Snarks

Workshop:

Impact of Women Mathematicians on Research and Education in Mathematics

BIRS

March 16-18, 2018

Kieka Mynhardt

University of Victoria

They sought it with thimbles, they sought it with care;
They pursued it with forks and hope;
They threatened its life with a railway-share;
They charmed it with smiles and soap.

From: "The Hunting of the Snark"
by
Lewis Carroll

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Connected, bridgeless cubic graph with chromatic index 4

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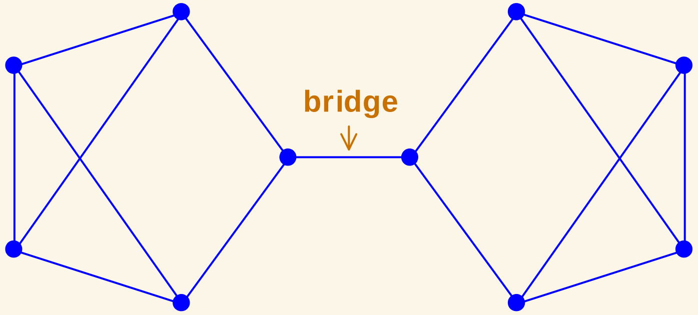
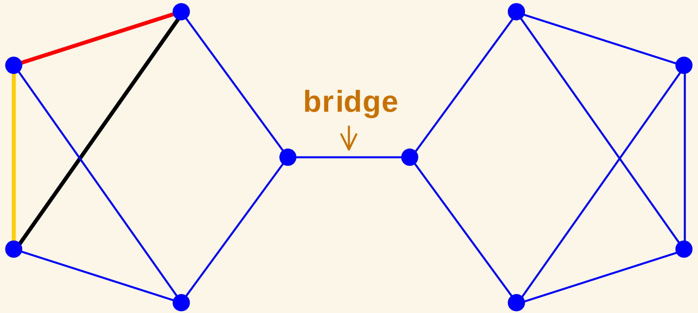
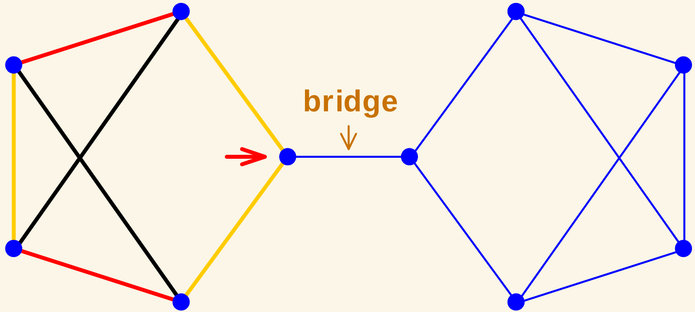
But: In 1880, there were no known snarks!



A connected cubic graph <u>with</u> a bridge

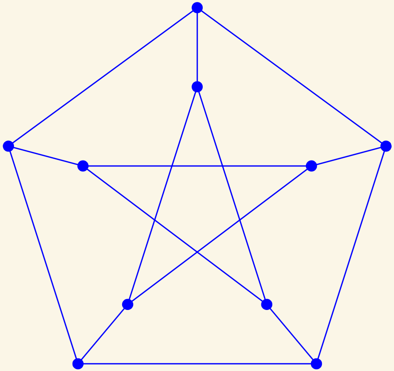


Try to colour the edges with three colours...



...It doesn't work!

The first snark discovered: **The Petersen Graph**



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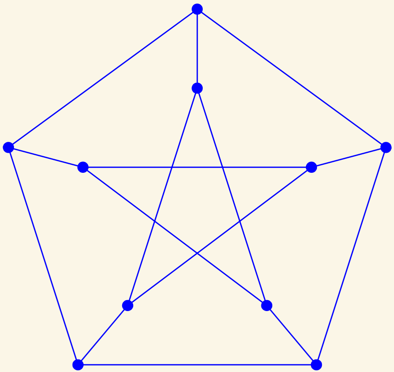

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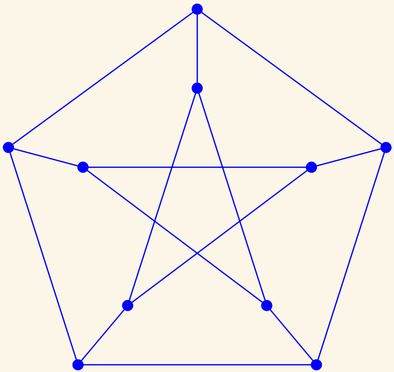

- Named after Danish mathematician **Julius Petersen**, who presented it in 1891 (some sources) or 1898 (other sources) as counterexample to Tait's claim that all cubic graphs were 3-edge colourable.
- But: **Alfred Bray Kempe** already mentioned this graph in 1886.

