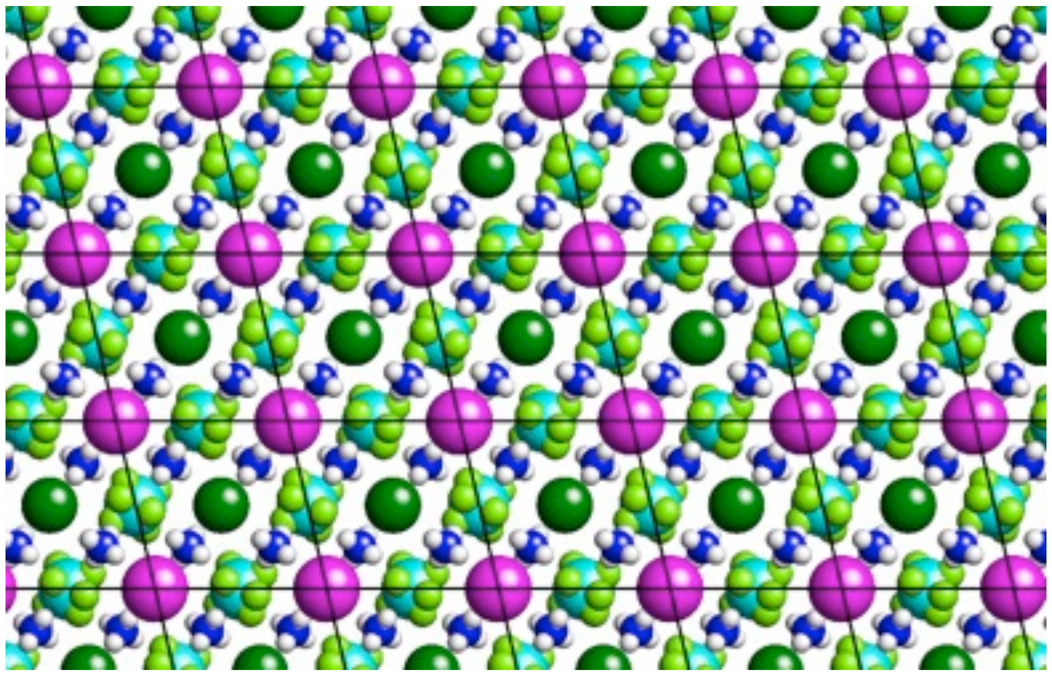
Symmetry Enriched Topological Phases

Lukasz Fidkowski

with N. Lindner and A. Kitaev

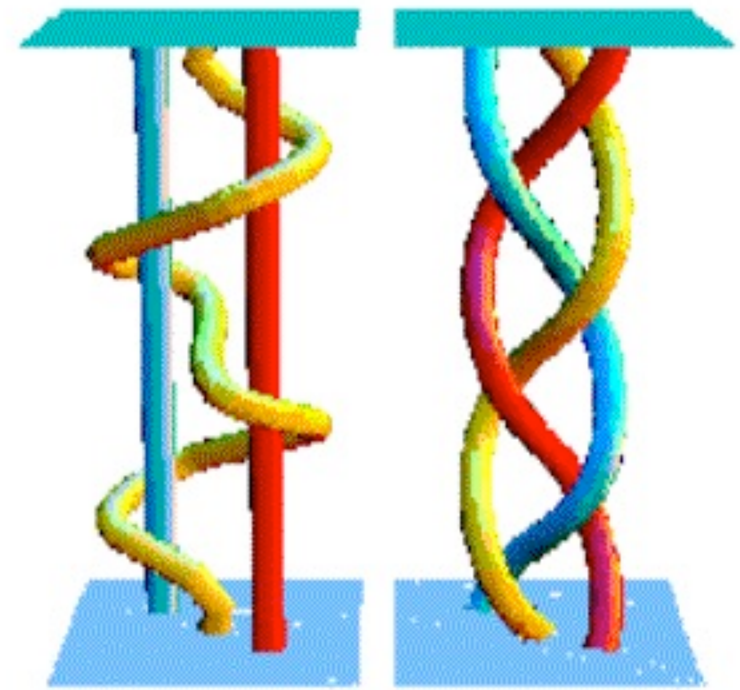
see also: Essin & Hermele arXiv:1212.0593
Mesaros & Ran, arXiv:1212.0835

Symmetry:



- Ginzburg-Landau classification
- local order parameters

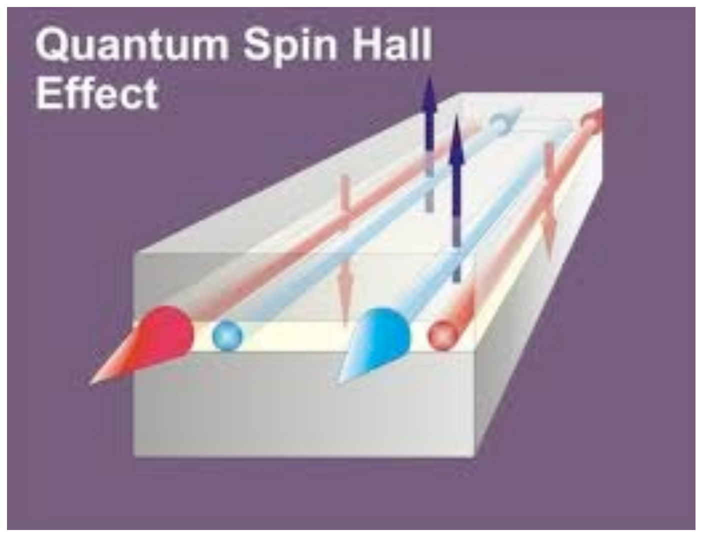
Topological Order:



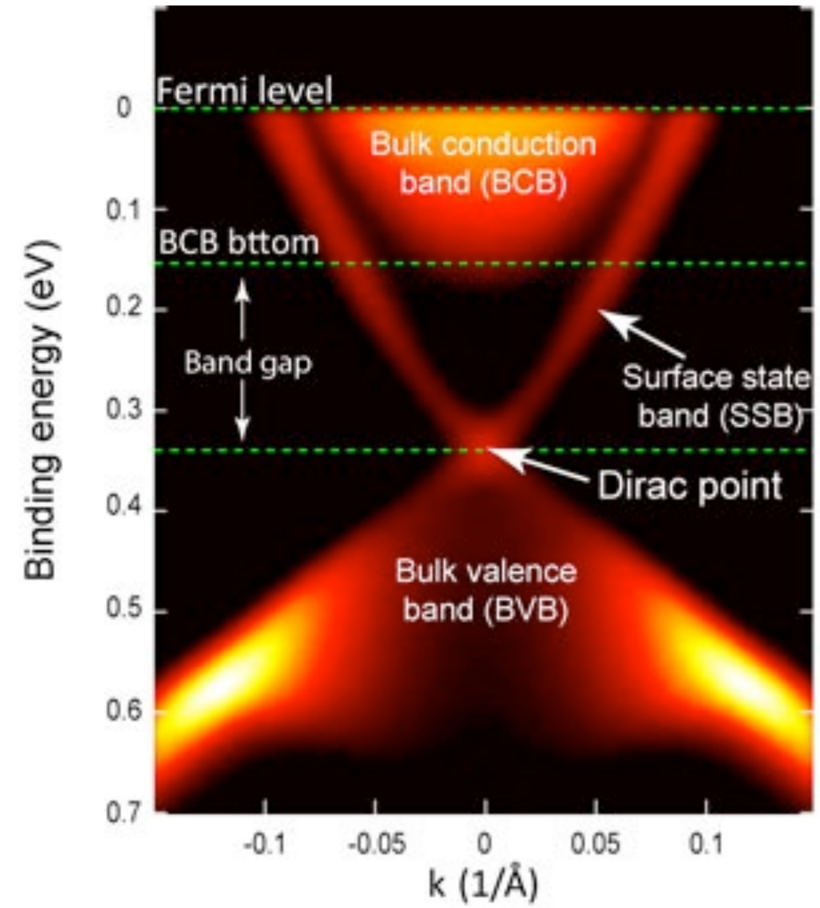
- exotic statistics
- ground state degeneracy
- gapless edge modes

1) topological insulators:

2d:



3d:

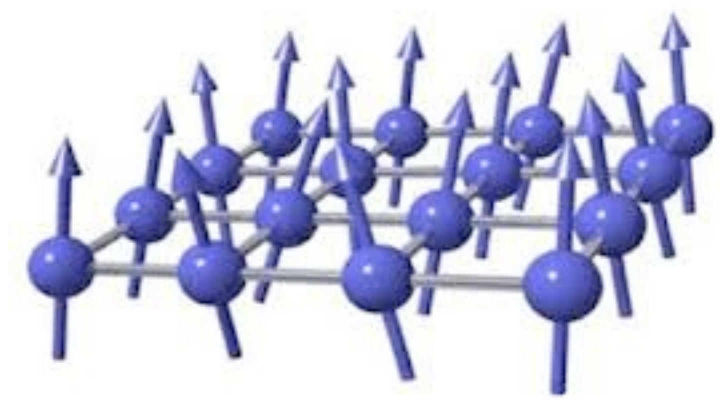


protected by time reversal symmetry

2) theoretical models of symmetry protected topological phases.

