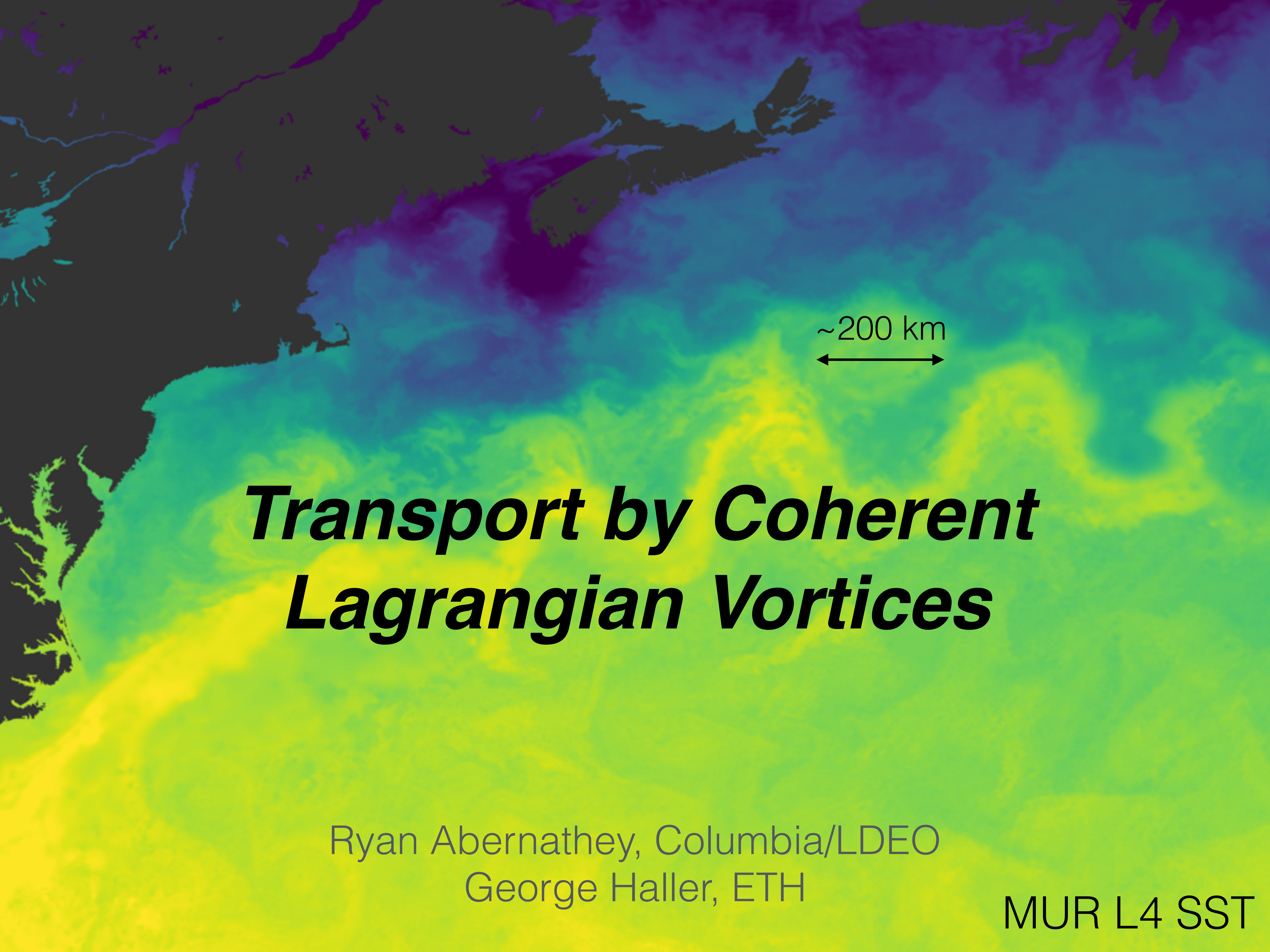


Transport by Coherent Lagrangian Vortices

Ryan Abernathey, Columbia/LDEO
George Haller, ETH

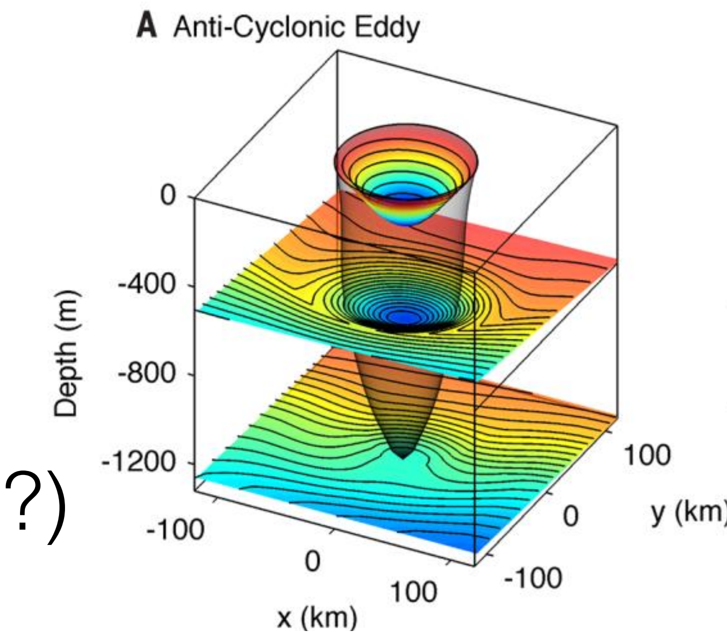
Two Paradigms of "Eddy Flux"

<u>Reynolds Flux</u>

$$\overline{v'\theta'} \qquad \overline{u'v'}$$

- Statistical
- Eulerian
- Flux due to correlations btw. fluctuating velocity and tracer
- Atmospheric science, linear theory, eddy parameterization, model diagnostics

<u>Coherent Eddy</u>



- Countable
- Lagrangian(?)
- Flux due to motion of coherent structure
- Dynamical systems, observational physical oceanography

Eddy fluxes from coherent structures

nature COMMUNICATIONS

ARTICLE

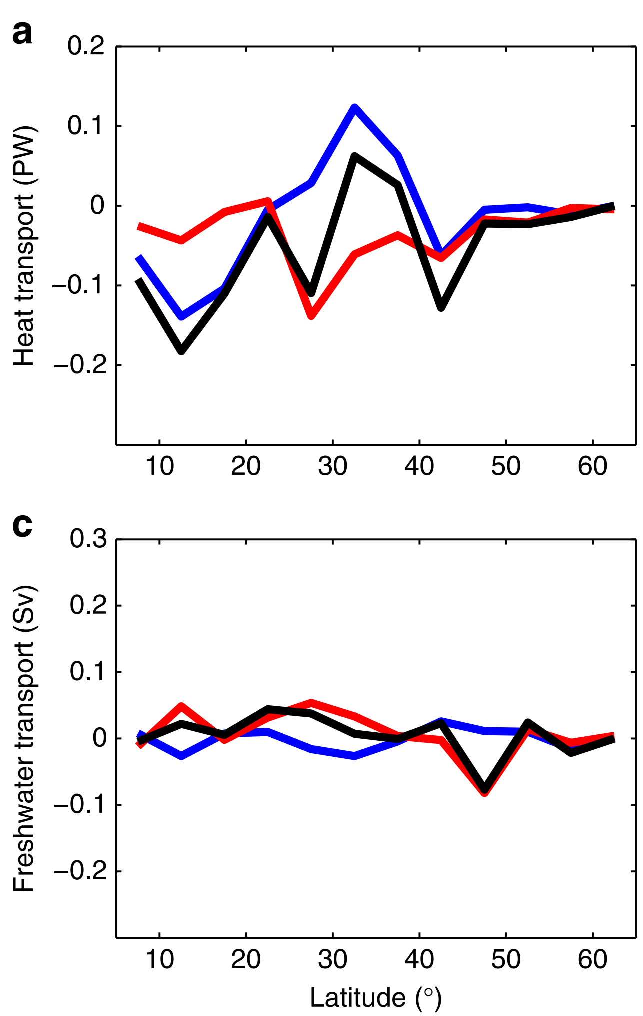
Received 23 Jun 2013 | Accepted 22 Jan 2014 | Published 18 Feb 2014 DOI: 10.1038/ncomms4294

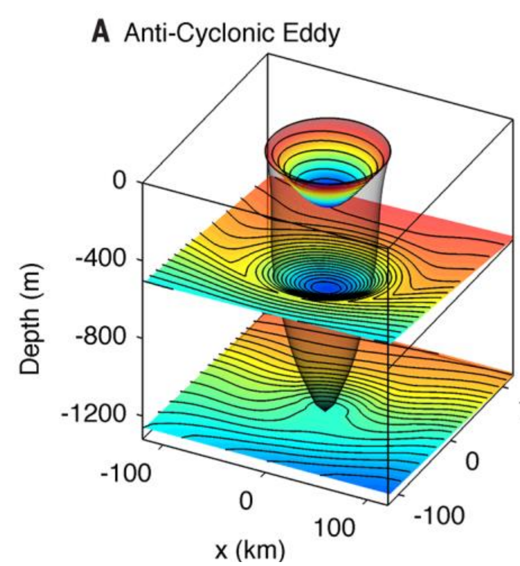
Global heat and salt transports by eddy movement

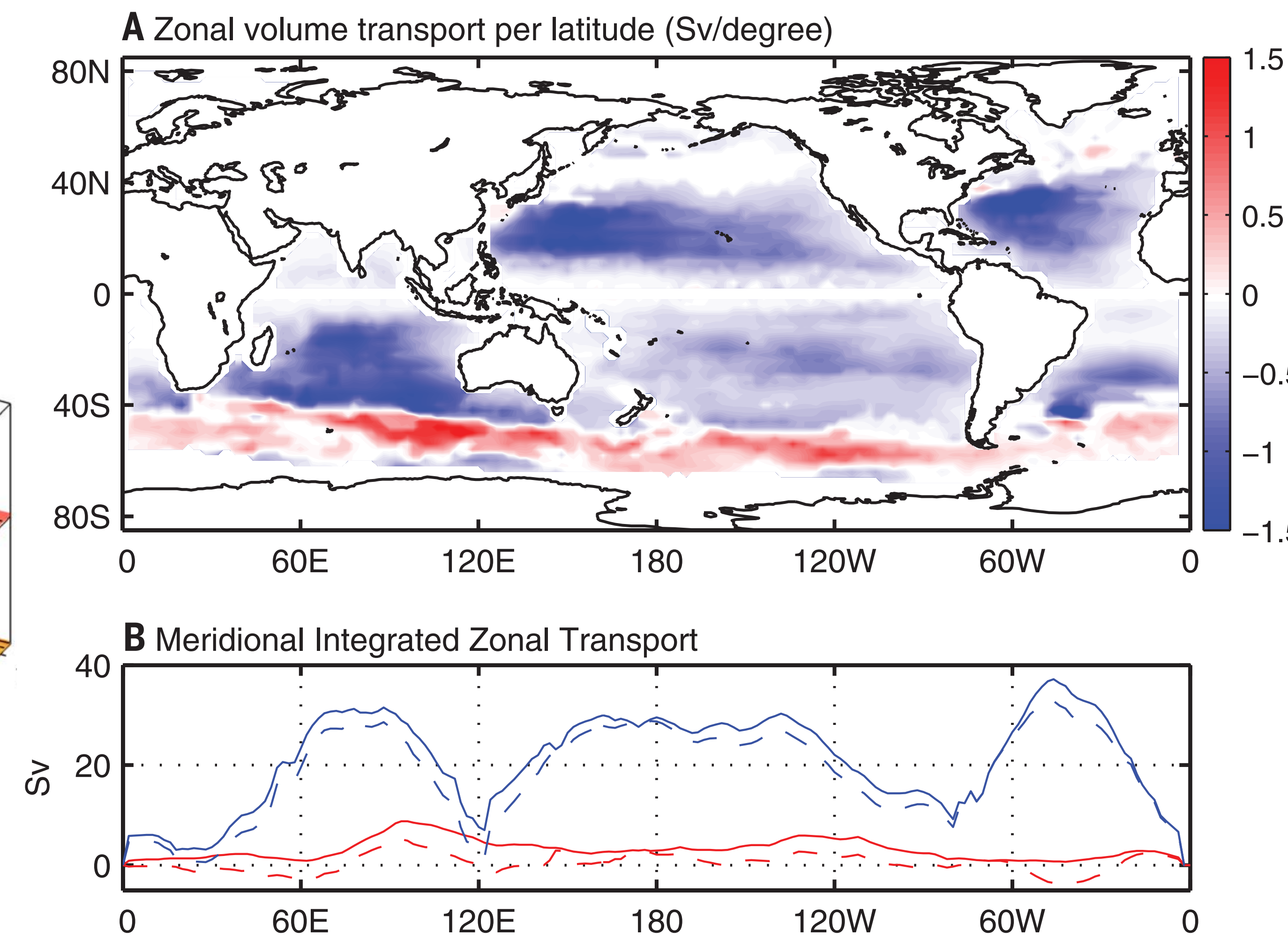
Changming Dong[1,2,3], James C. McWilliams[2], Yu Liu[3] & Dake Chen[1]

Science AAAS

Oceanic mass transport by mesoscale eddies
Zhengguang Zhang *et al.*
Science **345**, 322 (2014);
DOI: 10.1126/science.1252418







"…eddy heat and salt transports are mainly due to individual eddy movements."

Are all the "eddies" seen in altimetry actually materially coherent?

Lagrangian Dispersion

DIFFUSION BY CONTINUOUS MOVEMENTS

By G. I. Taylor.

[Received May 22nd, 1920.—Read June 10th, 1920.]

