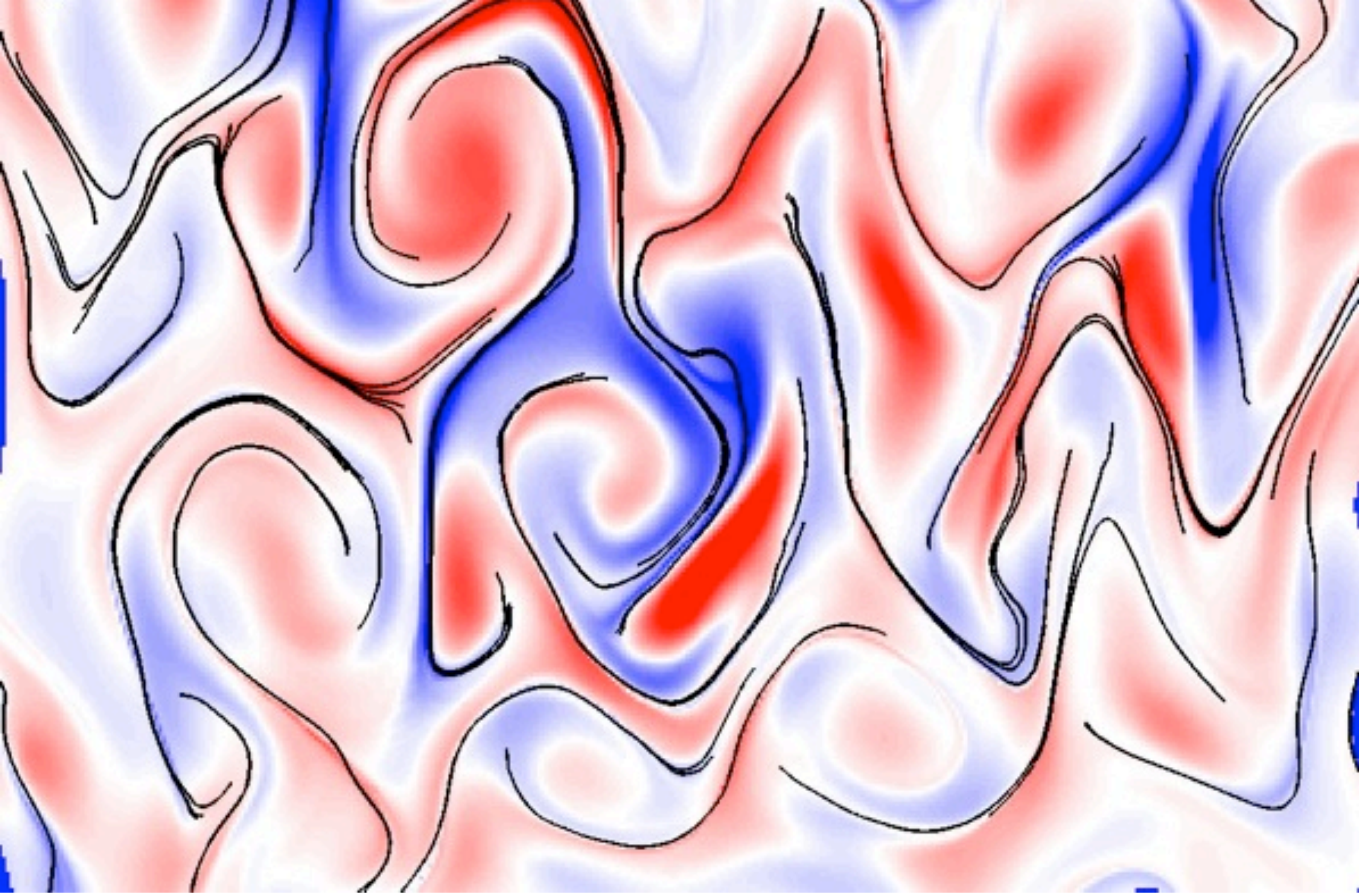


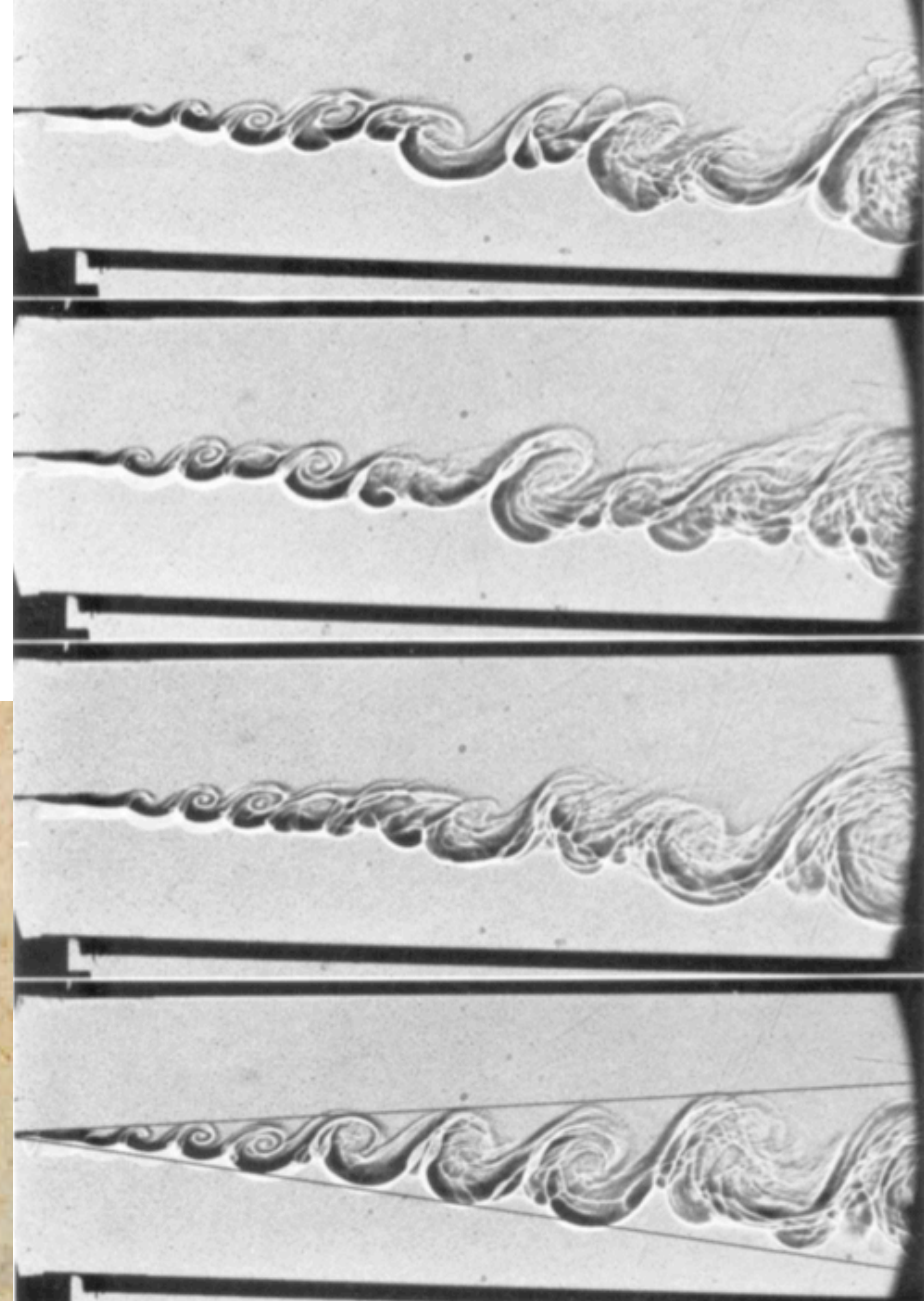
Connecting Spectral Dynamics to Coherent Structures

