A NUMERICAL INVESTIGATION OF NON-SYMMETRIC NONLINEAR WATER WAVES

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COWORKERS

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- inviscid, incompressible, irrotational
- gravity
- surface tension
- steady

NON SYMMETRIC WAVES....in two and three dimensions

- Periodic waves
- Solitary waves
- Generalised Solitary waves

flexural waves (thursday.....).

stability

PART 1

TWO-DIMENSIONAL FLOWS

FORMULATION

GRAVITY-CAPILLARY WAVES

$$\phi_{xx} + \phi_{yy} = 0$$

$$\phi_y = \phi_x \zeta_x \quad \text{on} \quad y = \zeta(x)$$

$$\frac{1}{2}(\phi_x^2 + \phi_y^2) + gy - \frac{T}{\rho}\kappa = B \quad \text{on} \quad y = \zeta(x)$$

$$\phi_y = 0 \quad \text{on} \quad y = -h$$

FLEXURAL WAVES

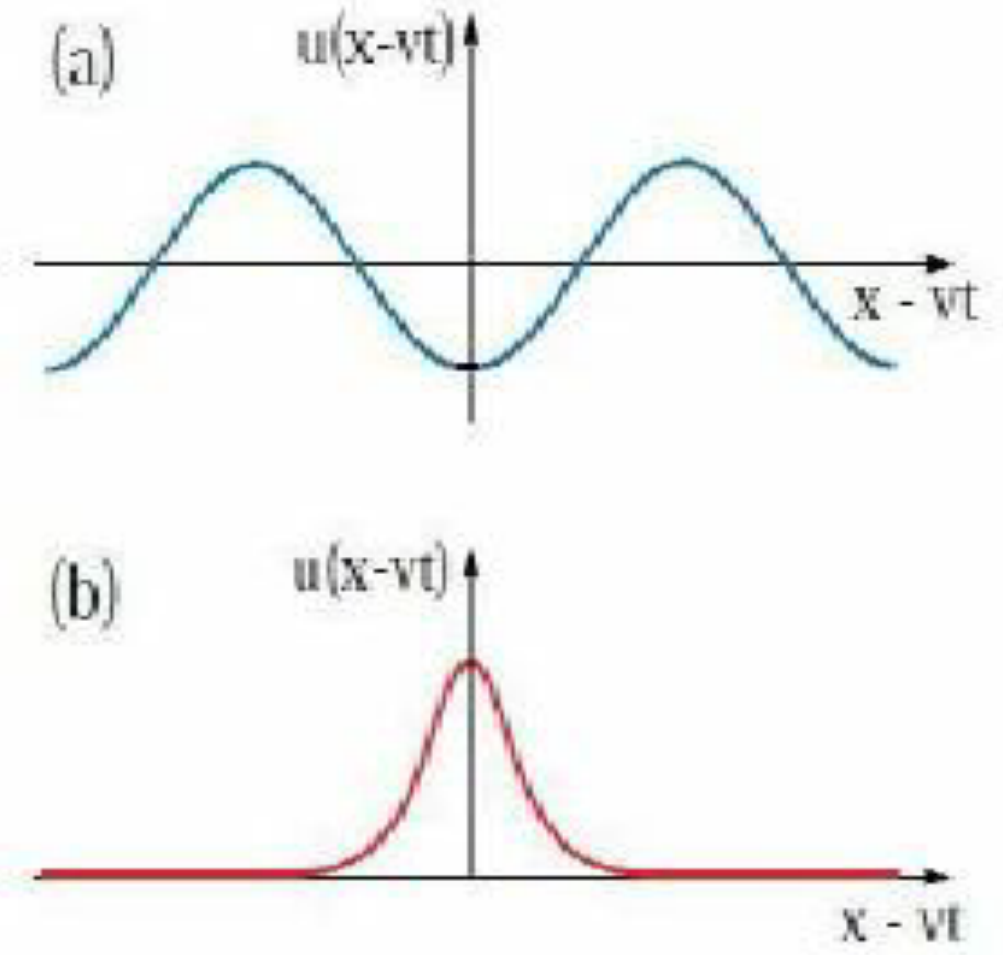
$$\frac{D}{\rho}(\partial_s^2 \kappa + \frac{1}{2}\kappa^3)$$

$T =$ surface tension, $D =$ flexural rigidity

$$\kappa = \frac{\zeta_{xx}}{(1 + \zeta_x^2)^{3/2}}$$

PERIODIC and SOLITARY waves

Gravity waves



Craig W. and Sternberg P. (1988)

NUMERICAL METHODS

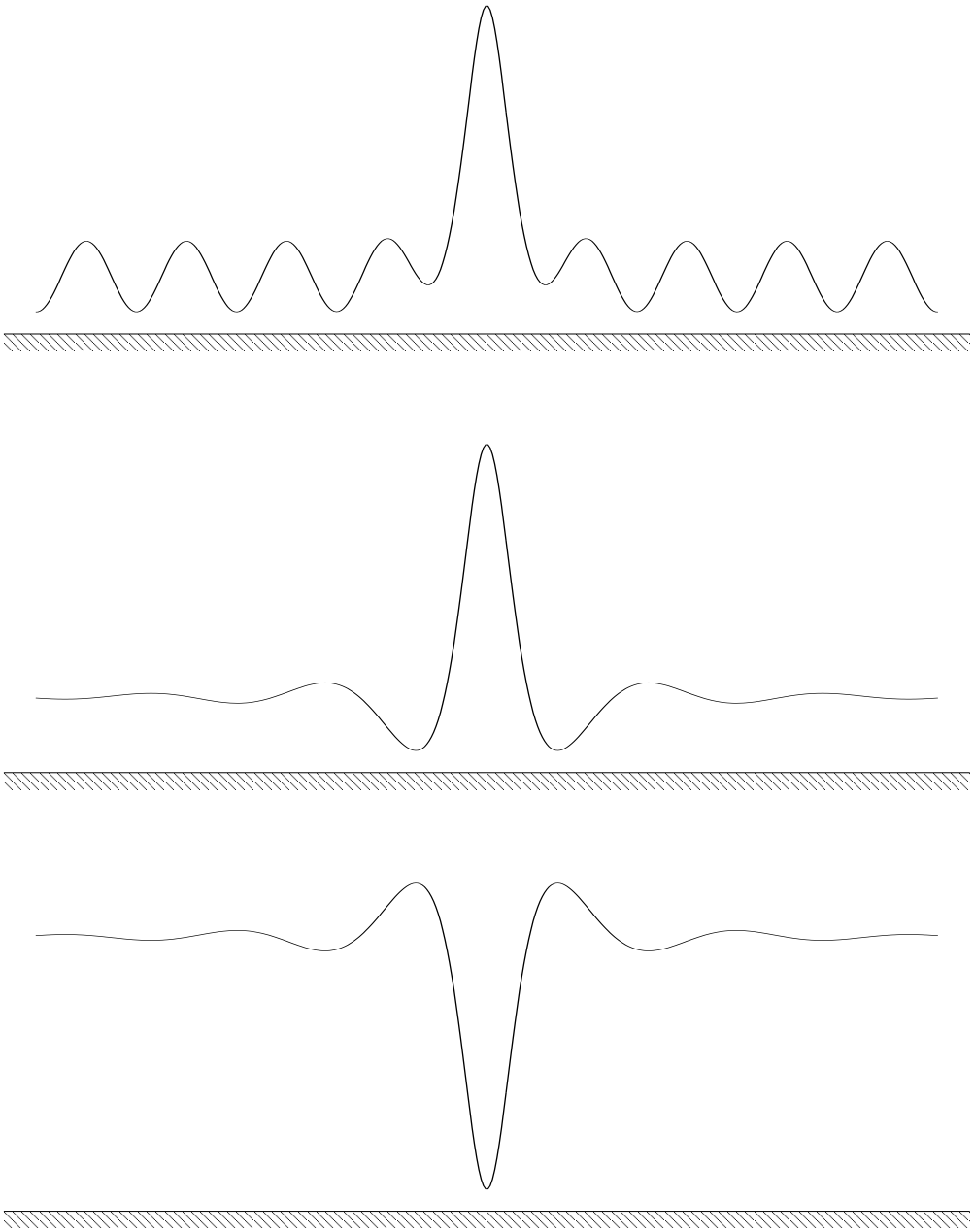
boundary integral equation methods, series truncation methods or ANY OTHER METHODS....

1. Iterations by using Newton's method

2. Continuation methods

3. INITIAL GUESS: bifurcations, symmetry breaking...

Gravity-capillary solitary waves



Dimensioless variables: $(\frac{T}{\rho g})^{1/2}$ (reference length), $(\frac{T}{\rho g^3})^{1/2}$ (reference time)

amplitude: A

phase velocity: c

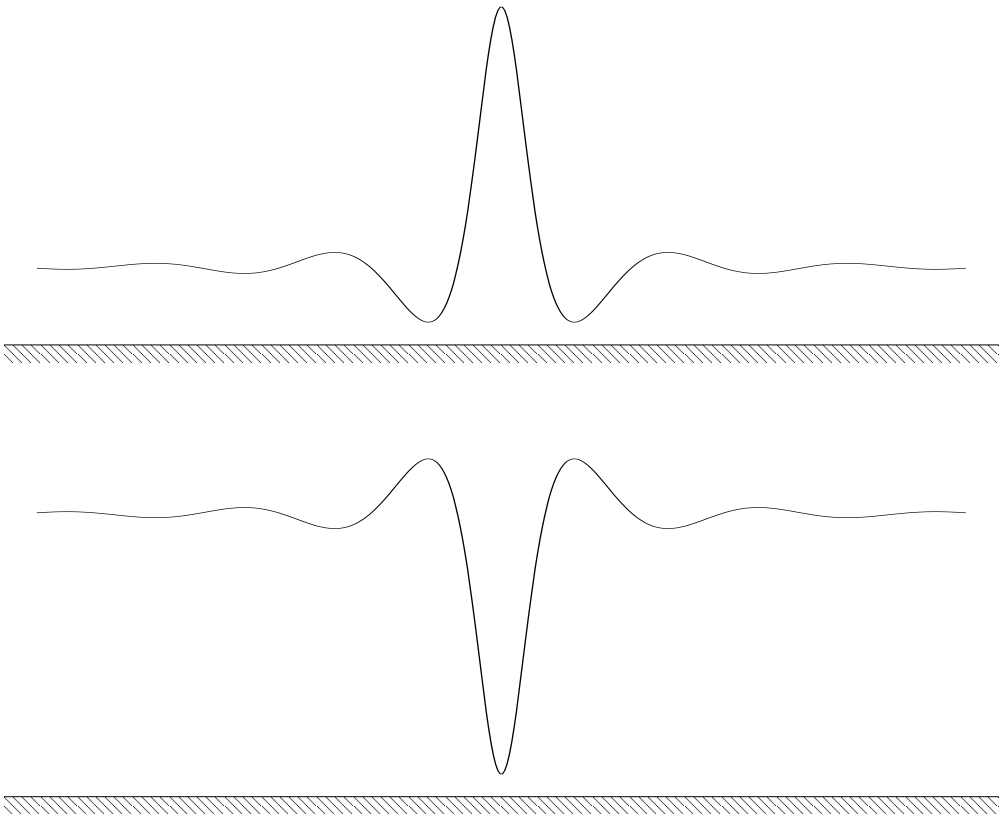
energy: E

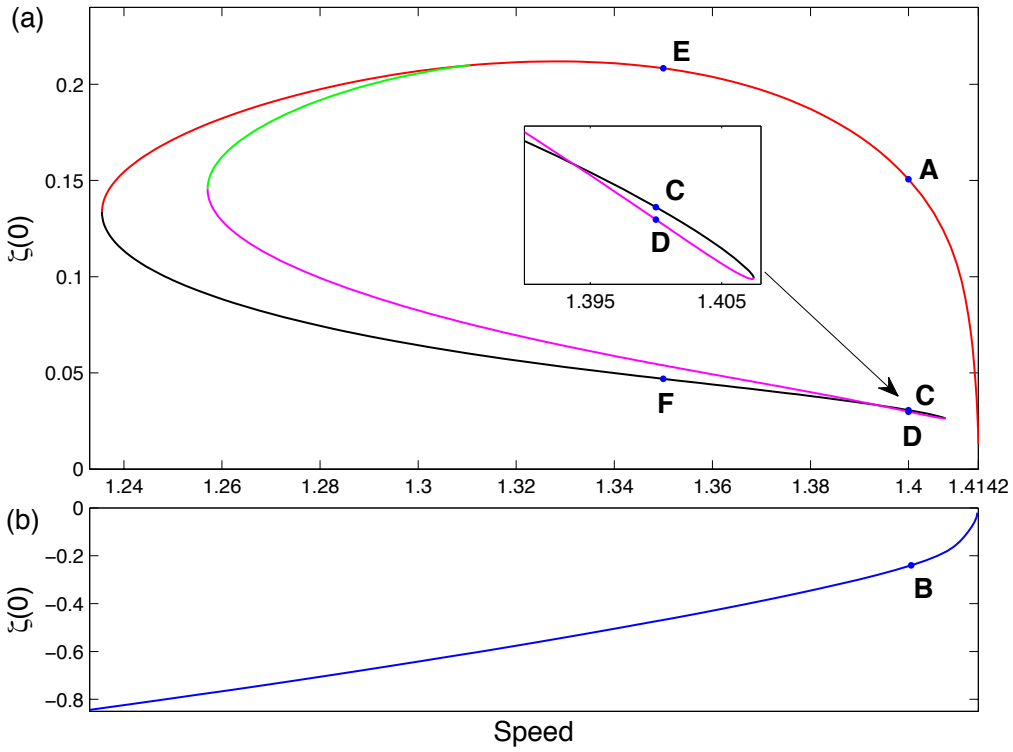
$$E = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\eta}(\phi_x^2 + \phi_y^2)dydx + \frac{1}{2}\int_{-\infty}^{\infty}\eta^2 dx$$
$$+ \int_{-\infty}^{\infty}(\sqrt{1+\eta_x^2} - 1)dx$$