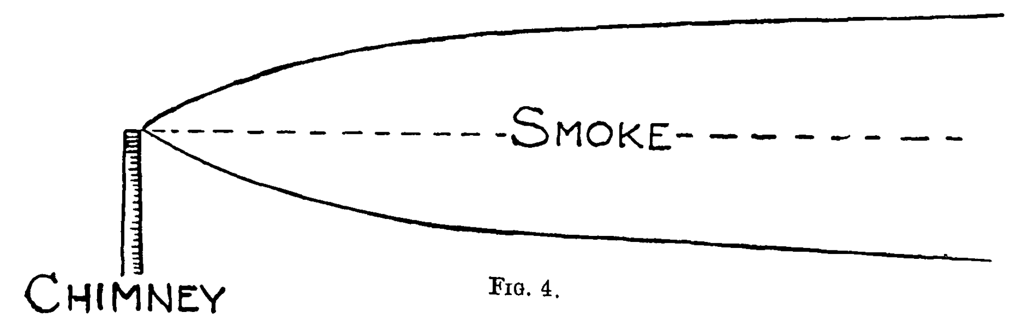


Fig. 4.

meridional heat flux $\overline{v'\theta'} = -K\frac{\partial\overline{\theta}}{\partial y}$

$$K = \frac{1}{2}\frac{\partial}{\partial t}\overline{(\Delta Y)^2}$$ eddy diffusivity

$$\Delta Y = Y(x,y,t) - Y(x,y,0)$$ single-particle Lagrangian dispersion

Relative dispersion

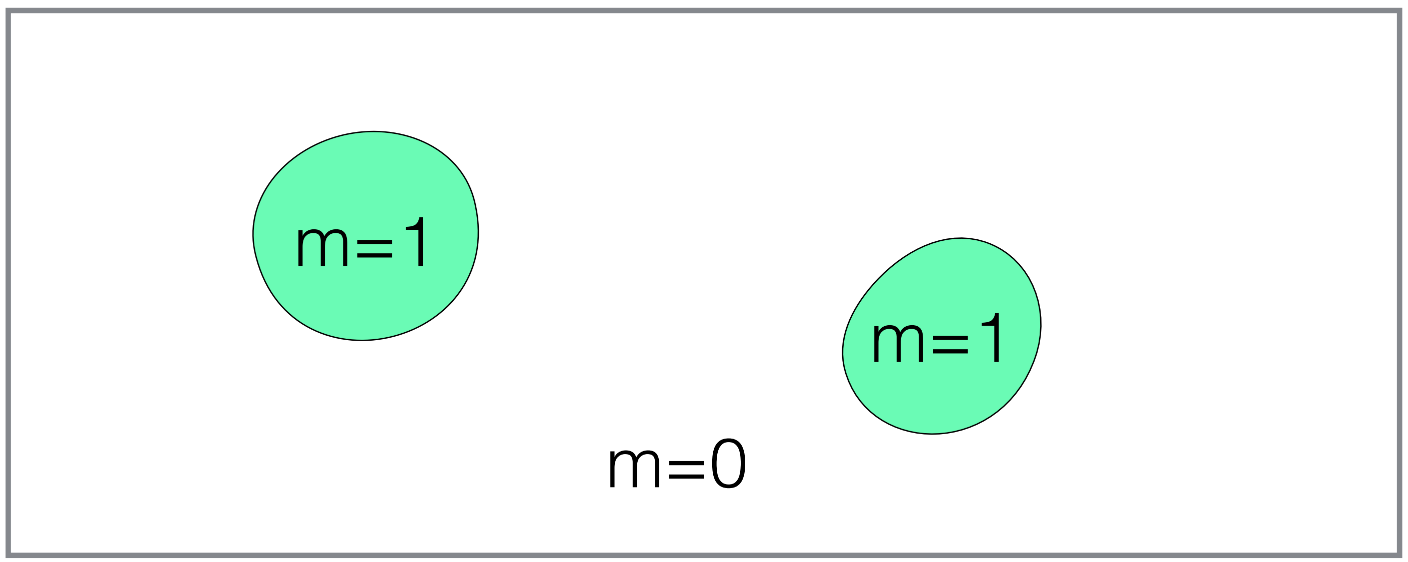
$$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(Y-\overline{Y})^2}$$

- Removes the dispersion due to "mean flow"
- Describes spreading of tracer patch
- Equivalent to K at large separation scales (i.e. long time scales)

Coherent Lagrangian Dispersion

Relative dispersion $$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(Y - \overline{Y})^2}$$

coherent structure mask



"Coherent" diffusivity $$K_{rel}^{CS} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(mY - \overline{Y})^2}$$

Assumes that the water outside the CSs is just moving with the mean flow

Coherent Lagrangian Dispersion

256,000 particles

Coherent Lagrangian Dispersion

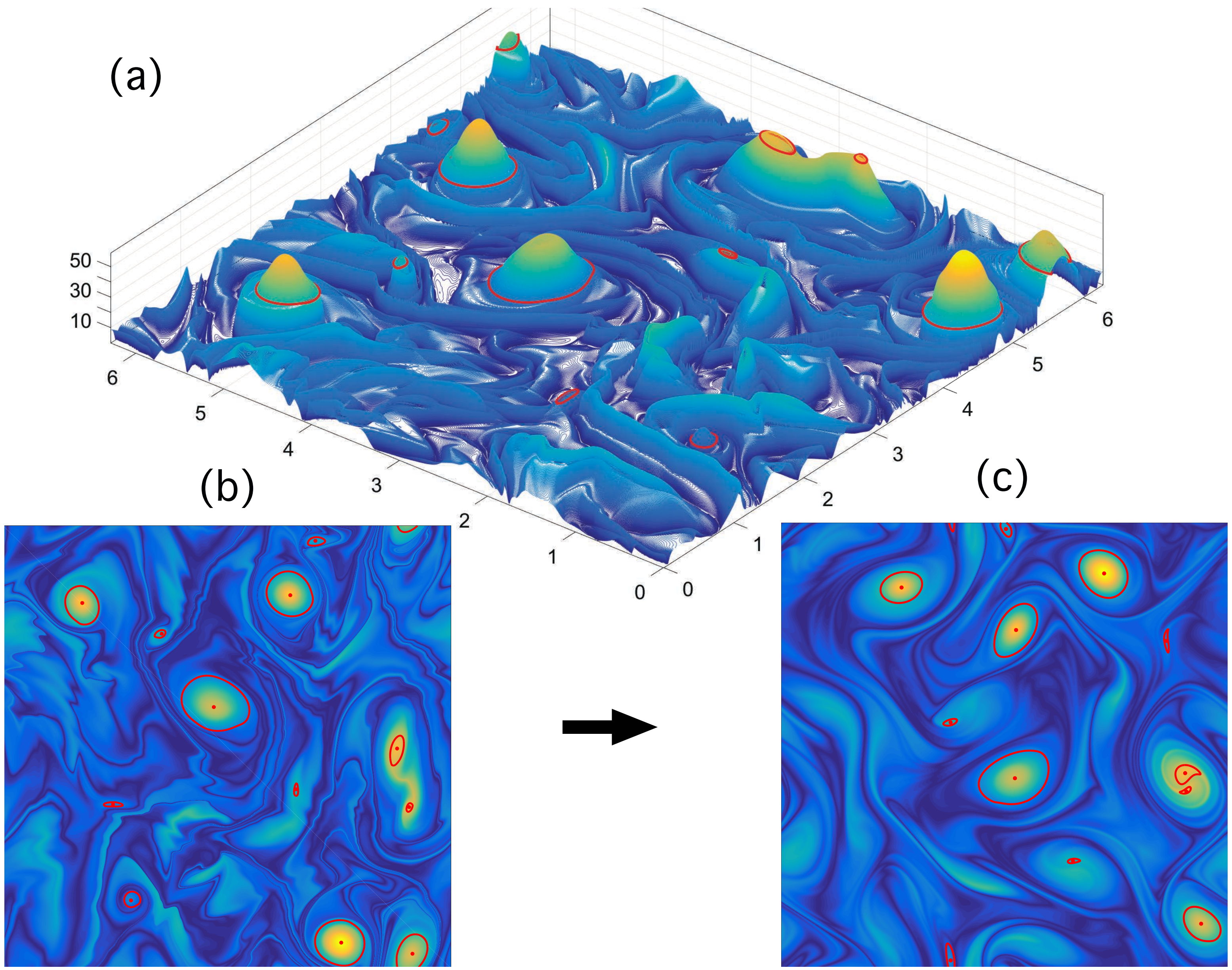
Rotationally Coherent Lagrangian Vortices

G. Haller, A. Hadjighasem, M. Farazamand & F. Huhn, 2016
Defining coherent vortices objectively from the vorticity. J. Fluid Mech.

Lagrangian-Averaged Vorticity Deviation:

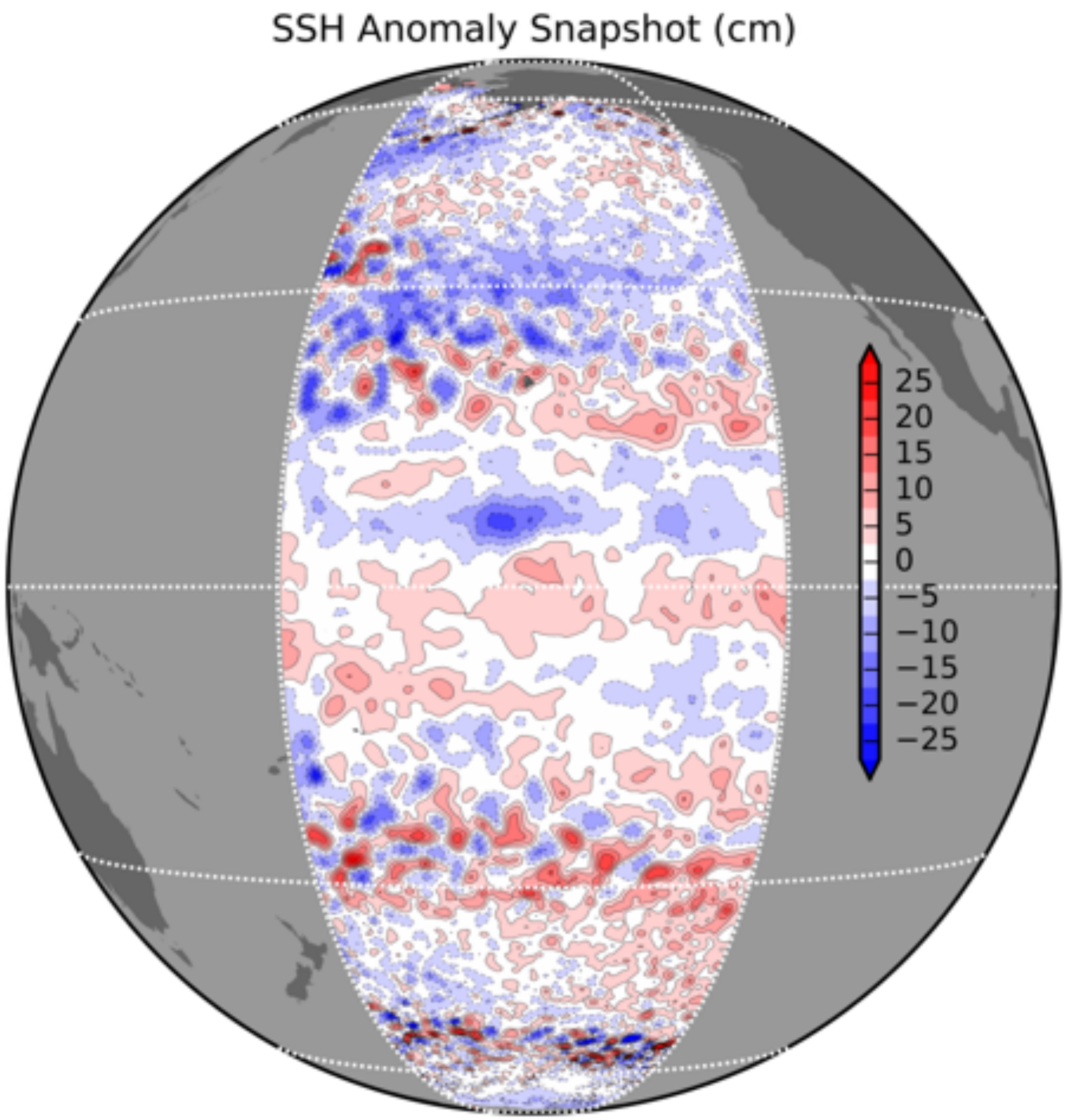
$$\mathrm{LAVD}_{t_0}^{t}(x_0) := \int_{t_0}^{t} |\omega(x(s;x_0),s) - \bar{\omega}(s)|\, ds$$

integral taken *along Lagrangian trajectory*



- Vortex centers: maxima of |LAVD|
- Vortex boundaries: outermost *convex* curves around maxima
- Coherence is defined over a *finite time interval*

