end{array} \right) \leftarrow \hat{\mu}^\times \left(\begin{array}{|c|c|c|} \hline \blacksquare & & \blacksquare \\ \hline & \blacksquare & \\ \hline \blacksquare & & \\ \hline \end{array} \right) \leftarrow \hat{\mu}^\times \left(\begin{array}{|c|c|c|} \hline & \blacksquare & \\ \hline \blacksquare & & \\ \hline & \blacksquare & \\ \hline \end{array} \right) \leftarrow \hat{\mu}^\times \left(\begin{array}{|c|c|c|} \hline & & \\ \hline \blacksquare & & \\ \hline & & \\ \hline \end{array} \right) \begin{array}{l} |g\rangle|e_i\rangle|e_j\rangle|f\rangle \\ \hat{\rho}^{(3)}(t_1 + t_2 + t_3) \\ \hat{\rho}^{(2)}(t_1 + t_2) \\ \hat{\rho}^{(1)}(t_1) \\ \hat{\rho}^{(0)}(0) \end{array}$$

$$\hat{H}_0 = \sum_{i,j=0}^1 \epsilon_{ij} |ij\rangle\langle ij| + \sum_{ij \in 01,10} \Delta_{ij,ji} |ij\rangle\langle ji|$$

$$J(\omega) = 2\lambda \frac{\omega/\omega_c}{(1+(\omega/\omega_c)^2)} = \frac{\pi}{2} \sum_i \frac{c_i^2}{\omega_i} \delta(\omega - \omega_i) \quad (\omega_c^{-1} = 300 \text{ fs})$$

$$+ \sum_{ij \in 01,10,11} \sum_{m=1}^M c_{ij}^m \hat{R}_m |ij\rangle\langle ij| + \sum_{m=1}^M \frac{1}{2} \left(\hat{P}_m^2 + \omega_m^2 \hat{R}_m^2 \right) \hat{1}$$

Coupled Dimer

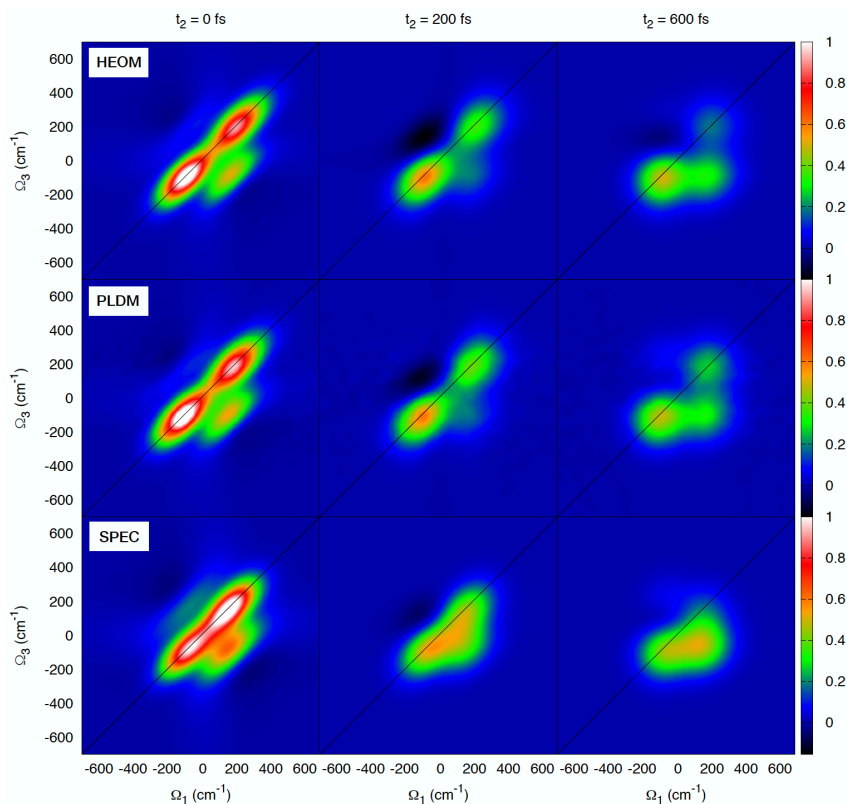
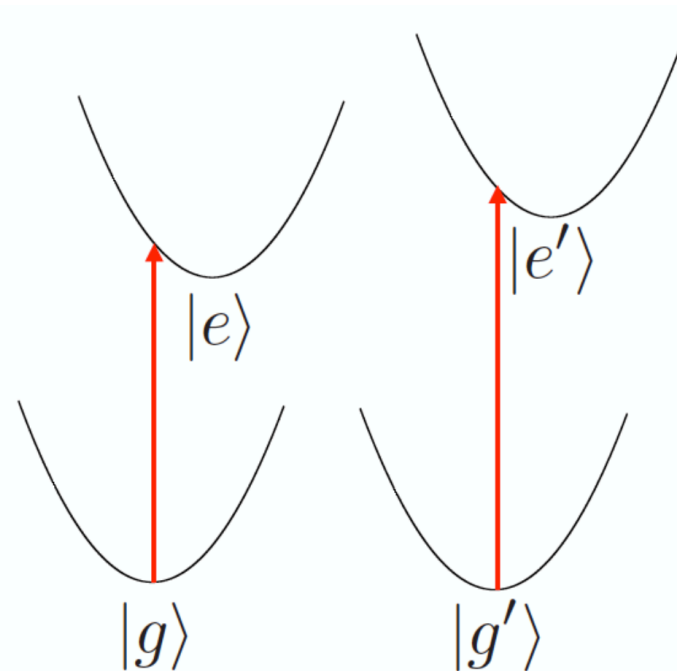
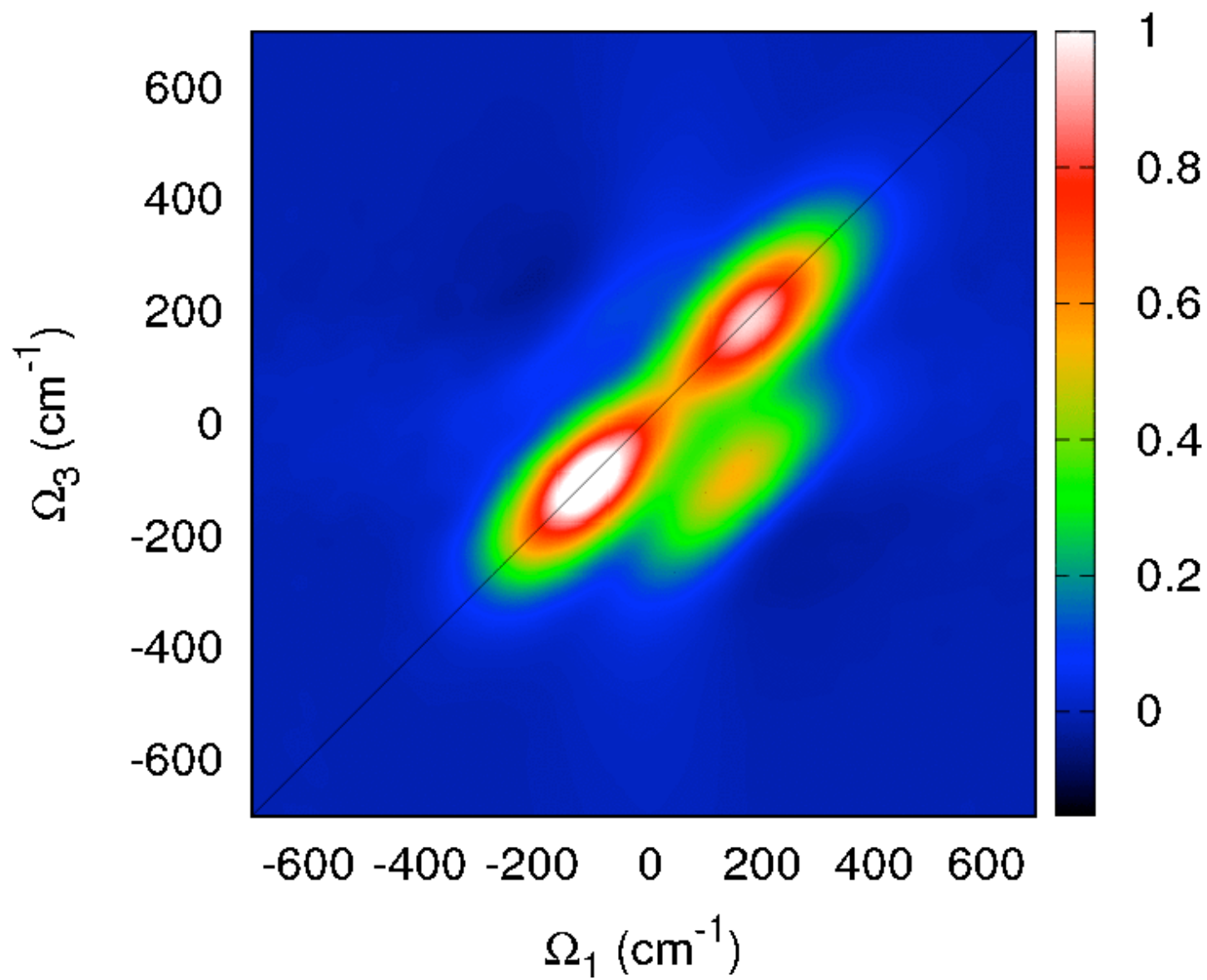


Figure 4: Comparison of HEOM, PLDM, and SPECTRON 2DES at different t_2 times for a coupled dimer at 300 K with $\epsilon_{10} - \epsilon_{01} = 100 \text{ cm}^{-1}$, $\Delta_{01,10} = 100 \text{ cm}^{-1}$, and $\mu_{00,01}/\mu_{00,10} = -0.2$. The bath has a cutoff frequency of $\omega_c = 18 \text{ cm}^{-1}$ and a reorganization energy of $\lambda = 50 \text{ cm}^{-1}$. The PLDM calculation presented here is averaged over only 60,000 trajectories initialized from each element of $(\mu \times \rho^{(0)})$ with nonzero amplitude.