N. T. Ouellette
D. H. Kelley, Y. Liao, M. R. Allshouse

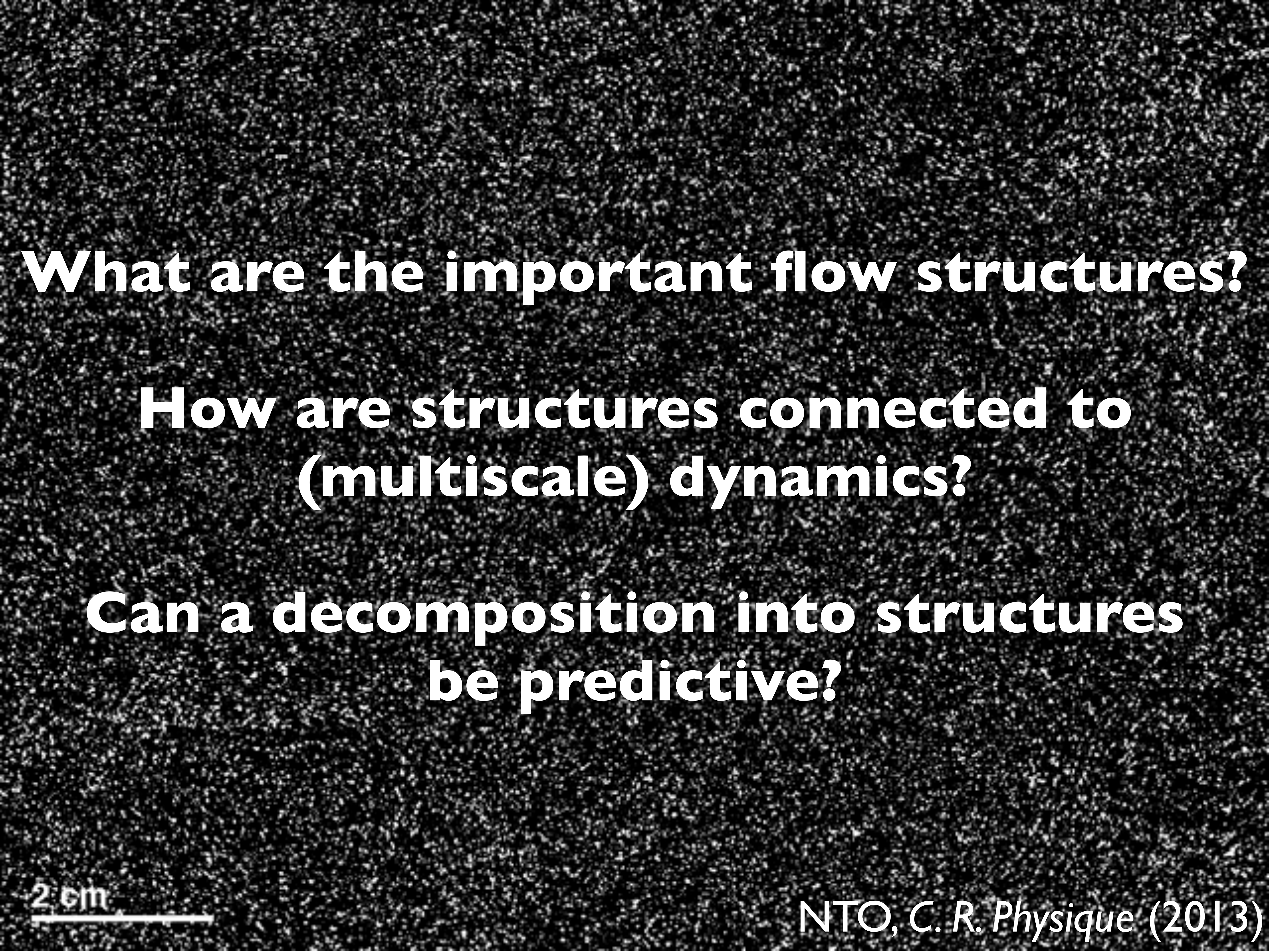
Flow Structures

Turbulent flows are full of structures!

Can the “right” definition of structures help us build better models?



G.L. Brown & A. Roshko, *J. Fluid Mech.* (1974)



What are the important flow structures?

How are structures connected to (multiscale) dynamics?

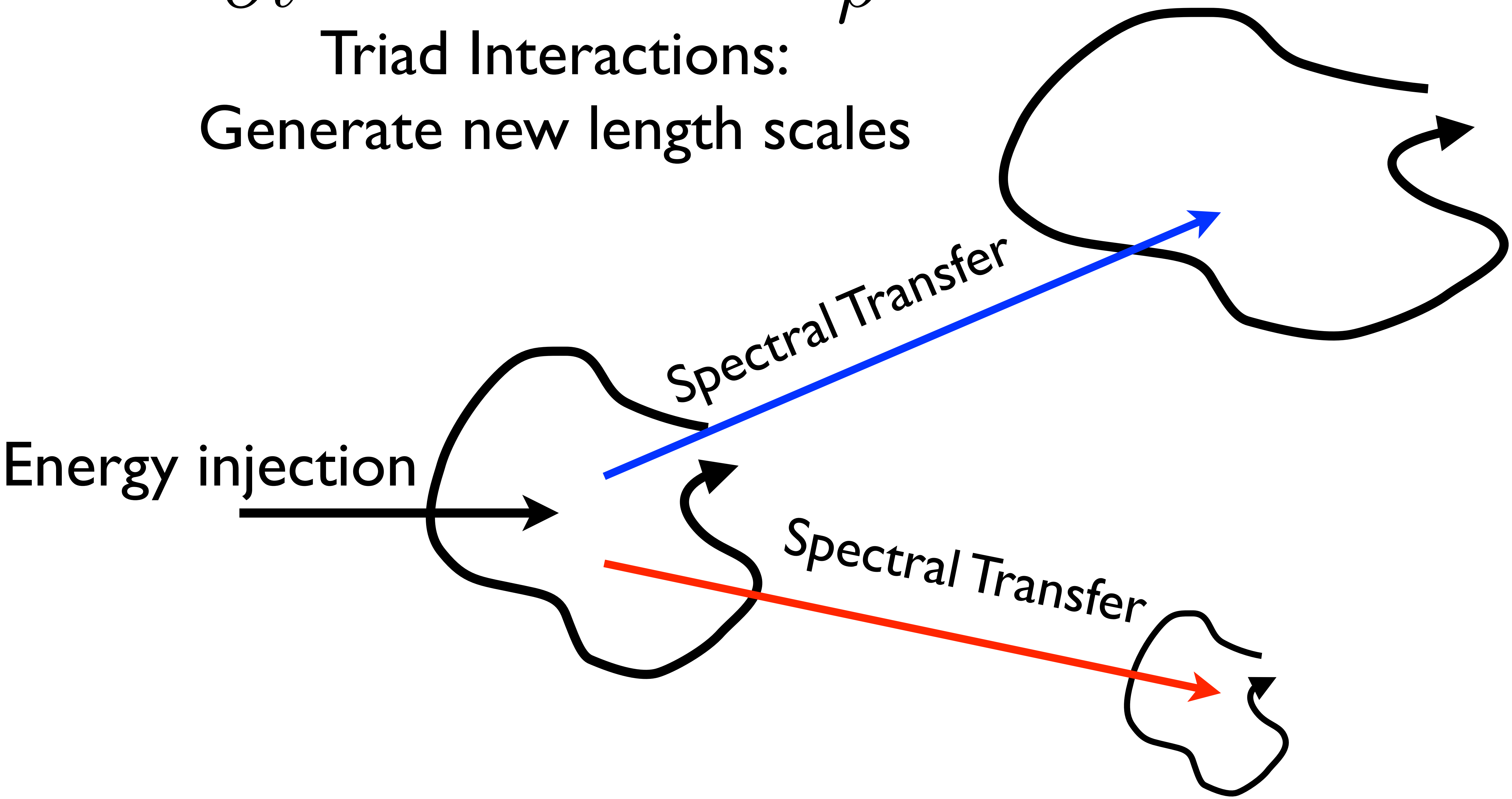
Can a decomposition into structures be predictive?

