Estimation of Conductances

C. Vich[1,★], R.W. Berg[2], S. Ditlevsen[3], A. Guillamon[4], R. Prohens[1] and A.E. Teruel[1]

1. Departament de Matemàtiques i Informàtica, Universitat de les Illes Balears
2. Department of Neuroscience and Pharmacology, University of Copenhagen
3. Department of Mathematical Sciences, University of Copenhagen
4. Departament de Matemàtiques, Universitat Politècnica de Catalunya

Introduction to the problem of the conductances' estimation

- Estimating the synaptic conductances impinging on a single neuron directly from its membrane potential is one of the open problems to be solved to understand the flow of information in the brain.
- In the literature, one can find some computational strategies that give circumstantial solutions assuming the absence of spiking regimes and nonlinear subthreshold ionic currents.
- The main constraint to provide strategies is related to the nonlinearity of the input-output curve and the difficulty to compute it.
- Anomalies in the subthreshold estimation have been detected when linear techniques are used.

In this work,

1. **We propose a quadratic method to estimate conductances in the subthreshold regime.**
2. **We propose an analytical method to estimate conductances in the spiking regime.**

Estimation in the subthreshold regime with activated ionic currents

[C.Vich, R.W.Berg, S.Ditlevsen, A.Guillamon; under review]

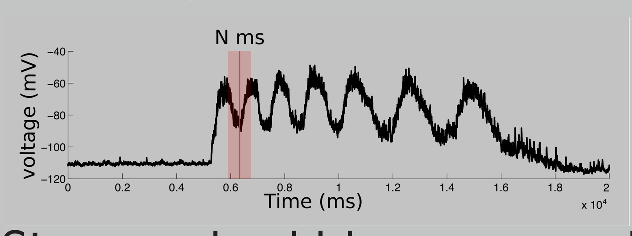
Subthreshold Estimation Procedure

Model: We consider the stochastic version of the **Quadratic Integrate and Fire (QIF) model**

$$C\frac{dV}{dt} = \alpha\,(V(t)-V_T)^2 - I_E(t) - I_I(t) - I_T + I_{app} + \eta(t)$$
$$I_E(t) = g_E(t)\,(V(t)-V_E),$$
$$I_I(t) = g_I(t)\,(V(t)-V_I)$$

Estimation Procedure: Write the system as the quadratic SDE $dV = (aV^2 + bV + c)dt + \sigma dW_t$ where a, b and c linearly depend on α, g_E and g_I. Then,

1. Using the Euler method, we discretize the diffusion process as $V_{n+1} = V_n + (aV_n^2 + bV_n + c)\Delta + \sigma\sqrt{\Delta}\xi_{n+1}$, where $\xi_{n+1} \sim \mathcal{N}(0,1)$ and $\Delta = t_{n+1} - t_n$.
2. In a sample window W of length N ms, we use the MLE to



i) Compute $\hat{a}$, $\hat{b}$, $\hat{c}$ in W
ii) And so, $\hat{\alpha}$, $\hat{g}_E$, $\hat{g}_I$ in W

3. Since α should be constant, let $\alpha = mean_t\hat{\alpha}(t)$ and use again the MLE to estimate g_E and g_I, suposing α known.