J. Provazza, F. Segatta, M. Garavelli, D.F. Coker
 J. Chem. Theor. Comput. 14, 856-866 (2018)

$t_2 = 0$ fs



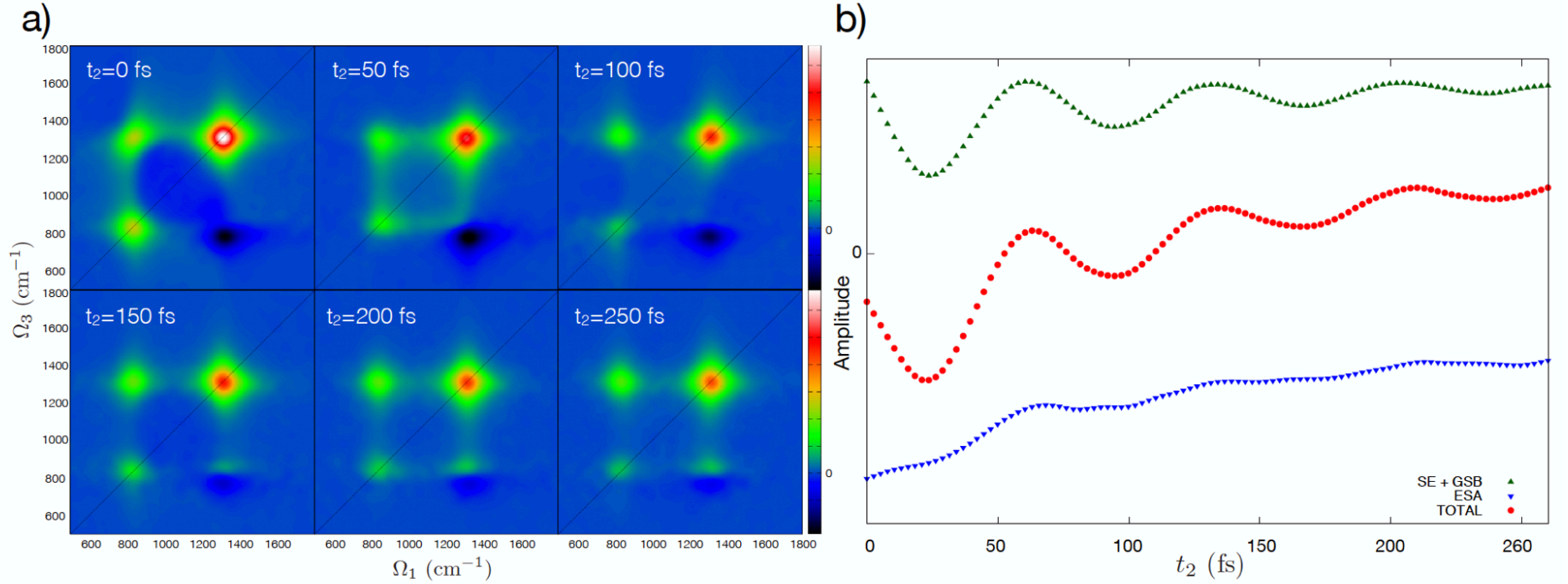
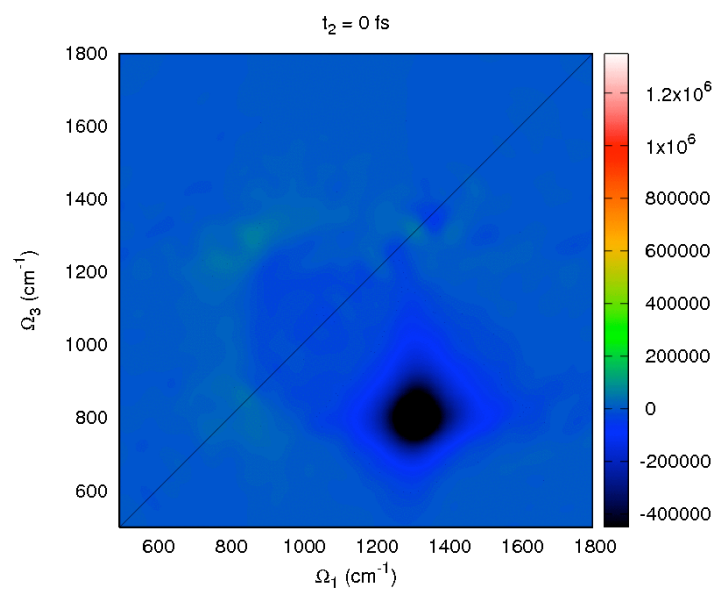
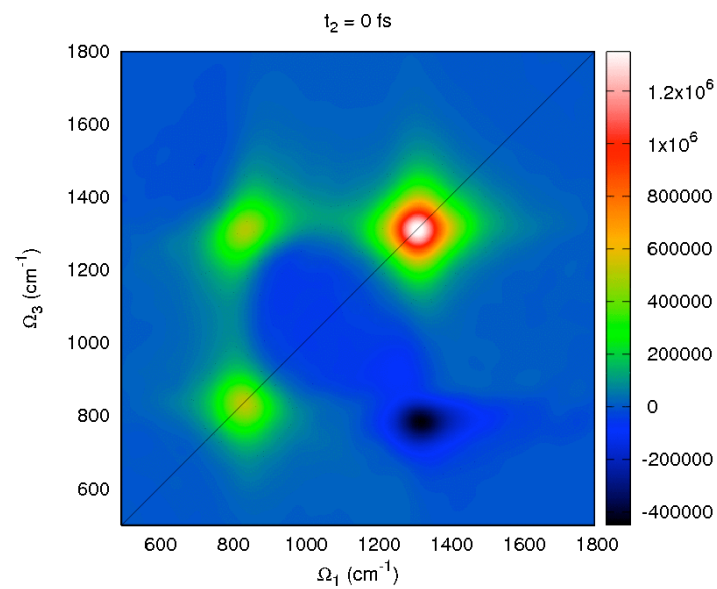


Figure 5: 2DES for a coupled dimer at 300 K with $\epsilon_{10} - \epsilon_{01} = 200 \text{ cm}^{-1}$, $\Delta_{01,10} = 200 \text{ cm}^{-1}$, and $\mu_{00,01}/\mu_{00,10} = 2.4$. The spectral density parameters employed were $\lambda = 50 \text{ cm}^{-1}$, $\omega_c = 200 \text{ cm}^{-1}$. Panel a) shows the spectra for varying t_2 waiting time. Panel b) shows the amplitude of the energy transfer-related cross-peak at $(\Omega_1, \Omega_3) \approx (1325 \text{ cm}^{-1}, 875 \text{ cm}^{-1})$. The amplitude evolves with coherent oscillations resonant with the energy difference between exciton states.