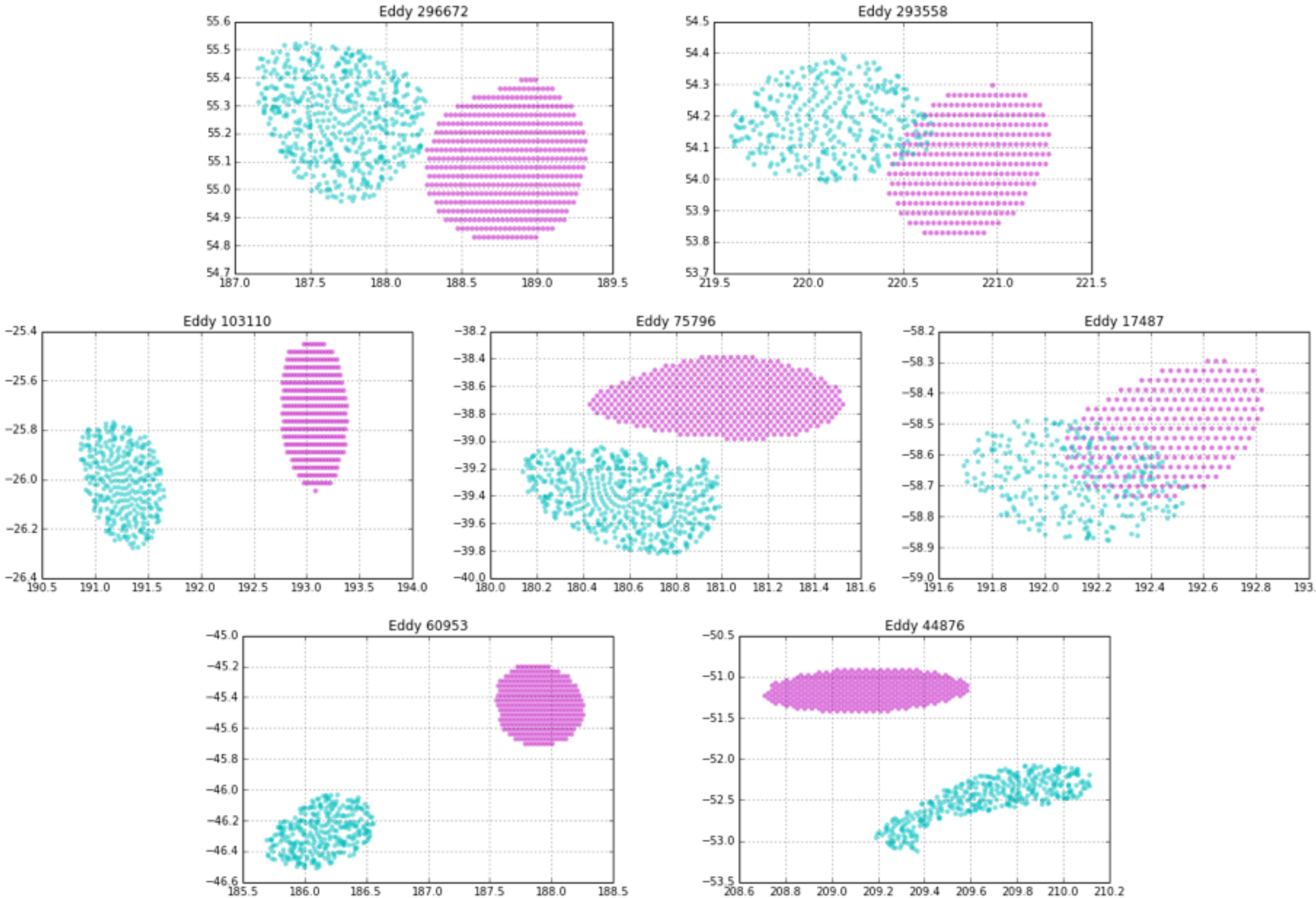
East Pacific Sector



$$\hat{\boldsymbol{k}} \times \boldsymbol{v}_g = -\frac{g}{f}\nabla\eta$$

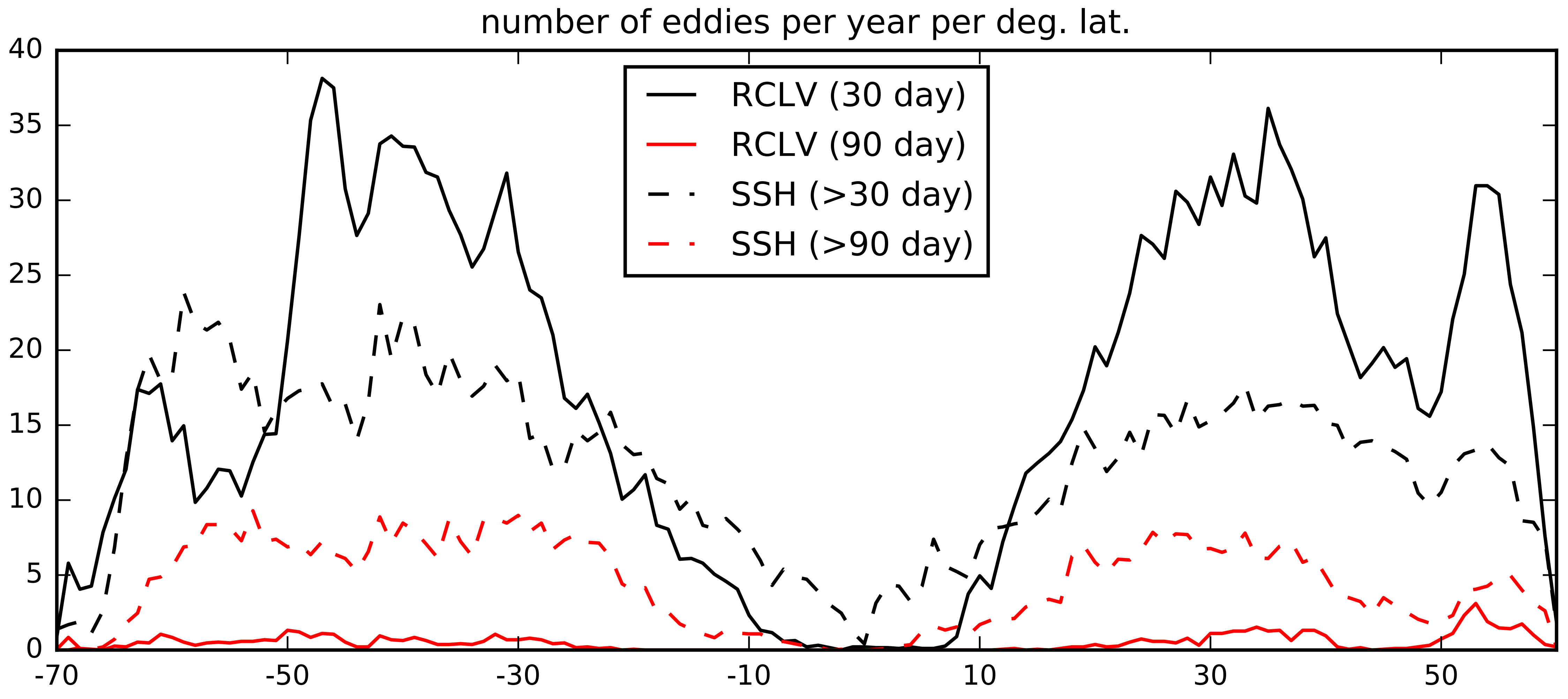
- Very little land, mostly zonally symmetric statistics
- Eulerian eddy fluxes are very well studied (Abernathey & Marshall, 2013; Klocker & Abernathey, 2014; Abernathey & Wortham, 2015)
- Use AVISO altimetric velocities to advect Lagrangian particles (30, 90, 270 days)
- 1/32-degree mesh is necessary to resolve RCLVs: ~8 million particles
- Track position, relative vorticity, of particles with daily sampling

90-day RLCVs



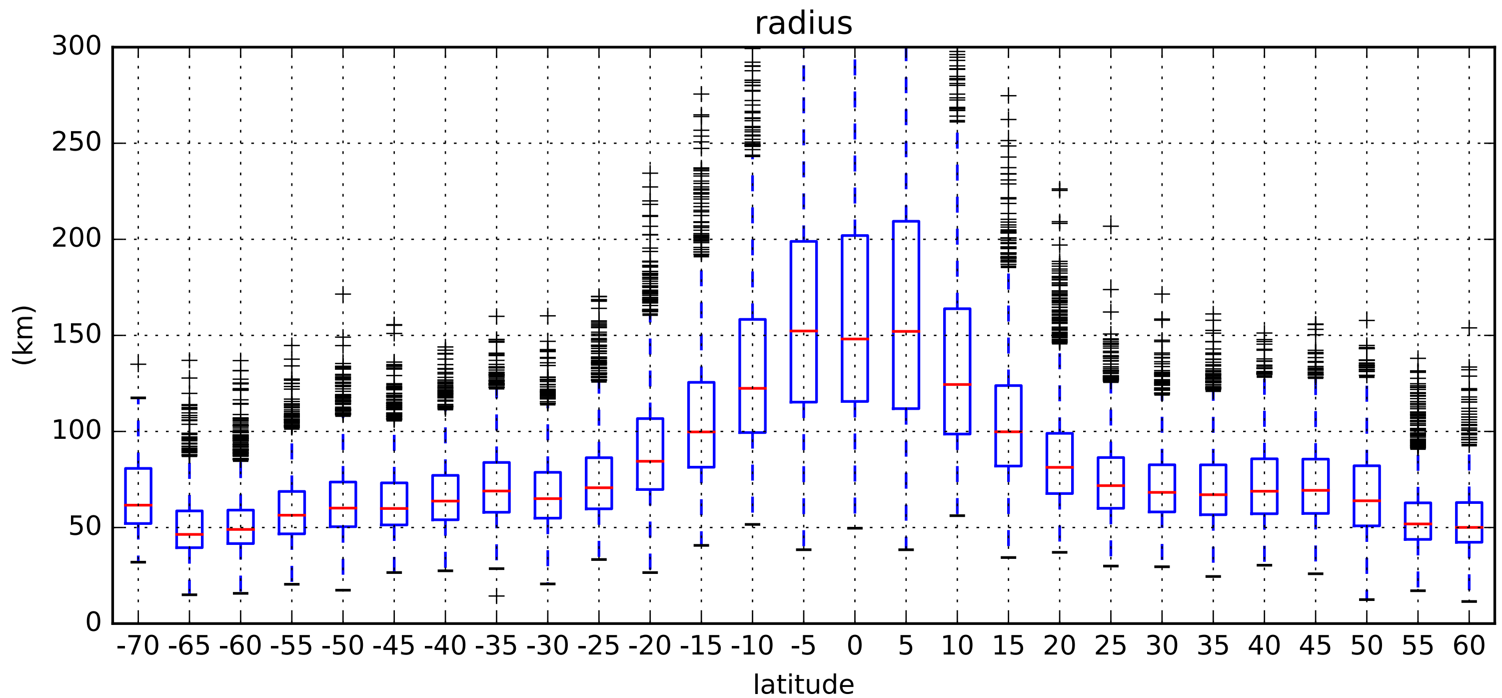
Eddy 296672
55.6
55.5
55.4
55.3
55.2
55.1
55.0
54.9
54.8
54.7
187.0
187.5
188.0
188.5
189.0
189.5
Eddy 293558
54.5
54.4
54.3
54.2
54.1
54.0
53.9
53.8
53.7
219.5
220.0
220.5
221.0
221.5
Eddy 103110
−25.4
−25.6
−25.8
−26.0
−26.2
−26.4
190.5
191.0
191.5
192.0
192.5
193.0
193.5
194.0
Eddy 75796
−38.2
−38.4
−38.6
−38.8
−39.0
−39.2
−39.4
−39.6
−39.8
−40.0
180.0
180.2
180.4
180.6
180.8
181.0
181.2
181.4
181.6
Eddy 17487
−58.2
−58.3
−58.4
−58.5
−58.6
−58.7
−58.8
−58.9
−59.0
191.6
191.8
192.0
192.2
192.4
192.6
192.8
193.0
Eddy 60953
−45.0
−45.2
−45.4
−45.6
−45.8
−46.0
−46.2
−46.4
−46.6
185.5
186.0
186.5
187.0
187.5
188.0
188.5
Eddy 44876
−50.5
−51.0
−51.5
−52.0
−52.5
−53.0
−53.5
208.6
208.8
209.0
209.2
209.4
209.6
209.8
210.0
210.2

Eddy Statistics: Frequency



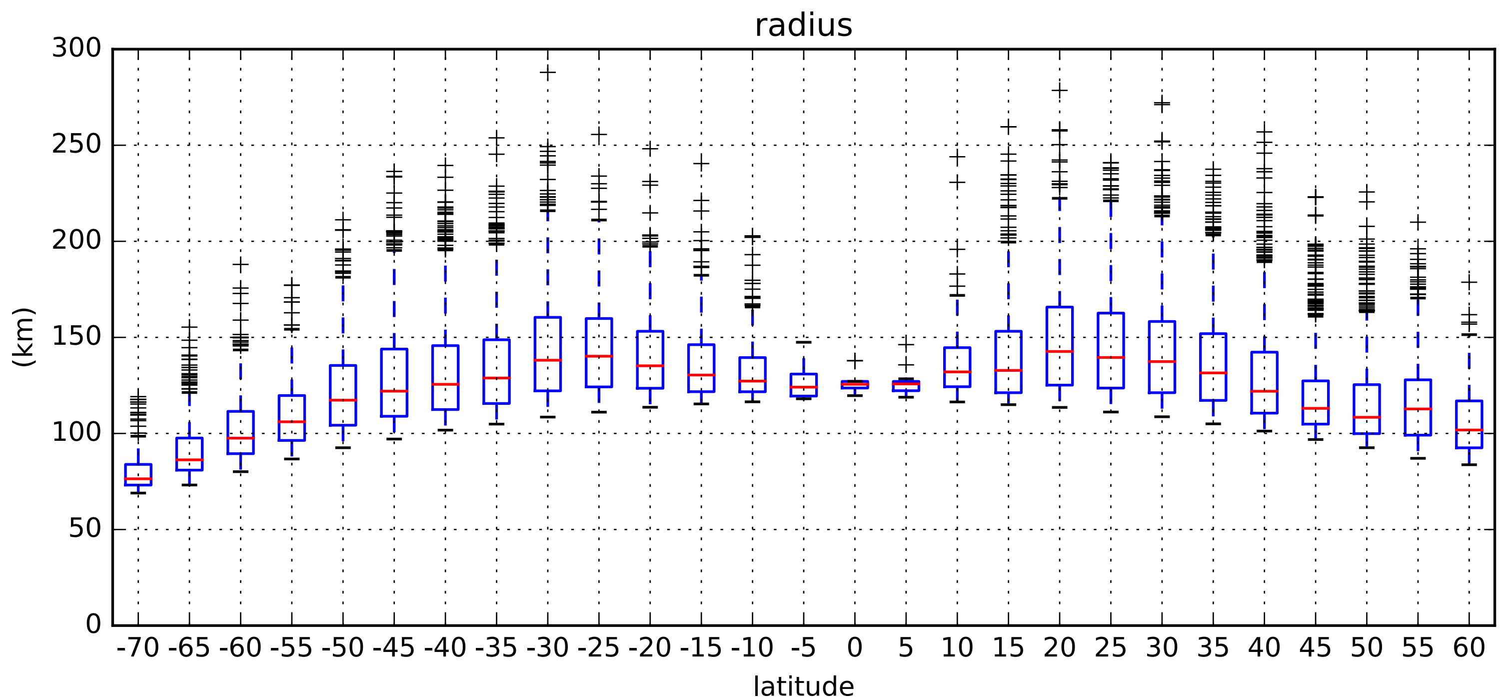
Eddy Statistics: Radius

SSH



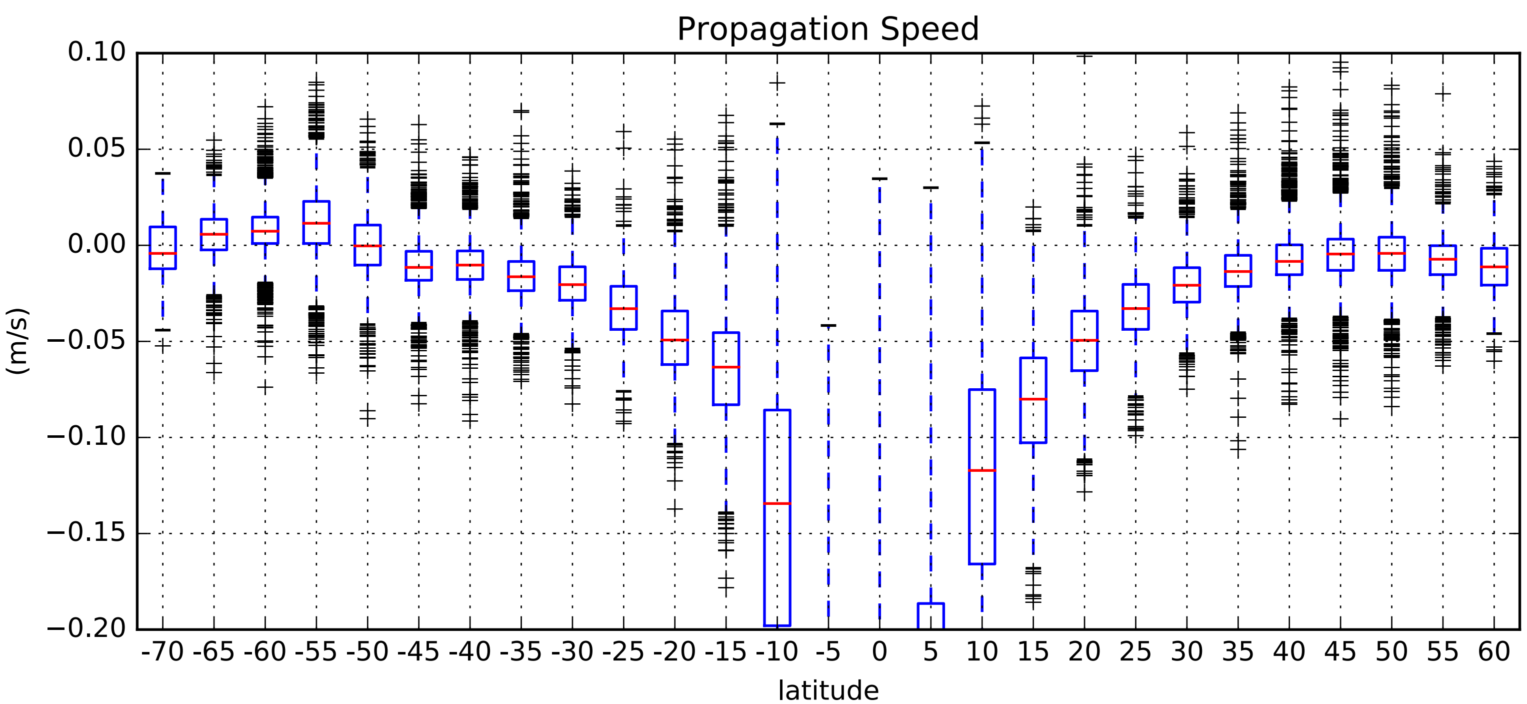
“mesoscale”