NTO, *C. R. Physique* (2013)

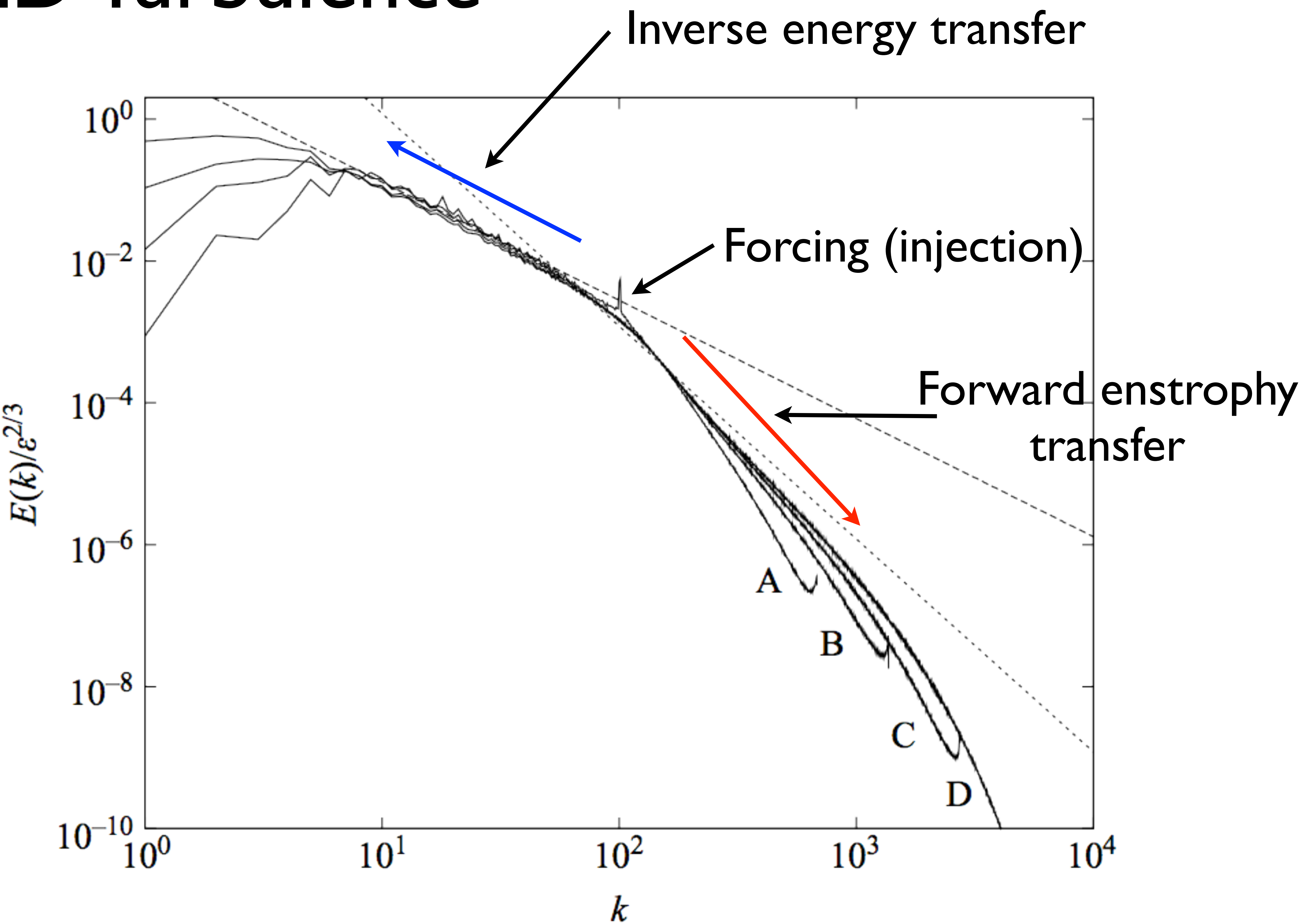
Dynamics

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u}$$

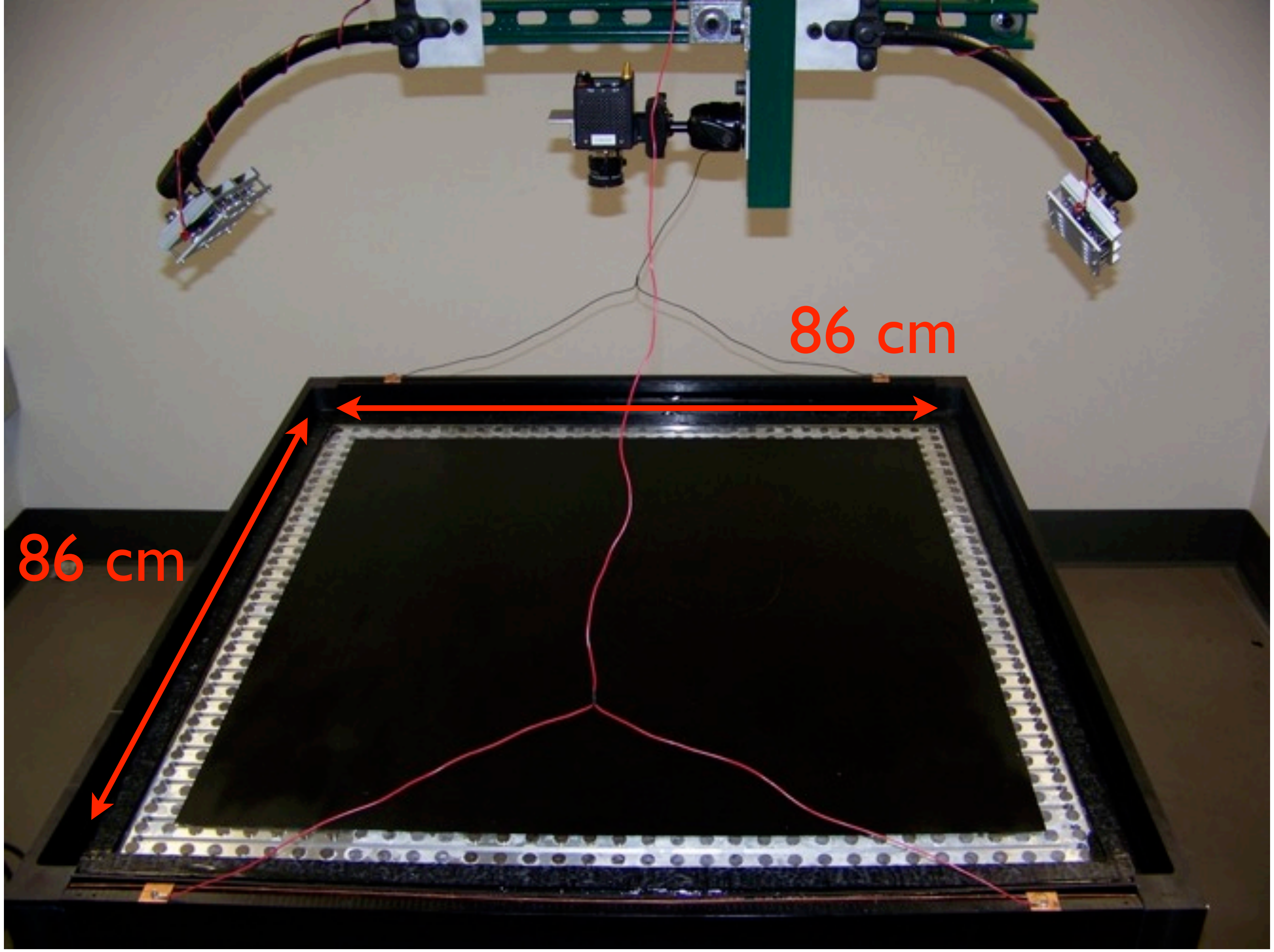
Triad Interactions:
Generate new length scales