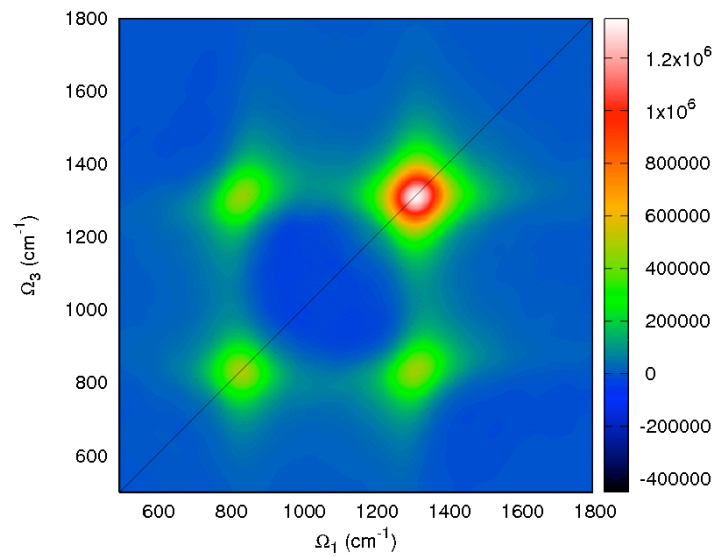
ESA



Total



$t_2 = 0$ fs



SE+GSB

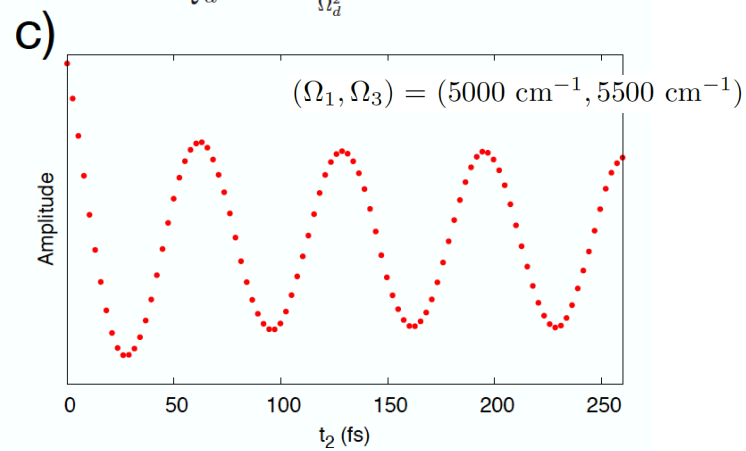
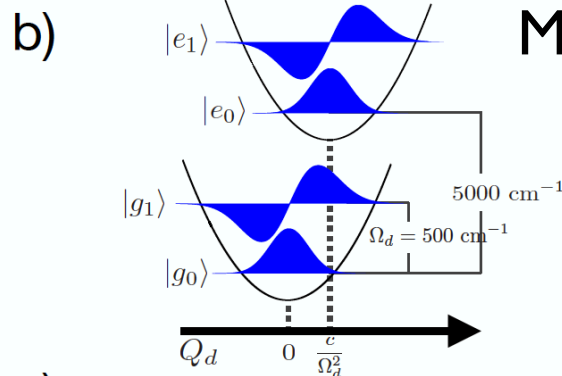
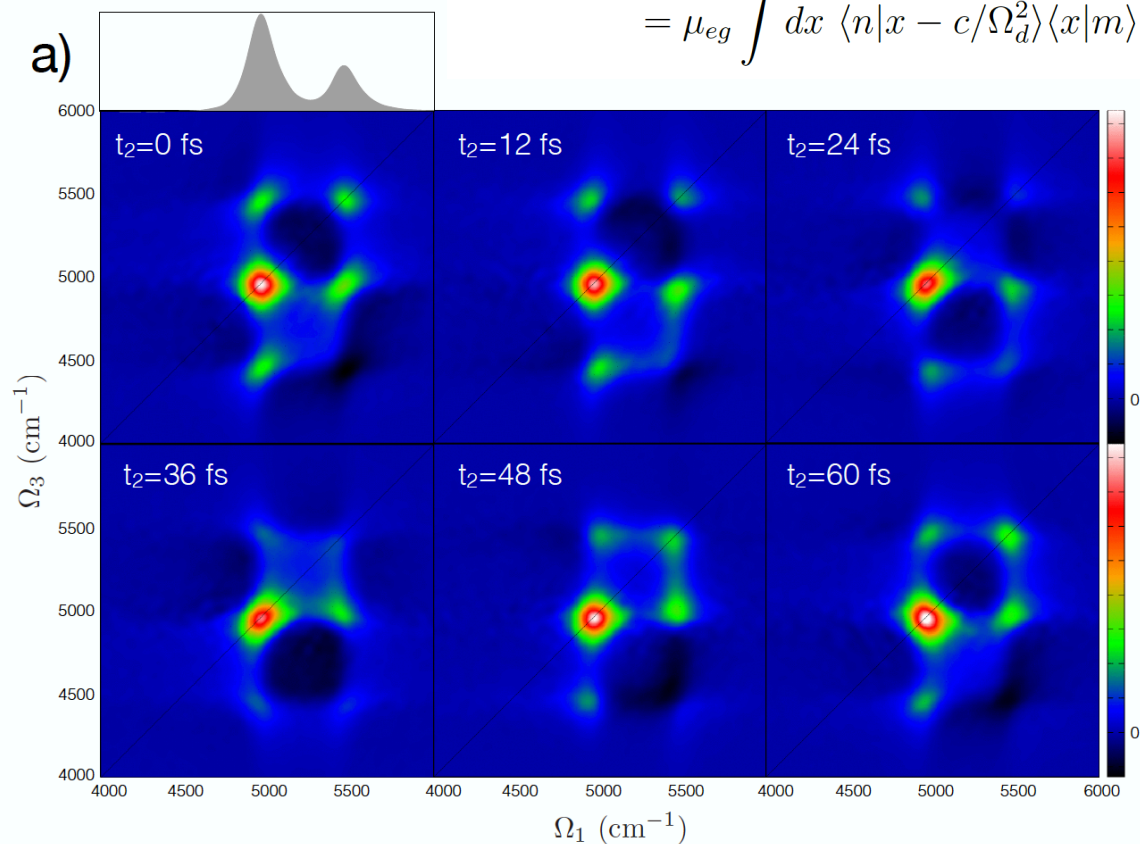
$$\hat{H}_0 = \sum_{n_u=0}^{\infty} \epsilon_g^{(n_u)} |g_{n_u}\rangle \langle g_{n_u}| + \sum_{n_s=0}^{\infty} \epsilon_e^{(n_s)} |e_{n_s}\rangle \langle e_{n_s}| + \sum_{n_s=0}^{\infty} \sum_{m=1}^{M_c} c_m \hat{R}_m |e_{n_s}\rangle \langle e_{n_s}| + \sum_{m=1}^{M_c} \frac{1}{2} \left(\hat{P}_m^2 + \omega_m^2 \hat{R}_m^2 \right) \hat{1}$$

$$\epsilon_g^{(n_u)} = \epsilon_g + \left(n_u + \frac{1}{2} \right) \Omega_d \quad \epsilon_e^{(n_s)} = \epsilon_e - \Lambda_d + \left(n_s + \frac{1}{2} \right) \Omega_d$$

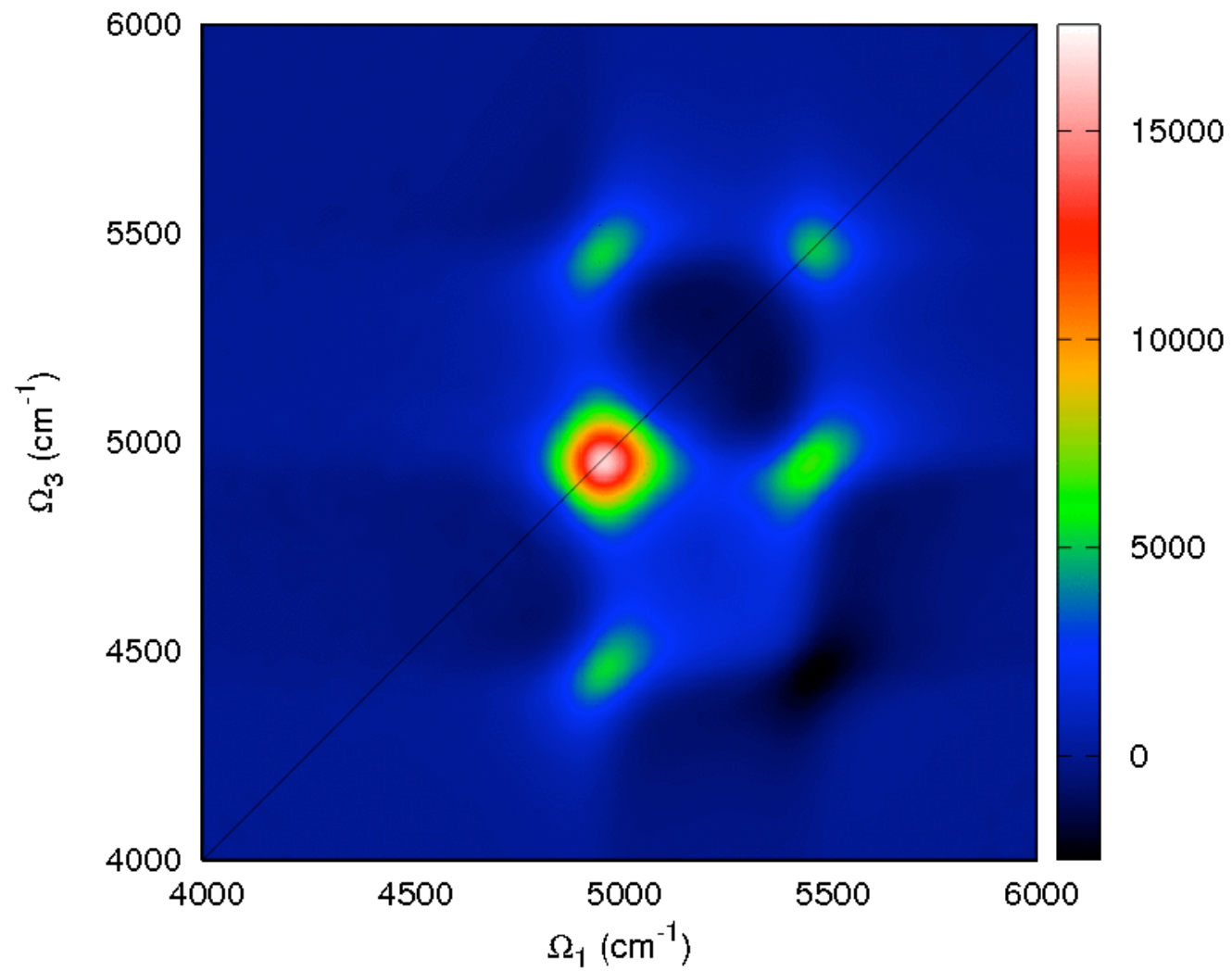
JCTC 14, 856-866 (2018) $\langle e_{n_s} | \hat{\mu} | g_{m_u} \rangle = \mu_{eg} \langle n_s | m_u \rangle$ $\Lambda_d = \frac{c_d^2}{2\Omega_d^2}$ $S = 0.45$ $S = \Lambda_d / \Omega_d$

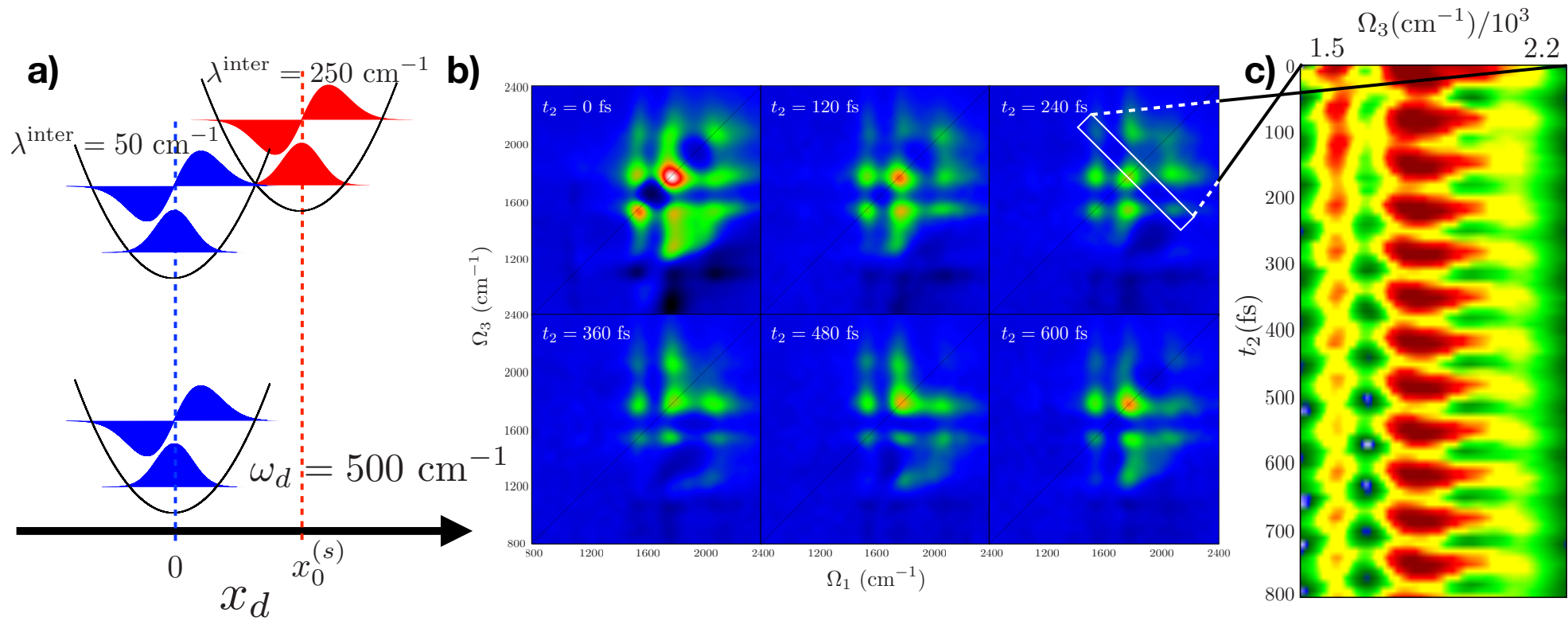
$$= \mu_{eg} \int dx \langle n | x - c/\Omega_d^2 \rangle \langle x | m \rangle$$