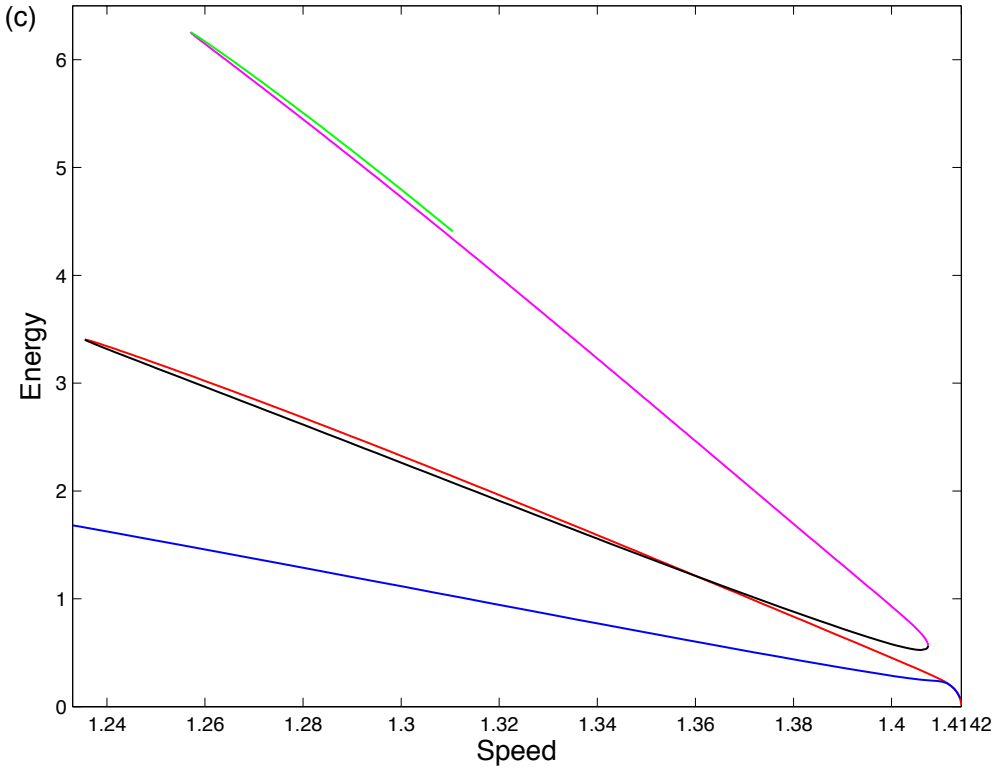
Boundary integral equation, Newton iterations, continuation

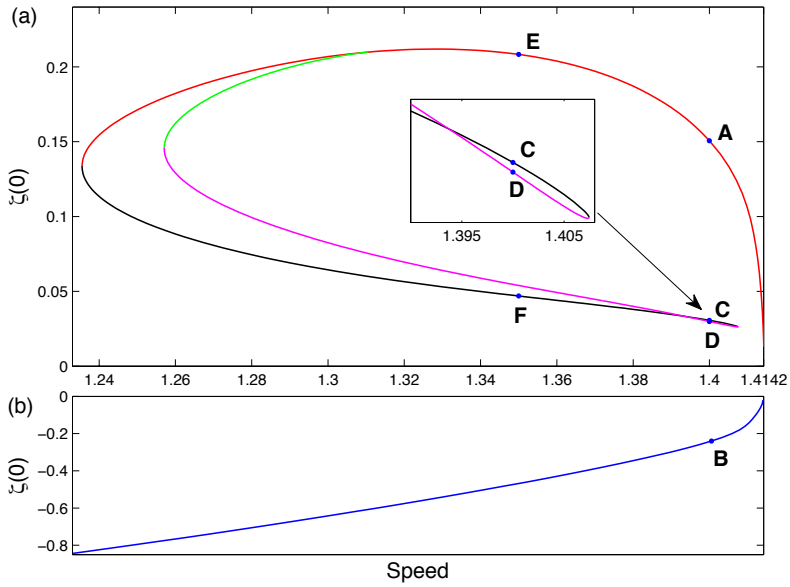
Gravity capillary solitary waves

infinite depth

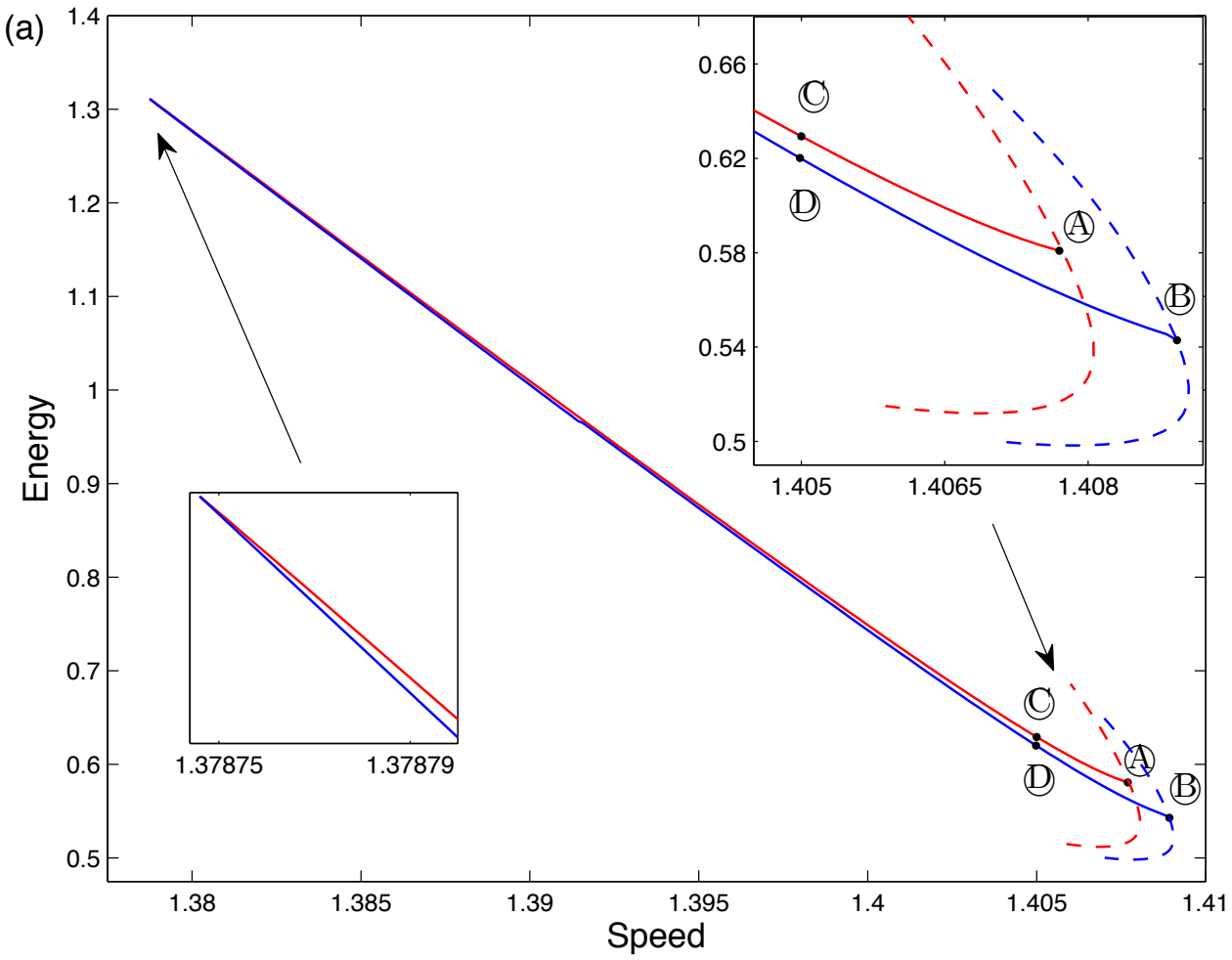


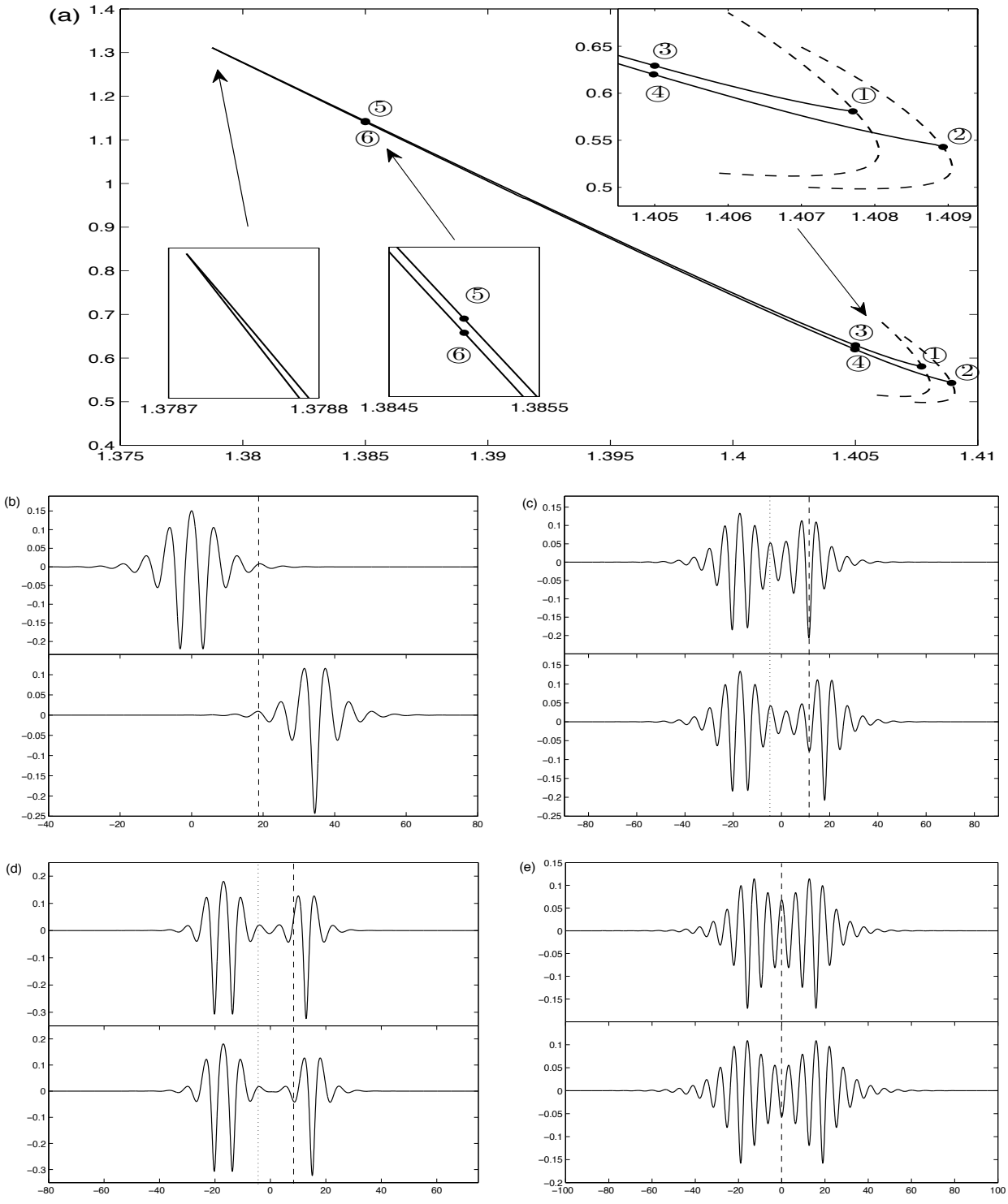

(a)
E
A
C
D
F
C
D
1.395
1.405
0.2
0.15
0.1
0.05
0
ζ(0)
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142
(b)
B
0
−0.2
−0.4
−0.6
−0.8
ζ(0)
Speed