How to distinguish phases with *both* topological order and symmetry?

- gapped
- bosonic degrees of freedom
- 2 dimensional
- symmetry group G:
  - finite (discrete),abelian
    (also translations)
- no spontaneous symmetry breaking



Projective representations of G

$$f = gh$$

$$U_f = U_g U_h$$ ← ordinary representation

$$U_f = e^{i\phi(g,h)} U_g U_h$$ ← projective representation

Example: $G = SO(3)$ (or $G = \mathbb{Z}_2 \times \mathbb{Z}_2 \subset SO(3)$)

ordinary reps: integer spins

projective reps: $(\text{integer} + \frac{1}{2})\,\text{spins}$

$$[\sigma_x \sigma_z = -\sigma_z \sigma_x]$$

Properties of projective reps:

$$U_{gh} = e^{i\phi(g,h)} U_g U_h$$

everything encoded in $\{\phi(g,h)\}$

- gauge transformation: $U_f \to e^{i\alpha(f)} U_f$

$$\phi'(g,h) = \phi(g,h) + \alpha(g) + \alpha(h) - \alpha(gh)$$

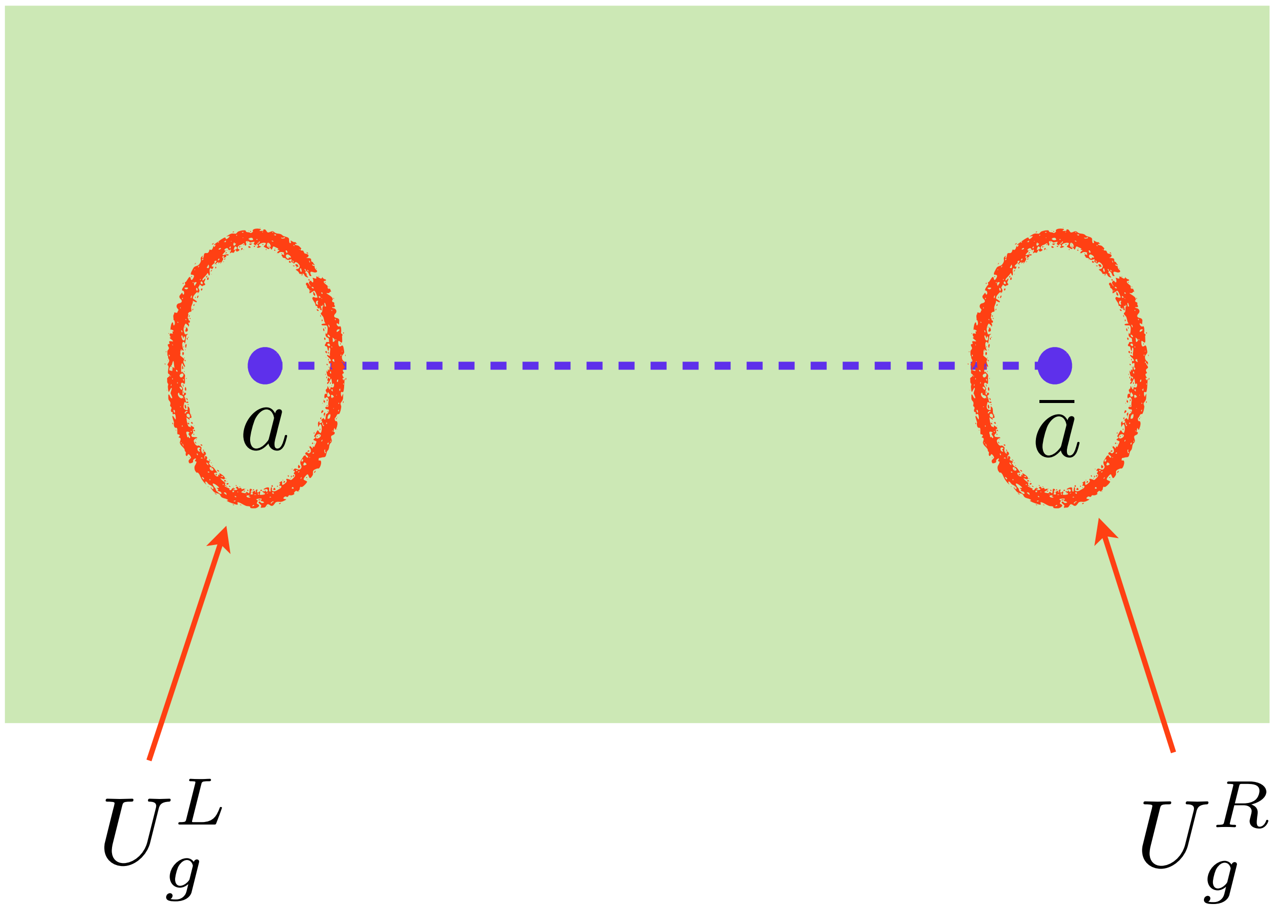
- constraint: $U_{fgh} = e^{i(\text{some phase})} U_f U_g U_h$