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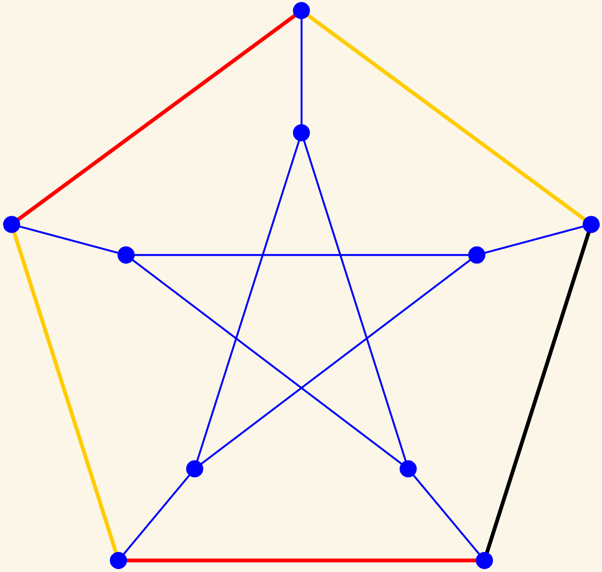

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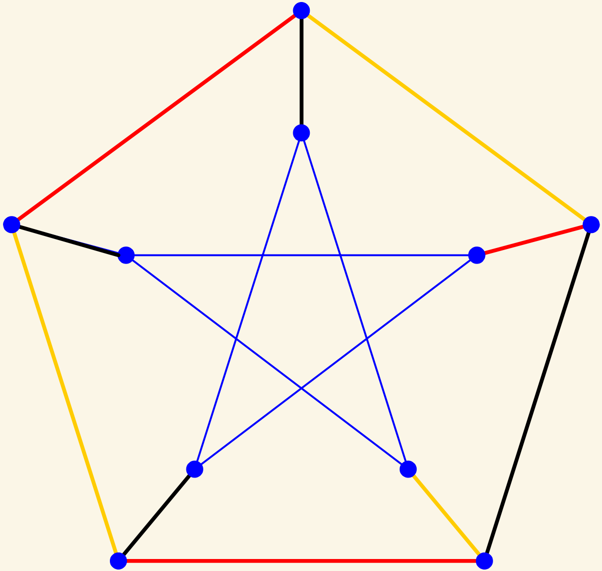
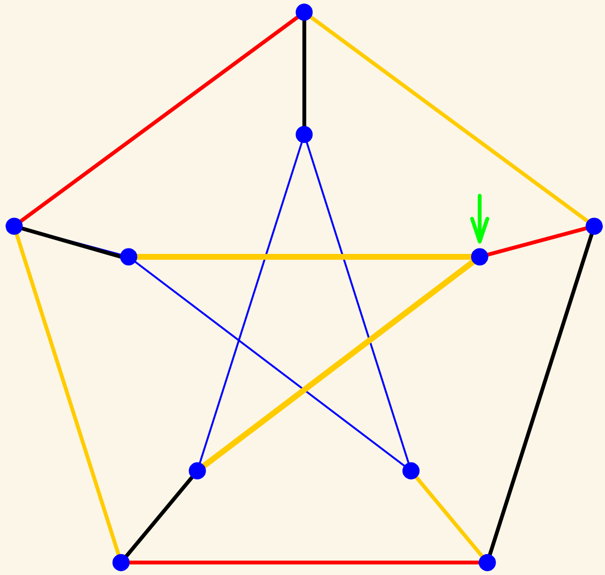


It can’t be done!