RCS
(30 day)

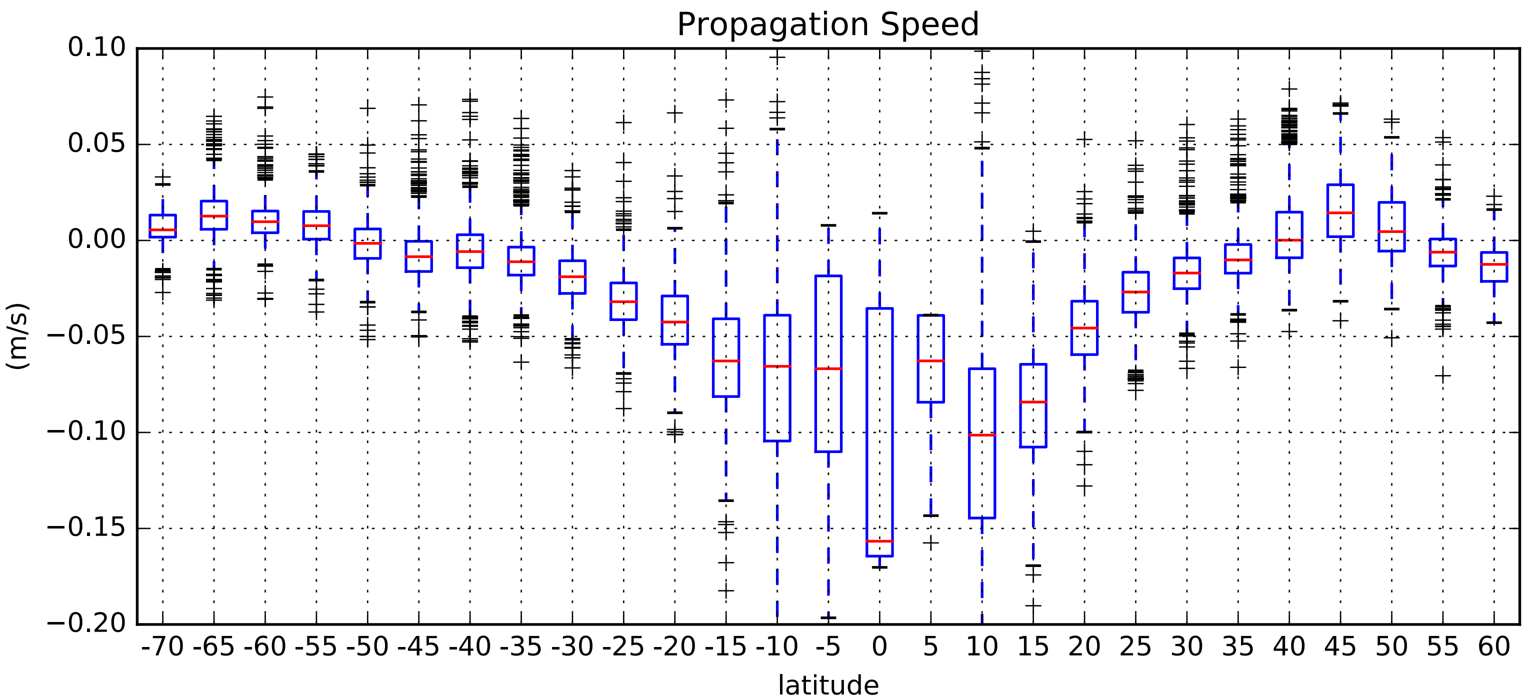


Eddy Statistics: Zonal Propagation

SSH

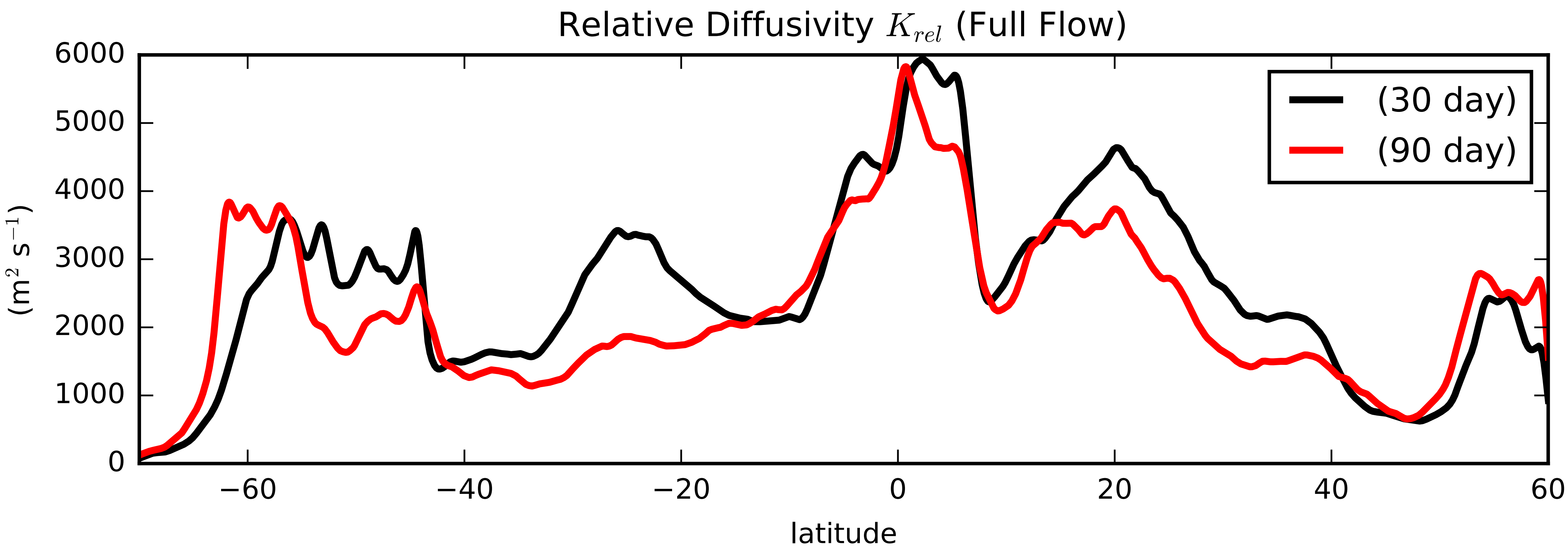


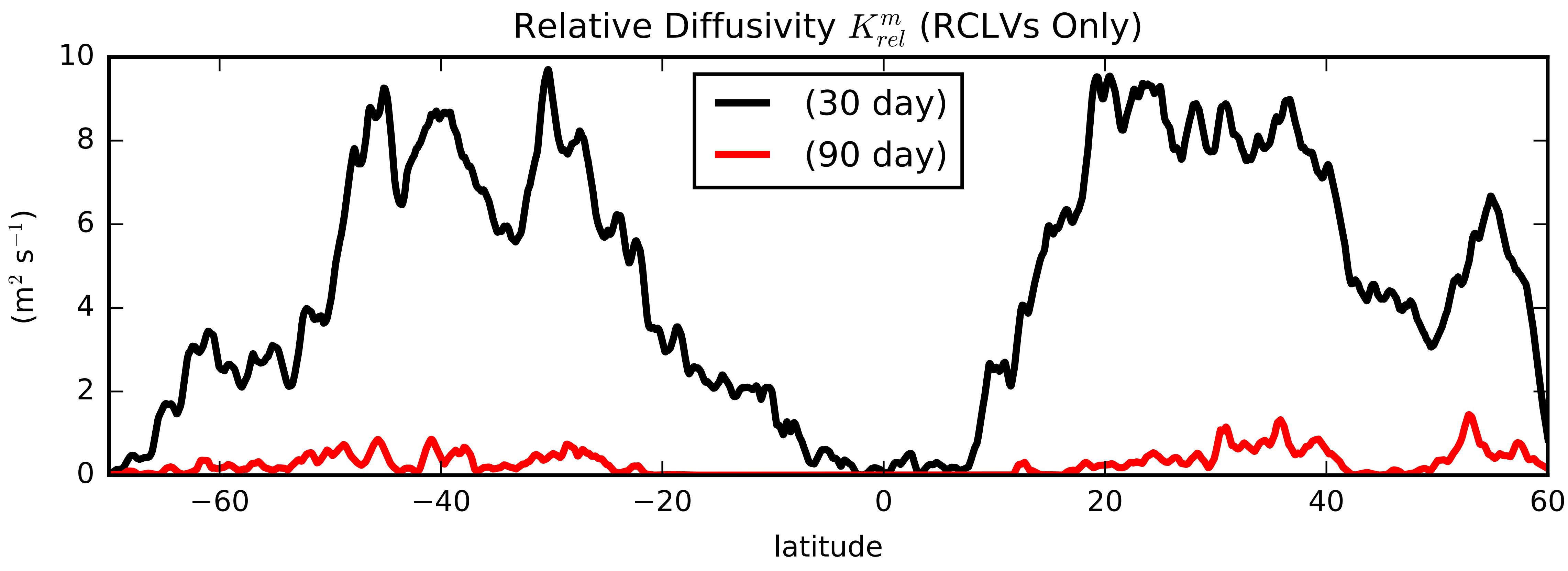
RCS
(30 day)



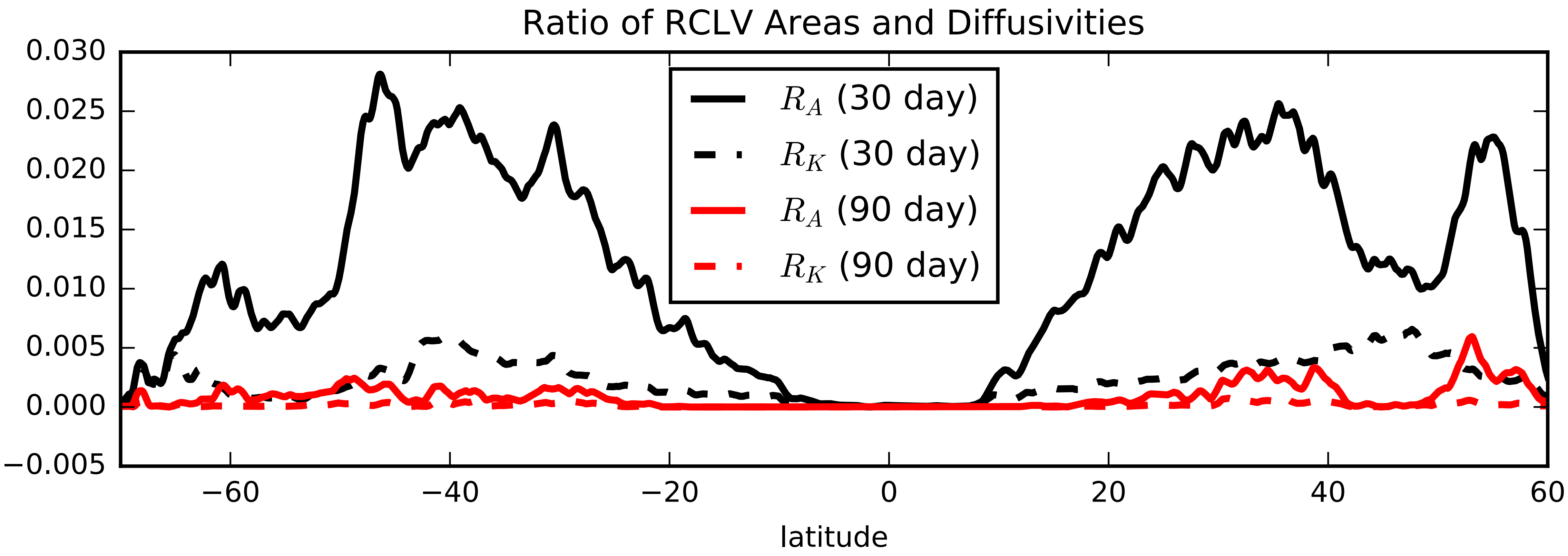
both “feel” Rossby wave dispersion relation