$$\phi(f,gh) + \phi(g,h) = \phi(fg,h) + \phi(f,g)$$

- analogous to flat gauge fields

- mathematical terminology: $H^2(G, U(1)) \equiv \{\phi(g,h)\}$

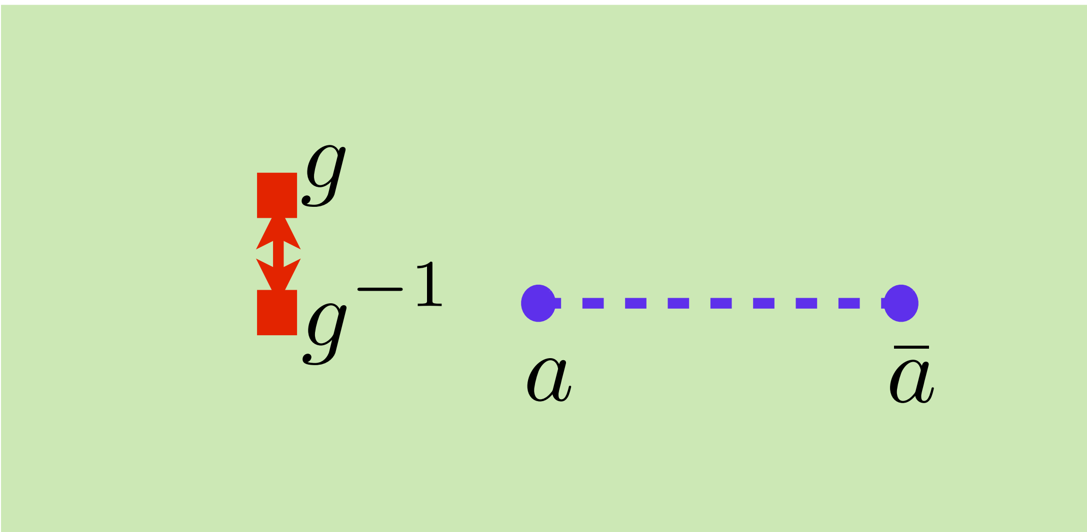
2d: anyons can become projective



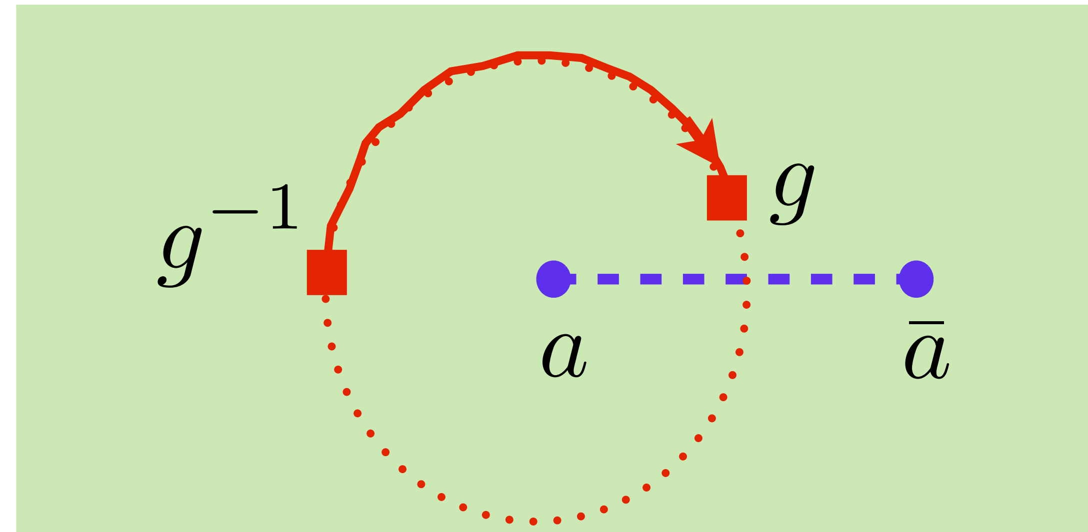
$$U_{gh}^L = e^{i\phi(g,h)} U_g^L U_h^L$$