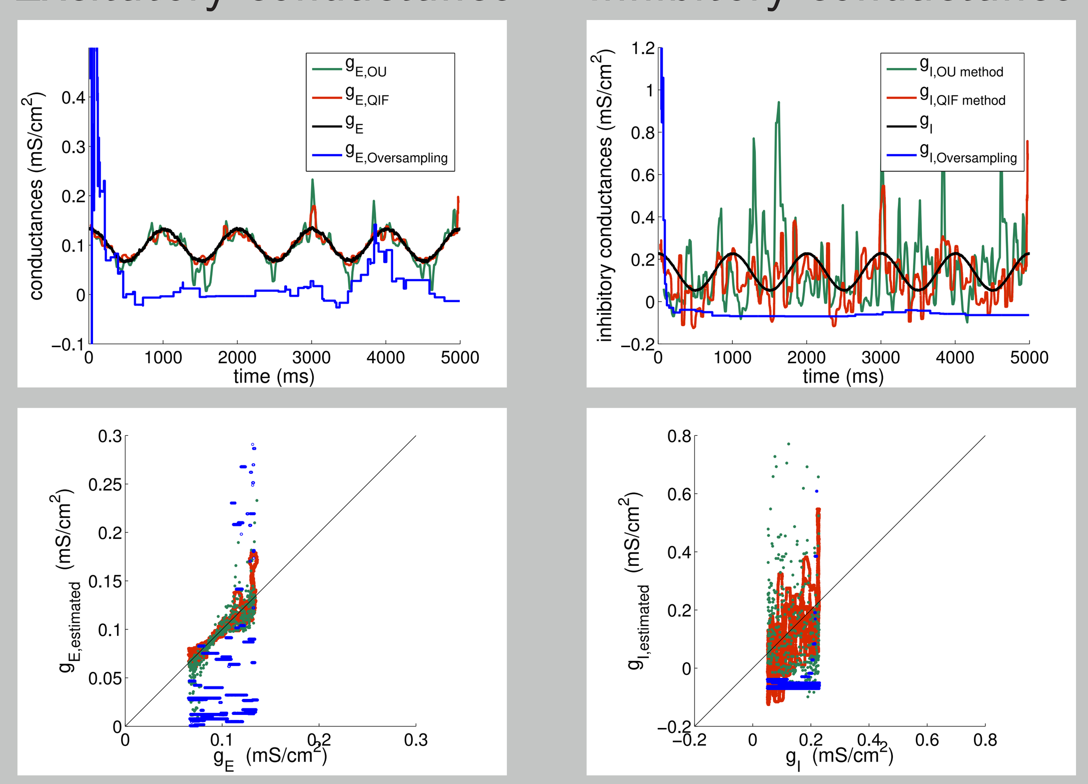
Results of the Estimation using in silico data

In silico data from a pyramidal cell model with I_T, I_{AHP}, I_{Na} and I_K currents

[Vich and Guillamon (2015)]

Results show an improvement when they are compared with different linear procedures.

Excitatory conductance | Inhibitory conductance



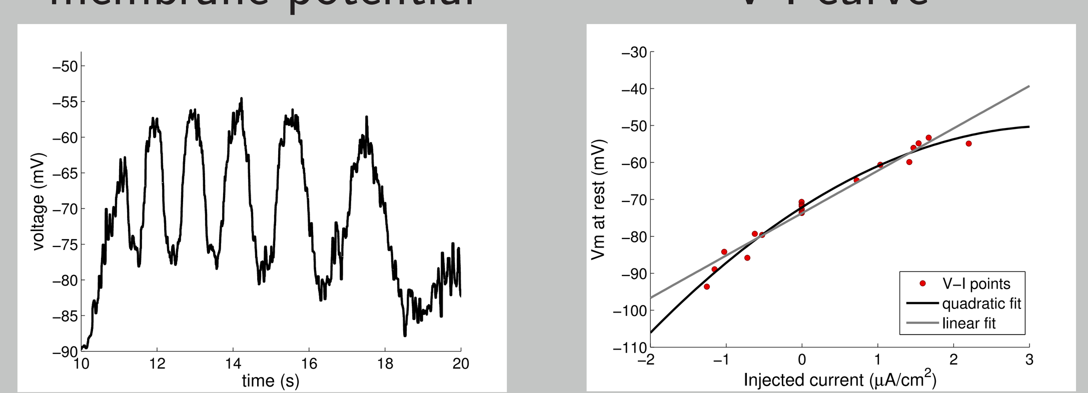
Legend: - Linear deterministic method
- Linear stochastic method
- Quadratic stochastic method

Results of the Estimation using in vivo data

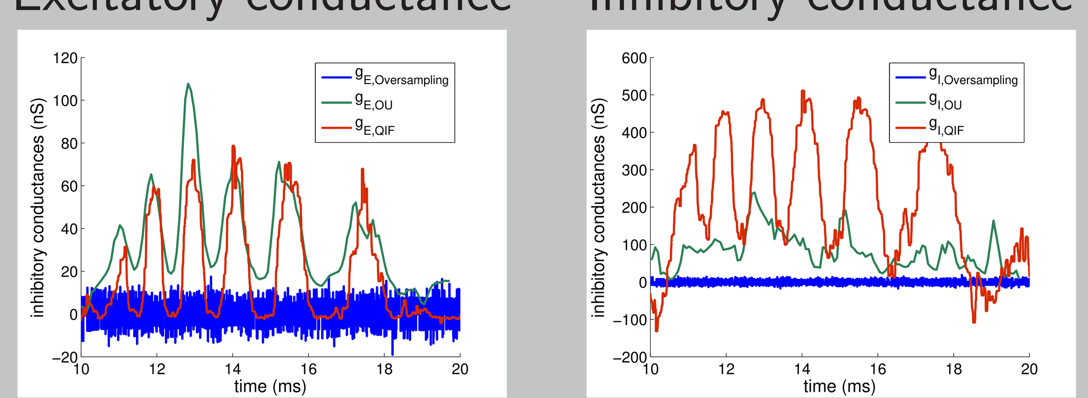
Intracellular recordings in current-clamp mode of spinal motoneurons of red-eared turtles