Second (and third) snarks discovered, 1946 – about 60 years later:

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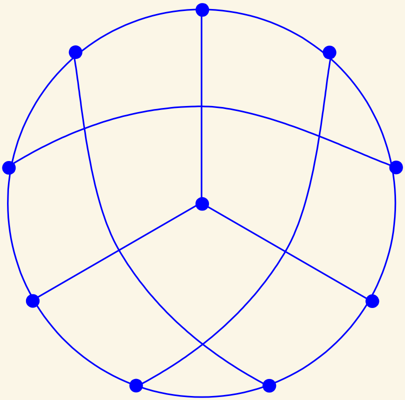

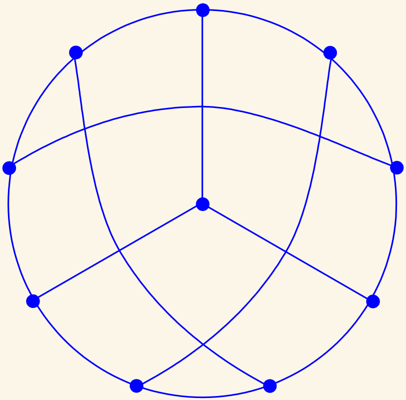

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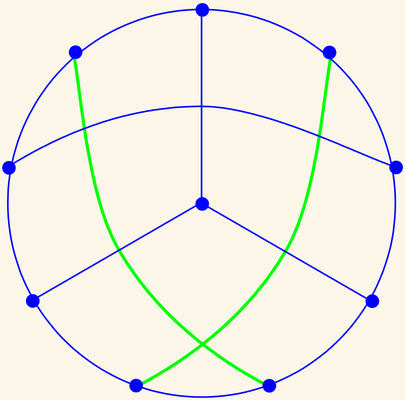

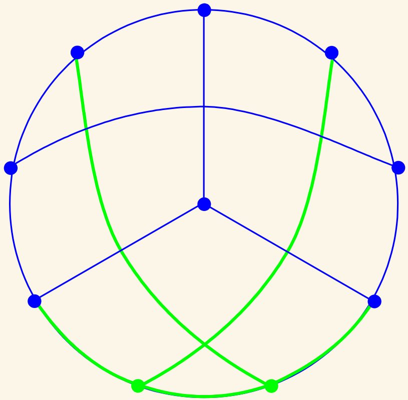

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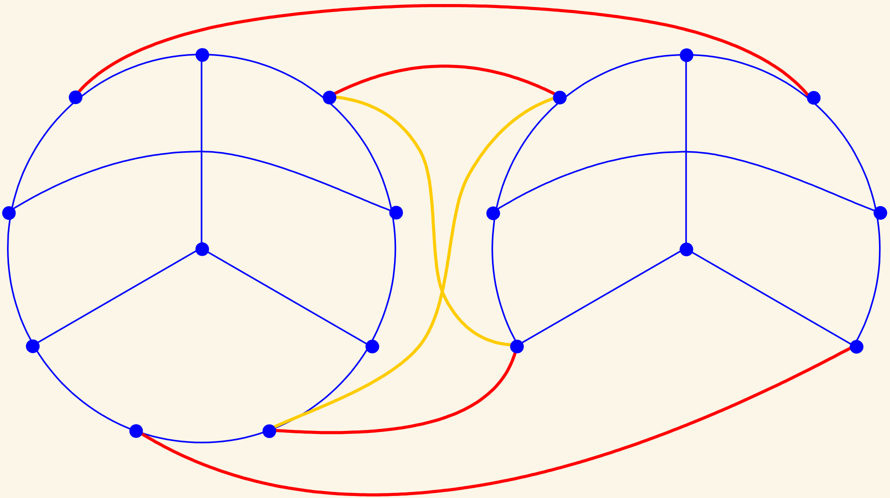