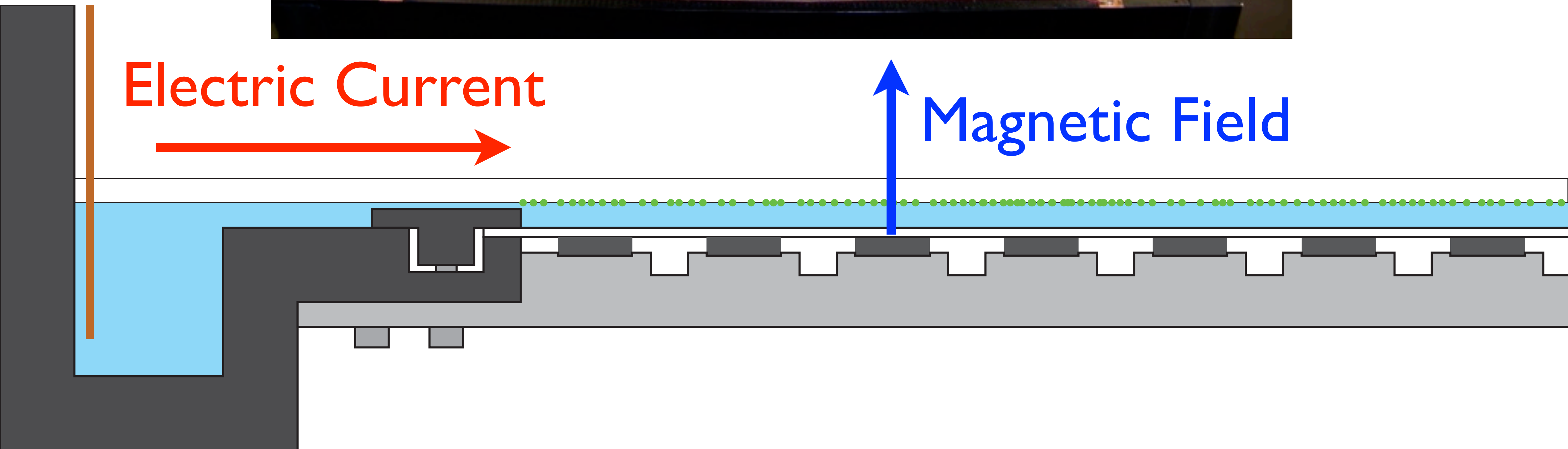


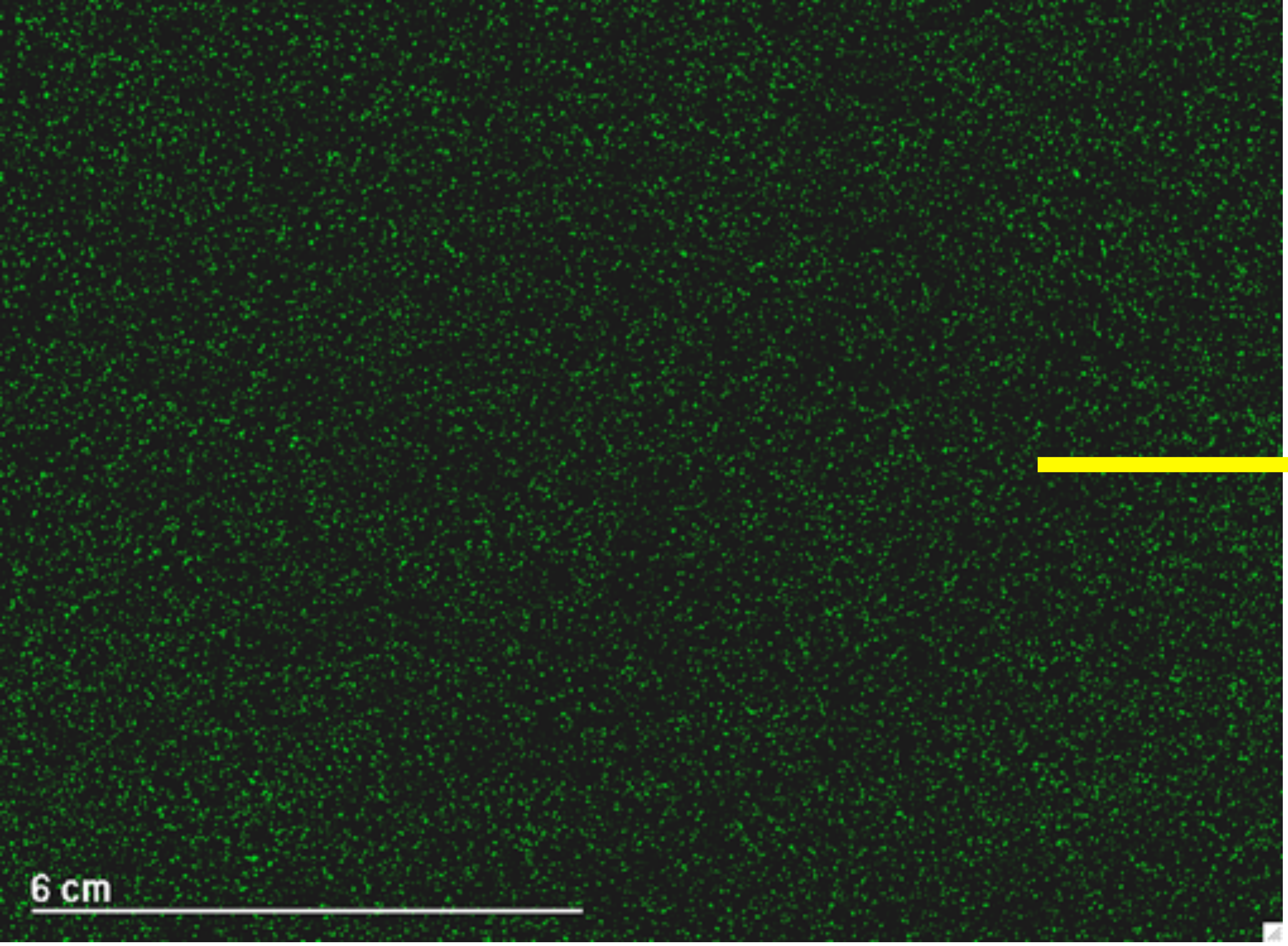
2D Turbulence

