# Nonlinear and Stochastic Problems in Atmospheric and Oceanic Prediction

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## **1** Overview of the Field

Advances on the theory of nonlinear dynamical systems, either deterministic or stochastic, as well as advances on the theory of nonlinear partial differential equations (PDEs) and their numerical treatment, have had a profound impact on the modeling and understanding of atmospheric and oceanic dynamics. With the rapid development of nonlinear and stochastic methods and their successful application in atmospheric and oceanic sciences, our capability of atmospheric and oceanic prediction has obtained significant improvement in the past decades. The success of numerical weather forecast is recognized as one of the most important achievements in science, technology and society in the 20th century.

The application of nonlinear and stochastic methods in atmospheric and oceanic prediction is ubiquitous. For example, the development of both control theory of nonlinear dynamic systems and estimation theory of stochastic systems makes it possible to optimally combine observations with numerical models (i.e., data assimilation), resulting in the revolutionary improvement in prediction initializations. The second example is the ensemble prediction that changes the traditional concept of prediction from single, deterministic prediction to multiple, probabilistic prediction, which deems from the Monte Carlo theory and the optimal unsteady theory of nonlinear dynamics systems. The ensemble prediction greatly addresses the uncertainties of predictions, and has been widely applied in operational prediction centers.

# 2 Recent Developments and Open Problems

Over past decades, significant progress has been made in applying nonlinear and stochastic methods onto atmospheric and oceanic prediction. A typical example is the ensemble prediction, which considers well the high degree of non-linearity and stochastic forcing of the atmosphere and ocean systems. The development of nonlinear and stochastic methods make the optimal perturbations available, which is fundamental in the construction of ensemble predictions. The other example is the data assimilation, which optimally use observations to initialize prediction and to constrain model parameter. Our success in weather forecast and climate prediction achieved in recent years can be to a great extent attributed to the advance of data assimilation. However, some specific challenges still emerge in atmospheric and oceanic predictions inherent to nonlinear

and stochastic theory, which can be summarized as following major scientific problems: (i) How can we estimate the model states for high-dimensional nonlinear and non-Gaussian systems, and which nonlinear and stochastic method will be best for the particular type of problem? (ii) What new nonlinear optimal methods or strategies need to be developed to detect the optimal prediction error growth? (iii) What new measurement metrics should be developed to optimally quantify the prediction utility for a nonlinear and stochastic system? (iv) How can we measure and differ the contribution of different error sources to predictability, and further reduce the errors? (v) How should nonlinear and stochastic methods complement the dynamical models used in the atmospheric and oceanic prediction community? (vi) How can full advantage be taken, in atmospheric and oceanic sciences, of the powerful tools that have been developed by mathematicians in the framework of the theory of dynamical systems (dynamic transitions, Lyapunov exponents and vectors, fluctuation-dissipation theorem, invariant and parameterizing manifolds).

It is clear that the atmospheric and oceanic predictions are cross-disciplinary and especially embodied in applications of mathematics in atmospheric and oceanic predictions. To push the collaboration of the researchers from atmosphere, ocean, and mathematics and explore the above major scientific problems in predictions and predictabilities of atmospheric and oceanic motions, The Banff International Research Station (BIRS) hosted the workshop Nonlinear and Stochastic Problems in Atmospheric and Oceanic Prediction during November 19 - 24, 2017. This workshop was co-convened by Prof. Youmin Tang (from Canada), Prof. Shouhong Wang (from America), Prof. Adam Mohana (from Canada), and Prof. Wansuo Duan (from China). And twenty-seven experts from China, Canada, US, and UK attended the workshop and presented talks on the problems mentioned above.

# **3** Presentation Highlights

According to the pre-mentioned major scientific problems, the workshop classified the talks in three themes: (i) nonlinear geophysical fluid dynamics, (ii) data assimilation, and (iii) predictability and prediction.

#### **3.1** Nonlinear Atmosphere and Ocean Dynamics

The atmosphere and ocean experiences significant changes in their nonlinear dynamics. To understand these changes, their mechanisms, and their regional implications a quantitative understanding of processes in the coupled ocean and atmosphere system is required. The presentations delivered under this theme shed light on such nonlinear dynamics of several atmospheric/ocean systems such as El-Nino Southern Oscillation (ENSO), Madan Julian Oscillation (MJO), turbulent systems and large geophysical flows. For example, Dr. Hai Lin discussed ensemble integrations using a primitive equation atmospheric model to investigate the atmospheric transient response to tropical thermal forcings that resemble El Nino and La Nina. He showed that such response develops in the North Pacific within one week after the integration and the signal in the North Atlantic and Europe is established by the end of the second week. Dr. Entcho Demirov talked about the role of dominant patterns of ocean and atmospheric variability which define coherent variations in physical characteristics over large areas. He applied Bayesian Gaussian mixture models to define four dominant subseasonal weather regimes. Based on his analysis, he developed a computationally efficient stochastic weather generator for analysis and prediction of the Subpolar North Atlantic atmospheric decadal variability. Dr. Chun-Hsiung Hsia applied a semi-discretized Euler scheme to solve three dimensional primitive equations.