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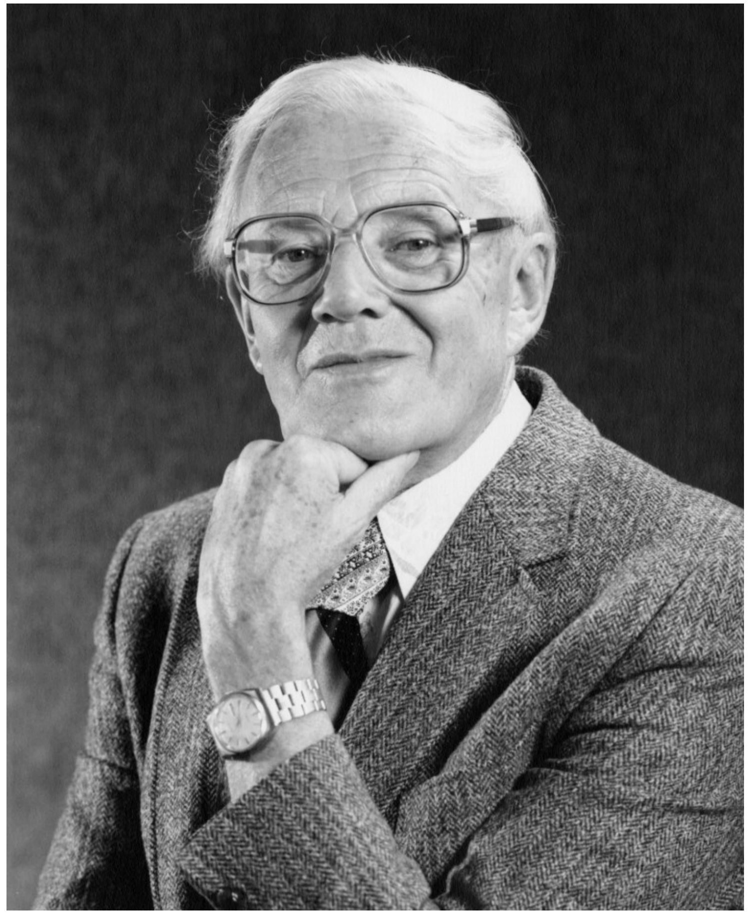
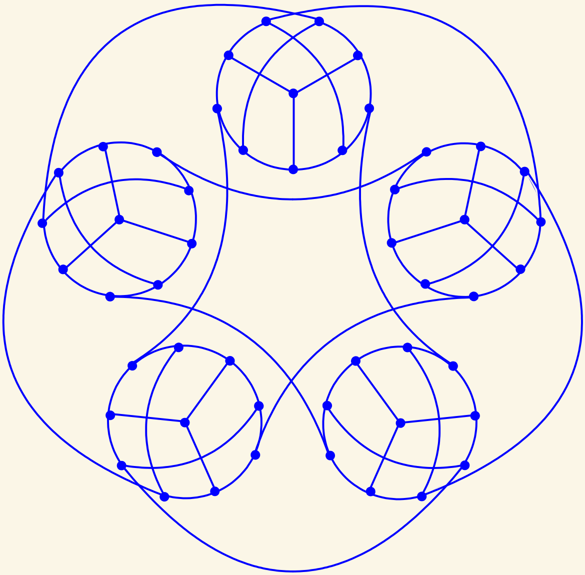
- **Blanche Descartes** was the collective pseudonym of R. Leonard Brooks, Arthur Harold Stone, Cedric Smith and William Tutte.



Bill Tutte, 1917 -2002

Fifth snark: 1973, **Szekeres**, 50 vertices



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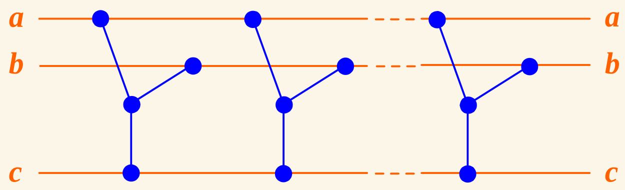

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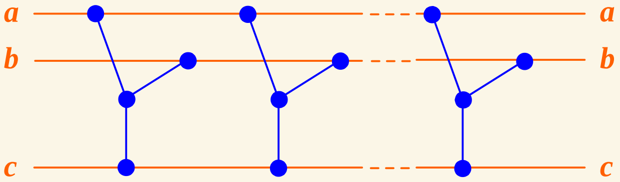

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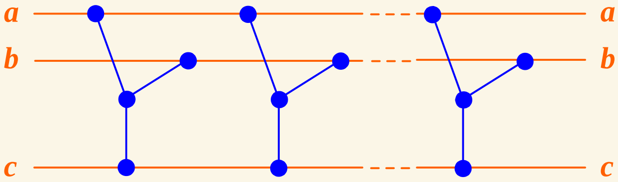

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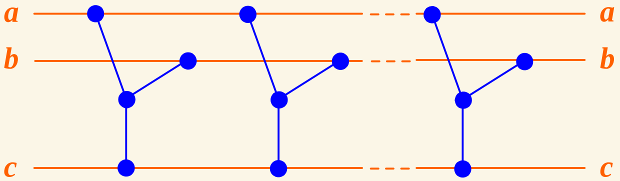