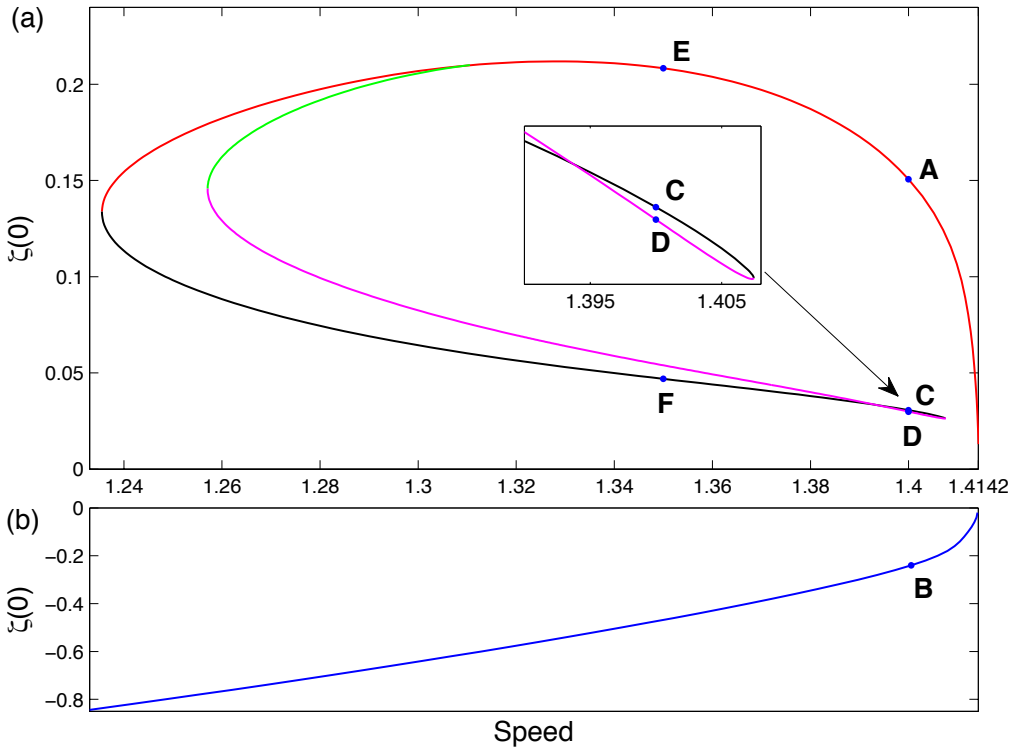

(c)
Energy
Speed
0
1
2
3
4
5
6
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142

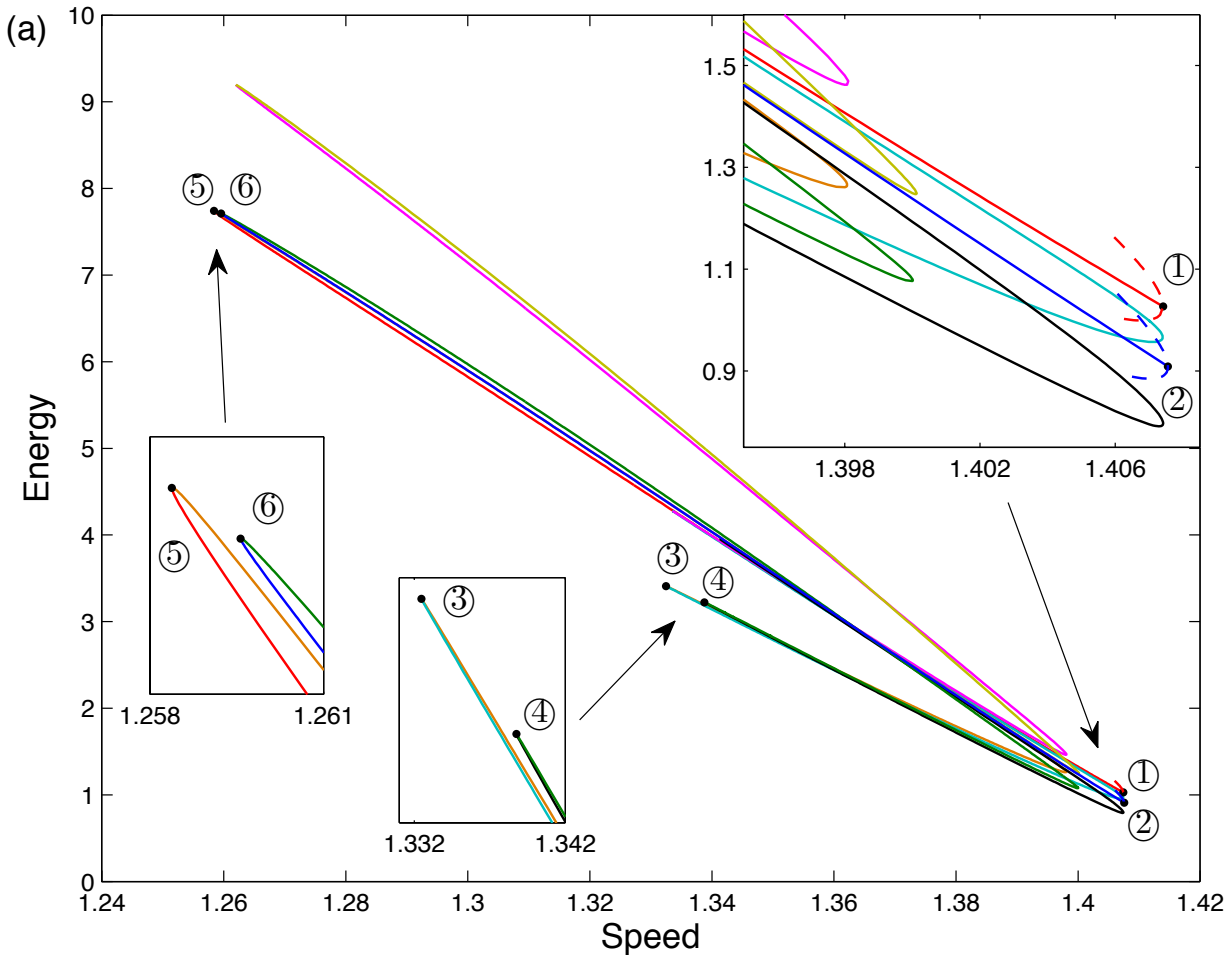


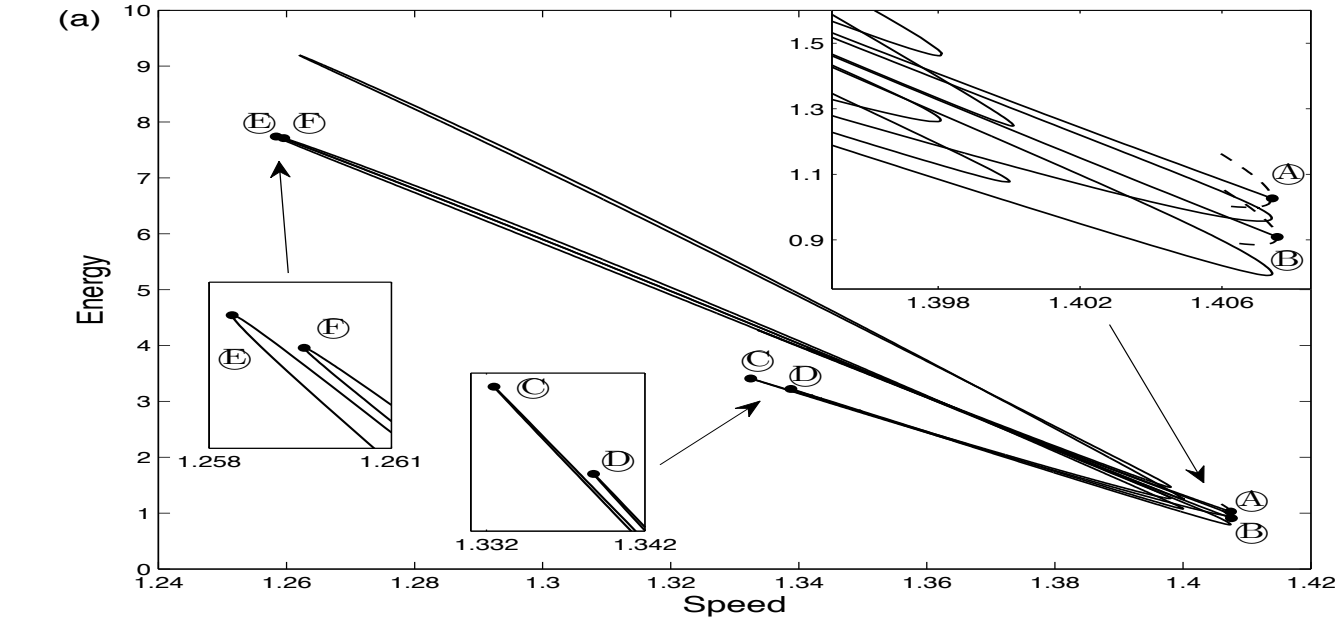
Zufiria (1987), Buffoni, Champneys and Toland (1996), Yang and Akylas (1997), Champneys and Groves (1997).....

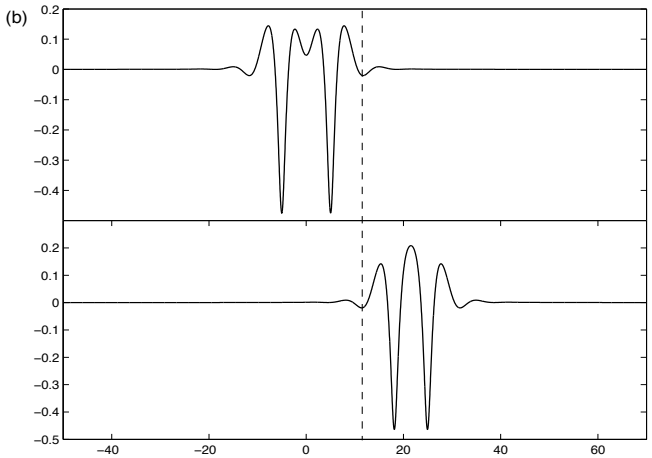

(a)
Energy
Speed
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
1.38
1.385
1.39
1.395
1.4
1.405
1.41
0.66
0.62
0.58
0.54
0.5
1.405
1.4065
1.408
1.37875
1.37879
C
D
A
B

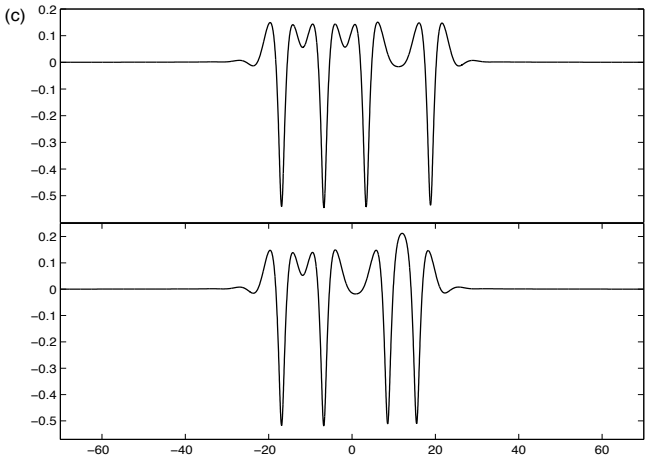

(a)
(b)
(c)
(d)
(e)
1.3787
1.3788
1.3845
1.3855

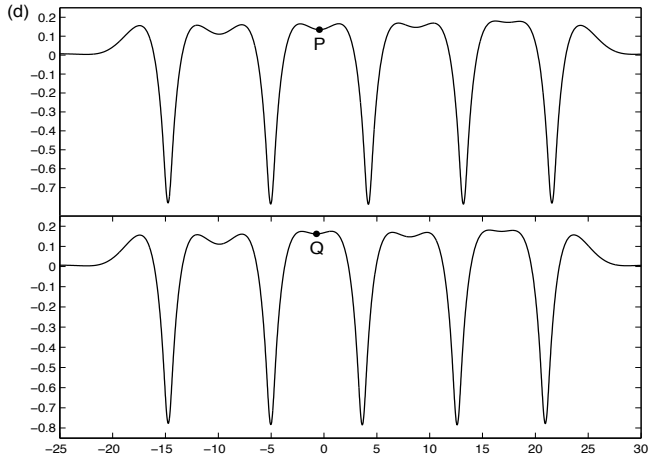

(a)
E
A
C
D
F
C
D
0.2
0.15
0.1
0.05
0
ζ(0)
1.395
1.405
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142
(b)
B
0
−0.2
−0.4
−0.6
−0.8
ζ(0)
Speed

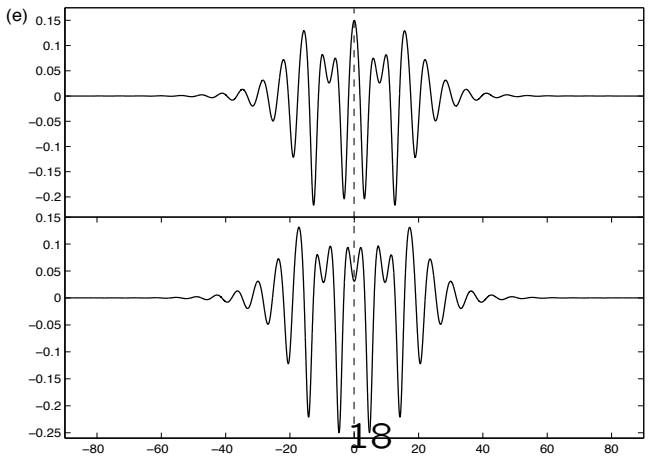

(a)
Energy
Speed
10
9
8
7
6
5
4
3
2
1
0
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.42
1.5
1.3
1.1
0.9
1.398
1.402
1.406
1.258
1.261
1.332
1.342
①
②
③
④
⑤
⑥