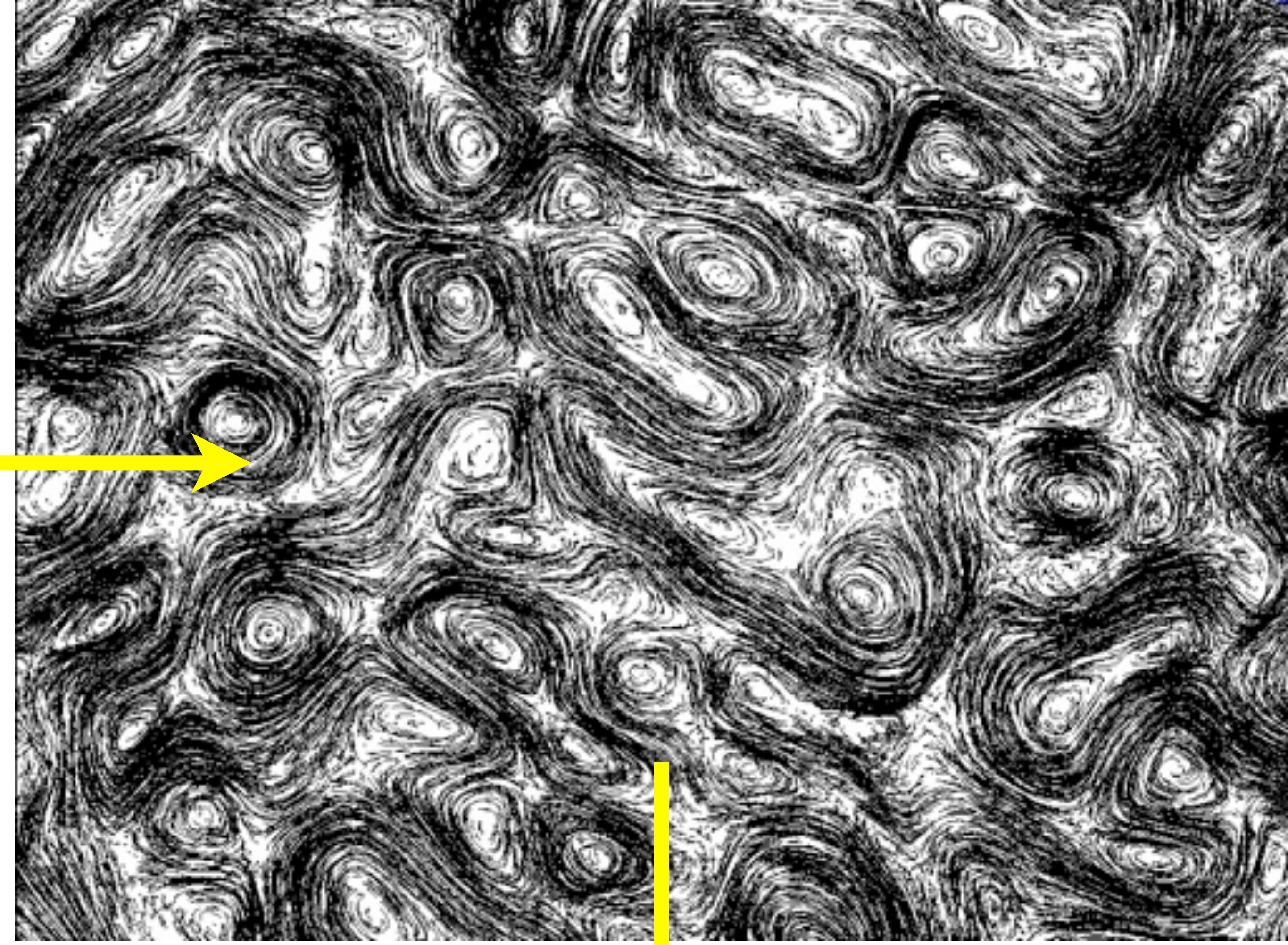


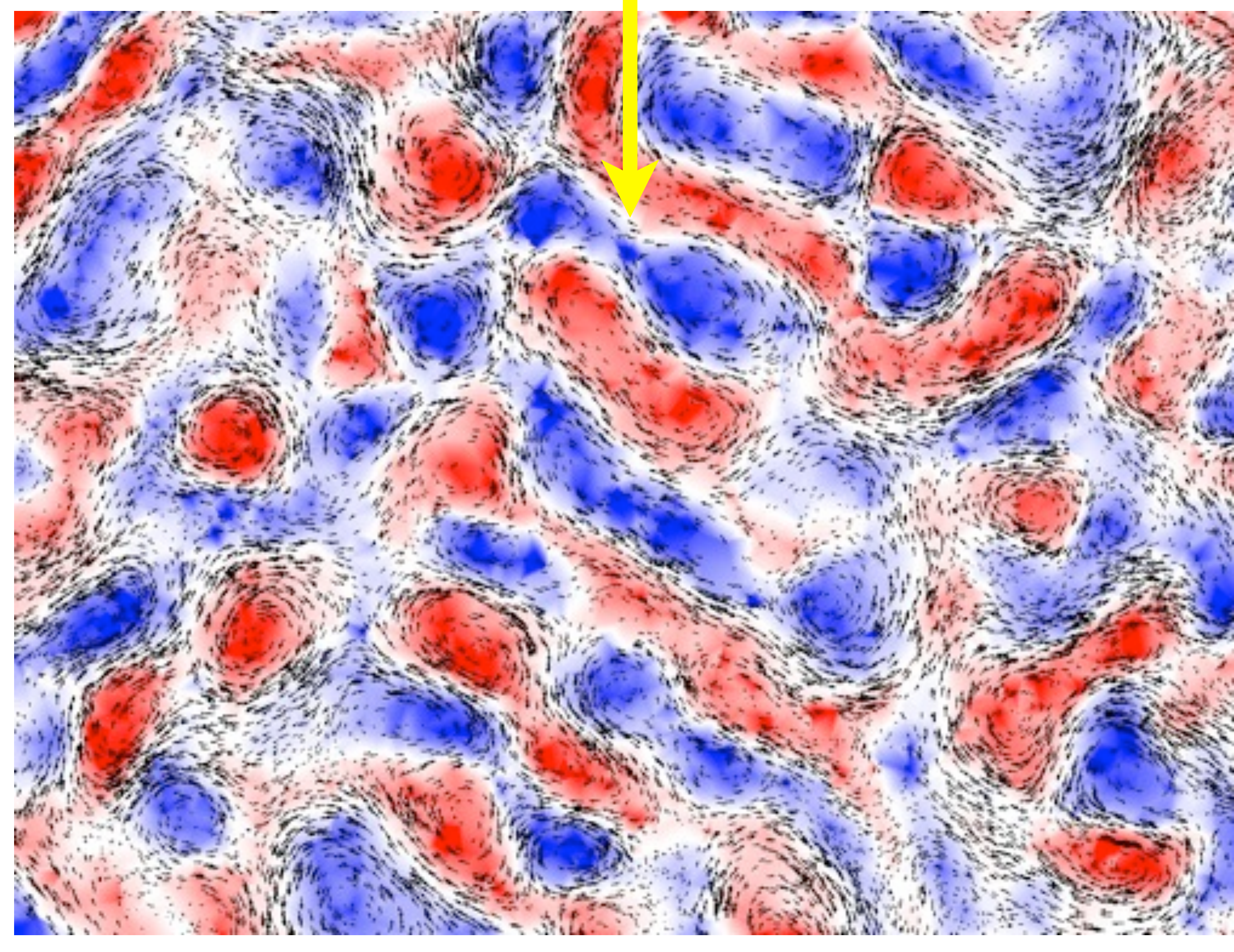
G. Boffetta, *J. Fluid Mech.* (2007)