Vibronic Monomer

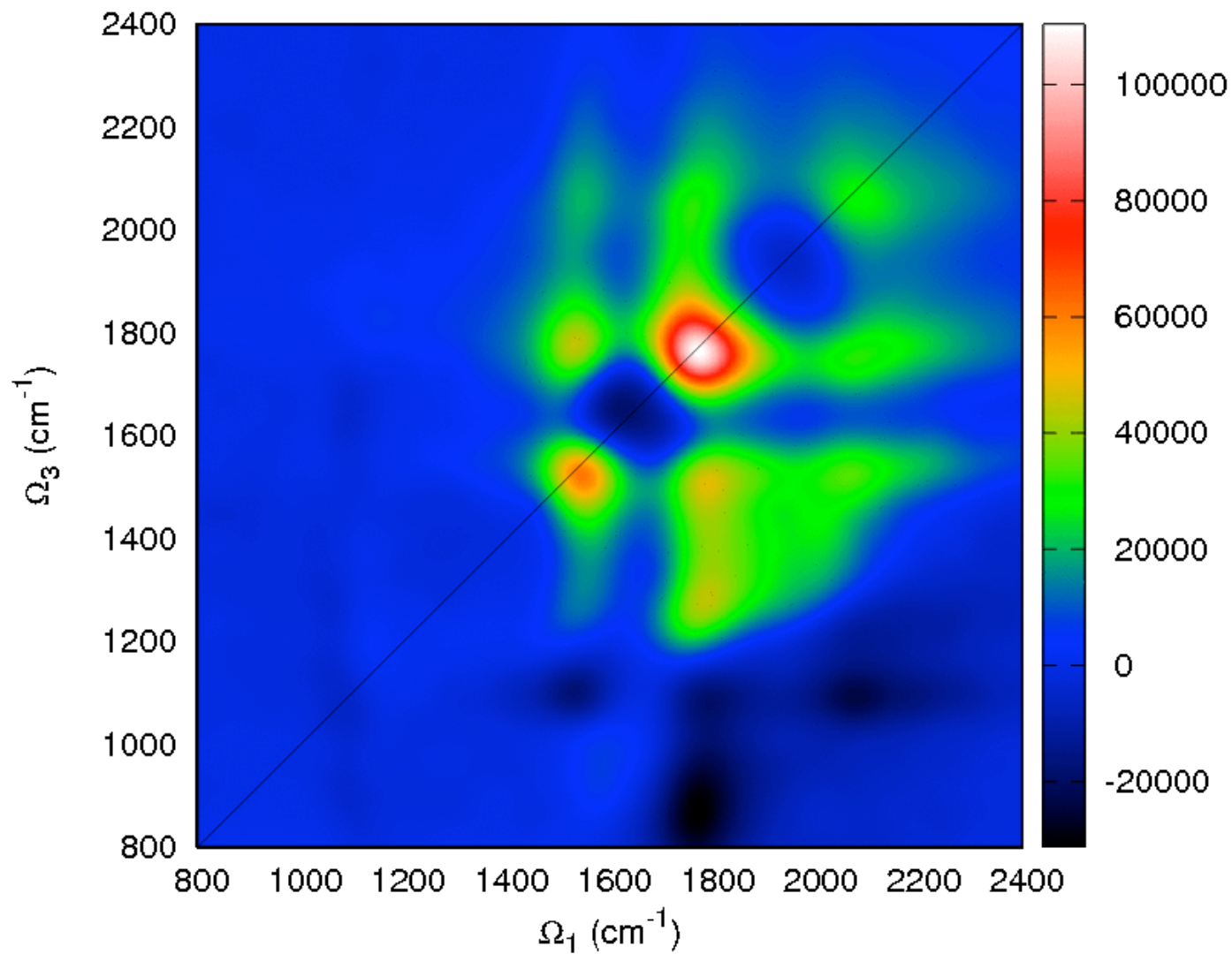


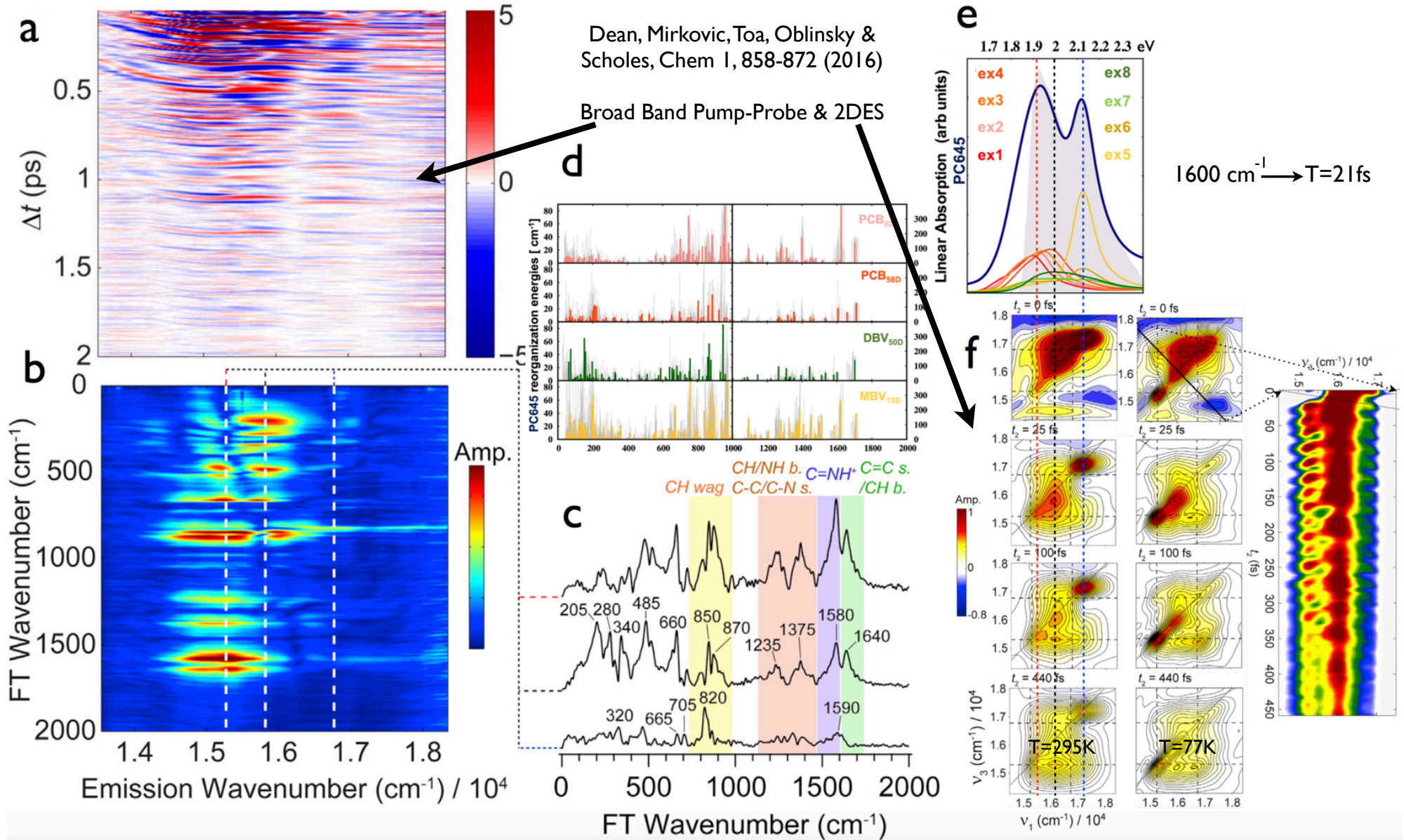
$t_2 = 0$ fs

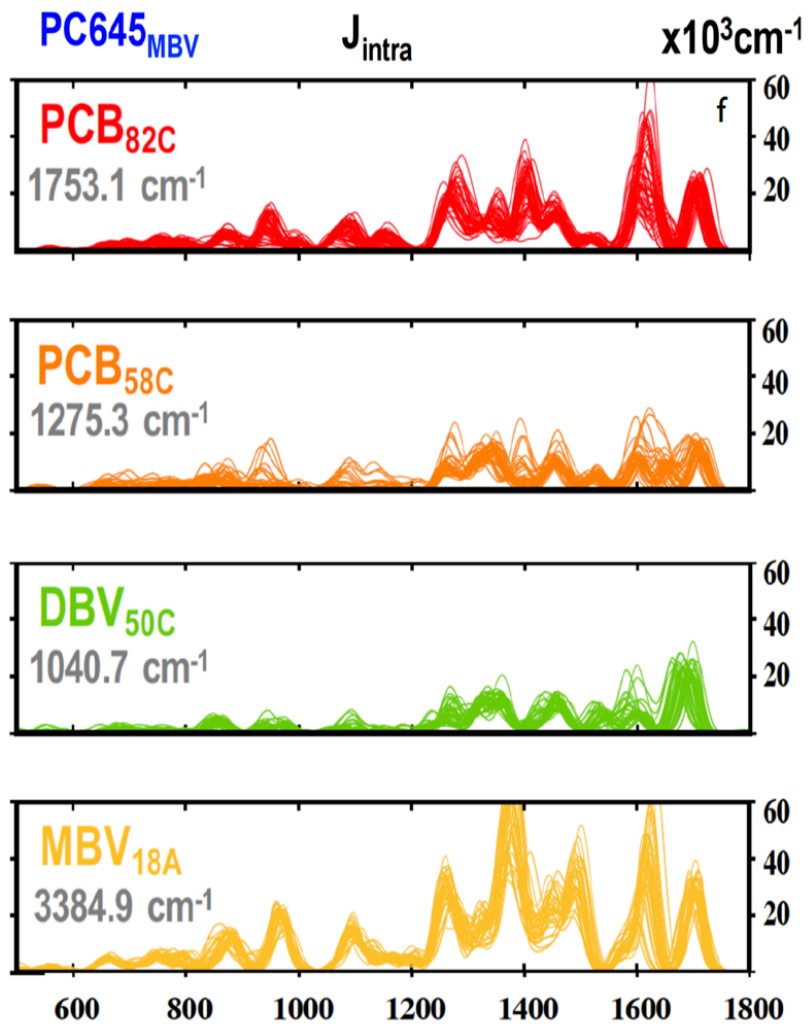
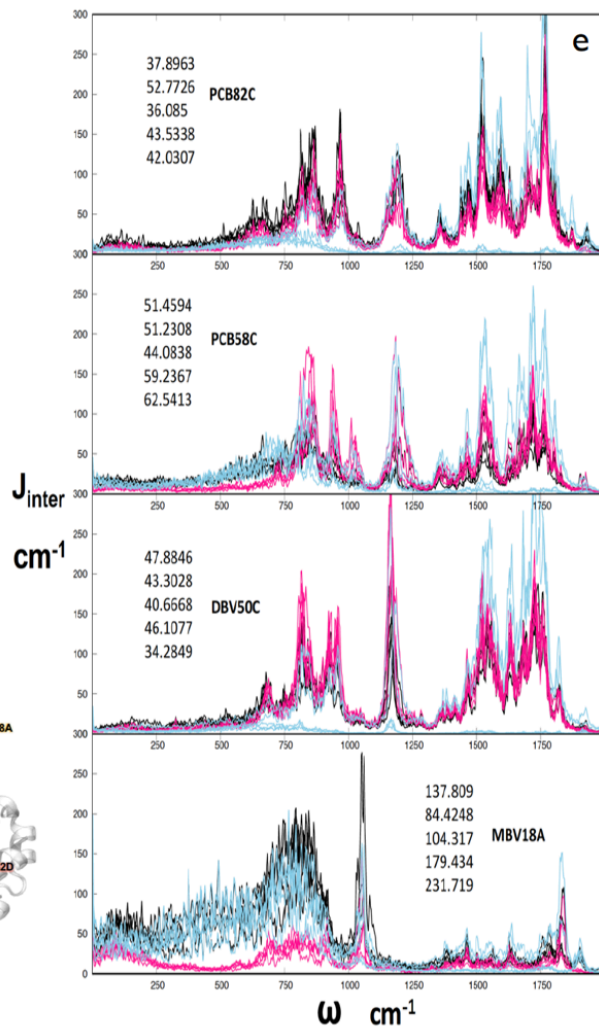
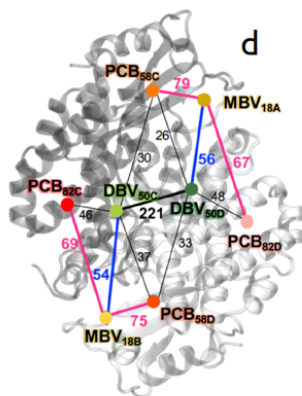
