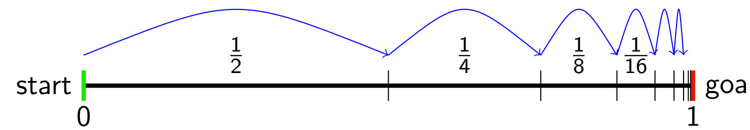
11.2 Explore Series

LEARNING GOALS:

- Define a series.
- Define a partial sum.

1 Recall Zeno's paradox:

"That which is in locomotion must arrive at the half-way stage before it arrives at the goal."



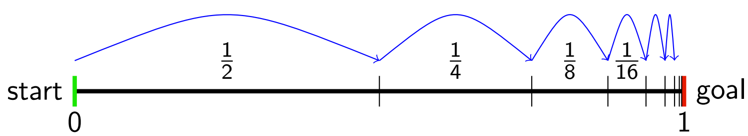
We get from 0 to 1 on the number line by an infinite *sequence* of intermediate steps. How long is the n^{th} step of Zeno's journey?

(a) $a_n = \frac{1}{2n}$ (b) $a_n = \frac{1}{2^n}$

(c) $a_n = \frac{1}{n^2}$ (d) $a_n = \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}$

2

"That which is in locomotion must arrive at the half-way stage before it arrives at the goal."



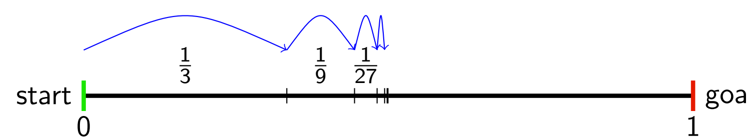
How far have we gone after n steps of Zeno's journey from 0 to 1?

(a) $S_n = a_1 + a_2 + \ldots + a_n$ (b) $S_n = \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}$

(c) $S_n = 1 - \frac{1}{2^n}$ (d) All of the above.

$\rightarrow a_n =$ the n^{th} step

3 Suppose Zeno had a slightly different insight, where each step is 1/3 the length of the previous step.



Think about it...
Would we make it all the way to the goal using this plan?

4 Let's consider our new infinite sum:

$$\frac{1}{3} + \frac{1}{9} + \ldots + \frac{1}{3^n} + \ldots$$

While we can't compute this sum by hand, we can compute a **partial sum**:

$$S_k = \frac{1}{3} + \frac{1}{9} + \ldots + \frac{1}{3^k}$$

Use Desmos to compute the third partial sum, S_3.

Enter your answer as a decimal, rounding to the nearest hundredths place (e.g., 2/3 would be entered as 0.67).

Desmos: partial sums

5 **Think about it...**
Desmos gives us a good indication that

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots + \frac{1}{3^n} + \ldots = \frac{1}{2}$$

Can we mathematically justify this is true?

Findings.

Given an infinite *sequence* of numbers:

$$a_1,\ a_2,\ a_3,\ \ldots$$

we can define an infinite sum called a **series** using a limit:

$$\lim_{k \to \infty} \sum_{n=0}^{k} a_n = \sum_{n=0}^{\infty} a_n$$

That is, a series is the limit of the sequence of **partial sums**:

$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ &\vdots \\ S_k &= a_1 + a_2 + a_3 + \ldots + a_k \end{aligned}$$

6 What questions do you have about this exploration? Is there anything interesting or confusing that you'd like to talk about in class?