Diffusivity

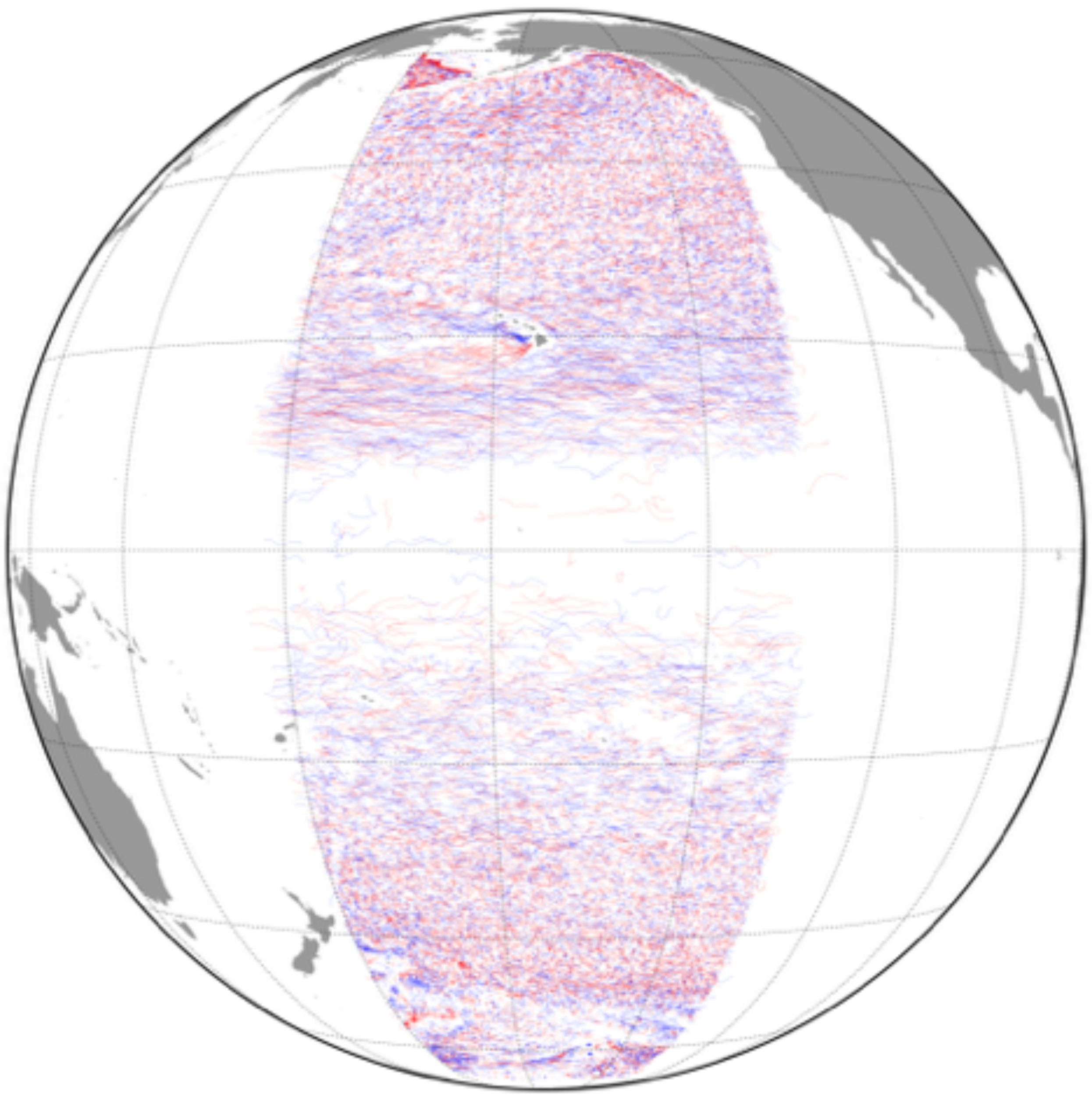




Area and Diffusivity Ratios



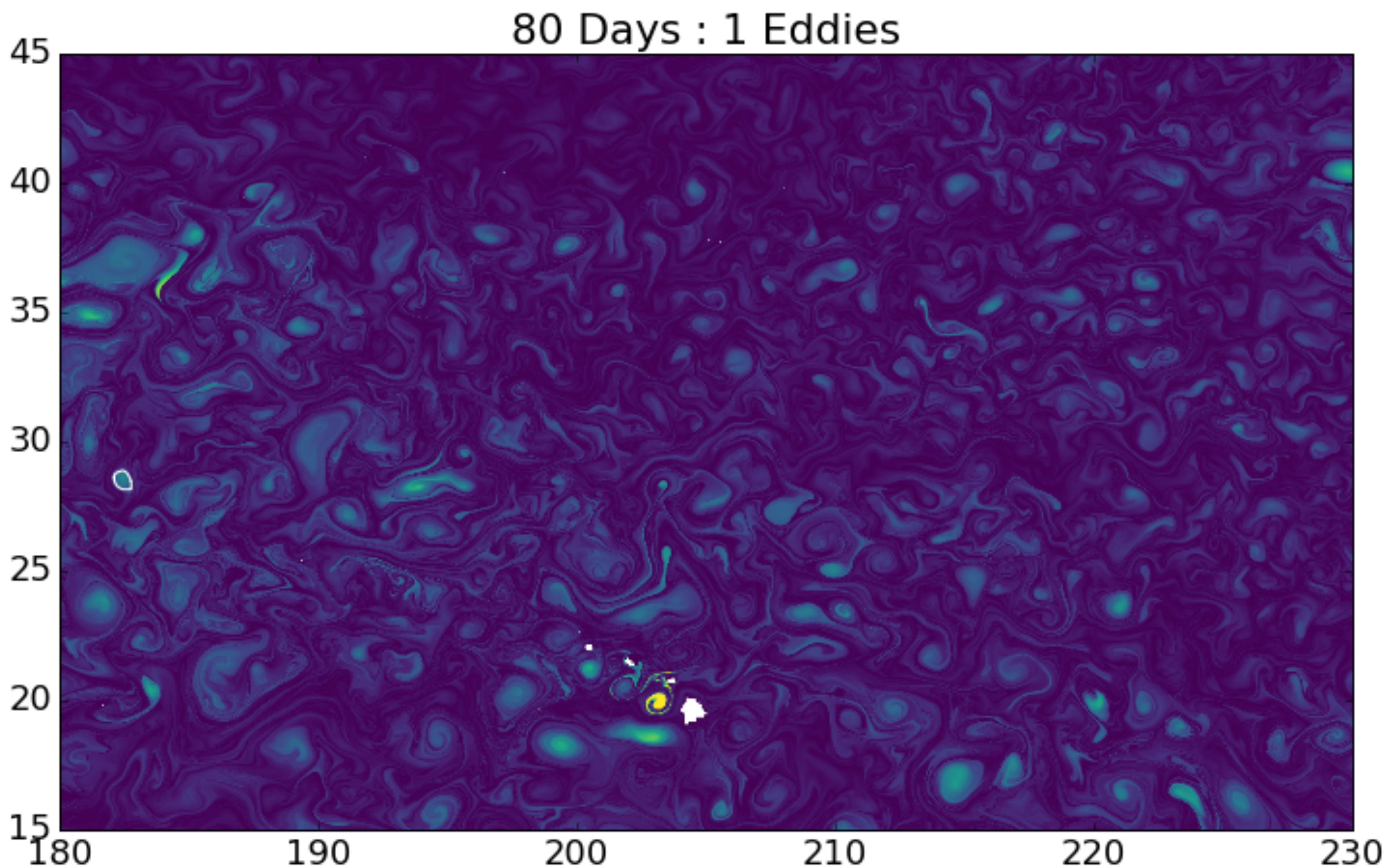
Conclusions



- It is feasible to perform a comprehensive "census" of RCLVs on a global scale using the entire altimetric dataset
- Certain statistics of RCLVs are broadly similar to SSH-eddies (e.g. propagation speed)…
- …But the RCLVs are much shorter lived
- Calculated the contributions RCLVs to net meridional dispersion (relative diffusivity): **totally negligible!**
- More validation and analysis in progress