Some interesting talks were delivered on dynamics of geophysical flows highlighting recent scientific progress. For example, Dr. Corentin Herbert discussed construction of a perturbative theory which describes the large-scale flow, relying on a technique known as stochastic averaging. Using direct numerical simulations in an idealized situation, he showed the validity and limitations of this approach by testing analytical predictions for the structure of the large-scale flow, such as the velocity profile or the eddy momentum flux profile. He further discussed the long term dynamics of large-scale geophysical flows and the abrupt transitions they undergo, using tools from large deviation theory. Dr. David Straub examined simulations of freely decaying turbulence in a non-hydrostatic Boussinesq model for which the base state stratification contains a tropopause. Dr. Xiaoming Wang discussed numerical algorithms that are able to capture the long-time statistical properties such as the climate, of large dissipative turbulent systems and applications to certain pro-

totype Geophysical fluid dynamics models. Dr. Shouhong Wang presented an overview of dynamic transition theory and its applications to various geophysical fluid flows.

#### **3.2 Data Assimilations**

In this theme, the difficulties of the assimilation approaches of ensemble Kalman filter (EnKF) and Particle filters (PF) were mainly discussed. These two approaches are both the numerical realization of the Bayes Theorem. Nevertheless, the former adopts the linear approximation to the observational operator and assume that the observational errors obey Gaussian distribution in statics while the latter avoids these linear approximations and assumptions and is much realistic. The former has been widely used in predictions of atmosphere and ocean but there are still limitations and difficulties in realistic predictions. The latter is most advanced in theory but presently under exploration. Clear, in-depth investigations are urgently needed for these two approaches. The presentations in this workshop provided some progresses on these two approaches. For example, to reduce the huge computational resource of EnKF in high resolution coupled earth systems, Prof. Shaoqing Zhang considered the combination of stationary, slow-varying and fast-varying filters and designed a high efficiency approximates EnKF (Hea-EnKF) to dramatically enhance the computational efficiency. He showed that it is the improved representation on stationary and slow-varying background statistics that makes the Hea-EnKF while only requiring a small fraction of computer resources be better than the standard EnKF that uses finite ensemble statistics. This also makes the Hea-EnKF practical to assimilate multi-source observations into any high-resolution coupled model intractable with current super-computing power for weather-climate analysis and predictions. For PF, Dr. Nan Chen introduced a conditional Gaussian framework for prediction of nonlinear turbulent dynamical systems, which allows efficient and analytically solvable conditional statistics that facilitates the real-time data assimilation and prediction. Especially, this method is able to beat the curse of dimensions in traditional particle methods. In addition, a challenge of PF is to avoid the filter degeneracy with limited computational resources, where the filter degeneracy is a situation in which almost every particle has negligible weight and contributes little to representing the PDFs. The filter degeneracy is still an open question. Dr. Zheqi Shen focused on this question and tried to give several strategies to overcome this filter degeneracy phenomenon, include using a hybrid scheme of PF and EnKF with a tunable parameter, and using vector weights to achieve localization, and finally thought that it is great potential to develop the feasible PF for large geophysical model systems with nonlinear/non-Gaussian observational systems.

#### 3.3 Predictability and Prediction

The predictability is a fundamental issue in both atmospheric and oceanic research, as well as numerical weather and climate prediction. It indicates the extent to which even minor imperfections in the knowledge of the current state or of the representation of the system limits knowledge of subsequent states. Study of predictability could provide useful information on reducing the prediction uncertainties for the atmospheric and oceanic motions. Study of predictability can include two aspects. The first is studying the reason and mechanism of the uncertainty of forecast result (e.g., forecast error) and the second is studying the way or methodology to reduce such kind of uncertainty. In the workshop, quite a few presentations focused on these two aspects and addressed progresses.