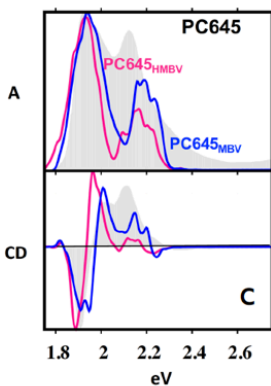
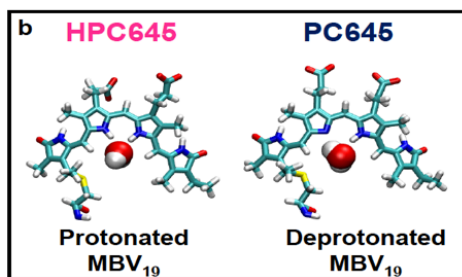
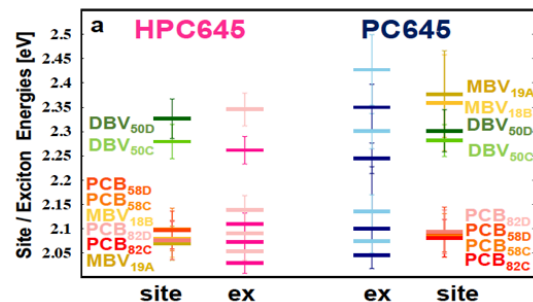


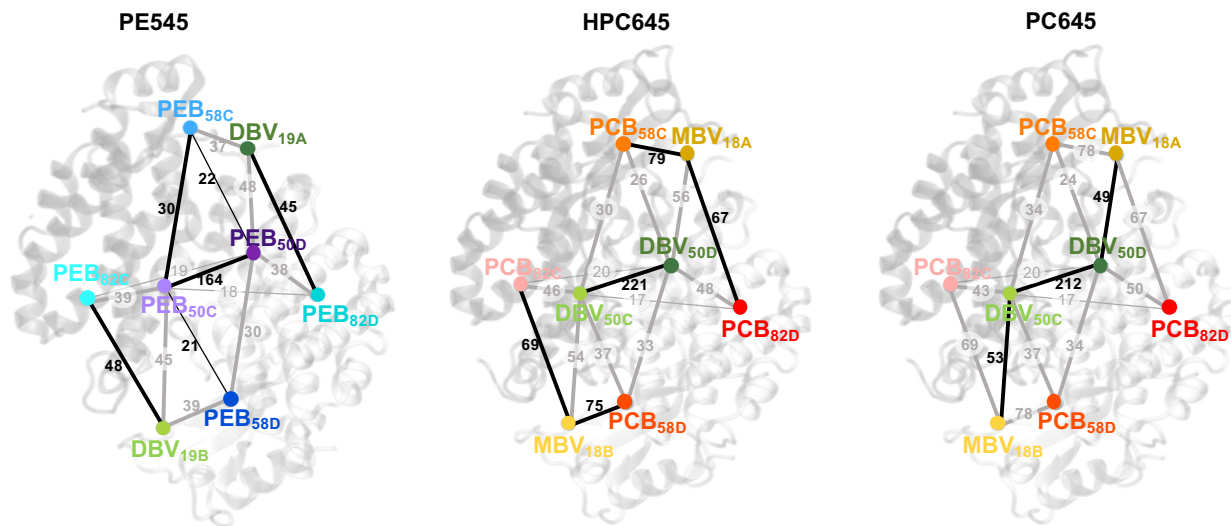
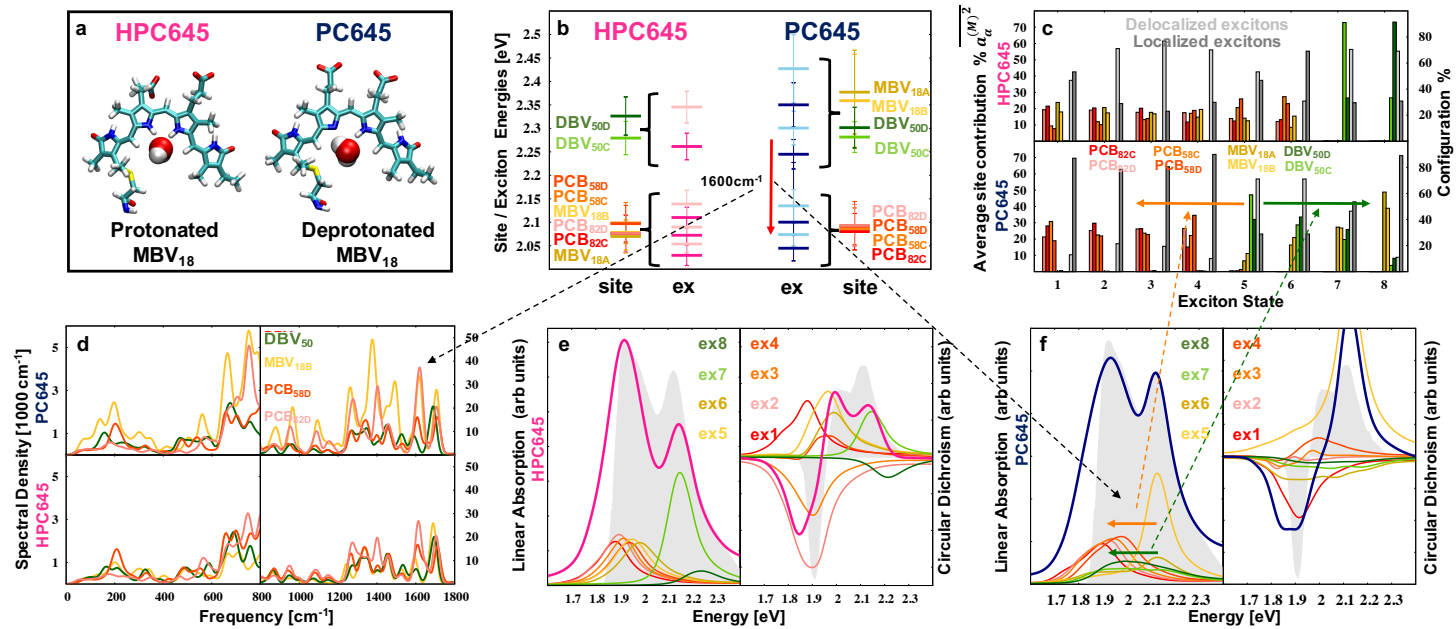