(a)
Energy
Speed
E
F
C
D
A
B
1.258
1.261
1.332
1.342
1.398
1.402
1.406


(b)

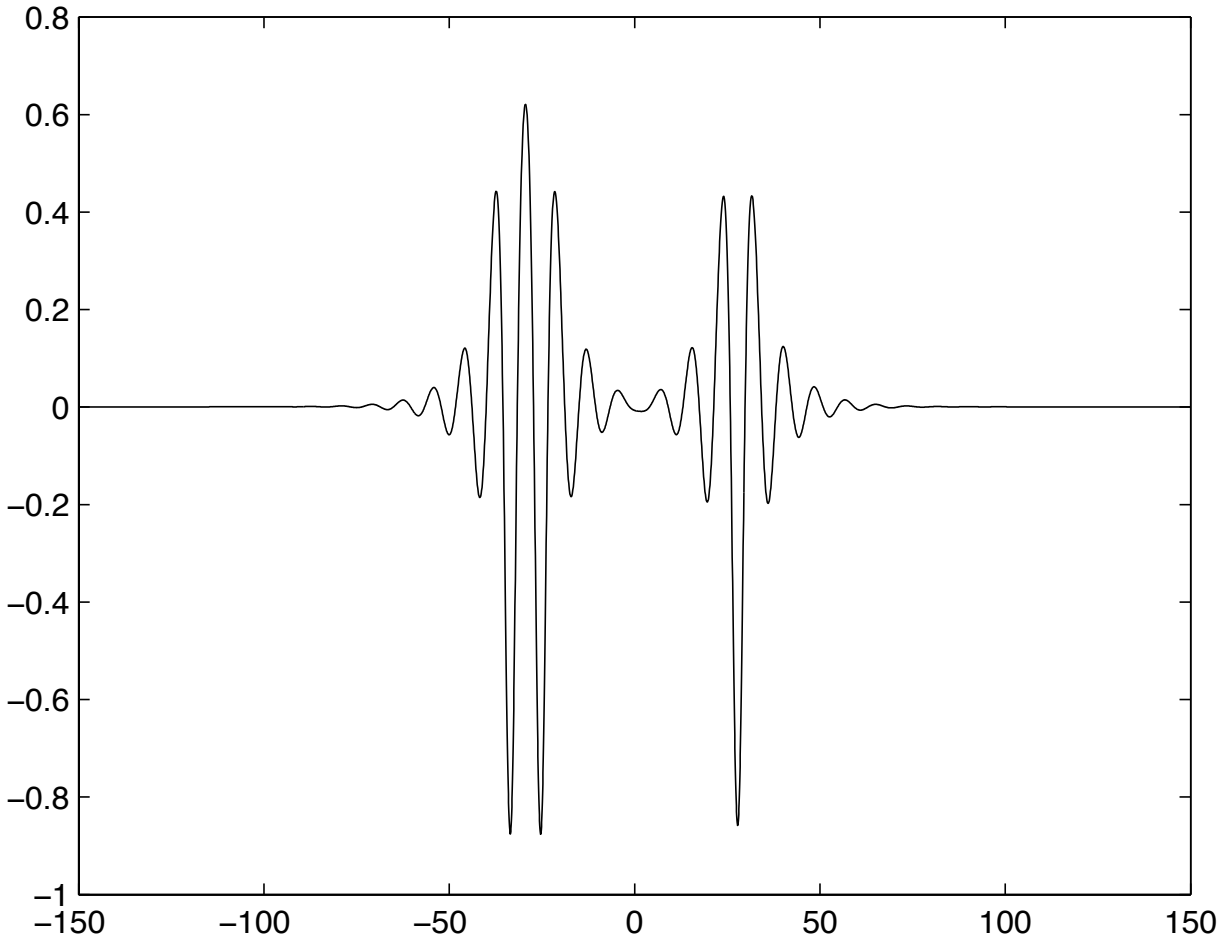

(c)


(d)
P
Q

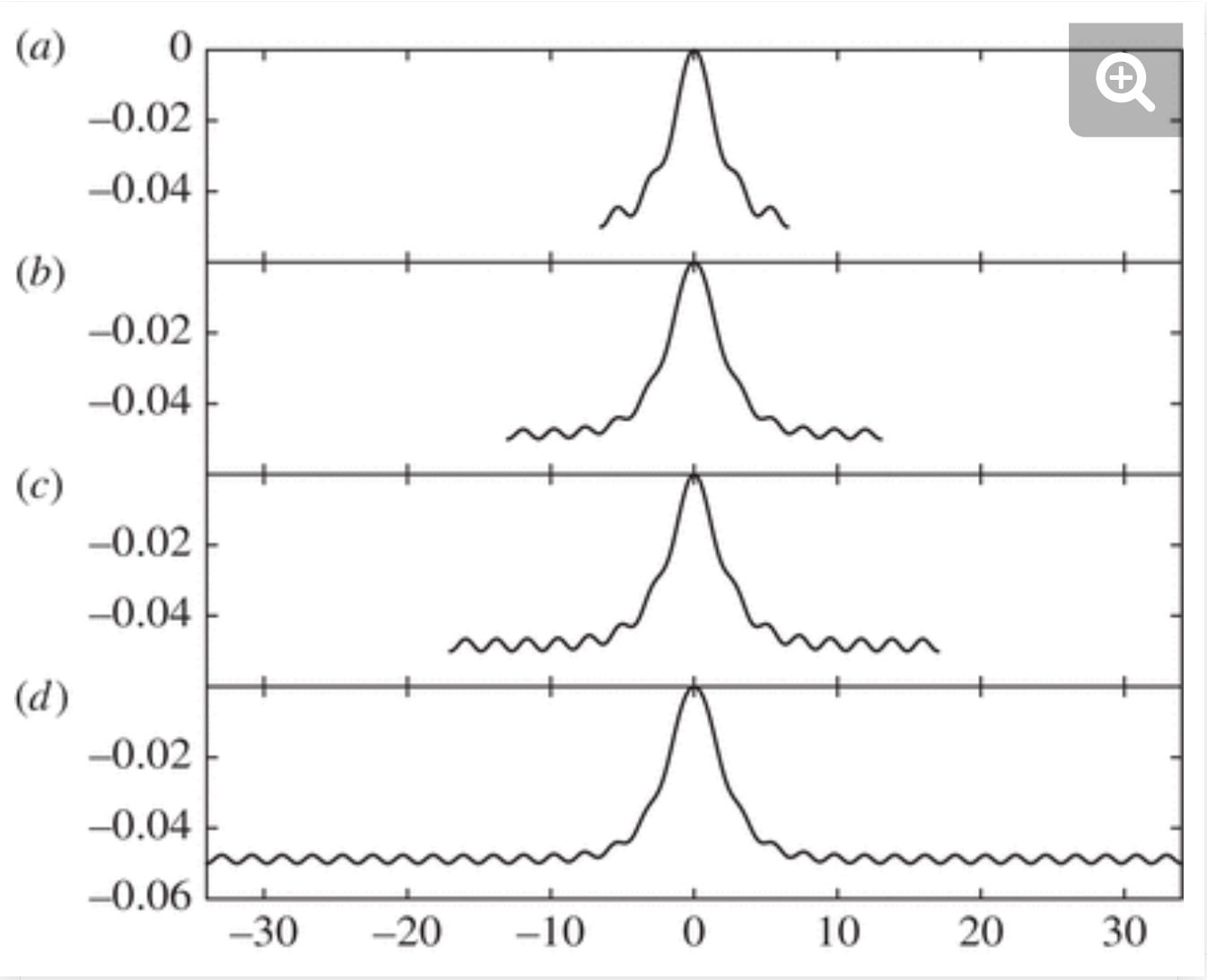

(e)