Identifying RLCVs

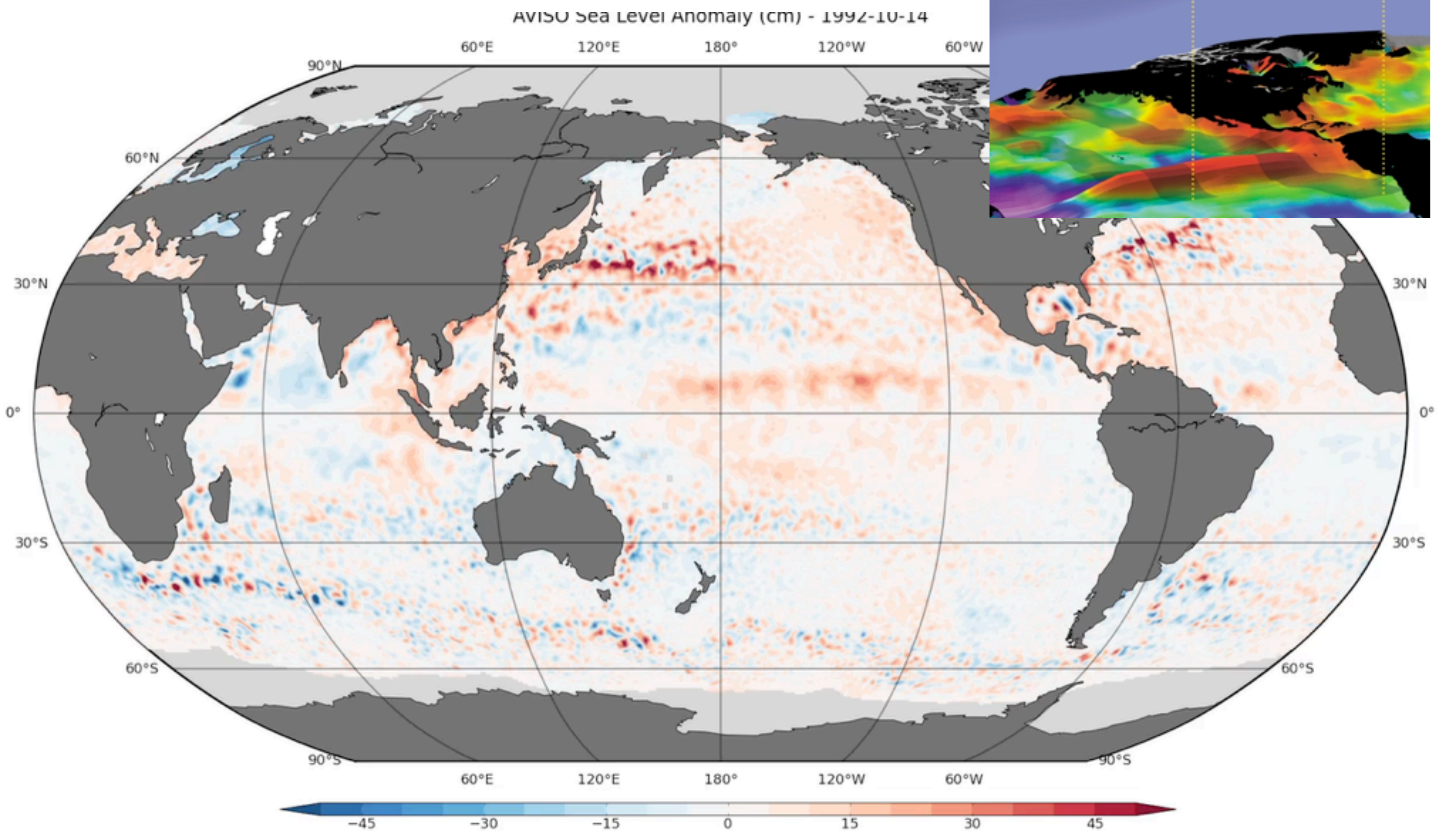
dependence on time interval

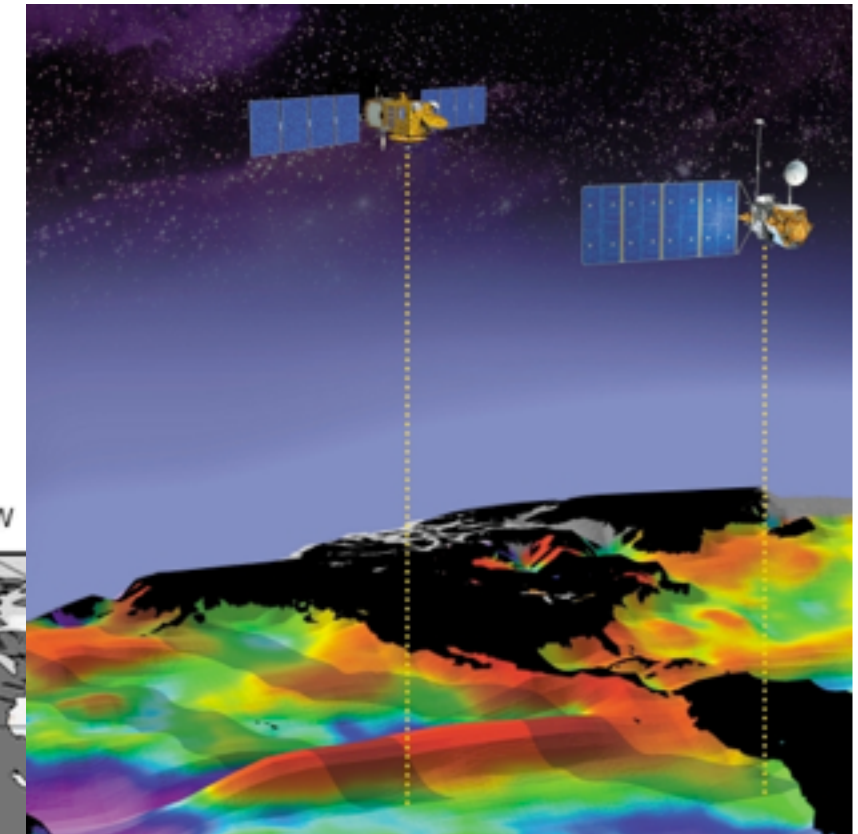


color = |LAVD|

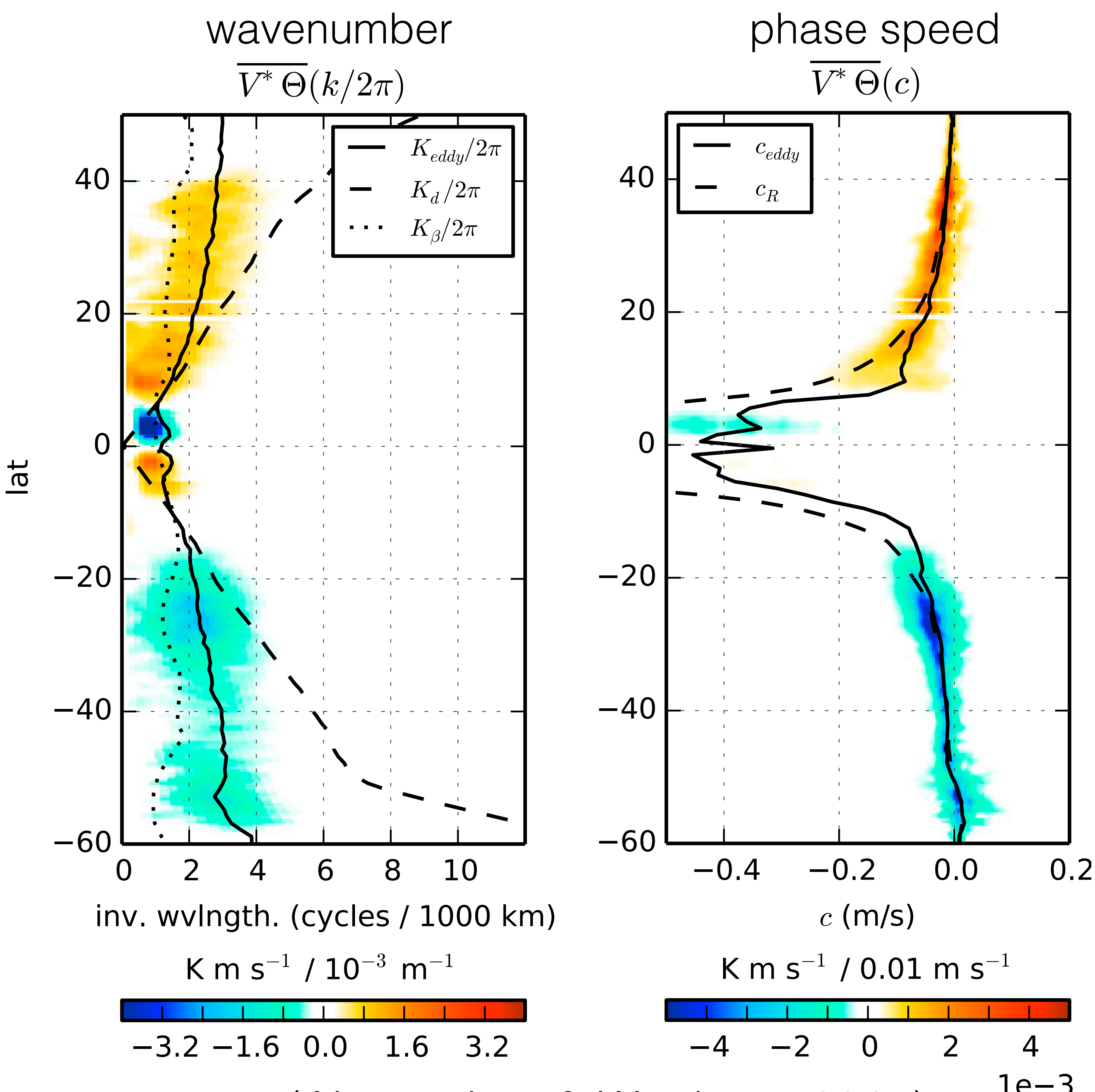
satellite altimetry
SSH is streamfunction
for geostrophic flow

$$\hat{\boldsymbol{k}} \times \boldsymbol{v}_g = -\frac{g}{f}\nabla\eta$$





East Pacific Sector - Heat Flux from Satellite Obs.



(Abernathey & Wortham, 2015)

- Cross-spectral analysis of SST and SSH-derived velocity
- Spectral peaks are aligned with the properties of tracked SSH eddies (Chelton)
- Implies an important role for coherent SSH "features" in eddy heat transport

Sea-surface-height eddies
(Chelton et al., 2011)

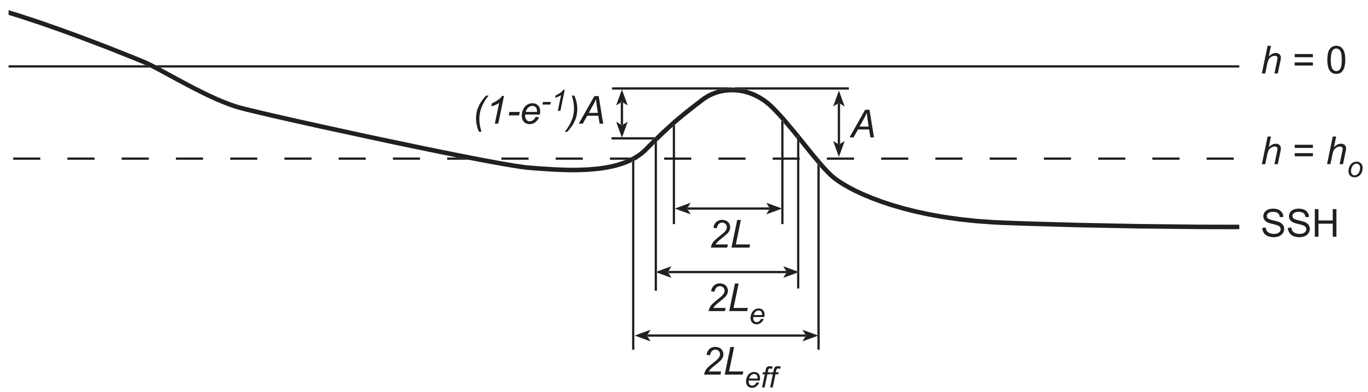
1. The SSH values of all of the pixels are above (below) a given SSH threshold for anticyclonic (cyclonic) eddies.

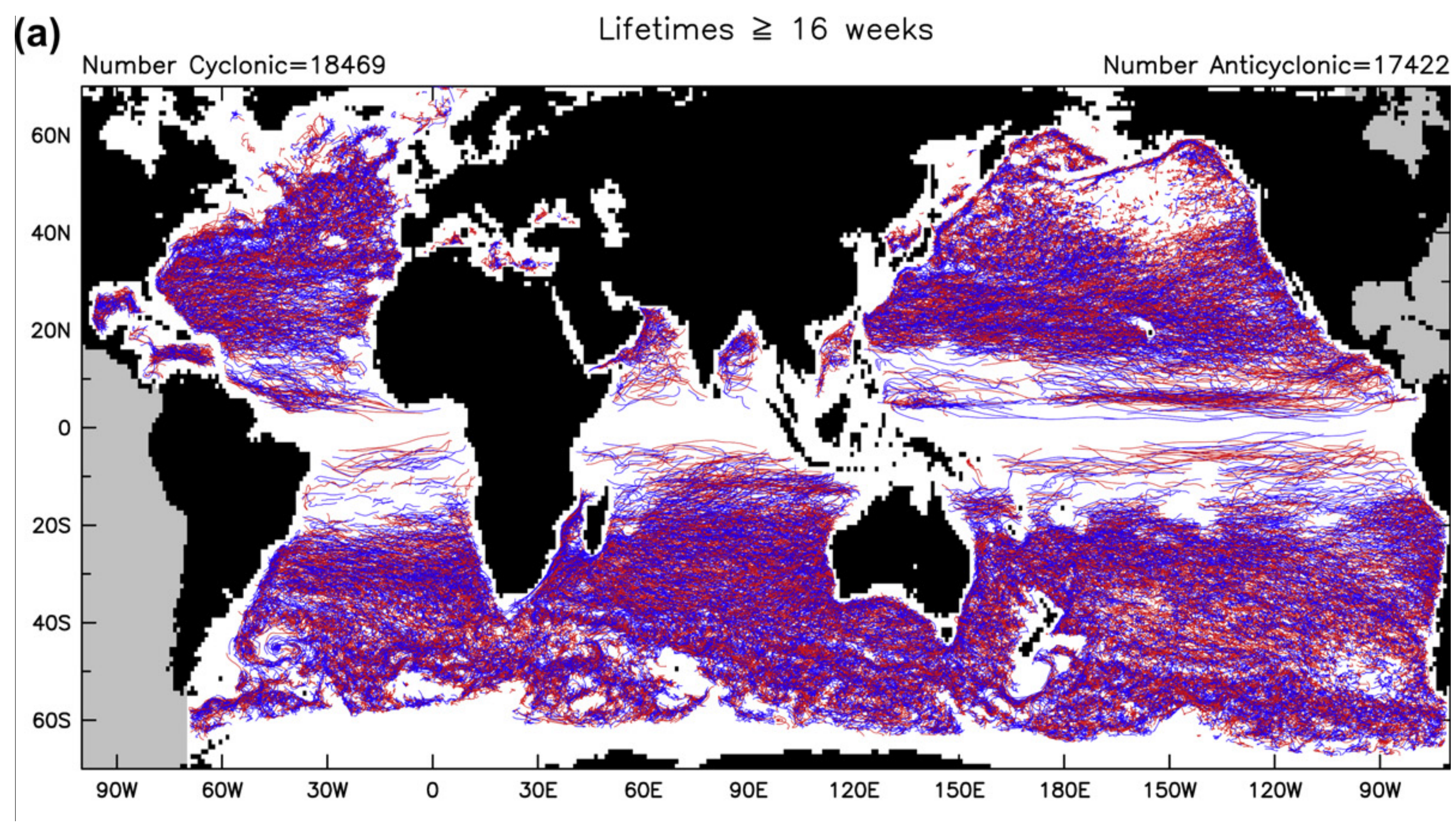
2. There are at least 8 pixels and fewer than 1000 pixels comprising the connected region.

3. There is at least one local maximum (minimum) of SSH for anticyclonic (cyclonic) eddies.

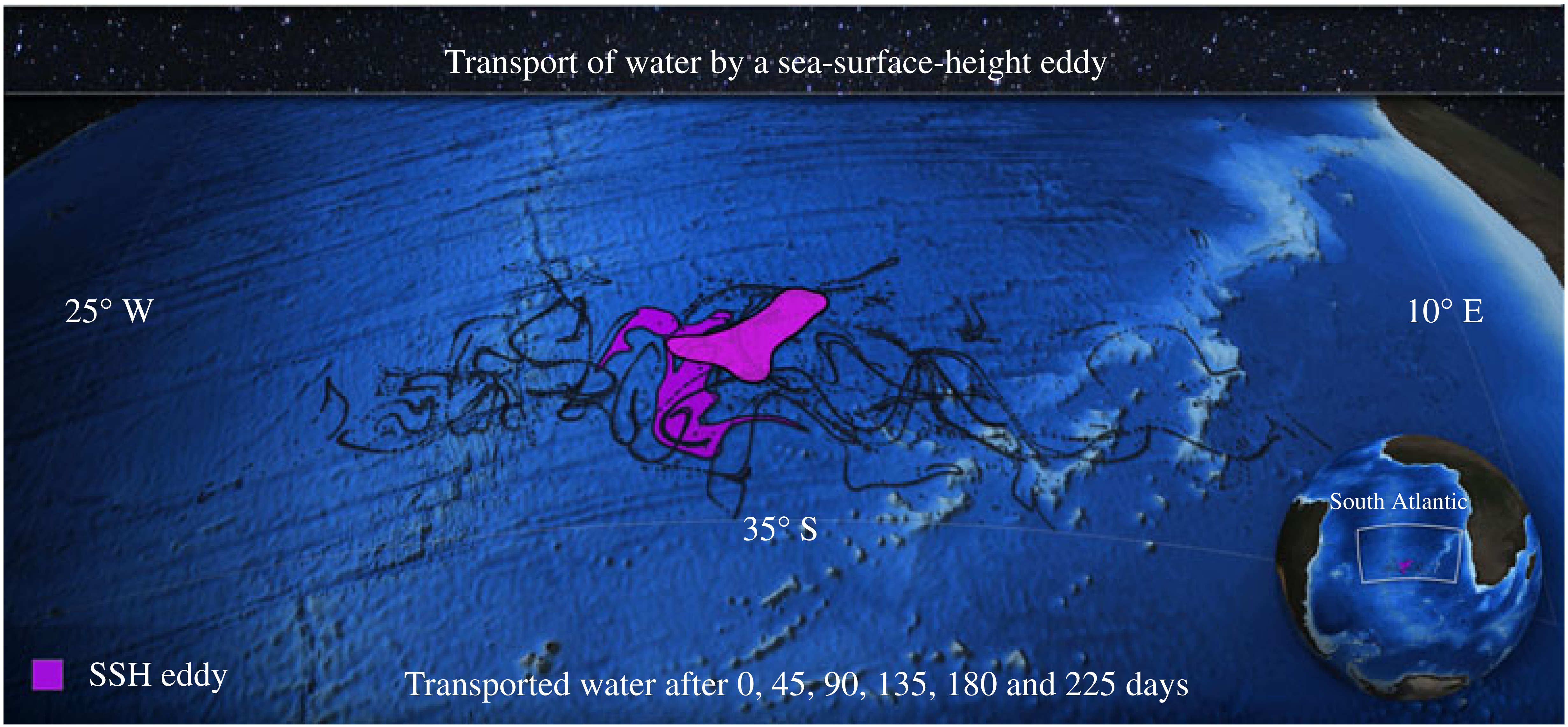
4. The amplitude of the eddy is at least 1 cm (see below).

5. The distance between any pair of points within the connected region must be less than a specified maximum.





…But SSH eddies are NOT materially coherent



"Coherently evolving velocity features tend to differ substantially from coherently moving fluid parcels…in unsteady flows"

Haller & Beron Vera (2013)

Mathematicians to the rescue…

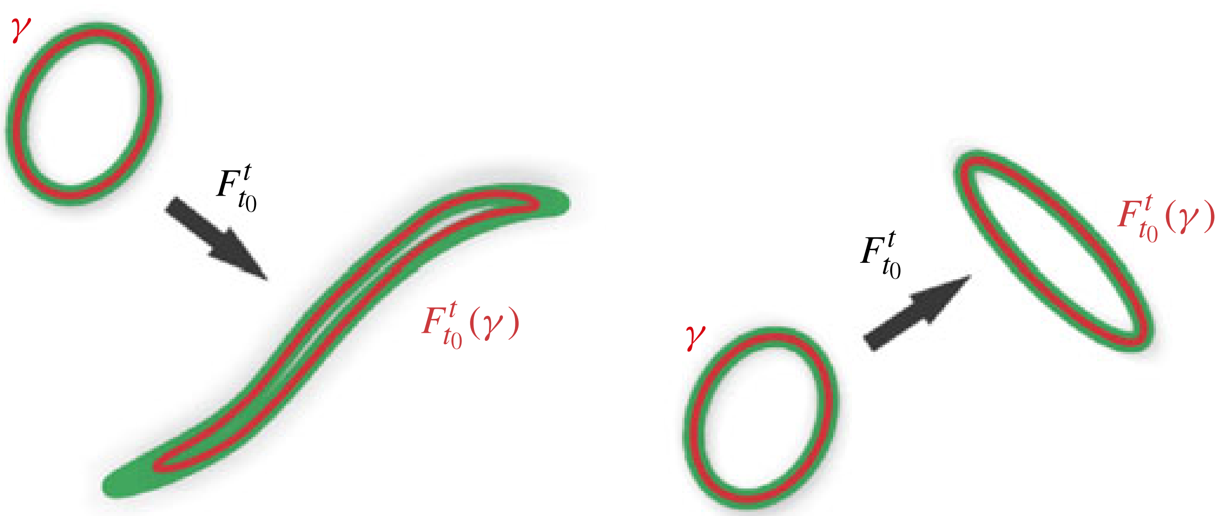
George Haller
ETH

Apply dynamical systems theory to the problem of eddy detection.