$t_2 = 0$ fs



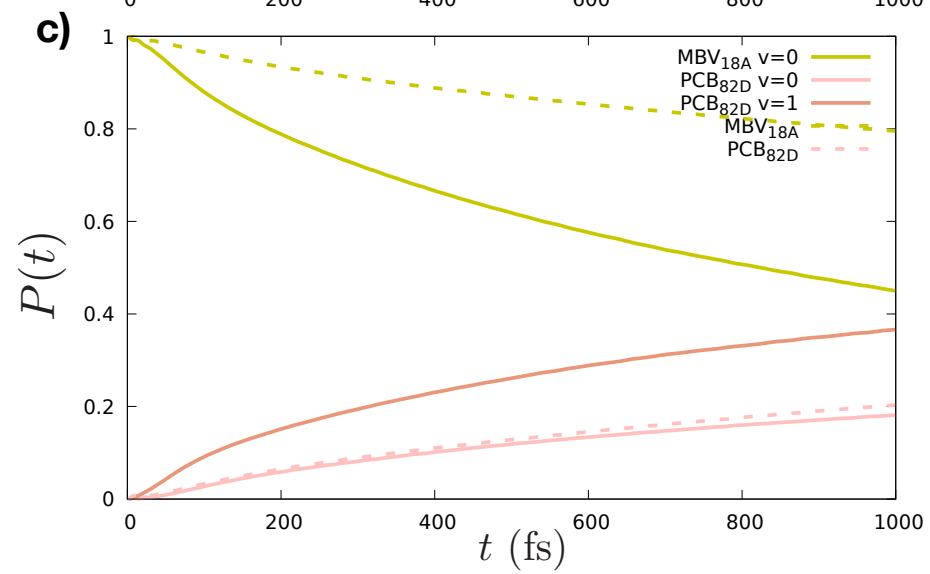
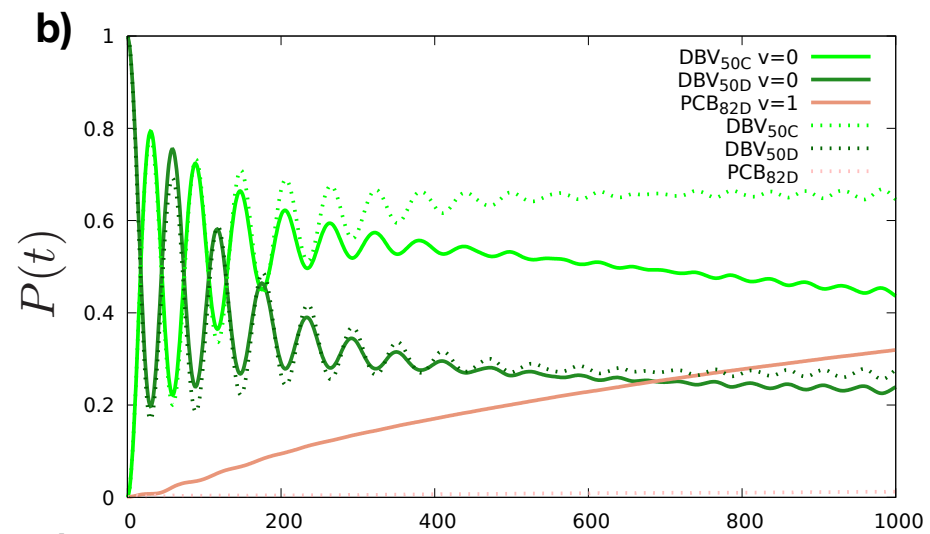
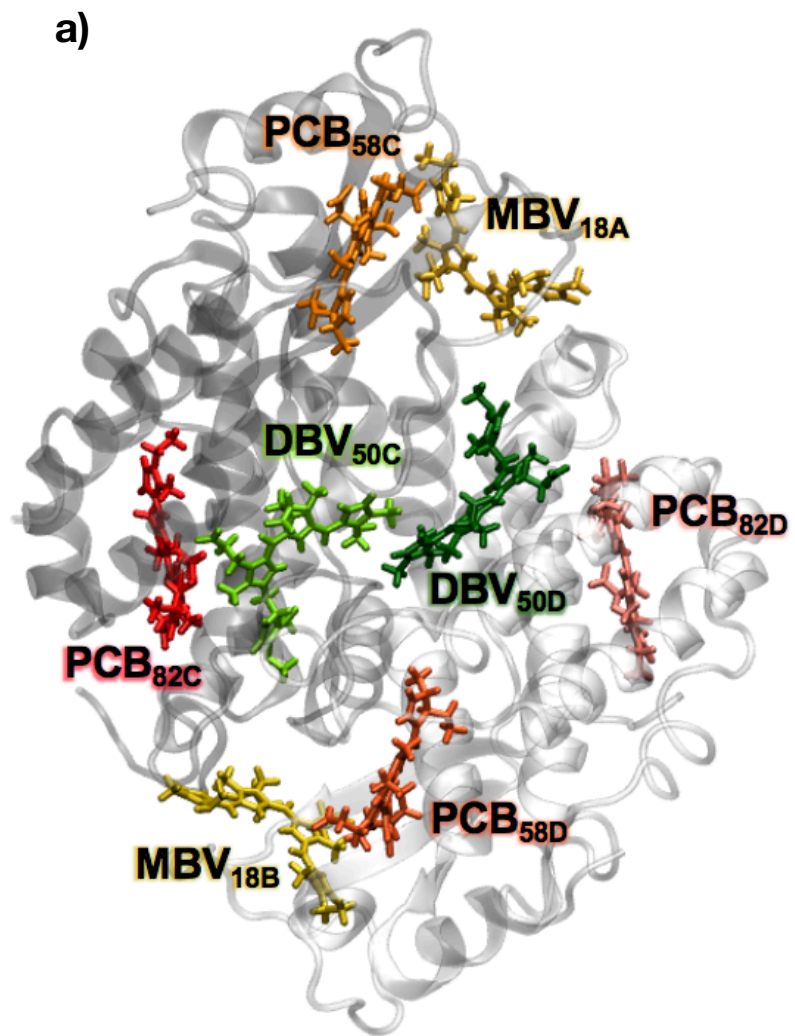






“Flickering” EET pathways

Protonation state control
of (de-)localization



OUTLINE:

- (1) Spectral Density Calculations
- (2) Excitation Energy Calculations
- (3) Dissipative Quantum Dynamics for General Regimes
- (4) Issues with Linearized Dynamics of Higher Frequency Modes
- (5) Spectra (PC645/HPC645) - Influence of Protonation and “Flickering” Pathways
- (6) Quantum Dynamics and Nonlinear Spectroscopy
- (7) Coherent State Density Matrix Dynamics (+PLDM)

$$\hat{H}_{\text{map}} = \sum_{k=1}^{N_{\text{osc}}} \left(\frac{\hat{P}_k^2}{2M_k} + \frac{1}{2} M_k \Omega_k^2 \hat{Q}_k^2 \right) + \frac{1}{2\hbar} \sum_{\alpha,\beta=1}^{N_{\text{st}}} h_{\alpha,\beta}(\{\hat{Q}_i\}) (\hat{q}_\alpha \hat{q}_\beta + \hat{p}_\alpha \hat{p}_\beta - \hbar \delta_{\alpha,\beta})$$

CSDM

$$e^{-\frac{i}{\hbar} \hat{H} t} \propto \int d\vec{q}_0 d\vec{p}_0 d\vec{Q}_0 d\vec{P}_0 |\vec{q}_t, \vec{p}_t\rangle |\vec{Q}_t, \vec{P}_t\rangle C_t e^{\frac{i}{\hbar} S_t} \langle \vec{q}_0, \vec{p}_0 | \langle \vec{Q}_0, \vec{P}_0 | \quad \gamma_i = m_i \omega_i / 2\hbar$$

$$\langle x | q, p \rangle = \exp[-\gamma(q-x)^2 + ip(x-q)/\hbar] \quad C_t = \det \left[\frac{1}{2} \left(\frac{\partial \vec{q}_t}{\partial \vec{q}_0} + \frac{\partial \vec{p}_t}{\partial \vec{p}_0} - i 2\hbar \gamma \frac{\partial \vec{q}_t}{\partial \vec{p}_0} + \frac{i}{2\hbar \gamma} \frac{\partial \vec{p}_t}{\partial \vec{q}_0} \right) \right]^{\frac{1}{2}}$$

$$C_t = \det \left[\frac{1}{2} \begin{pmatrix} C_t^{QQ} & C_t^{Qq} \\ C_t^{qQ} & C_t^{qq} \end{pmatrix} \right]^{\frac{1}{2}} \quad C_t^{QQ} = \det \left[\frac{1}{2} \left(\frac{\partial Q_t}{\partial Q_0} + \frac{\partial P_t}{\partial P_0} - im\omega \frac{\partial Q_t}{\partial P_0} + \frac{i}{m\omega} \frac{\partial P_t}{\partial Q_0} \right) \right]^{\frac{1}{2}}$$

$$a_{cl}(Q, P) = \sqrt{\frac{M\Omega}{2\hbar}} \left(Q + \frac{i}{M\Omega} P \right) \quad = \det \left[\frac{1}{2} \left(\frac{\partial a_{cl}(Q_t, P_t)}{\partial (\text{Re}(a_{cl}^\dagger(Q_0, P_0)))} + i \frac{\partial a_{cl}(Q_t, P_t)}{\partial (\text{Im}(a_{cl}^\dagger(Q_0, P_0)))} \right) \right]^{\frac{1}{2}}$$

$$V(Q) = \frac{1}{2} M \Omega^2 Q^2 + \Gamma(t) Q$$

$$a_{cl}(Q_t, P_t) = a_{cl}(Q_0, P_0) e^{-i\Omega t} - \frac{i}{\sqrt{2M\Omega\hbar}} \int_0^t d\tau \Gamma(\tau) e^{-i\Omega(t-\tau)}$$

$$C_t \approx \det \left[\frac{1}{2} \begin{pmatrix} C_t^{QQ} & 0 \\ C_t^{qQ} & C_t^{qq} \end{pmatrix} \right]^{\frac{1}{2}} = \det \left[\frac{1}{2} C_t^{QQ} \right]^{\frac{1}{2}} \det \left[\frac{1}{2} C_t^{qq} \right]^{\frac{1}{2}} = \exp \left[-\frac{i}{2} \sum_{k=1}^{N_{\text{osc}}} \Omega_k t \right] \exp \left[-\frac{i}{2} \sum_{\alpha} \int_0^t d\tau h_{\alpha,\alpha}(\{\hat{Q}_i\}) \right]$$