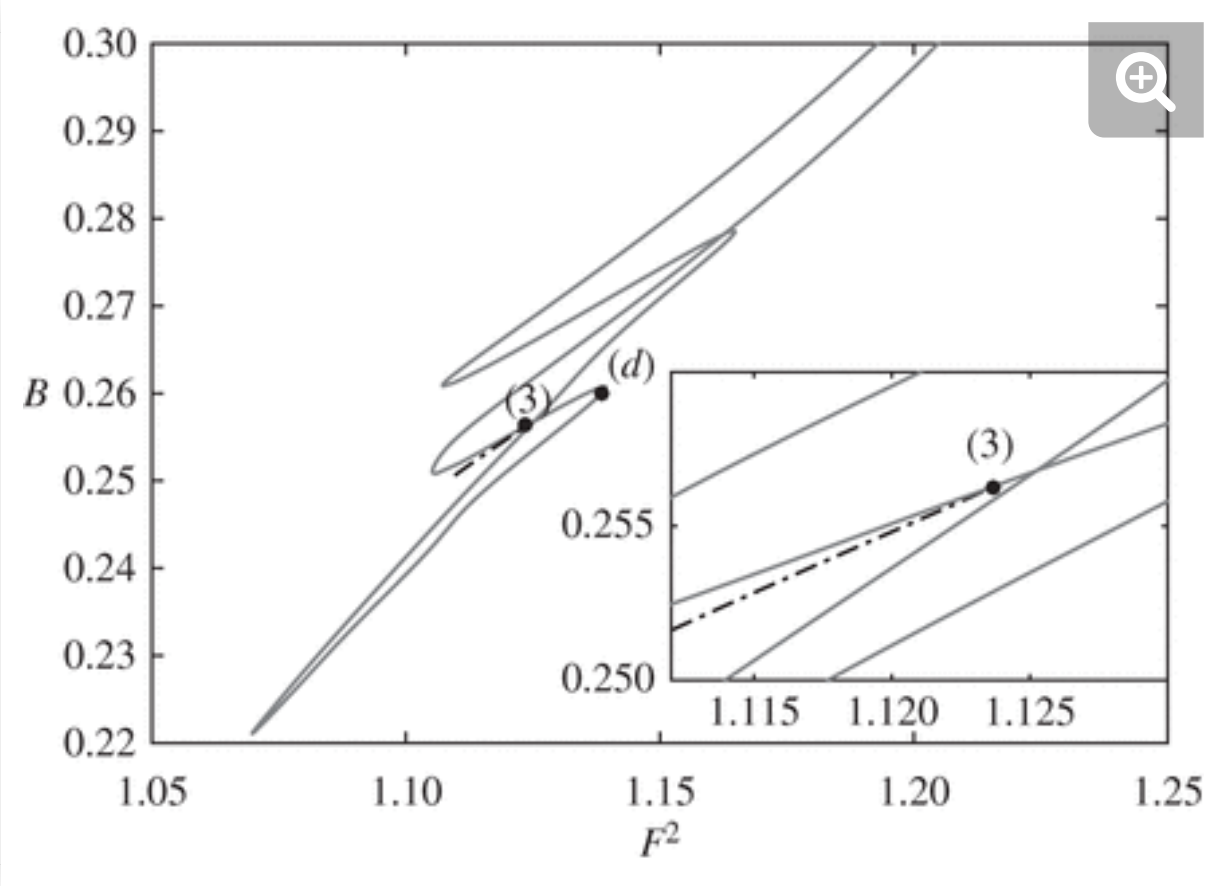
HYDROELASTIC WAVES

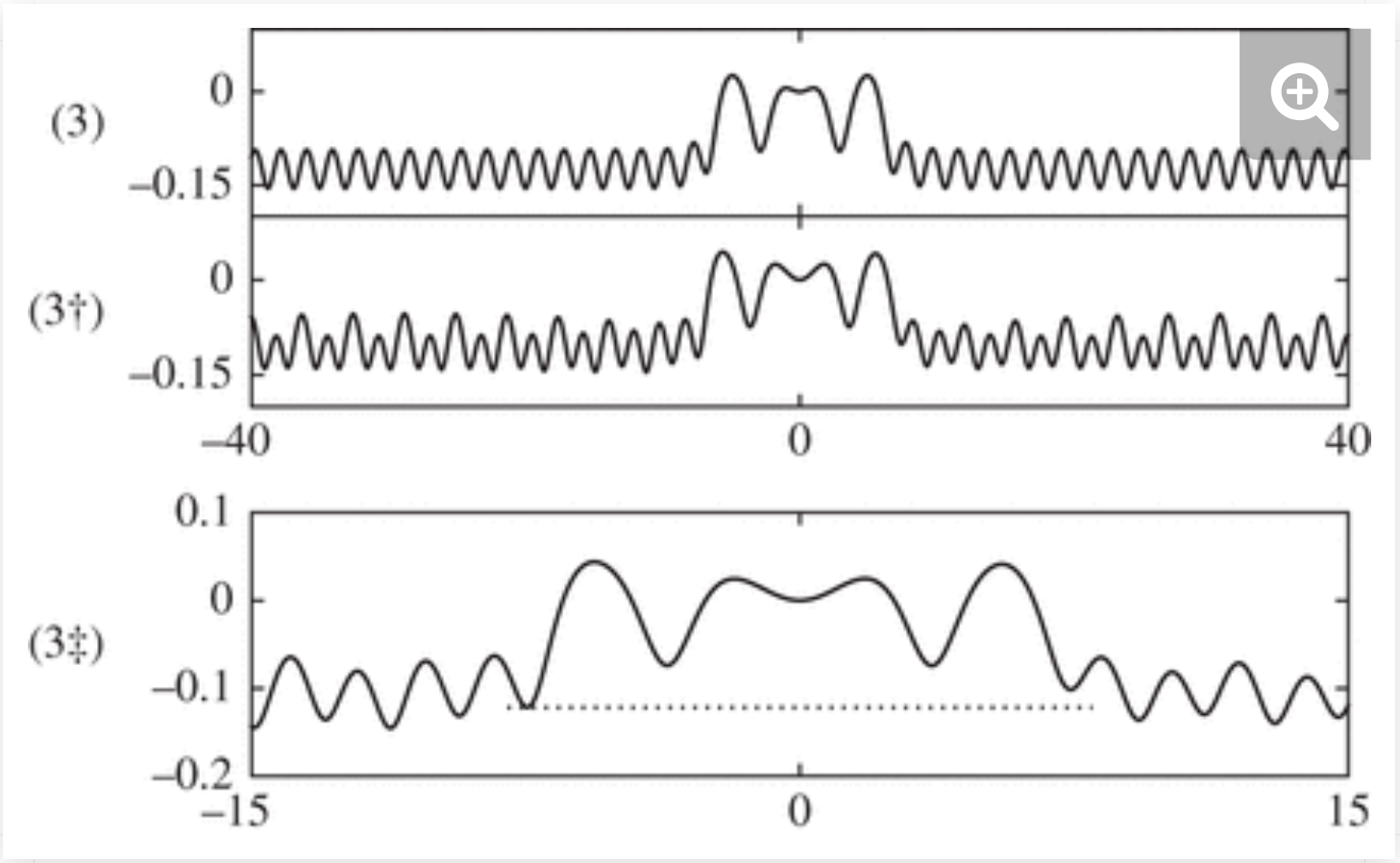
Tao Gao, Zhan Wang


0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−150
−100
−50
0
50
100
150

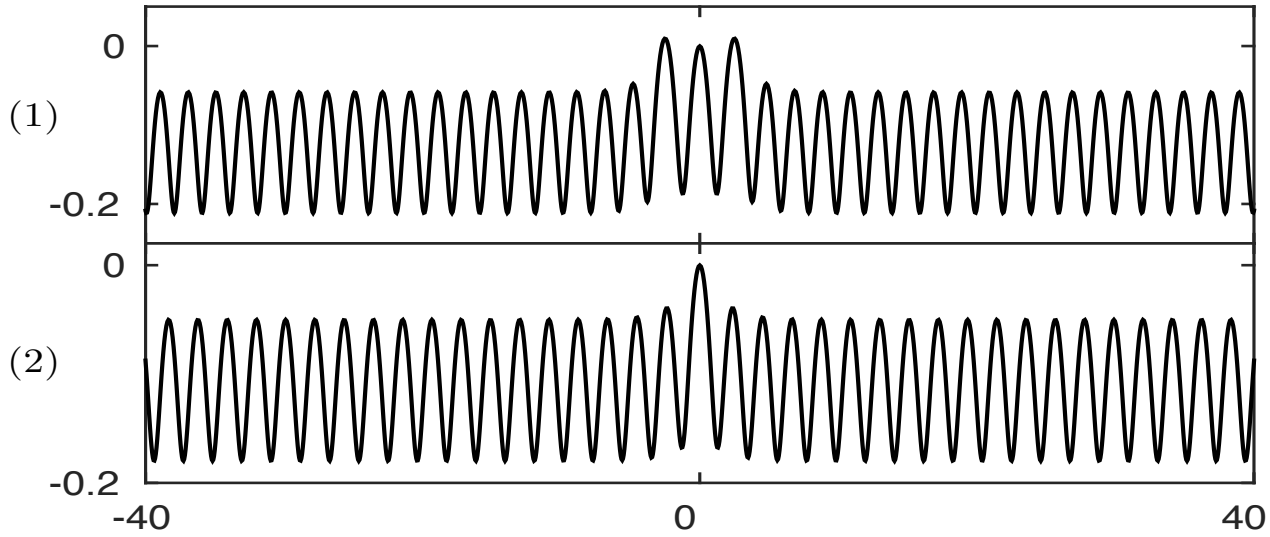
GENERALISED SOLITARY WAVES

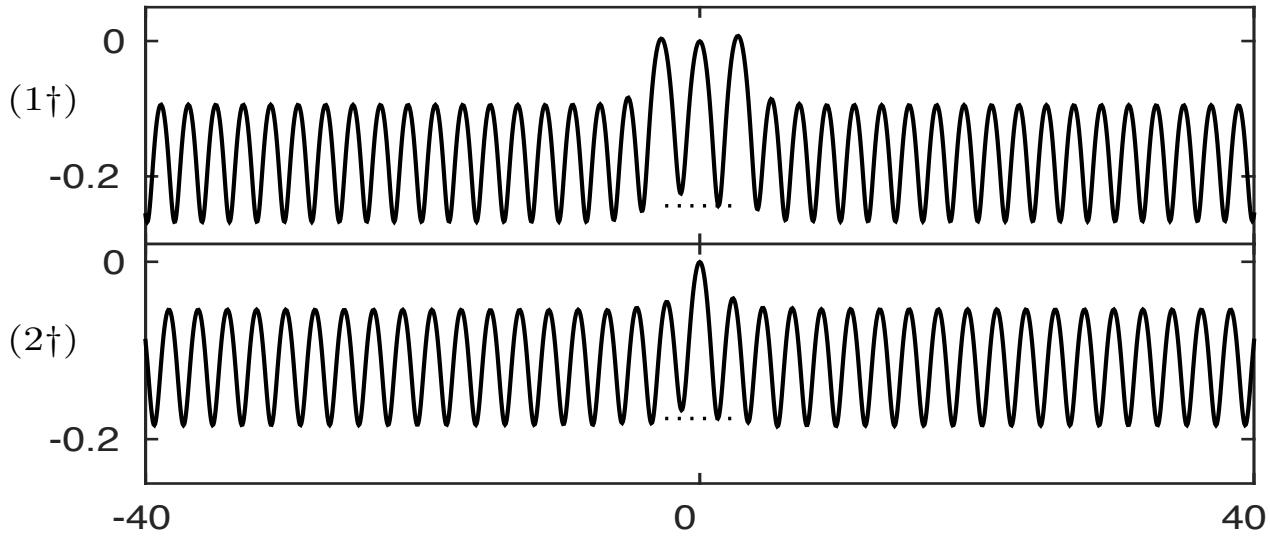

(a)
0
–0.02
–0.04
(b)
–0.02
–0.04
(c)
–0.02
–0.04
(d)
–0.02
–0.04
–0.06
–30
–20
–10
0
10
20
30

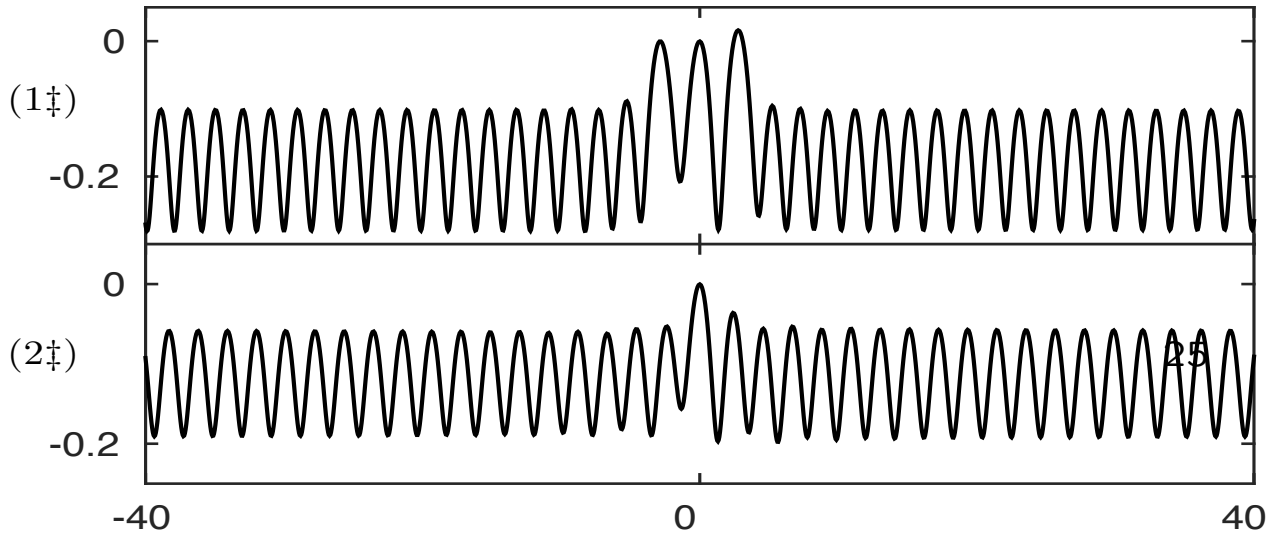

0.30
0.29
0.28
0.27
B 0.26
0.25
0.24
0.23
0.22
1.05
1.10
1.15
1.20
1.25
F^2
(3)
(d)
0.255
0.250
1.115
1.120
1.125