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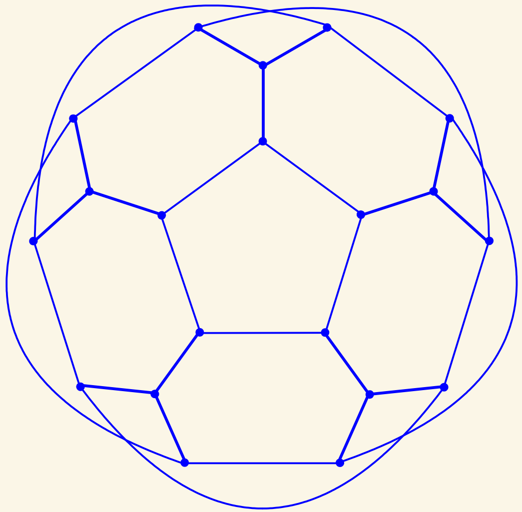
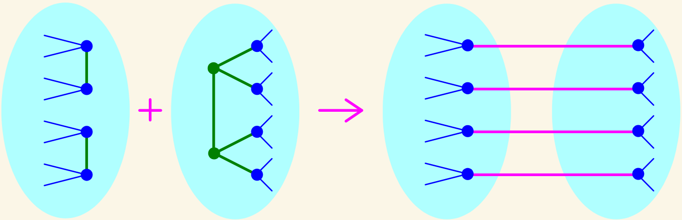
- This gives the **Flower snarks.**

Five copies of Y :



Isaacs' dot product: Join copies of known snarks as follows:



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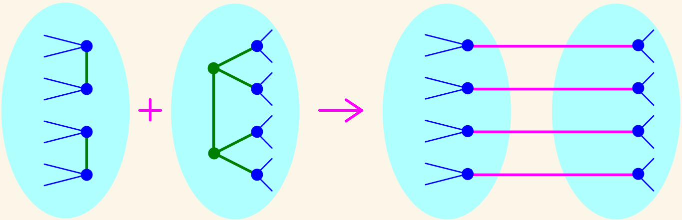
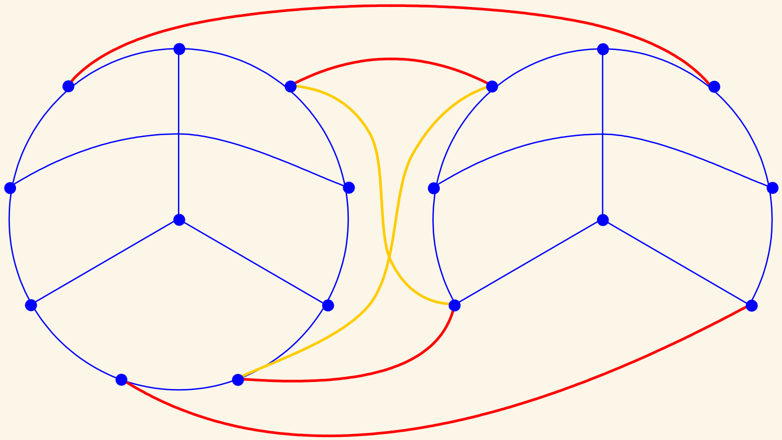
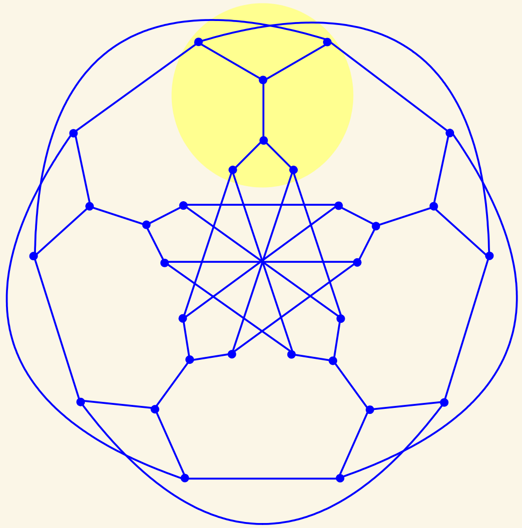


- Using copies of the Petersen graph, Isaacs constructed a new infinite class of snarks that contains all previously known snarks.

Isaacs' dot product gives the Blanuša Snarks:



Isaacs' double star snark:



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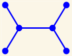
- But for double stars, let $D(n,k)$ denote the double star obtained from n copies of  , where the edges in the "inner ring" join every k^{th} copy. Then:

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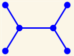
- But for double stars, let $D(n, k)$ denote the double star obtained from n copies of  , where the edges in the "inner ring" join every k^{th} copy. Then:

Theorem (Amanda Chetwynd, 1984)

The double star $D(n, k)$ is a snark if and only if it is one of $D(3, 1)$, $D(5, 2)$ (Isaacs' snark), or $D(n, k)$, where $n \equiv 0 \pmod 3$ and $\gcd(n, k) = n/3$.