86 cm
86 cm


Electric Current
Magnetic Field







Obtain velocity field with PTV

50 μm particles, ~35k per frame

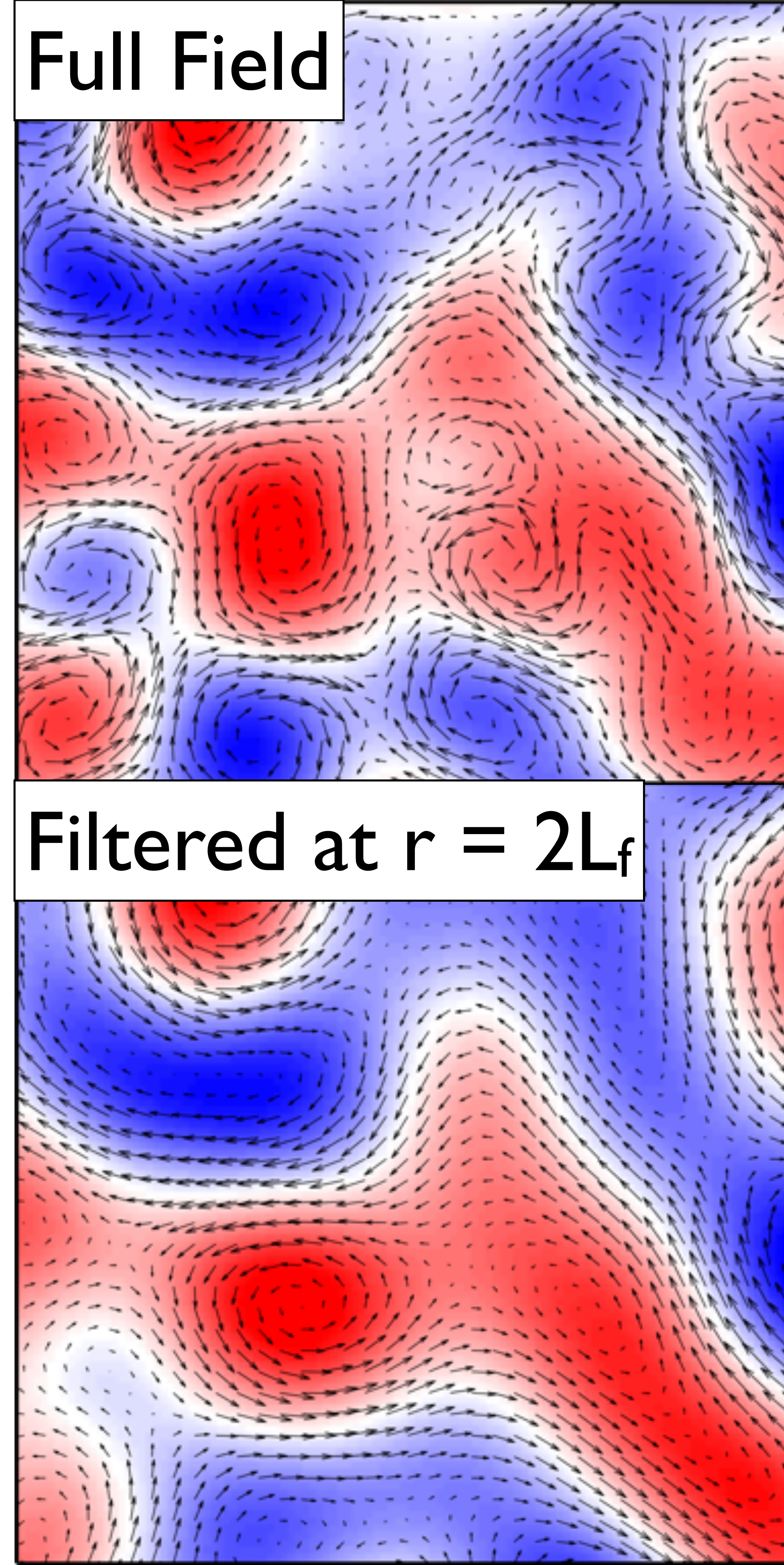
Advect virtual particles through field

NTO, H. Xu, & E. Bodenschatz, *Exp. Fluids* (2006)
NTO, P.J.J. O'Malley, & J.P. Gollub, *Phys. Rev. Lett.* (2008)
D.H. Kelley & NTO, *Phys. Fluids* (2011)

Spatially Resolved Spectral Fluxes

Convolve velocity field with spectral low pass filter:

$$\boldsymbol{u}^{(r)} = \int G^{(r)}(\boldsymbol{x}-\boldsymbol{x}')\boldsymbol{u}(\boldsymbol{x})\mathrm{d}\boldsymbol{x}'$$



M. Germano, *J. Fluid Mech.* (1992)
S. Liu, C. Meneveau, & J. Katz, *J. Fluid Mech.* (1994)
G.L. Eyink, *J. Stat. Phys.* (1995)
M.K. Rivera et al., *Phys. Rev. Lett.* (2003)

Equation of motion for filtered energy:

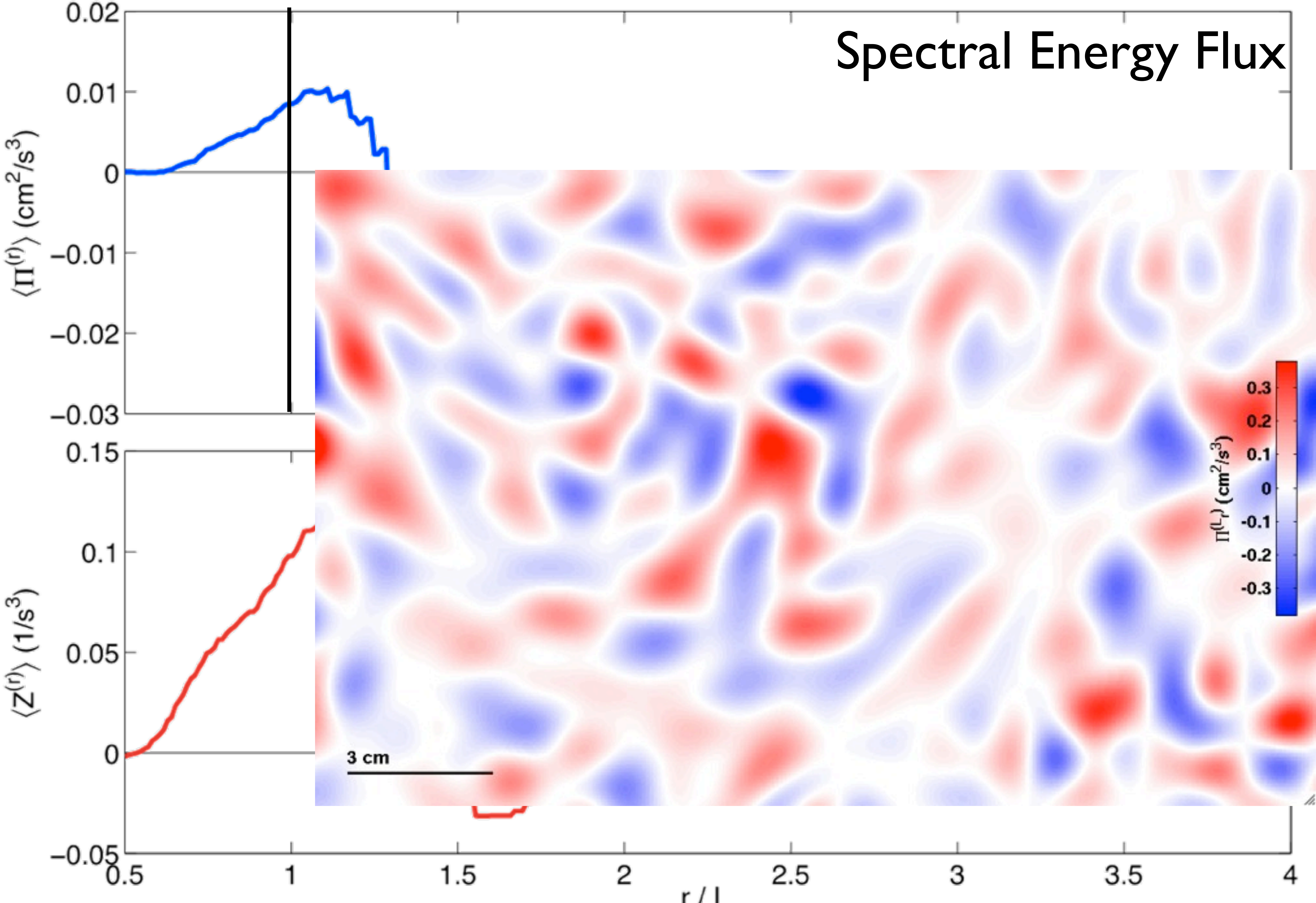
$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j}\frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point	Spatial transport	Viscous dissipation	Coupling to other scales

$$\Pi^{(r)} = -\left[(u_i u_j)^{(r)} - u_i^{(r)} u_j^{(r)}\right] \frac{\partial u_i^{(r)}}{\partial x_j}$$