Wang Z., Parau E.I. , Milewski P.A. and Vdb (2014) Proc. Roy. Soc. A 470


0
(1)
-0.2
0
(2)
-0.2
-40
0
40


0
(1†)
-0.2
0
(2†)
-0.2
-40
0
40

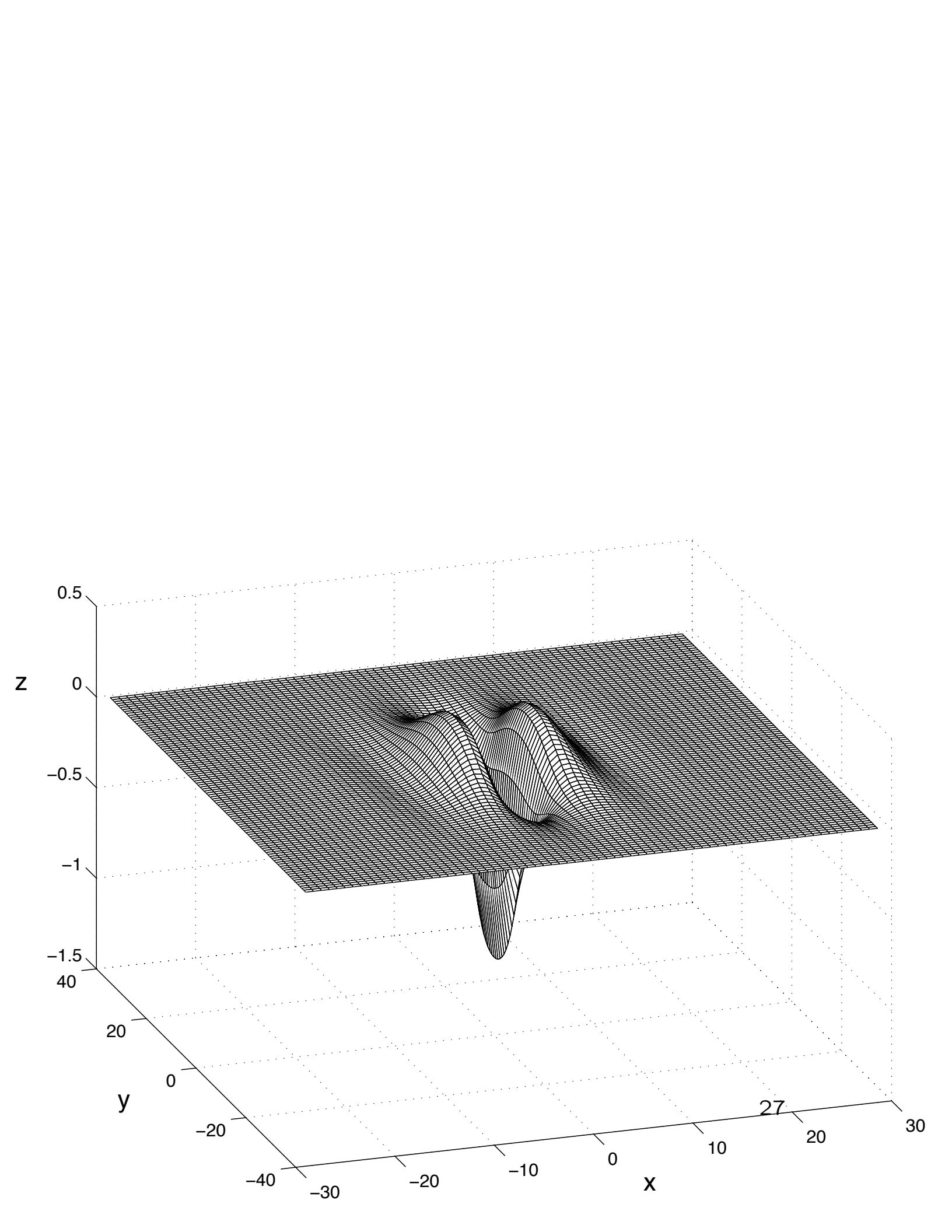

0
(1‡)
-0.2
0
(2‡)
-0.2
-40
0
40

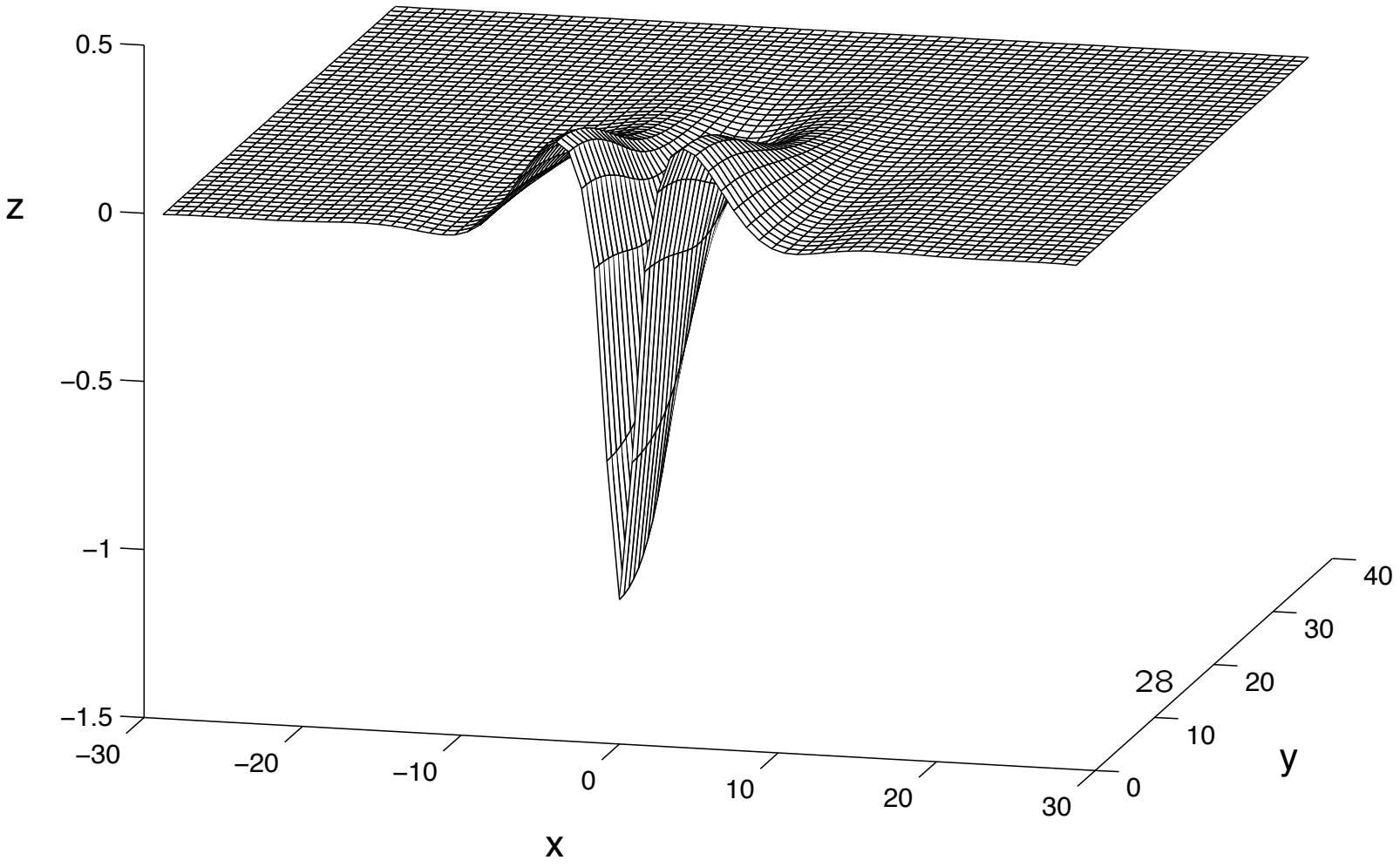
THREE-DIMENSIONAL FLOWS

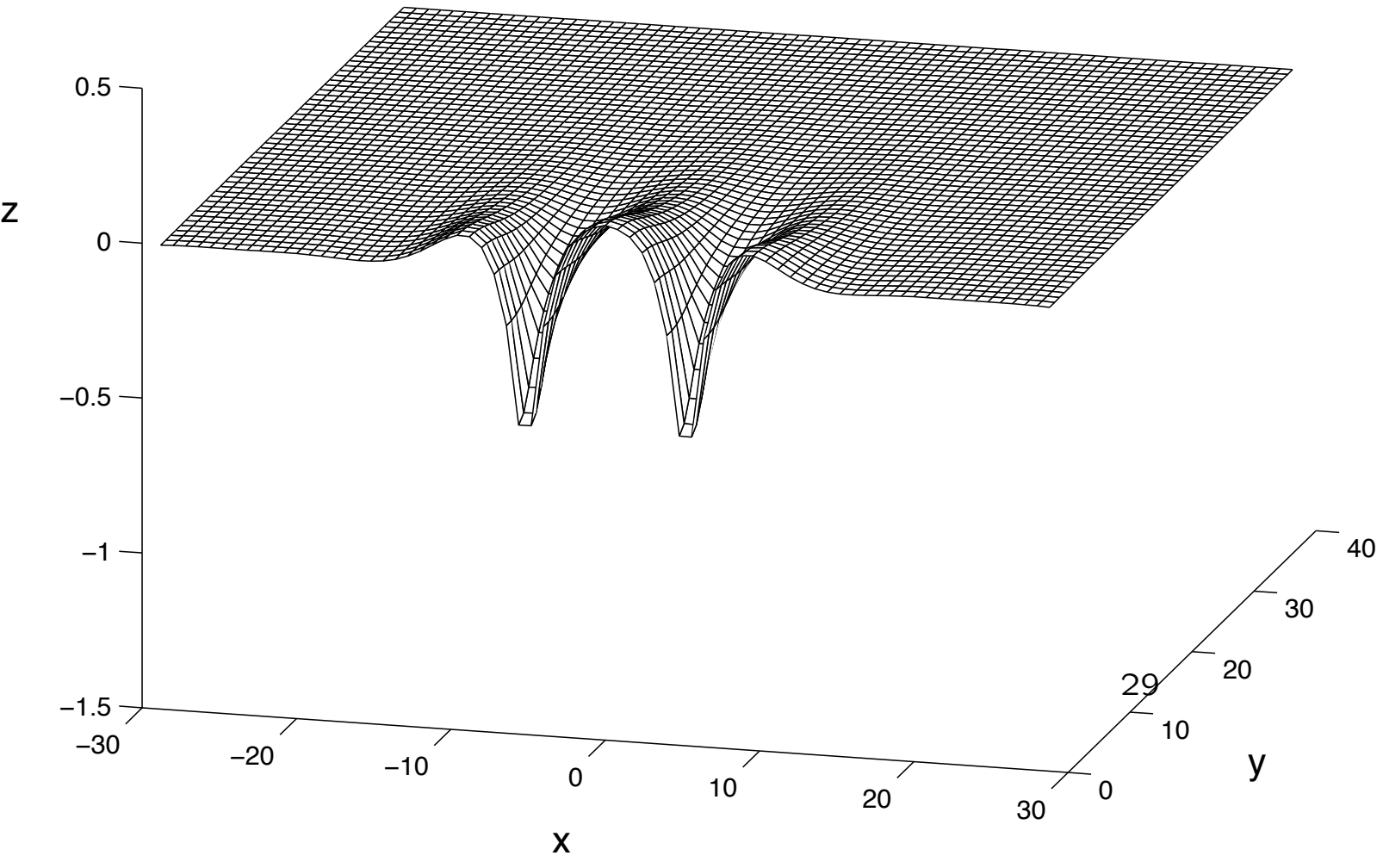
Use Green's theorem instead of Cauchy integral equation formula.