Amanda Chetwynd
Provost for Student Experience, Colleges and the Library
Lancaster University
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- Proof "announced" by Robertson, Sanders, Seymour, and Thomas (1999).

sarah-marie belcastro

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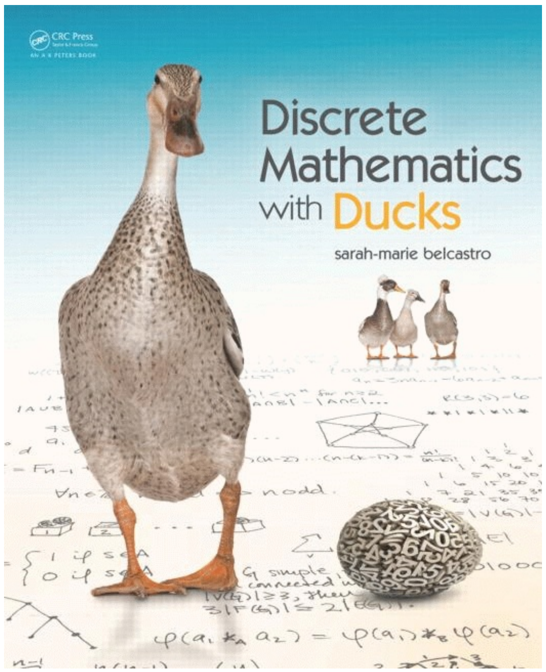
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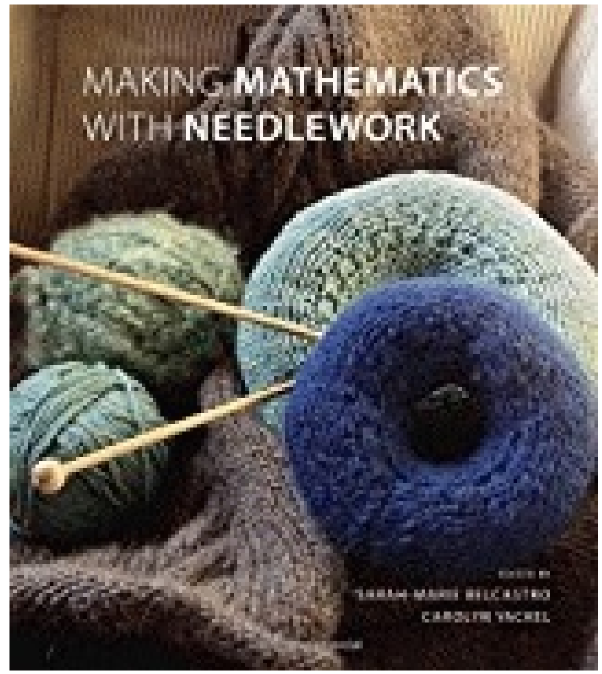
MAKING MATHEMATICS WITH NEEDLEWORK
SARAH-MARIE BELCASTRO
CAROLYN YACKEL

Henda Swart, 1939 - 2016

Female graph theory pioneer,
University of Kwazulu-Natal, South Africa

Henda Swart, inspiring teacher

These well-known graph theorists started their careers with Henda:

- **Ortrud Oellermann,** University of Winnipeg, Canada
- **Wayne Goddard,** Clemson University, South Carolina, USA
- **Mike Henning,** University of Johannesburg, South Africa
- **David Erwin,** University of Cape Town, South Africa
- **Jacques Verstraete,** University of California, San Diego, USA
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Let G be a cubic graph that has been 3-edge coloured in colours 1, 2 and 3. If an edge-cut of n edges contains n_i edges of colour i, $i = 1, 2, 3$, then

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- **Bridgeless:**
  - **Parity Lemma:** any cubic graph **with** a bridge has chromatic index 4. So NOT a snark because of triviality.