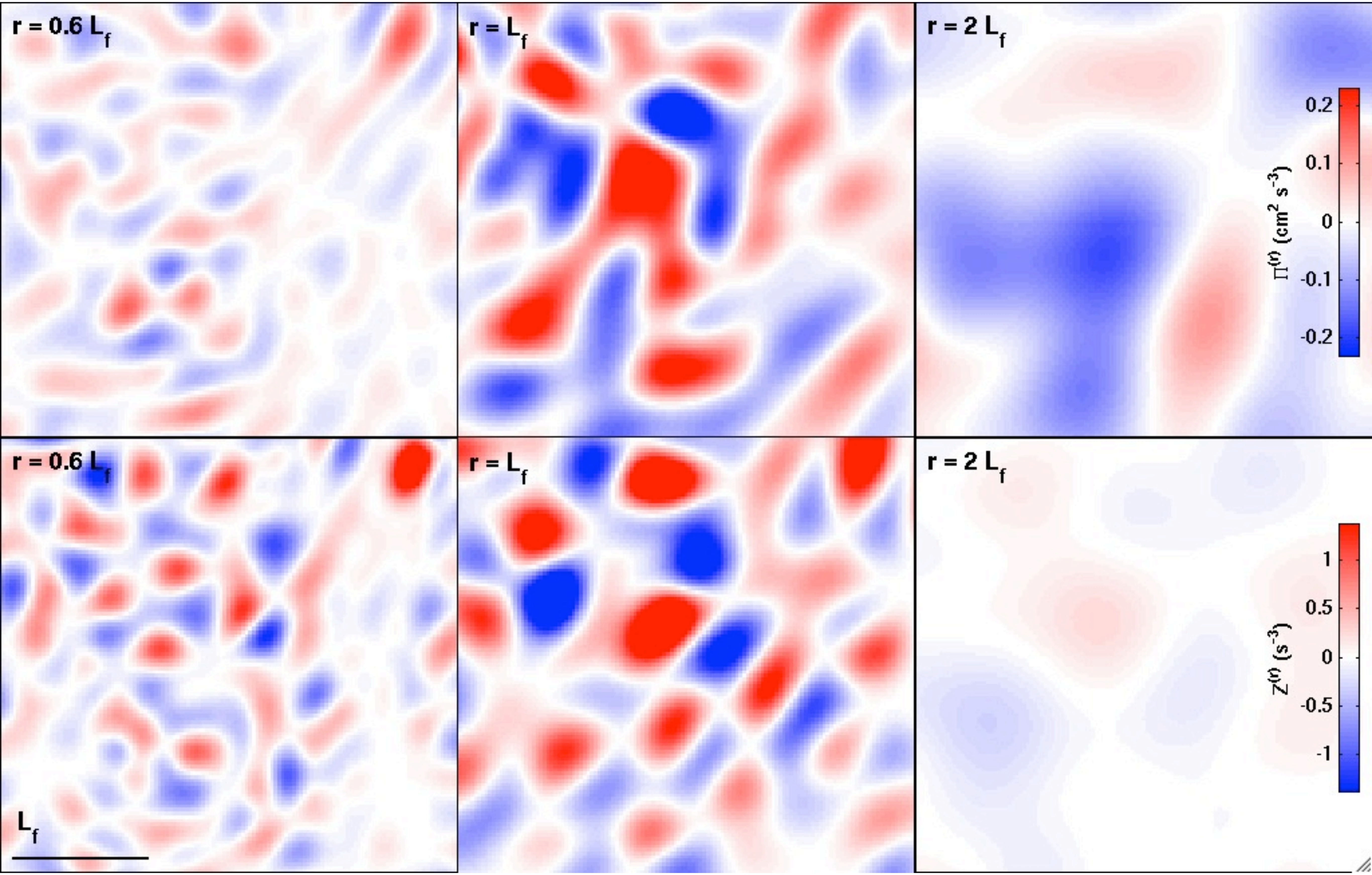
G.L. Eyink, *J. Stat. Phys.* (1995)
M.K. Rivera et al., *Phys. Rev. Lett.* (2003)

Spectral Energy Flux



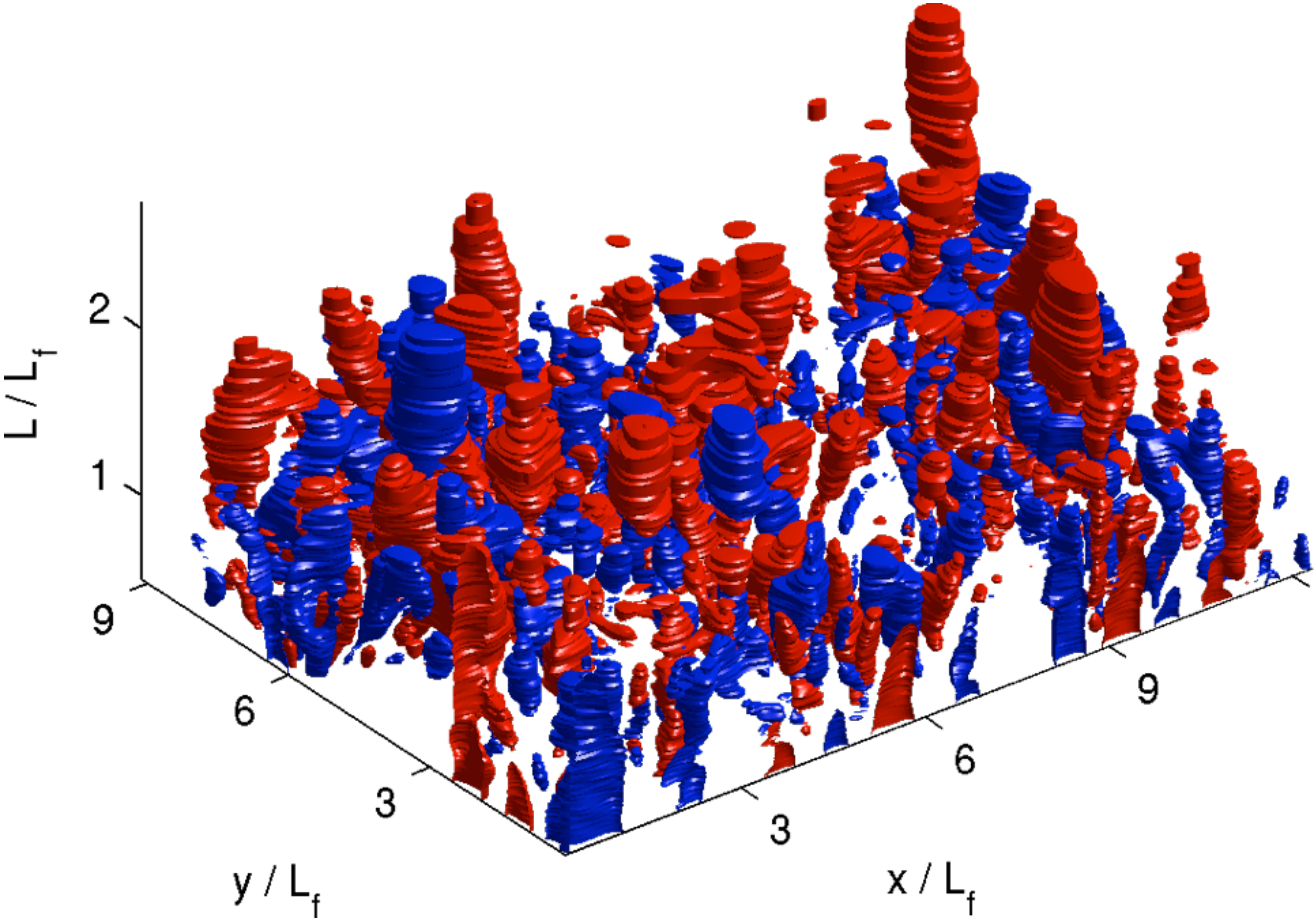
D.H. Kelley & NTO, *Phys. Fluids* (2011)