For the first aspect, it is important to estimate the predictability limit. In this field, Prof. Frank Kwasniok showed that the predictability of large-scale atmospheric flow is characterized by finite-time Lyapunov exponents and covariant Lyapunov vectors. Prof. Jianping Li considered this limitation and reported a nonlinear local Lyapunov exponent approach (NLLE). He explored the basic theory of the NLLE and then applied this approach to the predictability studies, and finally showed the usefulness of the NLLE in estimating the predictability time for weather and climate events and determining the sensitive area for targeting observations. Subsequently, Prof. Ruiqiang Ding presented another application of NLLE in ensemble forecast and reported the approach of orthogonal nonlinear local Lyapunov vector (NLLV). He used the NLLV to generate the initial perturbations of the ensemble forecast and demonstrated the applicability of NLLV in ensemble forecast. Singular vectors (SVs) is also an approach to estimating the limit of predictability. SVs have been successfully applied in operational weather forecast in ECWMF. In the presentation of Prof. Youmin Tang, he proposed the concept of climatological SVs.

tangent model in calculating SVs. Dr. Siraj Ul Islam investigated the error growth dynamics of SVs and showed that when the SVs are used as an initial perturbation, the forecast skill of key atmospheric variables over South Asian Monsoon region is significantly improved. Further, he demonstrated that the predictions with the singular vector have a more reliable ensemble spread, suggesting a potential merit for a probabilistic forecast. The linearity of SVs still limits the forecast skill. If considering effect of nonlinearity in SVs, the forecast skill generated by the SVs may be greatly improved. In the workshop, Prof. Wansuo Duan focused on this limitation of linear singular vector (LSV) and presented an approach of conditional nonlinear optimal perturbation (CNOP) to estimate the maximum prediction error. Prof. Duan applied this approach to the predictability of ENSO and revealed the difference between CNOP and LSV. Then he used this approach to present the optimal observational array for dealing with the challenge of ENSO predictions due to diversities of El Nino. By hindcast experiments, the target observations determined by the CNOP were illustrated to be useful in distinguishing the types of El Nino events in predictions. In addition, Dr. Guodong Sun also showed the usefulness of the CNOP in dealing with the effect of model parametric errors on predictability and emphasized the nonlinear interaction of multiply physical parameters is beneficial to improve the simulation ability and prediction skill of the soil moisture.

For the second aspect of predictability study, the presentations in the workshop reported several interesting approaches to improving the forecast skill for weather and climate. For example, Prof. Fei Zheng suggested a new stochastic perturbation technique to improve the prediction skills of ENSO through using an intermediate coupled model. Similar idea was existed in the presentation of Dr. Aneesh Subramanian. Dr. Aneesh Subramanian compared the stochastic perturbation scheme representing model errors and the super-parameterization strategy for including the effects of moist convection through explicit turbulent fluxes calculated from a cloud-resolving model (CRM) embedded within a global climate model (GCM), finally showing that the combination of the two approaches helps improve the reliability of forecasts of certain tropical phenomena, especially in regions that are affected by deep convective systems.

Another aspect to predictability is to identify the most predictable mode. Prof. Timothy M. Delsole focused on the degree to which interactive ocean circulations are important for making useful predictions of the next decade and identified the most predictable patterns of global sea surface temperature in coupled atmosphere-ocean models. He suggested that interactive ocean circulation is not essential for the spatial structure of multi-year predictability previously identified in coupled models and observations, but the time scale of predictability, and the relation of these predictable patterns to other climate variables, is sensitive to whether the model supports interactive ocean circulations or not, especially over the North Atlantic.

### 4 Scientific Progress Made

The participants learned several new techniques and scientific progress to understand and resolve atmospheric and oceanic dynamics. The group discussion touched on most of the challenges and scientific issues related to nonlinear geophysical fluid dynamics, data assimilation, and predictability and prediction. Participants discussed various ways to make dynamical ensemble prediction more skillful on season to decadal time scales along with the discussion of deficiencies in numerical modeling. Such discussion helped participants to design better experiments or better data collection protocols. All these discussions were very productive especially for the early career young scientists learning new and advance scientific techniques from the leading scientist working in climate dynamics.

### **5** Outcome of the Meeting

This meeting brought together various mathematical and atmospheric backgrounds. The meeting was very timely and useful, and will surely have a strong impact on the further developments of predictability and data assimilation and related subjects. The organizers were in particular pleased to observe the many young people interested in the subject; some of them have already made substantial contributions (as witnessed by the talks given by junior researchers) and will surely continue to advance the subject.

The meeting provided a great overview of the field, with a number of excellent talks describing some recent exciting results in predictability and data assimilation. It is quite certain that some new interesting results will emerge as a result of the workshop.

# References