Emilian Parau, Mark Cooker

Olga Trichtchenko


0.5
0
−0.5
−1
−1.5
z
40
20
0
−20
−40
y
−30
−20
−10
0
10
20
27
30
x


0.5
0
−0.5
−1
−1.5
z
−30
−20
−10
0
10
20
30
x
0
10
20
28
30
40
y

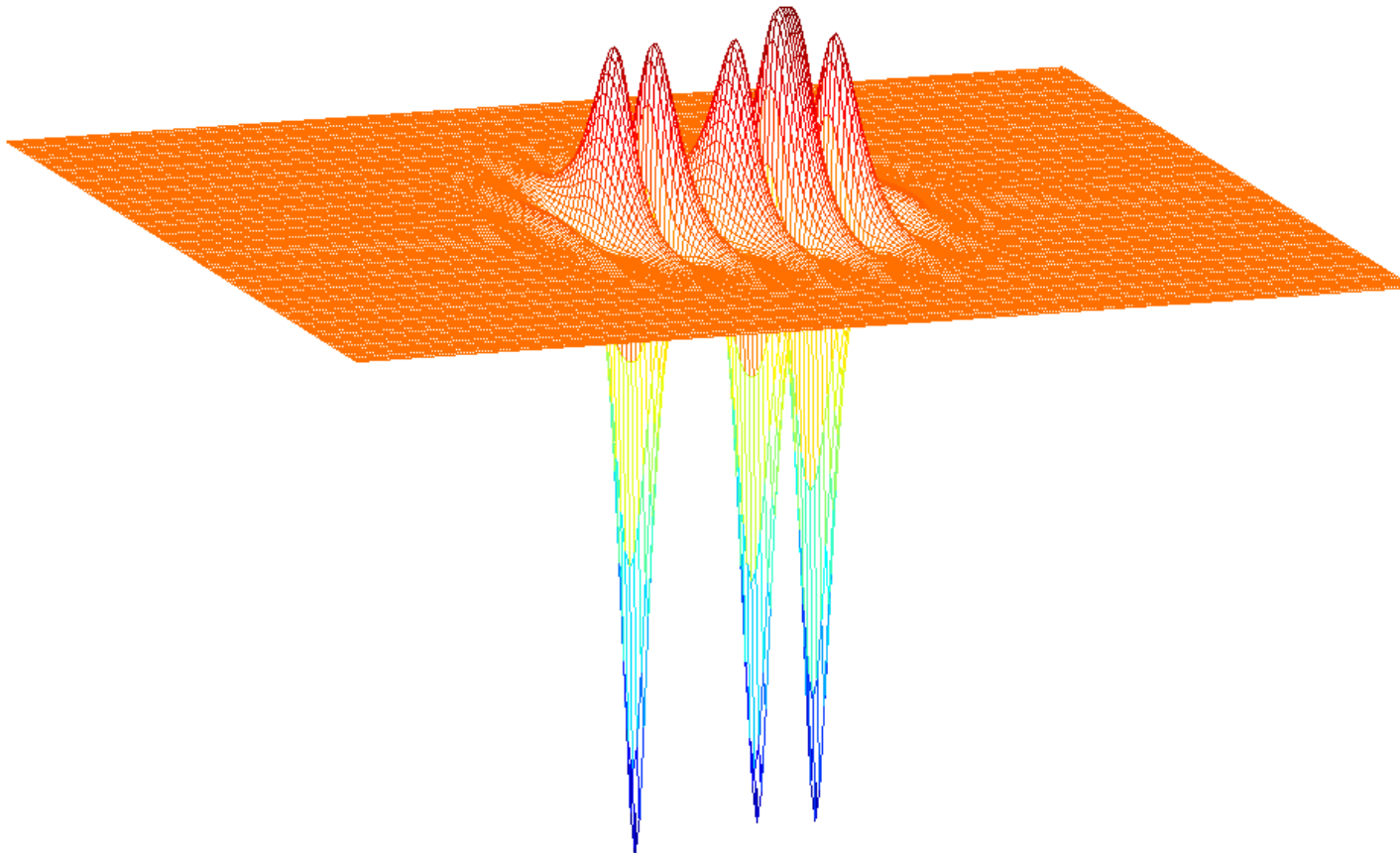

0.5
0
−0.5
−1
−1.5
z
−30
−20
−10
0
10
20
30
x
0
10
20
29
30
40
y

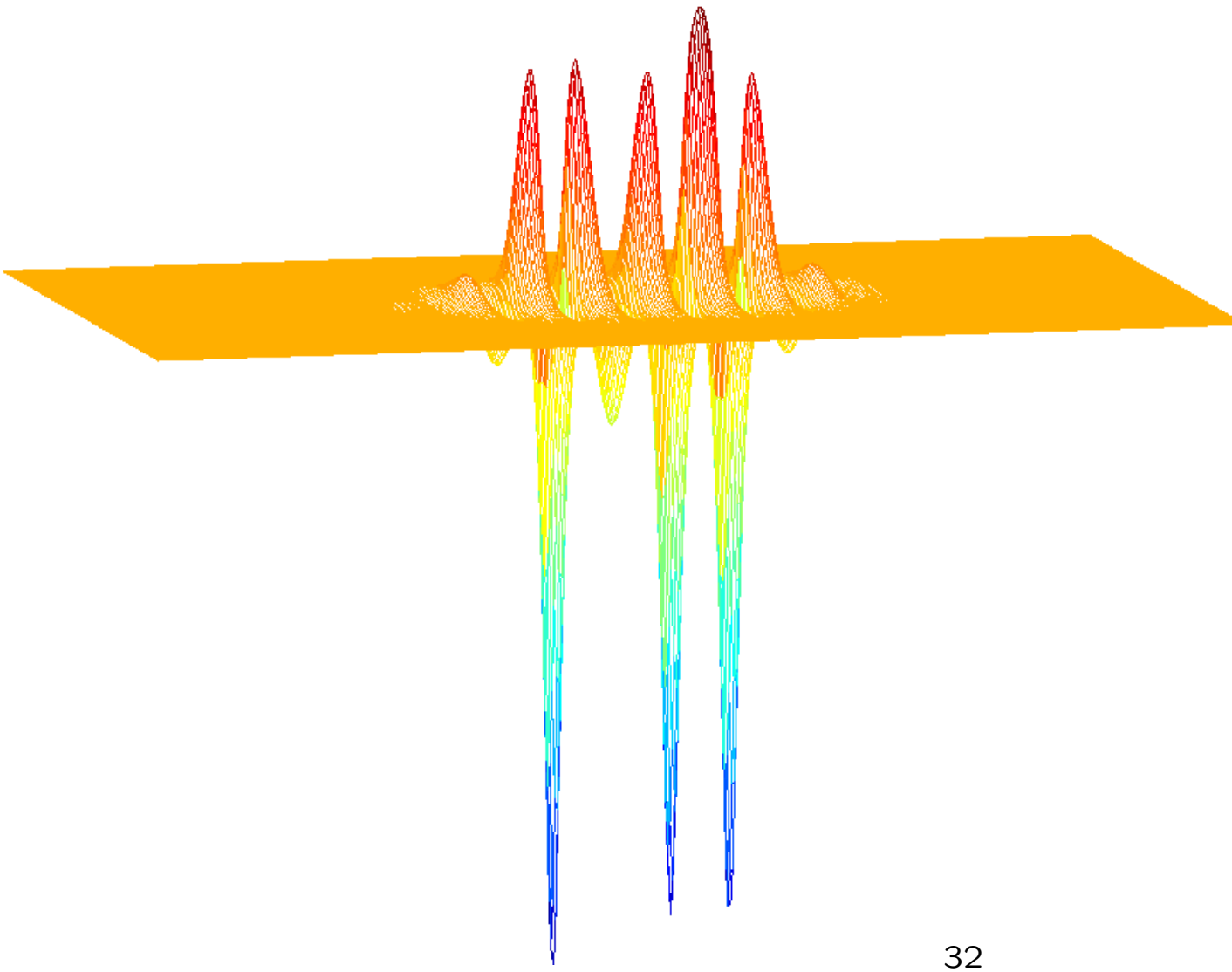
NON-SYMMETRIC 3D WAVES

Model: Akers and Milewski (2009)

$$u_t + \frac{\sqrt{2}}{2} u_x - \frac{\sqrt{2}}{4} H[u - u_{xx} - 2u_{yy}] + \alpha (u^2)_x = 0$$

Zhan Wang





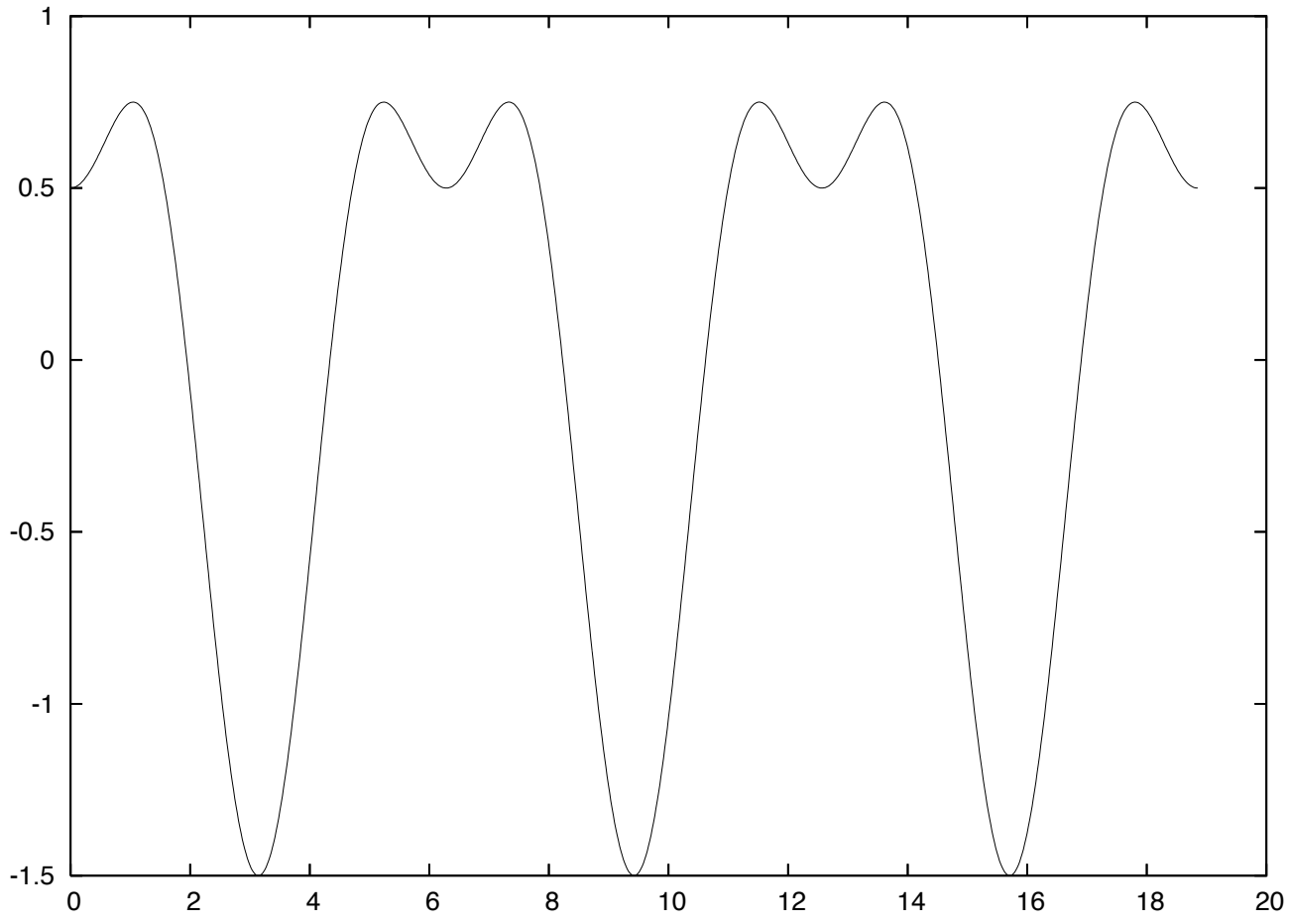
Non-symmetric PERIODIC gravity-capillary waves

Tao Gao and Zhan Wang

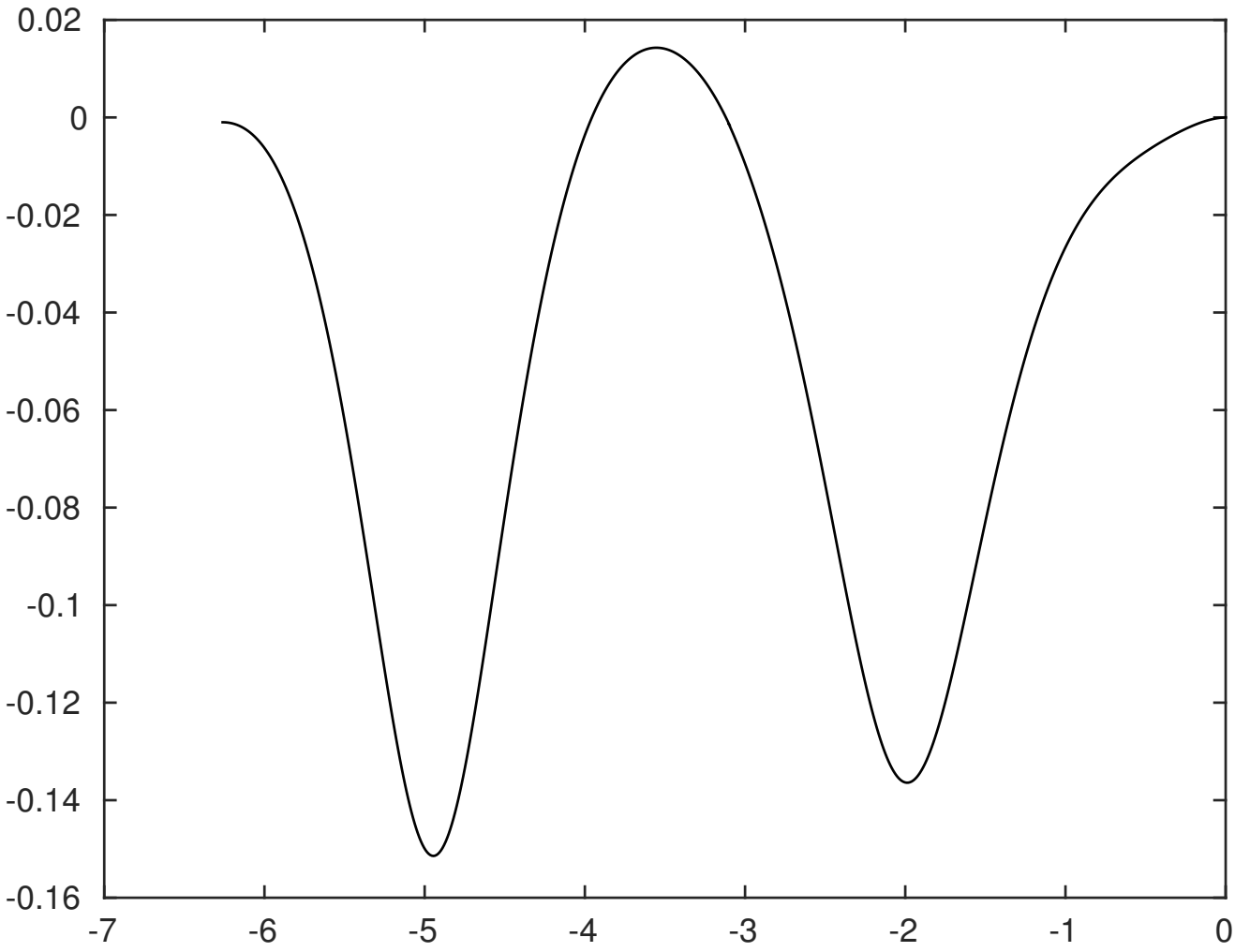
Zufiria (1987)

Shimizu ans Shoji (2012)