[R.W. Berg].

membrane potential | V-I curve



Excitatory conductance | Inhibitory conductance



Estimation in the oscillatory regime

[A.Guillamon, R.Prohens, A.E.Teruel, C.Vich; under review]

Oscillatory Estimation Procedure

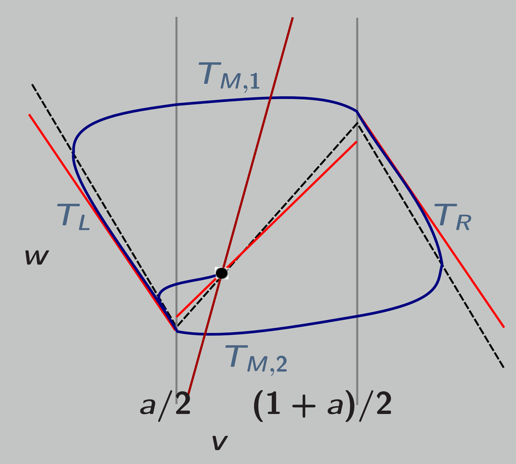
Model: We consider the **McKean model** given by

$$\begin{cases} C\dot{v} = f(v) - w - w_0 + I - I_{syn}, \\ \dot{w} = v - \gamma w - v_0, \end{cases}$$

where

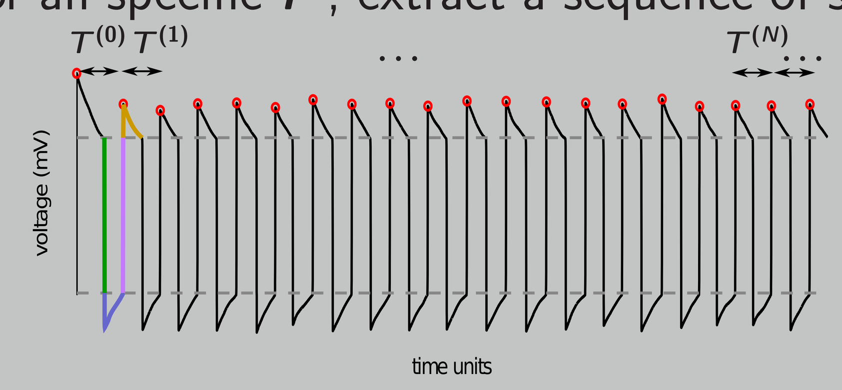
$$f(v) = \begin{cases} -v, & v < a/2, \\ v - a, & a/2 \leq v \leq (1+a)/2, \\ 1 - v, & v > (1+a)/2. \end{cases}$$

$$I_{syn} = g_{syn}(v - v_{syn})$$



Estimation Procedure: Using approximated expressions for T_L, $T_{M,1}$, T_{M_2}, and T_R, which have a nonlinear dependence on g_{syn},