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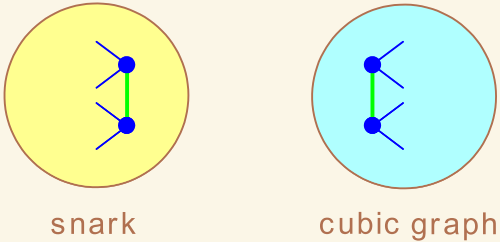

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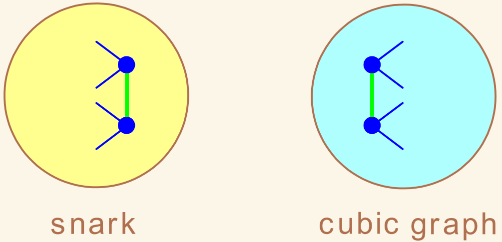

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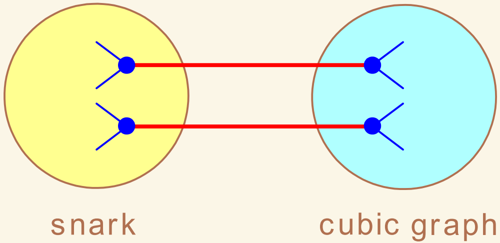

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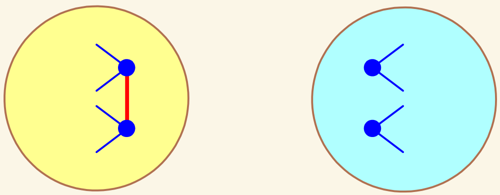

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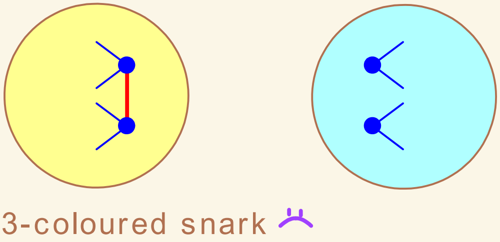

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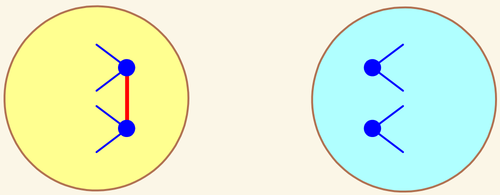

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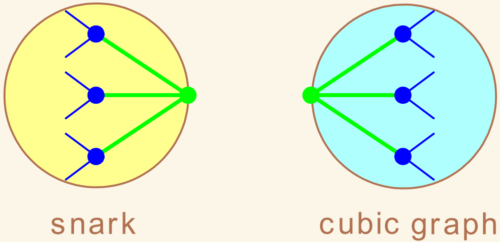

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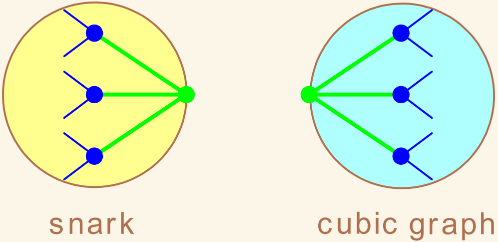

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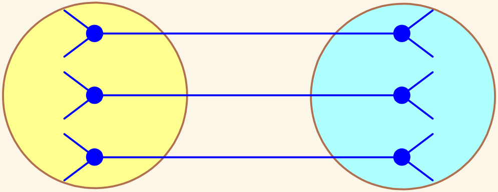

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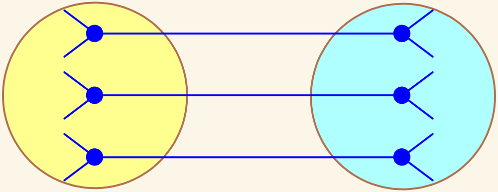

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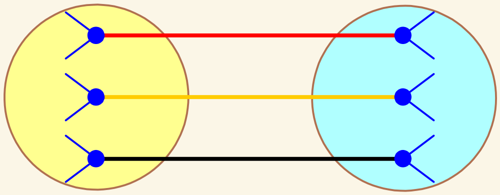

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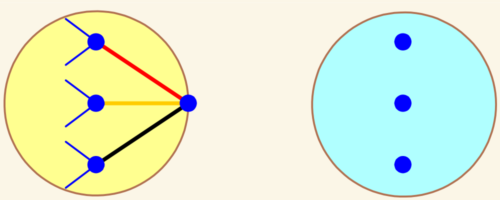

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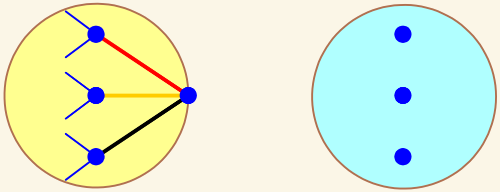