$$\langle q, p | \nu \rangle_{\gamma = \frac{m\omega}{2\hbar}} = \frac{1}{\sqrt{\nu!}} \left(a_{cl}^\dagger(q, p) \right)^\nu \langle q, p | 0 \rangle_{\gamma = \frac{m\omega}{2\hbar}} \quad \langle q, p | 0 \rangle_{\gamma = \frac{m\omega}{2\hbar}} = \left(\frac{\pi\hbar}{m\omega} \right)^{\frac{1}{4}} \exp \left[-\frac{m\omega}{4\hbar} q^2 - \frac{1}{4\hbar m\omega} p^2 + \frac{i}{2\hbar} pq \right]$$

$$\begin{aligned}
\langle n_f, \vec{\nu}_f | \hat{\rho}(t) | n'_f, \vec{\nu}'_f \rangle &\approx \int d\vec{q}_0 d\vec{p}_0 d\vec{q}'_0 d\vec{p}'_0 d\vec{Q}_0 d\vec{P}_0 d\vec{Q}'_0 d\vec{P}'_0 \langle \vec{Q}_0 \vec{P}_0 | 0 \rangle \langle 0 | \vec{Q}'_0, \vec{P}'_0 \rangle \langle \vec{q}_0 \vec{p}_0 | 0 \rangle \langle 0 | \vec{q}'_0, \vec{p}'_0 \rangle \\
&\times \langle 0 | \vec{Q}_t \vec{P}_t \rangle \langle \vec{Q}'_t, \vec{P}'_t | 0 \rangle \langle 0 | \vec{q}_t \vec{p}_t \rangle \langle \vec{q}'_t, \vec{p}'_t | 0 \rangle e^{i(\tilde{S}_t[\vec{q}, \vec{p}, \vec{Q}, \vec{P}] - \tilde{S}_t[\vec{q}', \vec{p}', \vec{Q}', \vec{P}'])} \\
&\times \sum_{n_0, n'_0=1}^{N_{st}} \rho_s^{(n_0, n'_0)} T_s^{(n_f, n_0)}(q_{n_{ft}}, p_{n_{ft}}, q_{n_{00}}, p_{n_{00}}) T_s^{*(n'_f, n'_0)}(q'_{n'_{ft}}, p'_{n'_{ft}}, q'_{n'_{00}}, p'_{n'_{00}}) \\
&\times \sum_{\vec{\nu}_0, \vec{\nu}'_0} \rho_b^{(\vec{\nu}_0, \vec{\nu}'_0)} T_b^{(\vec{\nu}_f, \vec{\nu}_0)}(\vec{Q}, \vec{P}) T_b^{*(\vec{\nu}'_f, \vec{\nu}'_0)}(\vec{Q}', \vec{P}')
\end{aligned}$$

$$\tilde{S}_t[\vec{q}, \vec{p}, \vec{Q}, \vec{P}] = \int_0^t d\tau \vec{p} \cdot \dot{\vec{q}} + \vec{P} \cdot \dot{\vec{Q}} - H_{\text{map}} - \frac{1}{2} \sum_{\alpha} h_{\alpha, \alpha}(\{Q_i\})$$

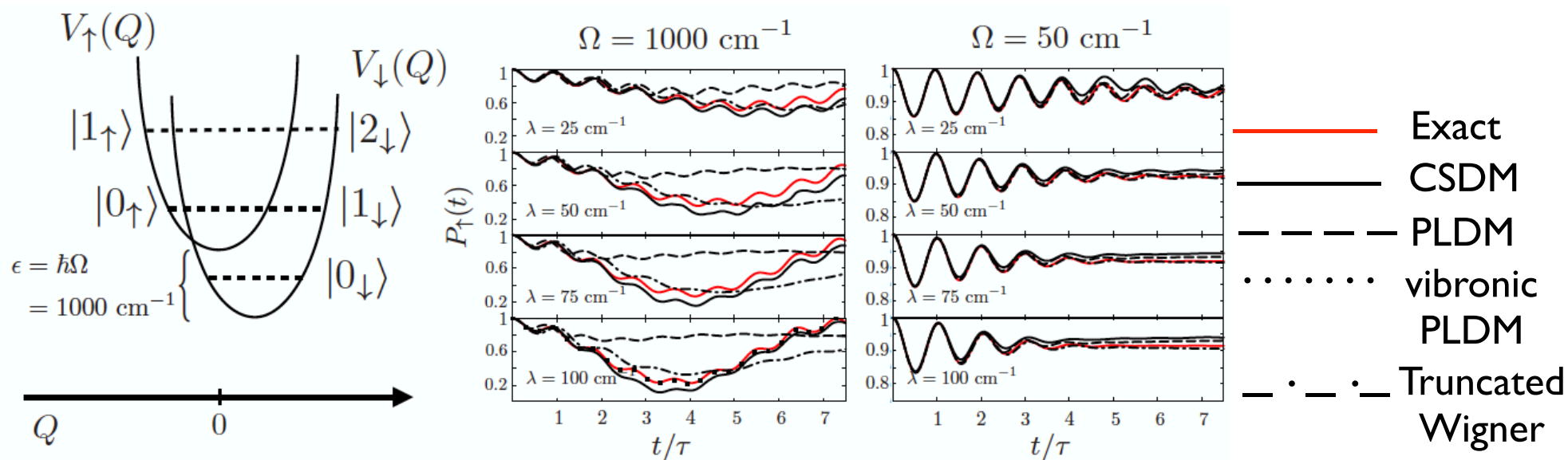
$$T_s^{(n_f, n_0)}(q_{n_{ft}}, p_{n_{ft}}, q_{n_{00}}, p_{n_{00}}) = \frac{1}{2} (q_{n_{ft}} + ip_{n_{ft}})(q_{n_{00}} - ip_{n_{00}})$$

$$T_b^{(\vec{\nu}_f, \vec{\nu}_0)}(\vec{Q}, \vec{P}) = \prod_{k=1}^{N_{osc}} \left(\frac{1}{\sqrt{\nu_0^{(k)}}} a_{cl}^{\dagger}(Q_{k0}, P_{k0}) \right)^{\nu_0^{(k)}} \frac{1}{\sqrt{\nu_f^{(k)}}} (a_{cl}(Q_{kt}, P_{kt}))^{\nu_f^{(k)}}$$

$$\begin{aligned}
\langle n_f, \vec{\nu}_f | \hat{\rho}(t) | n'_f, \vec{\nu}'_f \rangle &\propto \int d\vec{q}_0 d\vec{p}_0 d\vec{q}'_0 d\vec{p}'_0 d\vec{Q}_0 d\vec{P}_0 d\vec{Q}'_0 d\vec{P}'_0 \\
&\times \mathcal{G}(\vec{q}_0, \vec{p}_0) \mathcal{G}(\vec{q}'_0, \vec{p}'_0) \mathcal{F}(\vec{Q}_0, \vec{P}_0) \mathcal{F}(\vec{Q}'_0, \vec{P}'_0) e^{\frac{i}{2}(\Theta_t[\vec{Q}, \vec{P}, \vec{p}, \vec{q}] - \Theta_t^*[\vec{Q}', \vec{P}', \vec{p}', \vec{q}'])} \\
&\times \sum_{n_0, n'_0=1}^{N_{st}} \rho_s^{(n_0, n'_0)} T_s^{(n_f, n_0)}(q_{n_{f_t}}, p_{n_{f_t}}, q_{n_{0_0}}, p_{n_{0_0}}) T_s^{*(n'_f, n'_0)}(q'_{n'_{f_t}}, p'_{n'_{f_t}}, q'_{n'_{0_0}}, p'_{n'_{0_0}}) \\
&\times \sum_{\vec{\nu}_0, \vec{\nu}'_0} \rho_b^{(\vec{\nu}_0, \vec{\nu}'_0)} T_b^{(\vec{\nu}_f, \vec{\nu}_0)}(\vec{Q}, \vec{P}) T_b^{*(\vec{\nu}'_f, \vec{\nu}'_0)}(\vec{Q}', \vec{P}') \\
\Theta_t[\vec{Q}, \vec{P}, \vec{p}, \vec{q}] &= \int_0^t d\tau \sum_{k=1}^{N_{osc}} \sum_{\alpha, \beta=1}^{N_{st}} \sqrt{\frac{2}{M_k \Omega_k}} a_{cl}^\dagger(Q_k, P_k) \nabla_{Q_k} h_{\alpha, \beta}(\{Q_i\}) \frac{1}{2}(q_\alpha q_\beta + p_\alpha p_\beta - \delta_{\alpha, \beta})
\end{aligned}$$

$$\begin{aligned}
\langle n_f | \hat{\sigma}(t) | n'_f \rangle &\propto \int d\vec{q}_0 d\vec{p}_0 d\vec{q}'_0 d\vec{p}'_0 d\vec{Q}_0 d\vec{P}_0 d\vec{Q}'_0 d\vec{P}'_0 \\
&\times \mathcal{G}(\vec{q}_0, \vec{p}_0) \mathcal{G}(\vec{q}'_0, \vec{p}'_0) \mathcal{B}_\beta(\vec{Q}_0, \vec{P}_0, \vec{Q}'_0, \vec{P}'_0) e^{\frac{i}{2} \Phi_t[\vec{Q}, \vec{P}, \vec{Q}', \vec{P}', \vec{q}, \vec{p}, \vec{q}', \vec{p}']} \\
&\times \sum_{n_0, n'_0=1}^{N_{st}} \rho_s^{(n_0, n'_0)} T_s^{(n_f, n_0)}(q_{n_{f_t}}, p_{n_{f_t}}, q_{n_{0_0}}, p_{n_{0_0}}) T_s^{*(n'_f, n'_0)}(q'_{n'_{f_t}}, p'_{n'_{f_t}}, q'_{n'_{0_0}}, p'_{n'_{0_0}})
\end{aligned}$$