Method must be:

- Objective / frame-invariant (observers in all references frames would find the same eddies)
- Free of arbitrary “tuning parameters”

Solution: use Lagrangian “flow map” to identify exceptionally coherent material lines ⇒ *Lagrangian Coherent Structures* (LCS)



Typical material belt   Coherent material belt

flow map:

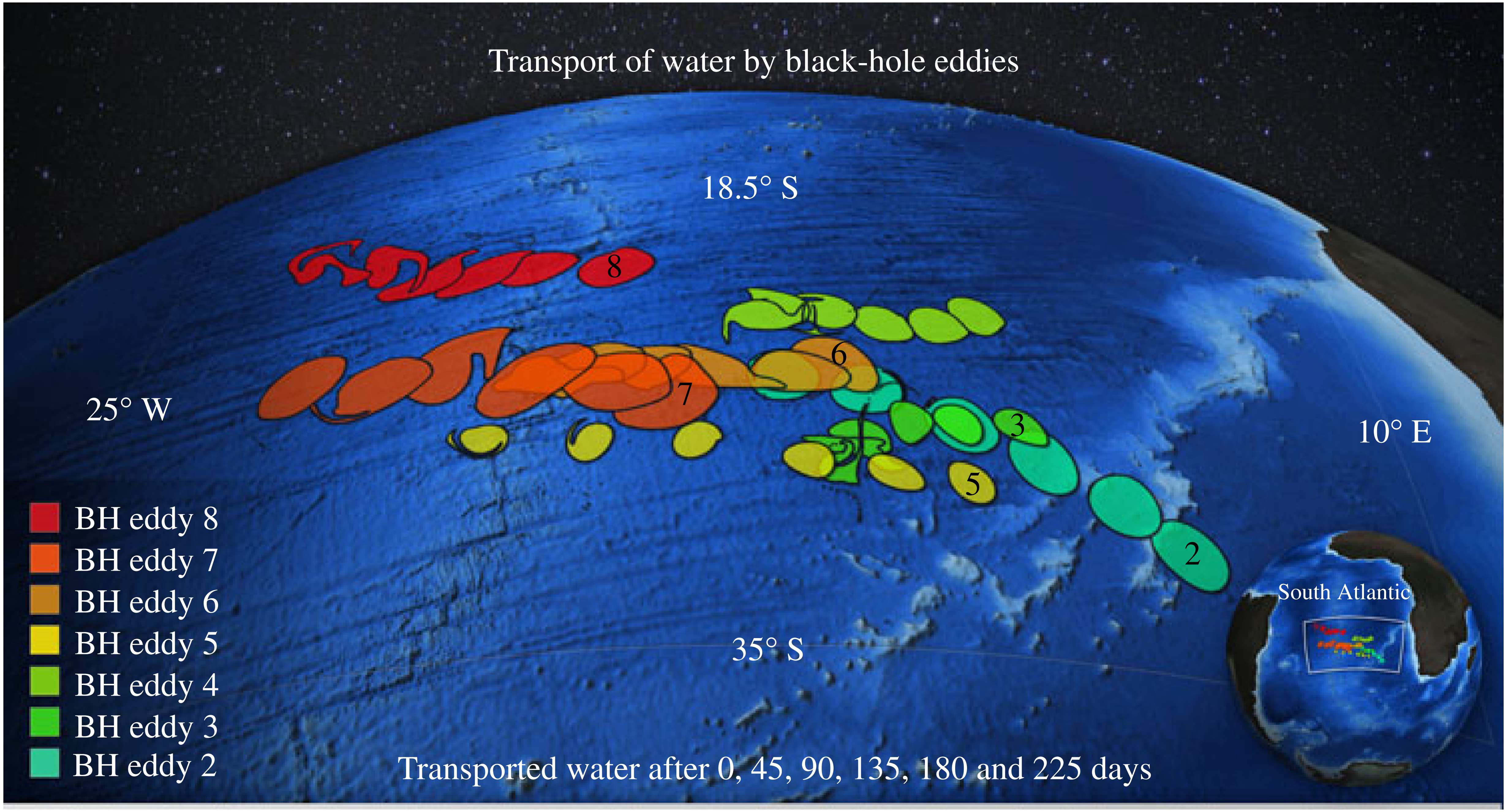
$$F_{t_0}^t(x_0) := x(t; t_0, x_0), \qquad x \in U, \quad t \in [t_0, t_1],$$

Cauchy-Green strain tensor:

$$C(x_0) = \left[\nabla F_{t_0}^{t_1}(x_0)\right]^T \nabla F_{t_0}^{t_1}(x_0).$$

Haller, G., 2015: Lagrangian Coherent Structures. Ann. Rev. of Fluid Mech., 47, 137–162.

Mathematicians to the rescue…



Computing The Flow Map

initial position $\boldsymbol{x}_0 \rightarrow \boldsymbol{x}_1$ *final position*

- Initialize a dense “mesh” of Lagrangian particles over the whole domain (~8 million particles)
- Solve the equation $\frac{d}{dt}\boldsymbol{x} = \boldsymbol{v}_g$ for each particle.
- Run forward for 30, 90, 270 days.
- Repeat for each segment of the ~22 year dataset (1993 - 2015)
- ~10 TB of data generated

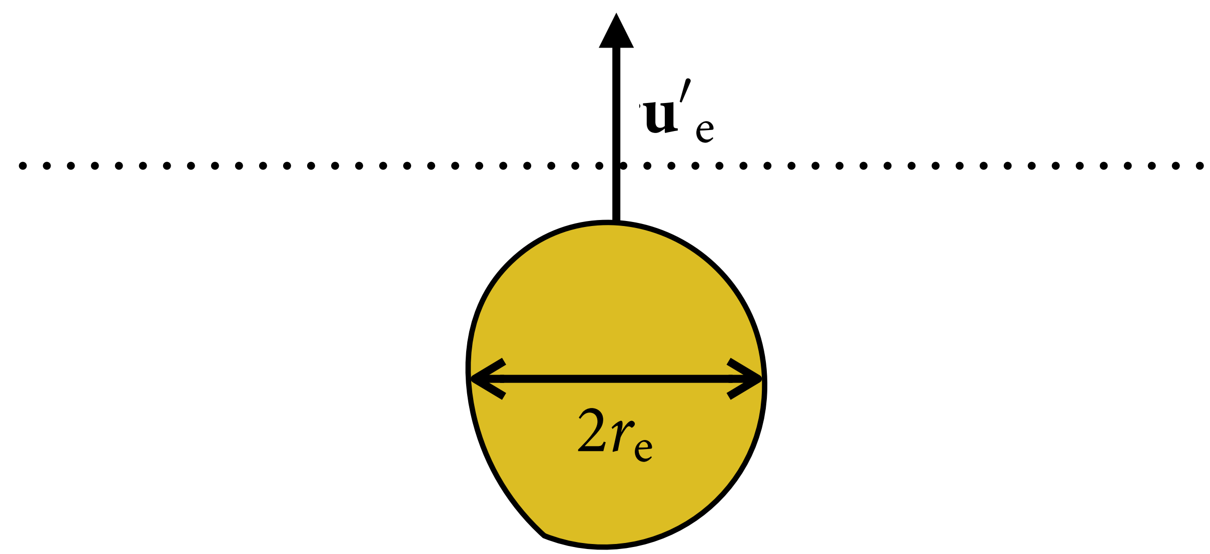
“Transport” by a coherent eddy

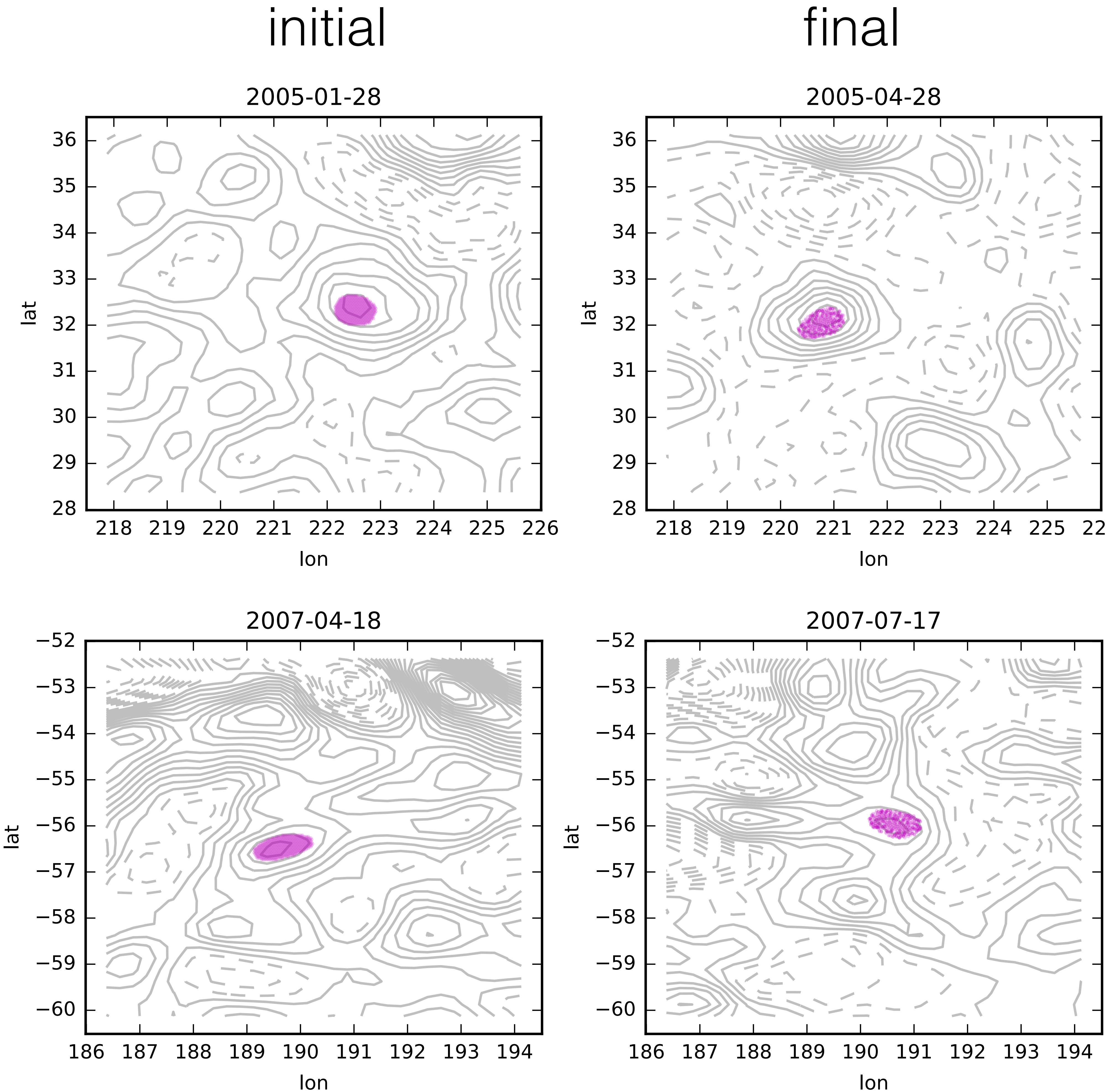
$$\mathbf{Q}_{\mathrm{eh}} = s\mathbf{u}'_{\mathrm{e}} \int dz \rho_{\mathrm{o}} C_{\mathrm{po}} (2r_{\mathrm{e}}) T'_{\mathrm{e}},$$

$$\mathbf{T}_h = N_{\mathrm{e}} \langle \mathbf{Q}_{\mathrm{eh}} \rangle,$$

“…eddy heat and salt transports are mainly due to individual eddy movements.”

Dong et al. (2014)





initial
final
2005-01-28
2005-04-28
2007-04-18
2007-07-17
lat
lon
36
35
34
33
32
31
30
29
28
218
219
220
221
222
223
224
225
226
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−53
−54
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186
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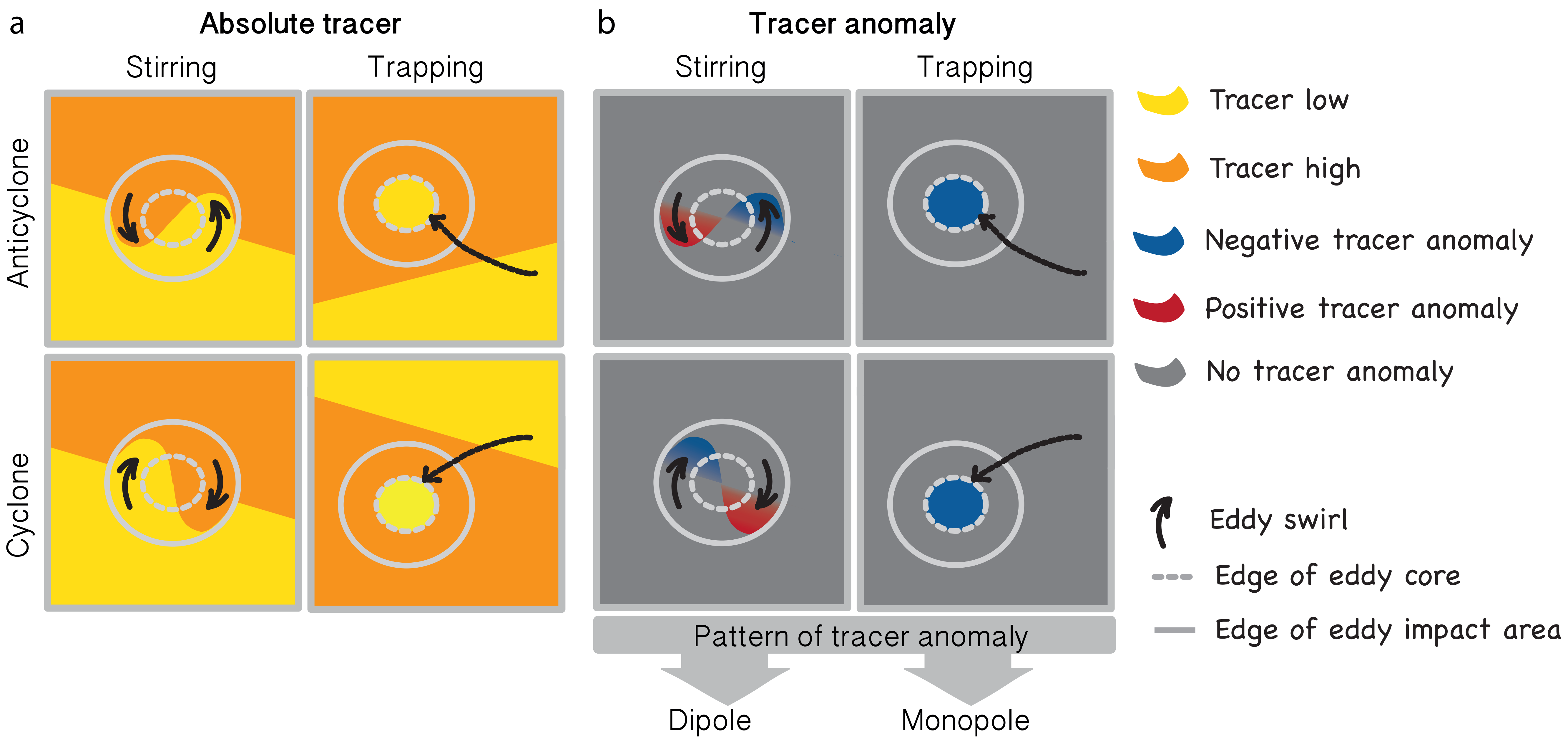
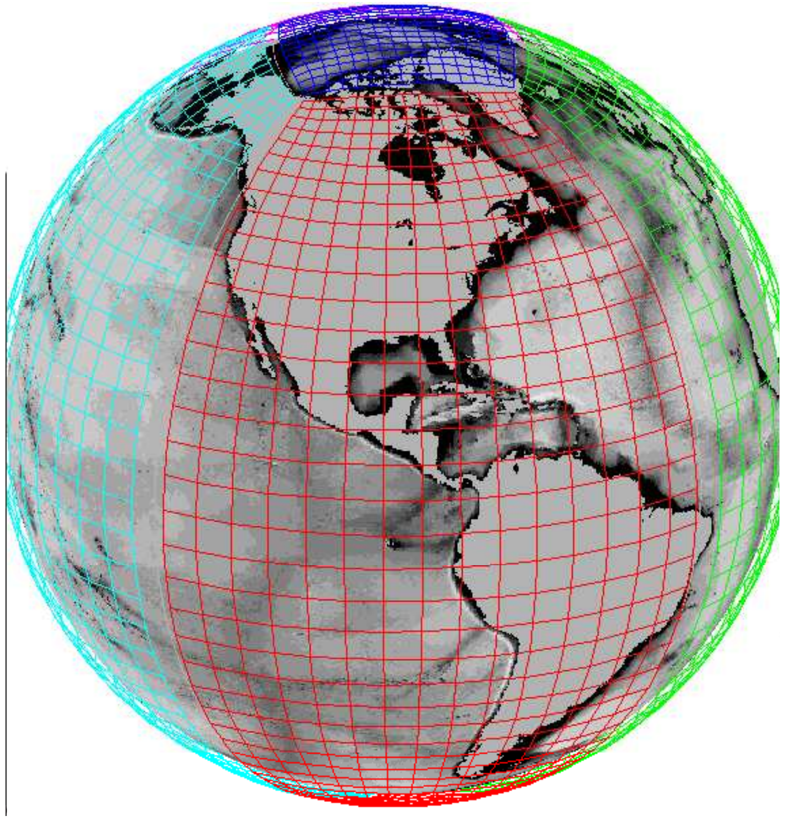
194

