

# Hochschild cohomology of noncommutative quadrics and planes

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# Automorphism groups of elliptic triples

divisor

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1. elliptic curve
2. cuspidal cubic
3. nodal cubic
4. three lines in general position
5. three concurrent lines
6. conic and a line
7. conic and a tangent line
8. triple line
9. double line and a line



# Automorphism groups of elliptic triples

divisor	$\text{Aut}_{\mathcal{L}_0, \mathcal{L}_1}(C)$
1. elliptic curve	$(\mathbb{Z}/3\mathbb{Z})^{\oplus 2}$
2. cuspidal cubic	$1$
3. nodal cubic	$\mathbb{Z}/3\mathbb{Z}$
4. three lines in general position	$\mathbf{G}_m^2 \rtimes \text{Cyc}_3$
5. three concurrent lines	$\mathbf{G}_a^2 \rtimes \text{Sym}_3$
6. conic and a line	$\mathbf{G}_m$
7. conic and a tangent line	$\mathbf{G}_a$
8. triple line	$\mathbf{G}_a^2 \rtimes \text{PGL}_2$
9. double line and a line	$\mathbf{G}_a \times (\mathbf{G}_a \rtimes \mathbf{G}_m)$



## $HH^\bullet$ of noncommutative $\mathbb{P}^2$

	divisor	$\dim_k HH_{ab}^1(\text{qgr } A)$	$\dim_k HH_{ab}^2(\text{qgr } A)$
1.	elliptic curve	0	2
2.	cuspidal curve	0	2
3.	nodal curve	0	2
4.	three lines in general position	2	4
5.	three lines through a point	2	4
6.	conic and line in general position	1	3
7.	conic and tangent line	1	3
8.	triple line	5	7
9.	double line and line	3	5



# Automorphism groups of elliptic quadruples

divisor	$\text{Aut}_{\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2}(C)$
1. elliptic curve	
2. cuspidal curve	
3. nodal curve	
4. two conics in general position	
5. two tangent conics	
6. conic and two lines in a triangle	
7. conic and two lines intersecting in one point	
8. quadrangle	
9. twisted cubic and bisecant	
10. twisted cubic and tangent line	
11. double conic	
12. two double lines	
13. double line and two lines in general position	



# Automorphism groups of elliptic quadruples

divisor	$\text{Aut}_{\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2}(C)$
1. elliptic curve	$\mathbb{Z}/2\mathbb{Z}^{\oplus 2}$
2. cuspidal curve	1
3. nodal curve	$\mathbb{Z}/2\mathbb{Z}$
4. two conics in general position	$\mathbf{G}_m \times \mathbb{Z}/2\mathbb{Z}$
5. two tangent conics	$\mathbf{G}_a$
6. conic and two lines in a triangle	$\mathbf{G}_m$
7. conic and two lines intersecting in one point	$\mathbf{G}_a^2 \rtimes \mathbb{Z}/2\mathbb{Z}$
8. quadrangle	$\mathbf{G}_m^2 \rtimes \mathbb{Z}/2\mathbb{Z}$
9. twisted cubic and bisecant	$\mathbf{G}_m$
10. twisted cubic and tangent line	$\mathbf{G}_a$
11. double conic	$\text{PGL}_2$
12. two double lines	$\mathbf{G}_a^2 \rtimes \mathbf{G}_m$
13. double line and two lines in general position	$(\mathbf{G}_a^2 \times \mathbf{G}_m) \rtimes \text{Sym}_2$



# HH<sup>•</sup> of noncommutative $\mathbb{P}^1 \times \mathbb{P}^1$

	divisor	$\dim_k \text{HH}_{\text{ab}}^1(\text{qgr } A)$	$\dim_k \text{HH}_{\text{ab}}^2(\text{qgr } A)$
1.	elliptic curve	0	3
2.	cuspidal curve	0	3
3.	nodal curve	0	3
4.	two conics in general position	1	4
5.	two tangent conics	1	4
6.	conic and two lines in a triangle	1	4
7.	conic and two lines through a point	2	5
8.	quadrangle	2	5
9.	twisted cubic and a bisecant	1	4
10.	twisted cubic and a tangent line	1	4
11.	double conic	3	6
12.	two double lines	3	6
13.	double line and two lines in general position	3	6