High frequency harmonic vibrations treated with coherent state density matrix (CSDM) dynamics



$$\hat{H} = \frac{\epsilon}{2} \hat{\sigma}_z + \Delta \hat{\sigma}_x + \frac{\hat{P}^2}{2M} + \frac{1}{2} M \Omega^2 \left(\hat{Q} - \frac{c}{M \Omega^2} |\downarrow\rangle \langle \downarrow| \right)^2$$

$$\epsilon = \Omega = 1000 \text{ cm}^{-1}, \Delta = 200 \text{ cm}^{-1}, \text{ and } T = 77 \text{ K}$$

$$\hat{H}_{\text{map}} = \sum_{k=1}^{N_Q} \left(\frac{\hat{P}_k^2}{2M_k} + \frac{1}{2} M_k \Omega_k^2 \hat{Q}_k^2 \right) + \sum_{k=1}^{N_R} \left(\frac{\hat{P}_{Rk}^2}{2M_{Rk}} + V_0(\{\hat{R}_j\}) \right) \\ + \frac{1}{2\hbar} \sum_{\alpha, \beta=1}^{N_{st}} h_{\alpha, \beta}(\{\hat{Q}_i\}, \{\hat{R}_j\}) (\hat{q}_\alpha \hat{q}_\beta + \hat{p}_\alpha \hat{p}_\beta - \hbar \delta_{\alpha, \beta})$$

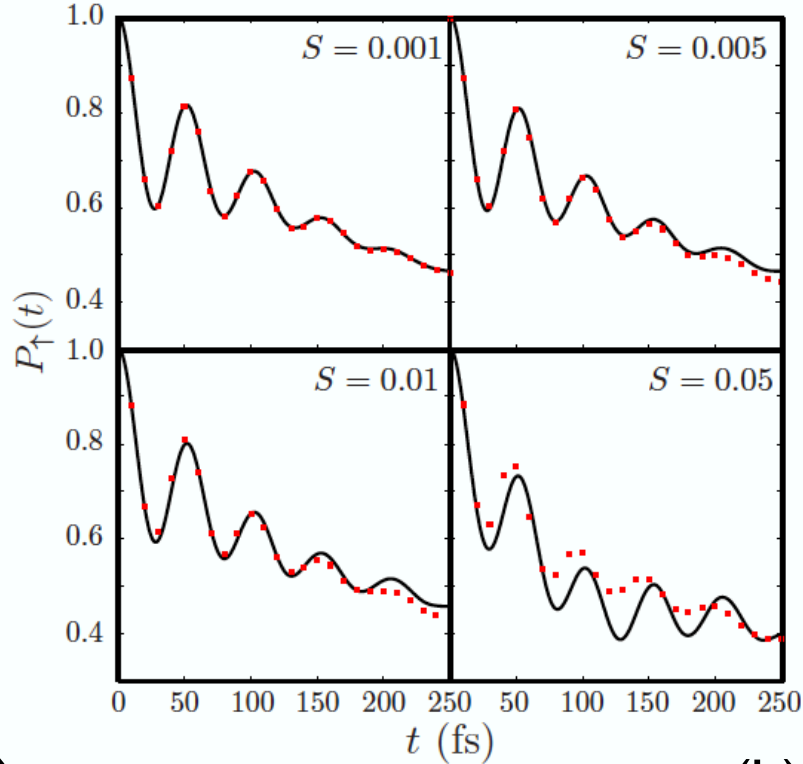
$$h_{\alpha, \beta}(\{\hat{Q}_i\}, \{\hat{R}_j\}) = \left(\tilde{\epsilon}_\alpha - \sum_{k=1}^{N_Q} c_k^{(\alpha)} \hat{Q}_k + V_{\alpha, \beta}(\{\hat{R}_j\}) \right) \delta_{\alpha, \beta} + \Delta(\{\hat{R}_j\})_{\alpha, \beta} (1 - \delta_{\alpha, \beta})$$

$$\langle n_f | \hat{\sigma}(t) | n'_f \rangle \propto \int d\vec{q}_0 d\vec{p}_0 d\vec{q}'_0 d\vec{p}'_0 d\vec{Q}_0 d\vec{P}_0 d\vec{Q}'_0 d\vec{P}'_0 d\vec{R}_0 \\ \times \mathcal{G}(\vec{q}_0, \vec{p}_0) \mathcal{G}(\vec{q}'_0, \vec{p}'_0) \mathcal{B}_\beta(\vec{Q}_0, \vec{P}_0, \vec{Q}'_0, \vec{P}'_0) e^{\frac{i}{2} \Phi_t[\vec{Q}, \vec{P}, \vec{Q}', \vec{P}', \vec{q}, \vec{p}, \vec{q}', \vec{p}']} \\ \times \sum_{n_0, n'_0=1}^{N_{st}} \rho_s^{(n_0, n'_0)} T_s^{(n_f, n_0)}(q_{n_{ft}}, p_{n_{ft}}, q_{n_{00}}, p_{n_{00}}) T_s^{*(n'_f, n'_0)}(q'_{n'_{ft}}, p'_{n'_{ft}}, q'_{n'_{00}}, p'_{n'_{00}}) \\ \times \left(\prod_{k=1}^{N-1} \int d\bar{R}_k \frac{d\bar{P}_{Rk}}{2\pi} \right) \rho_W^{n_0 n'_0}(\bar{R}_0, \bar{P}_{R_1}) \prod_{k=1}^{N-1} \delta \left(\frac{\bar{P}_{R_{k+1}} - \bar{P}_{R_k}}{\epsilon} - F_k \right) \prod_{k=1}^N \delta \left(\frac{\bar{R}_k - \bar{R}_{k-1}}{\epsilon} - \frac{\bar{P}_{R_k}}{M_R} \right)$$

$$F_k = -\frac{1}{2} \nabla_{\bar{R}_k} (\tilde{H}(\{Q_i\}, \{\bar{R}_j\}, \vec{q}, \vec{p}) + \tilde{H}(\{Q'_i\}, \{\bar{R}_j\}, \vec{q}', \vec{p}'))$$

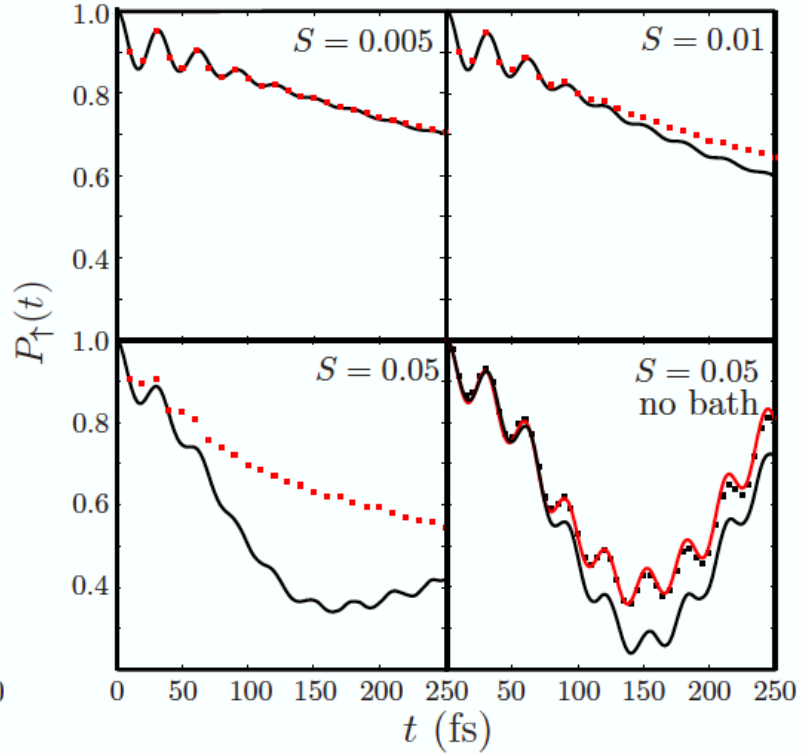
$$\tilde{H} = H_{\text{map}} + \frac{1}{2} \sum_{\alpha=1}^{N_{st}} h_{\alpha, \alpha}(\{Q_i\}, \{R_j\})$$

**Multi-Level
CSDM-PLDM**



(a)

$N_d = 1$ where $\epsilon = \Omega_1 = 500 \text{ cm}^{-1}$



(b)

$N_d = 2$ where $\epsilon = \Omega_1 = 1000 \text{ cm}^{-1}$ and $\Omega_2 = 500 \text{ cm}^{-1}$

$$\hat{H} = \frac{\epsilon}{2} \hat{\sigma}_z + \Delta \hat{\sigma}_x + \sum_{K=1}^{N_d} \frac{\hat{p}_K^2}{2M_K} + \frac{1}{2} M_K \Omega_K^2 \left(\hat{Q}_K - \frac{C_K}{M_K \Omega_K^2} |\downarrow\rangle \langle \downarrow| \right)^2 + \sum_{k_\downarrow=1}^{N_c} \frac{\hat{p}_{k_\downarrow}^2}{2m_{k_\downarrow}} + \frac{1}{2} m_{k_\downarrow} \omega_{k_\downarrow}^2 \left(\hat{q}_{k_\downarrow} - \frac{c_{k_\downarrow}}{m_{k_\downarrow} \omega_{k_\downarrow}^2} |\downarrow\rangle \langle \downarrow| \right)^2 + \sum_{k_\uparrow=1}^{N_c} \frac{\hat{p}_{k_\uparrow}^2}{2m_{k_\uparrow}} + \frac{1}{2} m_{k_\uparrow} \omega_{k_\uparrow}^2 \left(\hat{q}_{k_\uparrow} - \frac{c_{k_\uparrow}}{m_{k_\uparrow} \omega_{k_\uparrow}^2} |\uparrow\rangle \langle \uparrow| \right)^2$$

$$N_c = 100 \quad J(\omega) = 2\lambda \frac{\omega/\omega_c}{1+(\omega/\omega_c)^2} \text{ with } \lambda = 50 \text{ cm}^{-1} \text{ and } \omega_c = 200 \text{ cm}^{-1}$$

$$S = \frac{C_1^2}{2\hbar M_1 \Omega_1^3} = \frac{C_2^2}{2\hbar M_2 \Omega_2^3}$$

CSDM-PLDM

$\Delta = 200 \text{ cm}^{-1}$ at $T = 77 \text{ K}$

..... PLDM
 — CSDM
 — Exact
 Vibronic PLDM

OUTLINE:

- (1) Spectral Density Calculations
- (2) Excitation Energy Calculations
- (3) Dissipative Quantum Dynamics for General Regimes
- (4) Issues with Linearized Dynamics of Higher Frequency Modes
- (5) Spectra (PC645/HPC645) - Influence of Protonation and “Flickering” Pathways
- (6) Quantum Dynamics and Nonlinear Spectroscopy
- (7) Coherent State Density Matrix Dynamics (+PLDM)



Thank You!