Phenomenology of eddy flux



Frenger et al. (2015)

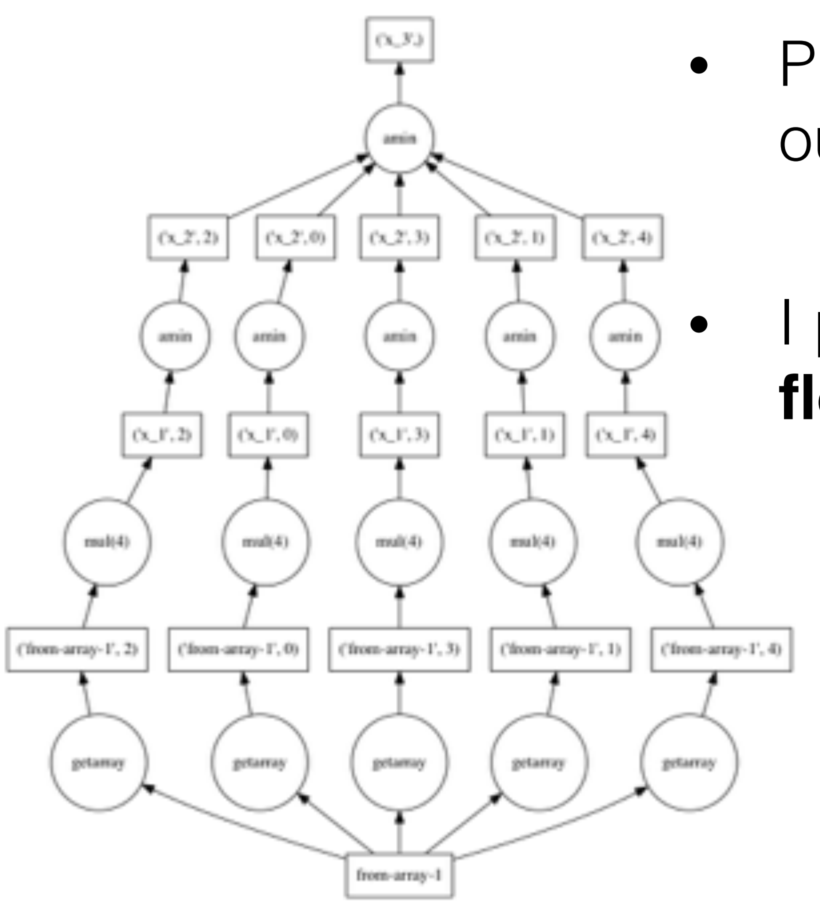
The Tools: MITgcm for Lagrangian Advection

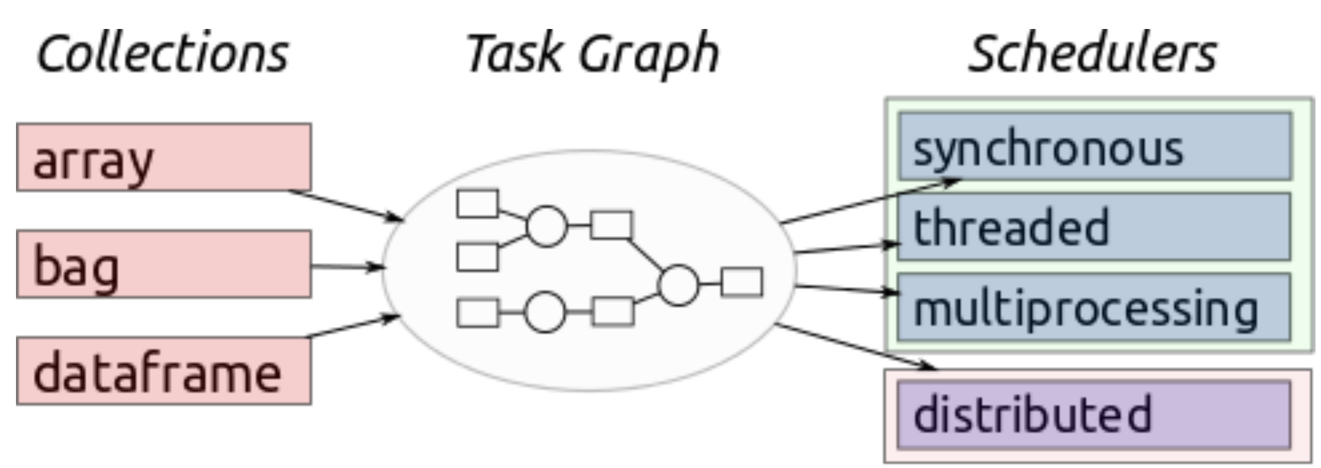


- Has a built in Lagrangian "particle tracking" via FLT package
- Originally designed to simulate ARGO floats
- Full 3D RK4 with trilinear interpolation in space
- "Turbulence" options, but I didn't use them
- Combined with "offline mode," which reads velocity fields from files
- Fully MPI parallelized, scales to 100s of processors
- IO bound

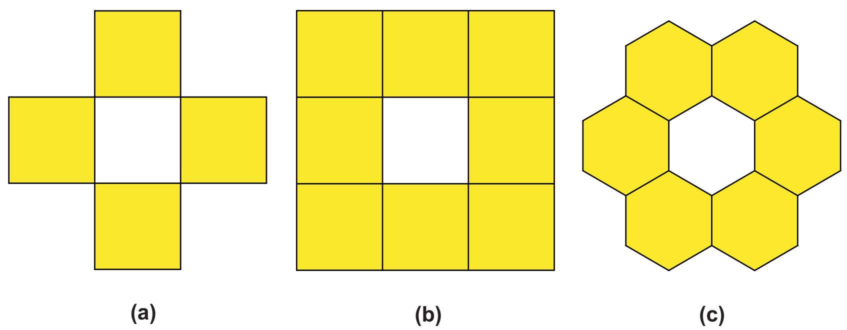
The Tools: PyTables and Dask for Post-Processing

- 8 million particles x 5 fields (X,Y,U,V,ω) x 90 days x 32 bits = ~40 GB
- This is “medium data”: fits on a hard drive, but probably not in RAM
- Binary output first converted to **index HDF5** via python PyTables package. Facilitates fast searching and efficient access.
- Processed using **dask**, a parallel task scheduler designed for out-of-core data analysis on multi-core systems
- I put these ingredients together in a lightweight package called **floater**: https://github.com/rabernat/floater



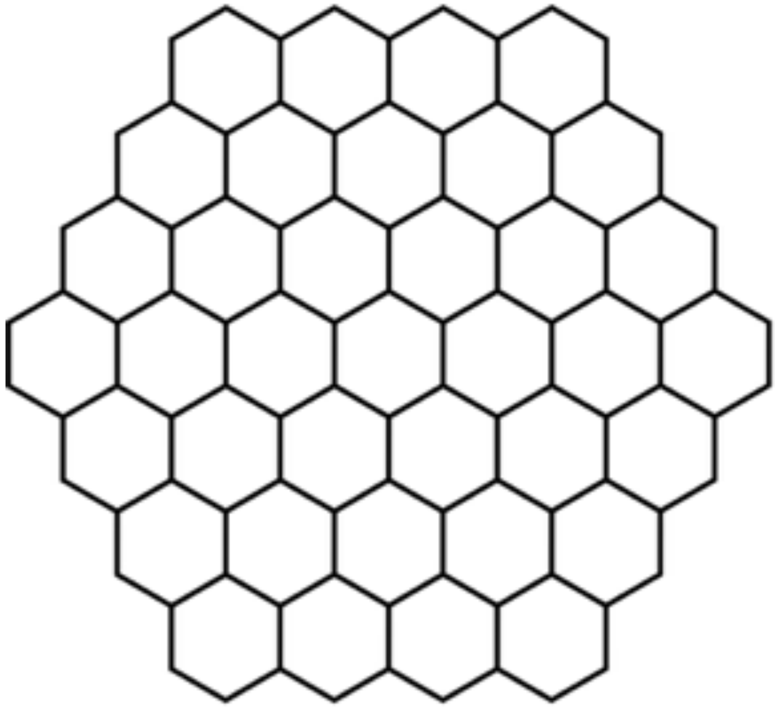


Identifying RLCVs



particles deployed using *hexagonal grid tessellation*
→ unambiguous identification of critical points (extrema and saddles)

- Start from maxima of LAVD
- Add neighboring points until region boundary is no longer convex
- Filter out RLCVs smaller than a certain size threshold (50 km)



Learning from Other Fields

- Global “dense” Lagrangian simulation in high-resolution OGCM would generate ~2 PB of trajectory data
- Compare to “dark sky simulation”
  http://darksky.slac.stanford.edu/
- “ds14_a is the first trillion particle simulation released to the public, and is being used to study the very largest scales in the Universe. It spans a cubical region 8 Gpc/h on a side, nearly 40 billion light years across. With 102403 (~1.07 trillion) particles, we follow the growth of the largest clusters of galaxies, weighing in at more than a quadrillion times the mass of our own sun.”
- Open data access through http
  http://darksky.slac.stanford.edu/edr.html

"Eddy Fluxes" (Eulerian View)

$$\theta = \theta(x, y, t)$$

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \boldsymbol{v} \cdot \nabla\theta$$

advection of a scalar (e.g. temperature)

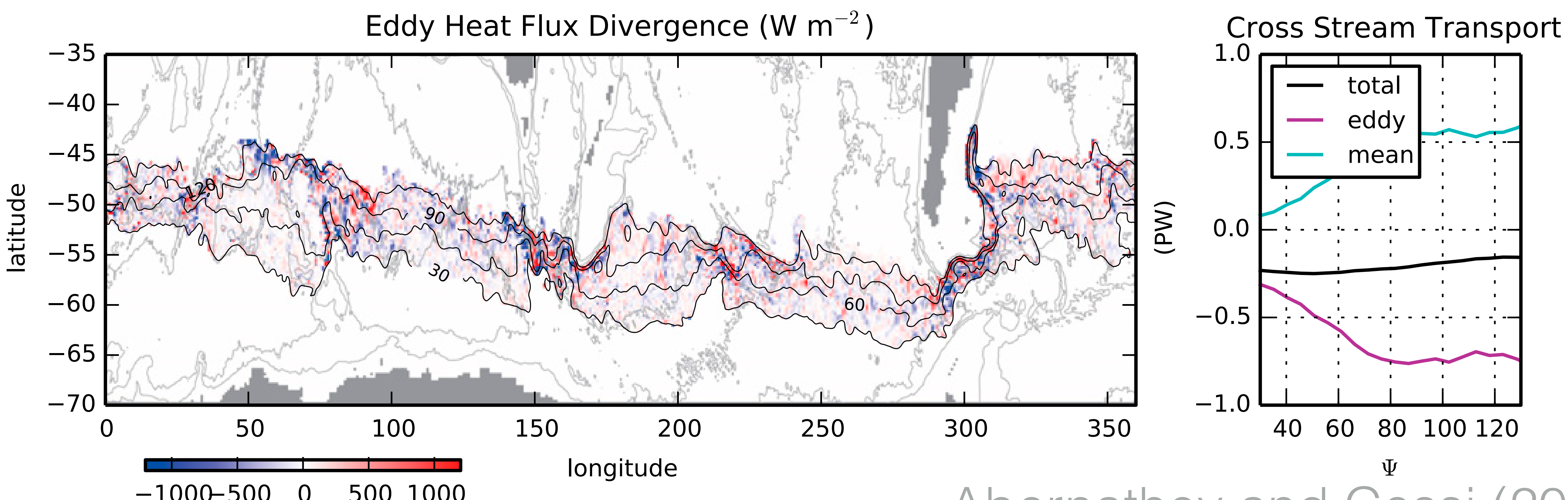
Reynolds average:

$\overline{(\ )} =$ time average

$(\ )' =$ fluctuation / "eddy"

$$\frac{\partial\overline{\theta}}{\partial t} + \overline{\boldsymbol{v}} \cdot \nabla\overline{\theta} = \nabla \cdot (\overline{\boldsymbol{v}'\theta'})$$

"eddy flux"



Abernathey and Cessi (2014)