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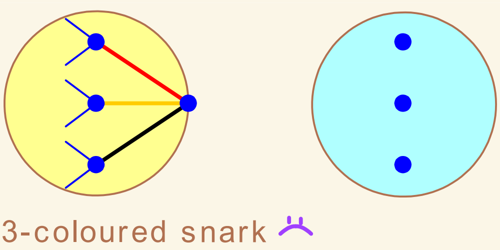

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- Terminology: A snark should be **cyclically 4-edge connected.**

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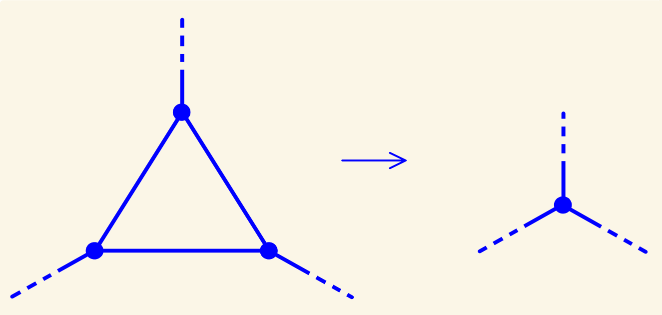

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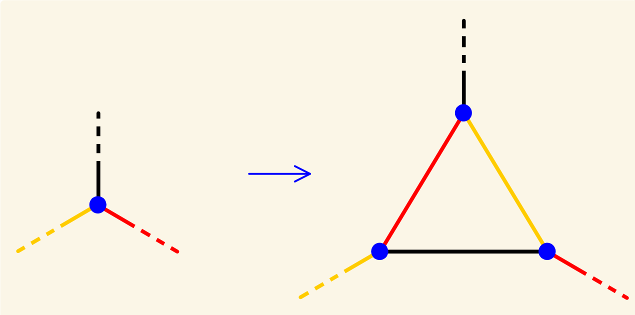

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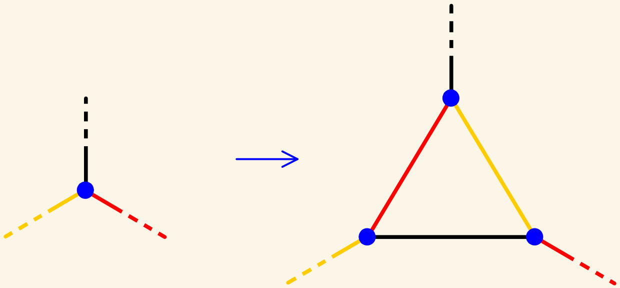

Hence we want snarks to be **triangle-free**.

(A snark with a triangle also has a 3-edge cut.)

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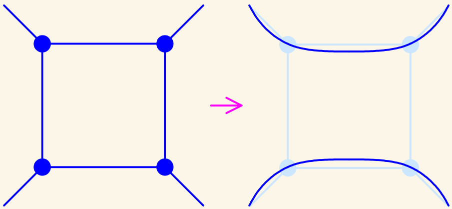

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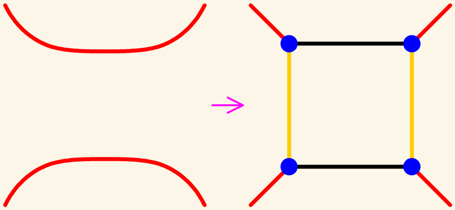

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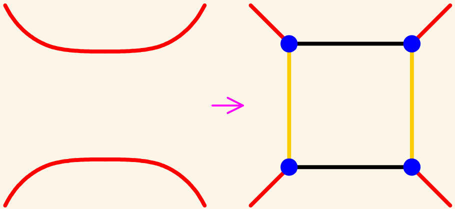

If the new graph is 3-edge colourable, then the original one is, too. 

Hence a snark with a 4-cycle can easily be obtained from a smaller snark.

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- **Good Question:**
  - Why are other operations, such as Isaacs' dot product, to make larger snarks from smaller ones, allowed?

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Let G be a bridgeless graph. Then for every cycle C in G there is a CDC that contains C.

- A minimum counterexample to either conjecture, if it exists, is a snark.
- Hence it is sufficient to prove the CDCC/SCDCC for snarks.

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A minimal counterexample must be a weak snark.

Illustration by Henry Holiday (1839-1927) for "The Hunting of the Snark"

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