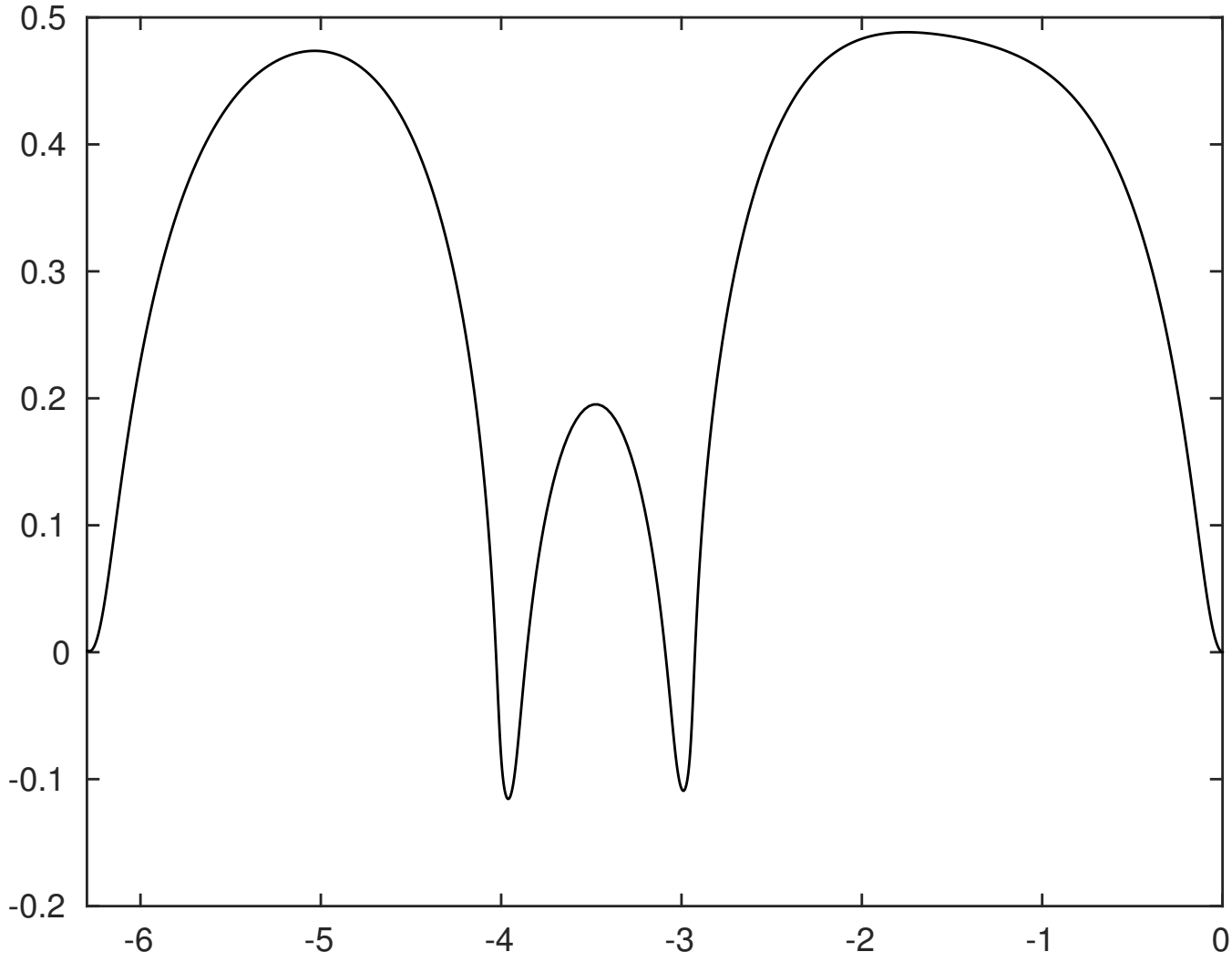
Symmetric waves



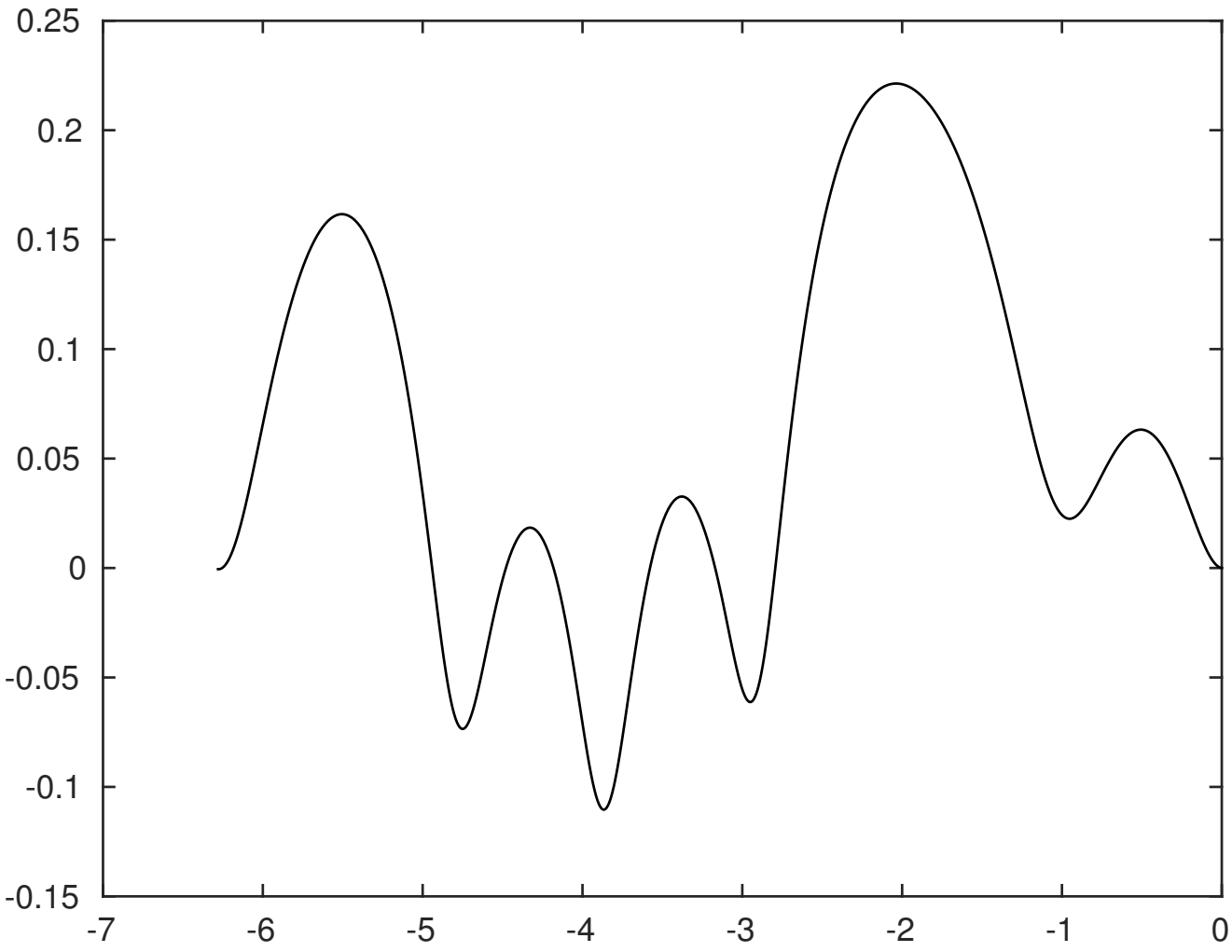
Non-symmetric waves



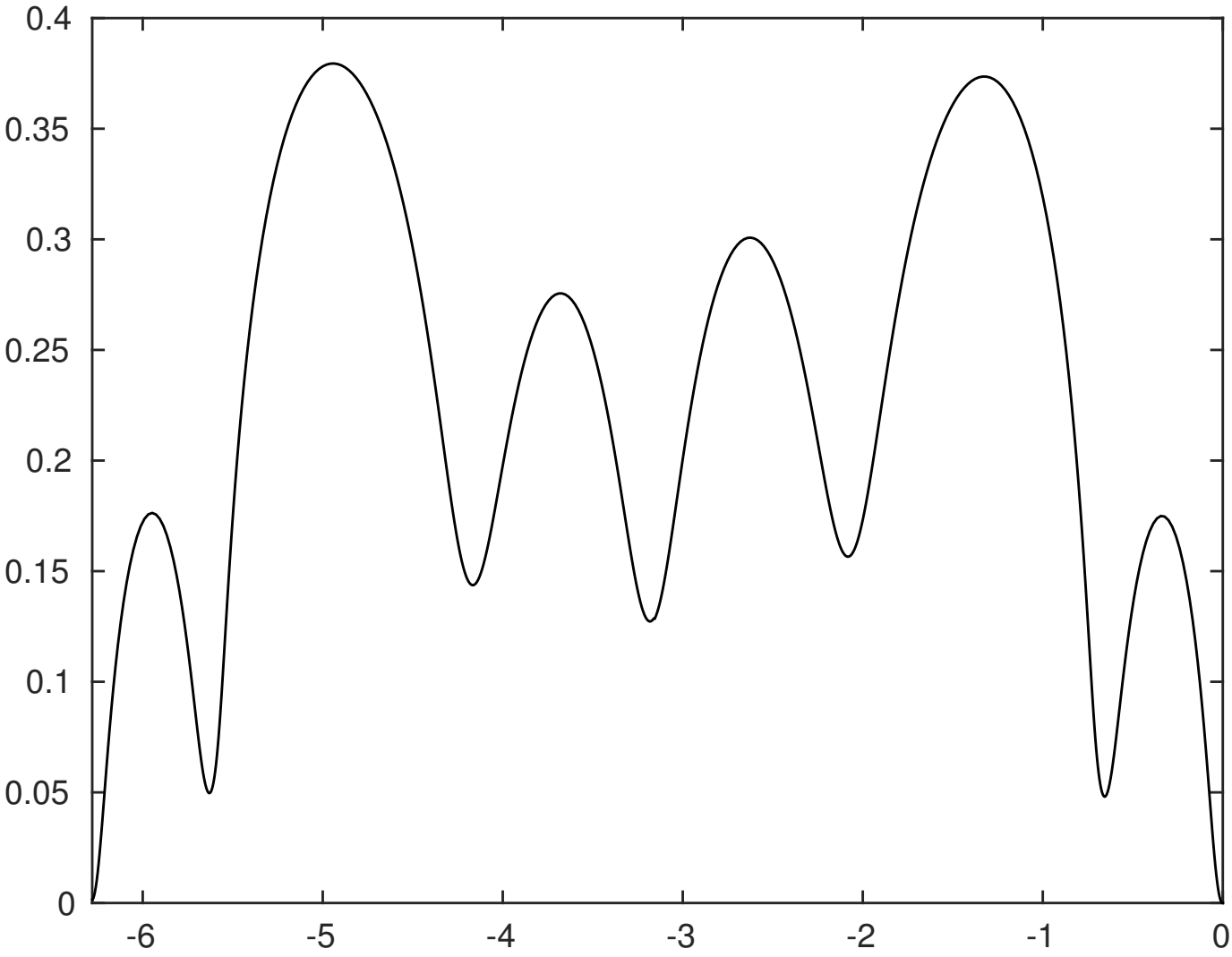
Non-symmetric waves

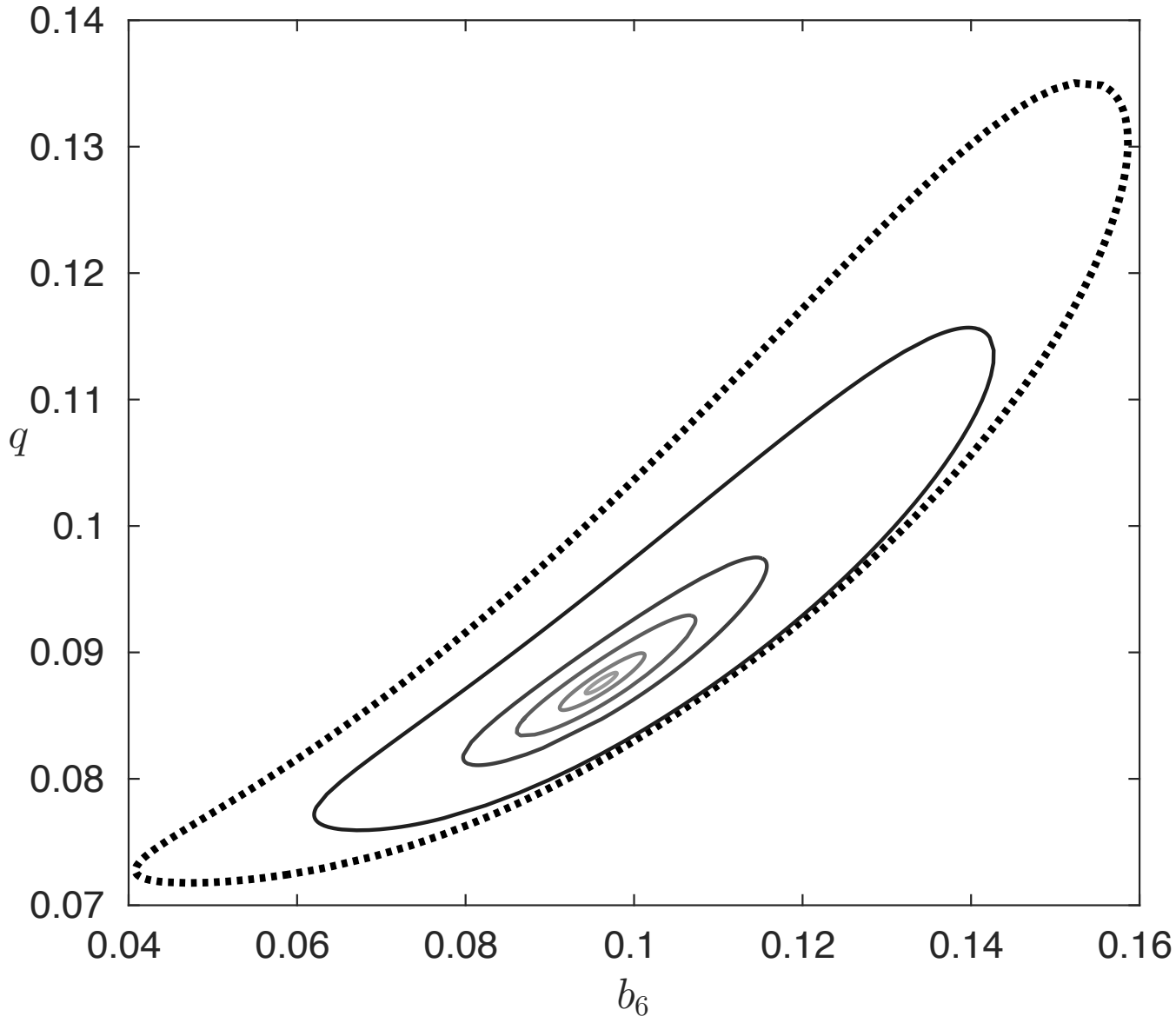


Non-symmetric waves



Non-symmetric waves




0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
q
0.04
0.06
0.08
0.1
0.12
0.14
0.16
b_6

Conclusions

New non-symmetric gravity-capillary waves for the Euler's equations in 2D (solitary waves)

New non-symmetric flexural waves for the Euler's equations in 2D (solitary waves)

New non-symmetric gravity-capillary waves for a model in 3D (solitary waves)

New non-symmetric generalised solitary waves in 2D

New non-symmetric periodic gravity-capillary waves in 2D

References

1. Wang Z. , Vanden-Broeck J.-M. and Milewski, PA., 2014, J. Fluid Mech. 759-770

2. Gao T., Wang Z. and Vanden-Broeck J.-M., 2016, J. Fluid Mech. 788, pp 469-491

3. Gao T., Wang Z. and Vanden-Broeck J.-M. 2016, Proc. Roy,. Soc. A (in press)

4. Gao T., Wang Z. and Vanden-Broeck J.-M. 2016, J. Fluid Mech. (in press)