Local action of G via adiabatic process:

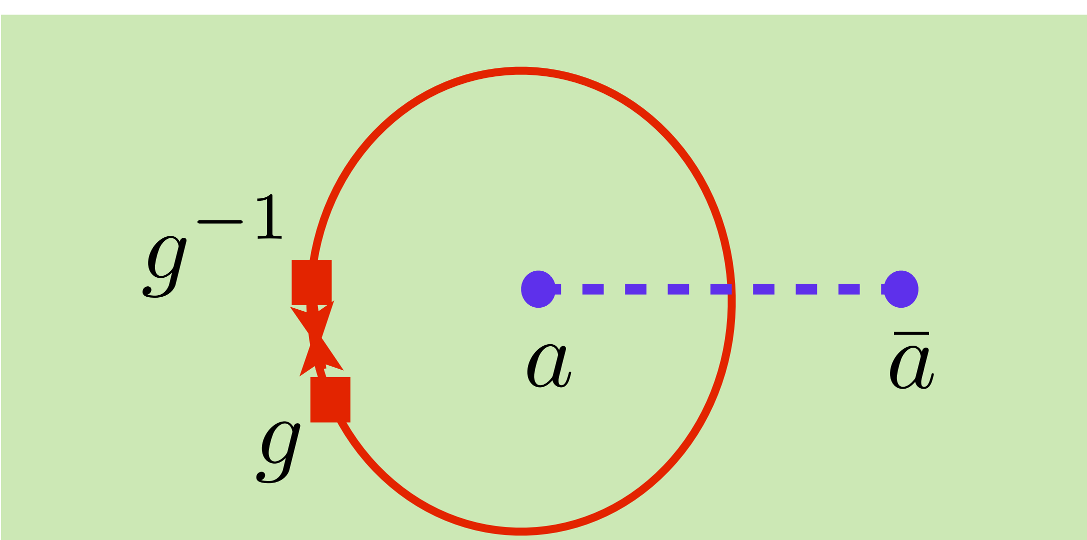
1. nucleate defect / anti-defect pair:



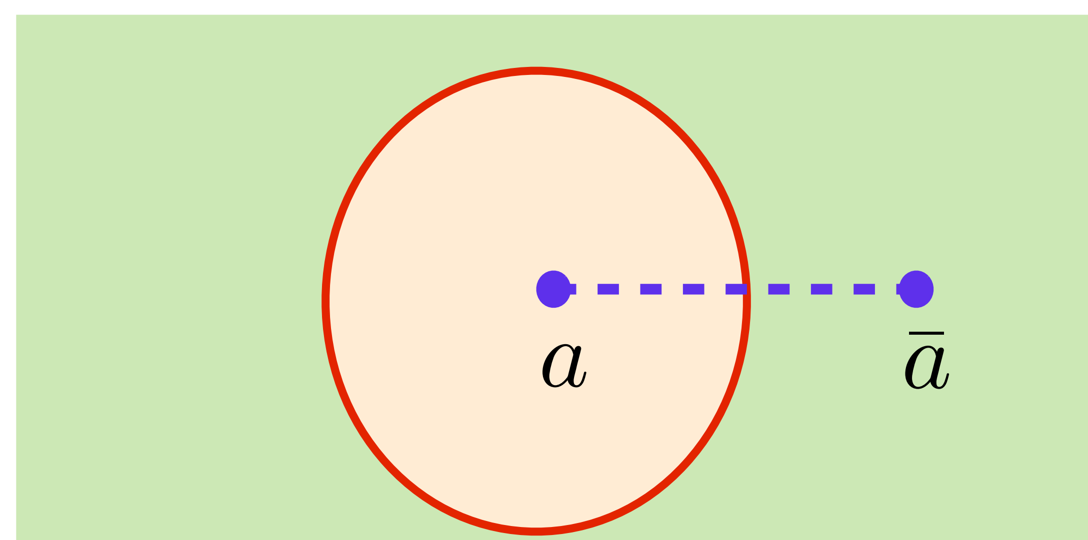
2. take one defect around anyon:



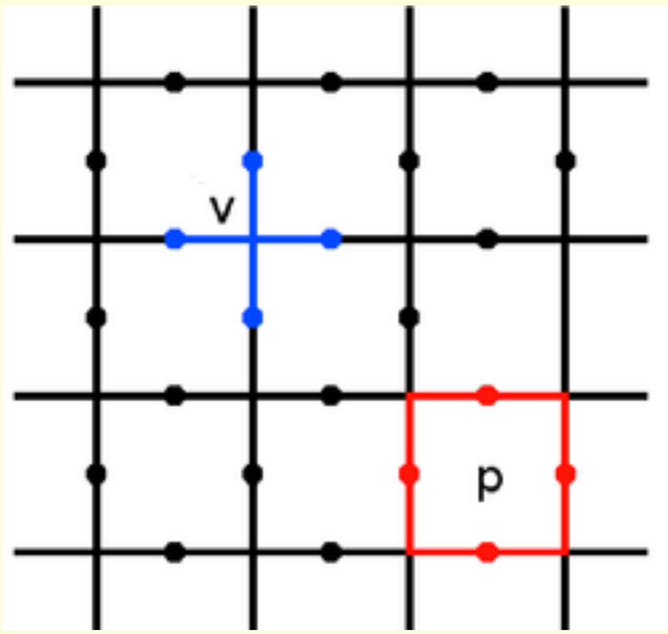
3. Annihilate defects:



3. Act with g on spins inside defect loop to remove branch cut



Example: "deformed" toric code:



$$A_v = \prod_{i \in v} \sigma_i^x, \;\; B_p = \prod_{i \in p} \sigma_i^z$$

$$H = -J \sum_v A_v + J \sum_p B_p, \;\; J > 0$$

note plus sign

G = spatial translations = $\mathbb{Z} \times \mathbb{Z}$

generators: T_x, T_y

Ground state of "deformed" toric code:

- ordinary toric code:

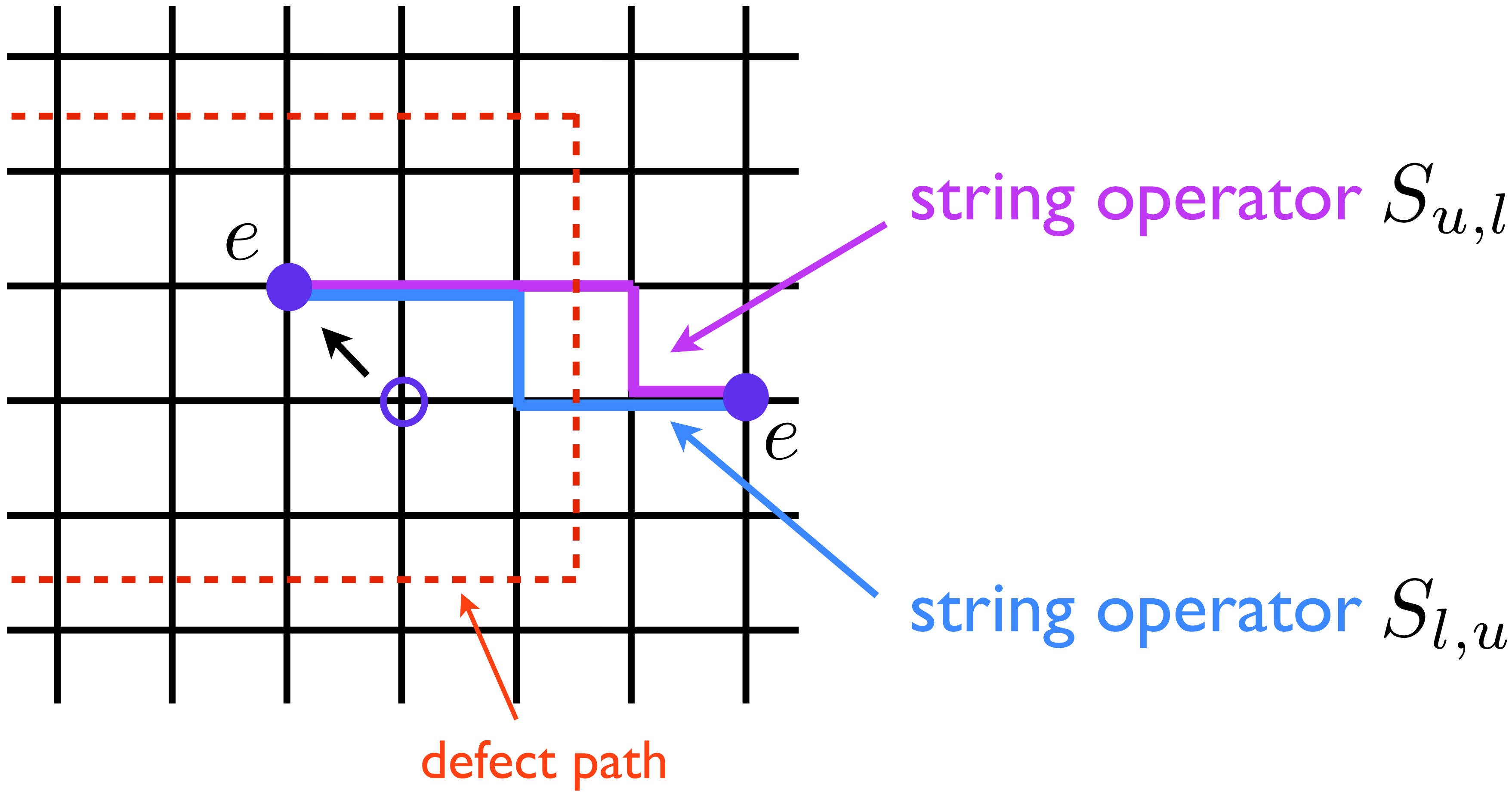
$$|\Psi\rangle = \sum_{\text{loop configs L}} |L\rangle$$