1. Given $v(t)$ for an specific I^*, extract a sequence of subperiods such that

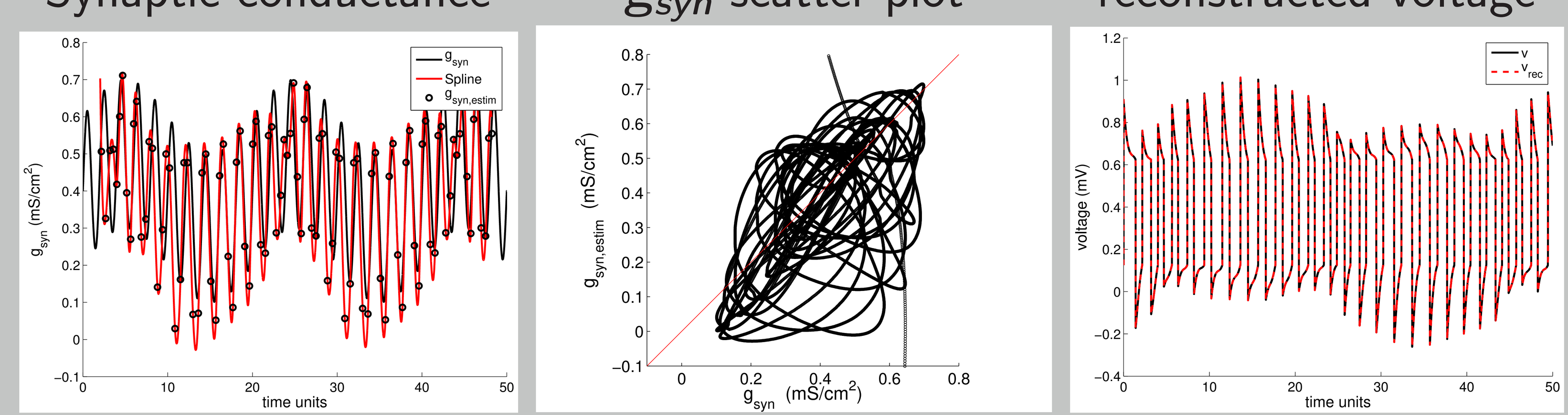


2. $\{T_L^{(k)},\ T_{M,1}^{(k)},\ T_R^{(k)},\ T_{M,2}^{(k)};\}_{k=1}^{N} \xrightarrow[\text{for all } i\in\{L;M,1;M,2;R\},0\leq k\leq N]{\text{Solve } \hat{T}_i(g_{syn})=T_i^{(k)}} \{g_{syn}^{(m)}\}_{m=1}^{4N}$
3. Interpolate $\{(t^{(m)}, g_{syn}^{(m)})\}_{m=1}^{4N}$ to obtain $g_{syn}(t)$.

Results of the Estimation using in silico data

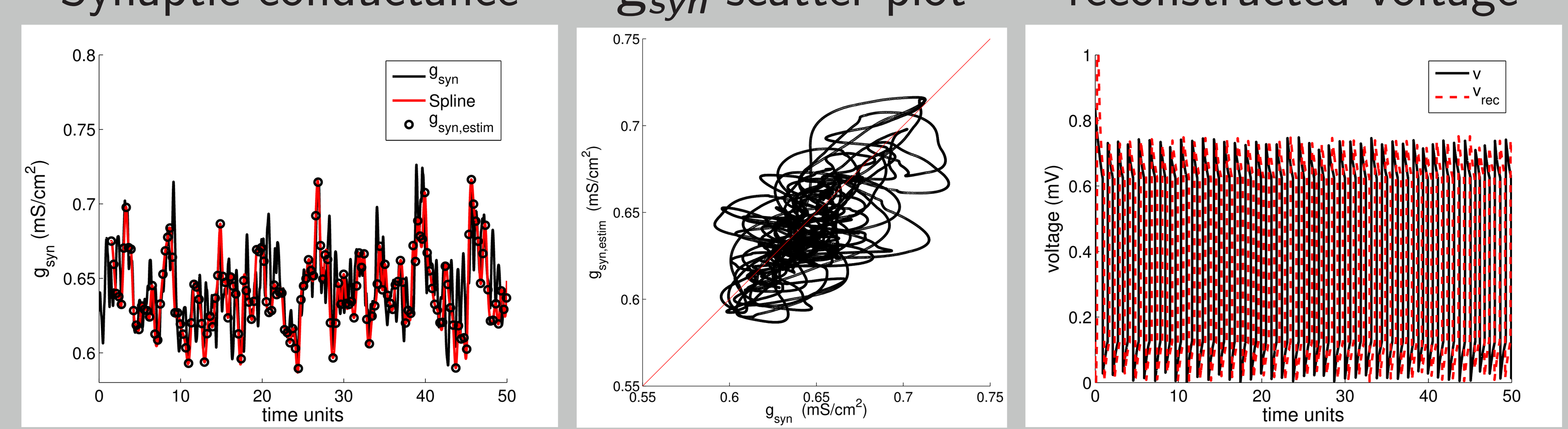
- Using prescribed conductances with doble frequency

Synaptic conductance | g_{syn} scatter plot | reconstructed voltage



- Using prescribed conductances from a V1 computational network [Tao et al. (2004)]

Synaptic conductance | g_{syn} scatter plot | reconstructed voltage



Conclusion

- In the subthreshold regime, the Quadratization procedure substantially improves the estimation.
- In the oscillatory regime, we have presented a useful method to estimate conductances when they vary slowly in time. even though they are not perfectly estimated through time for the netwok case, we do capture the accurate mean conductance. This estimation could be improved by using a non-autonomous system.

MTM2014-54275-P and MTM2015-71509-C2-2-R (AEI/FEDER, UE). Obra Social ''La Caixa''.
★ catalina.vich@uib.es