- "deformed" toric code:

$$|\Psi\rangle = \sum_{\text{loop configs L}} (-1)^{|L|} |L\rangle$$

area of loops in L (well defined with periodic boundary conditions and even # of sites)

Electric charge is projective under $\mathbb{Z} \times \mathbb{Z}$



$$U^L_{T_x} U^L_{T_y} |\Psi\rangle = S_{l,u}|0\rangle$$

$$U^L_{T_y} U^L_{T_x} |\Psi\rangle = S_{u,l}|0\rangle$$

but $S_{u,l}|0\rangle = -S_{l,u}|0\rangle$

Anyons carrying projective reps:

$$U_{gh}|a\rangle = e^{i\phi_a(g,h)}U_g U_h|a\rangle$$

a is any anyon

If $a \times b \to c$ then

$$\phi_a(g,h) + \phi_b(g,h) = \phi_c(g,h)$$

Example: $G = \mathbb{Z}_n$: fractional charges

$$U_{\frac{[k]}{n}}|a\rangle = e^{2\pi i \frac{[k]}{n} q}|a\rangle$$

a has charge q

[k] means (k mod n)

$$U_{\frac{[k+l]}{n}}|a\rangle = e^{2\pi i \frac{[k+l]}{n} q}|a\rangle$$

$$U_{\frac{[k]}{n}}U_{\frac{[l]}{n}}|a\rangle = e^{2\pi i \frac{[k]+[l]}{n} q}|a\rangle$$

phase mismatch when q is fractional

Suppose two G-symmetric gapped phases, 1 & 2, have the same anyons, and the anyons carry the same projective representations of G. Are 1 & 2 the same phase?

No: There exist 2d SPT's (non-trivial G-symmetric phases without anyons). Classified by:

$$H^3(G, U(1)) \longrightarrow \chi(f, g, h)$$

functions of 3 group elements....

Physically, $H^3(G, U(1))$ encodes fractional/projective charges carried by G-defects (i.e. G-fluxes)

$$U_{fg}|h\rangle = e^{i\chi(f,g,h)} U_f U_g |h\rangle$$

↑ h-flux

Gauge G:

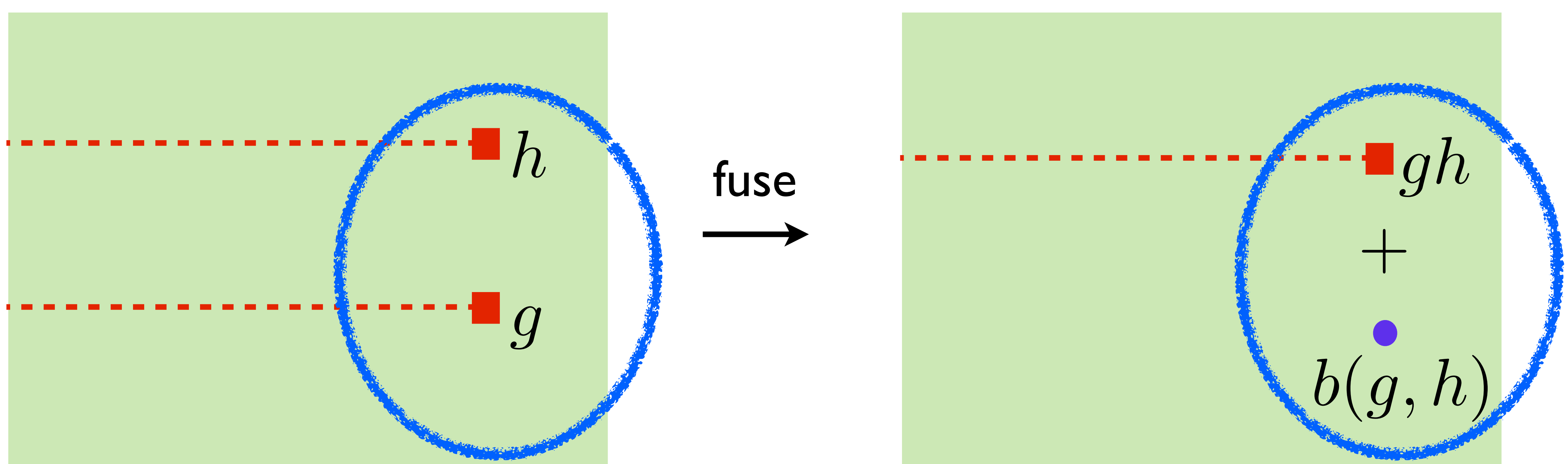
G-fluxes, G-charges become deconfined excitations:

quasiparticles = {original anyons, G-fluxes, G-charges}

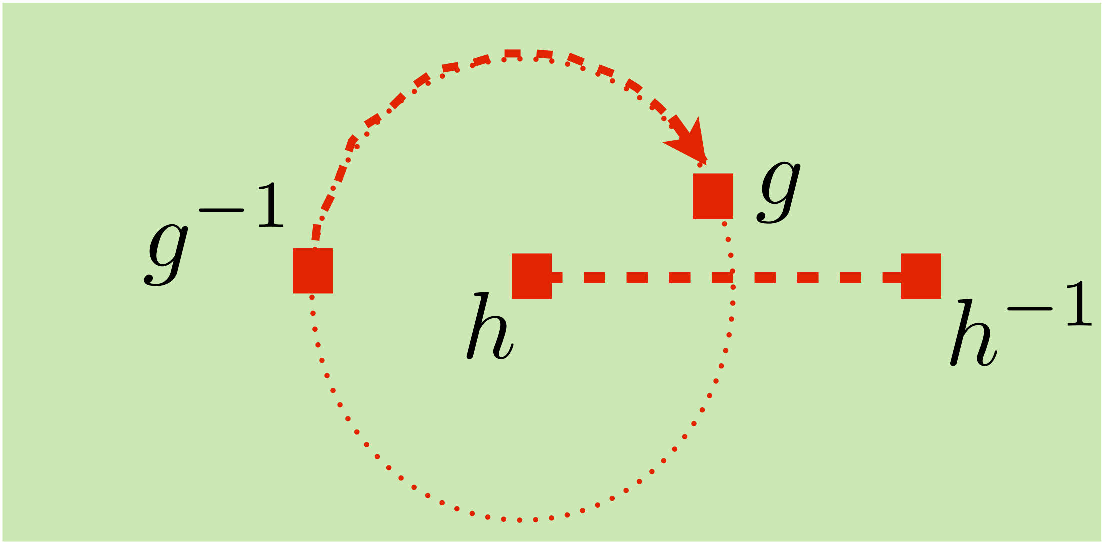
Then the projective character of the anyons and defects is reflected in the fusion/braiding structure of this enlarged theory.

(see also Levin & Gu, arXiv:1202:3120)

1) projective/fractional anyons => defect fusion rules



2) projective/fractional defects => defect braiding rules



Any consistent theory for fusion/braiding in enlarged theory must come from some some choice of these two invariants

(Etingof et al., Kirillov, Mueger)

(can also handle case when G changes anyon superselection sectors)

Connection to 3D SPT's:

Is every choice of $H^2(G, A)$ and $H^3(G, U(1))$ realized in some 2d theory?

No: sometimes there is an obstruction to a consistent choice of braiding and fusion rules in the enlarged theory

Example:

anyons = {1,a} (a=semion)

abelian theory, K=(2)

$\nu = 1/2$ bosonic FQHE

G = $\mathbb{Z}_2 \times \mathbb{Z}_2$, a is projective under G

Although this theory cannot be realized in 2d, we have an exactly solvable model which realizes it at the surface of a 3d SPT.

(with F. Burnell, X. Chen, A. Vishwanath)

In general, the obstruction is in $H^4(G, U(1))$, which classifies 3d SPT's.