Energy →



Enstrophy →

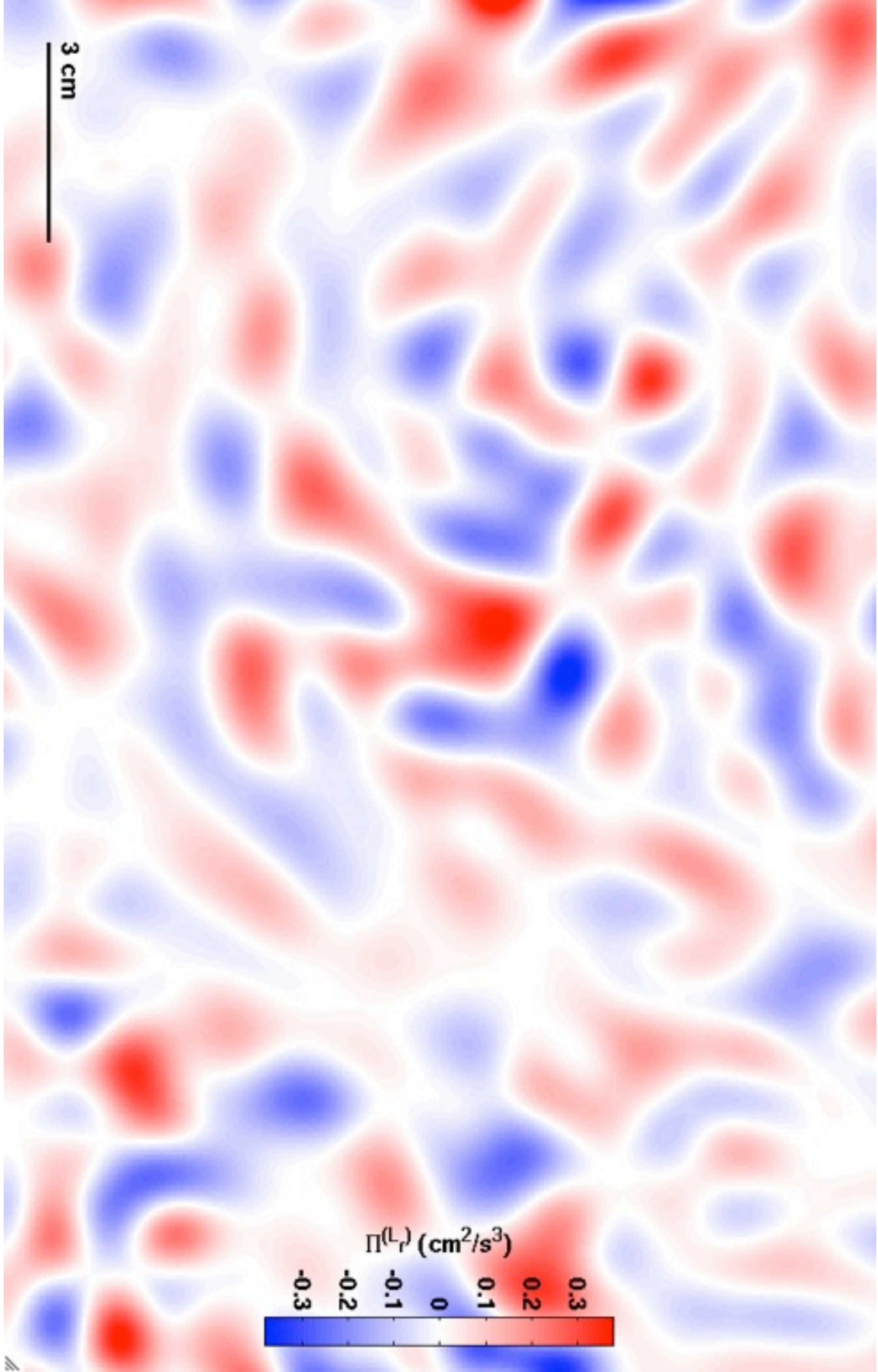
D.H. Kelley & NTO, *Phys. Fluids* (2011)



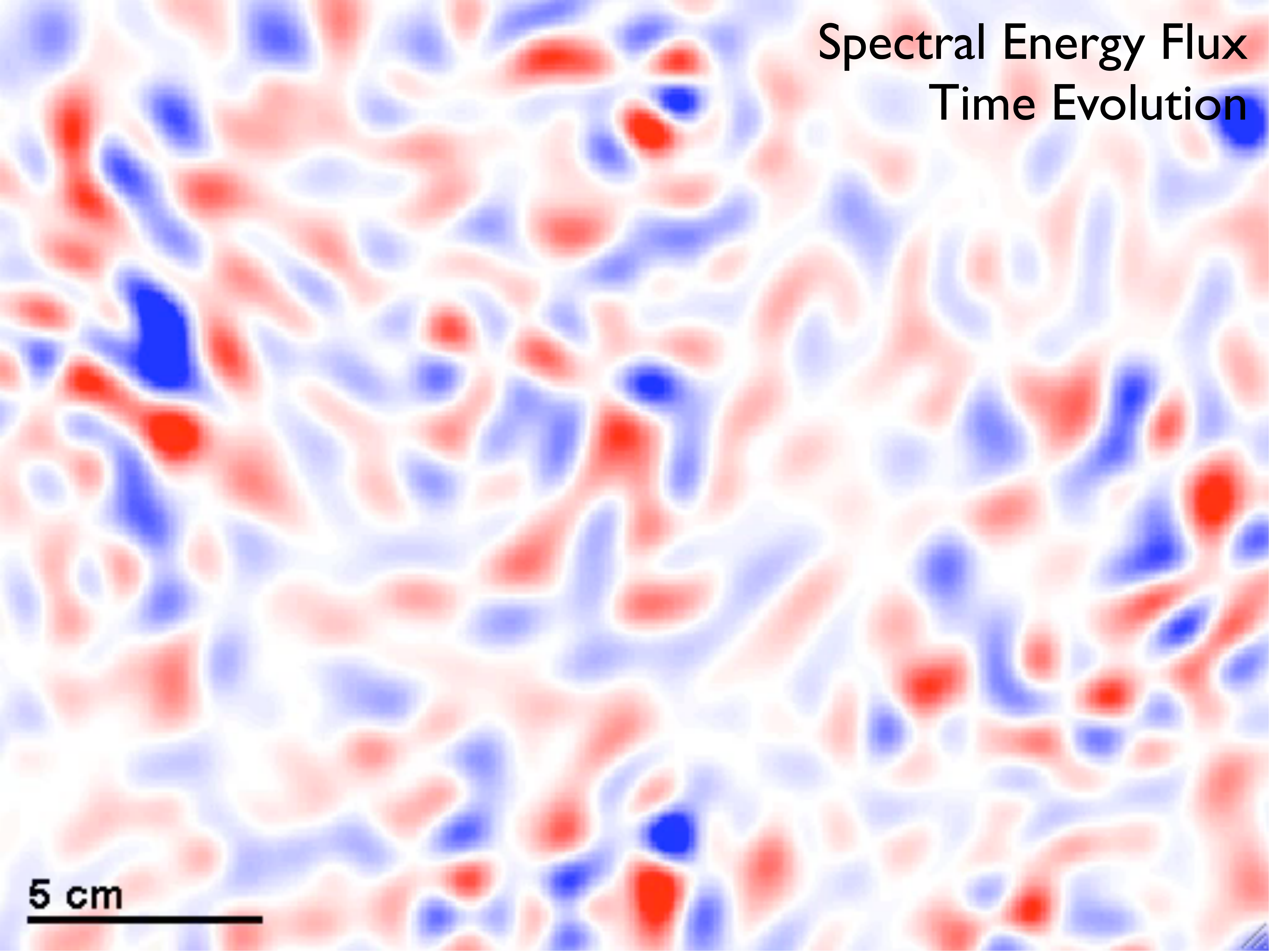
NTO, *C. R. Physique* (2012)

Spectral transfer is not constant in time!

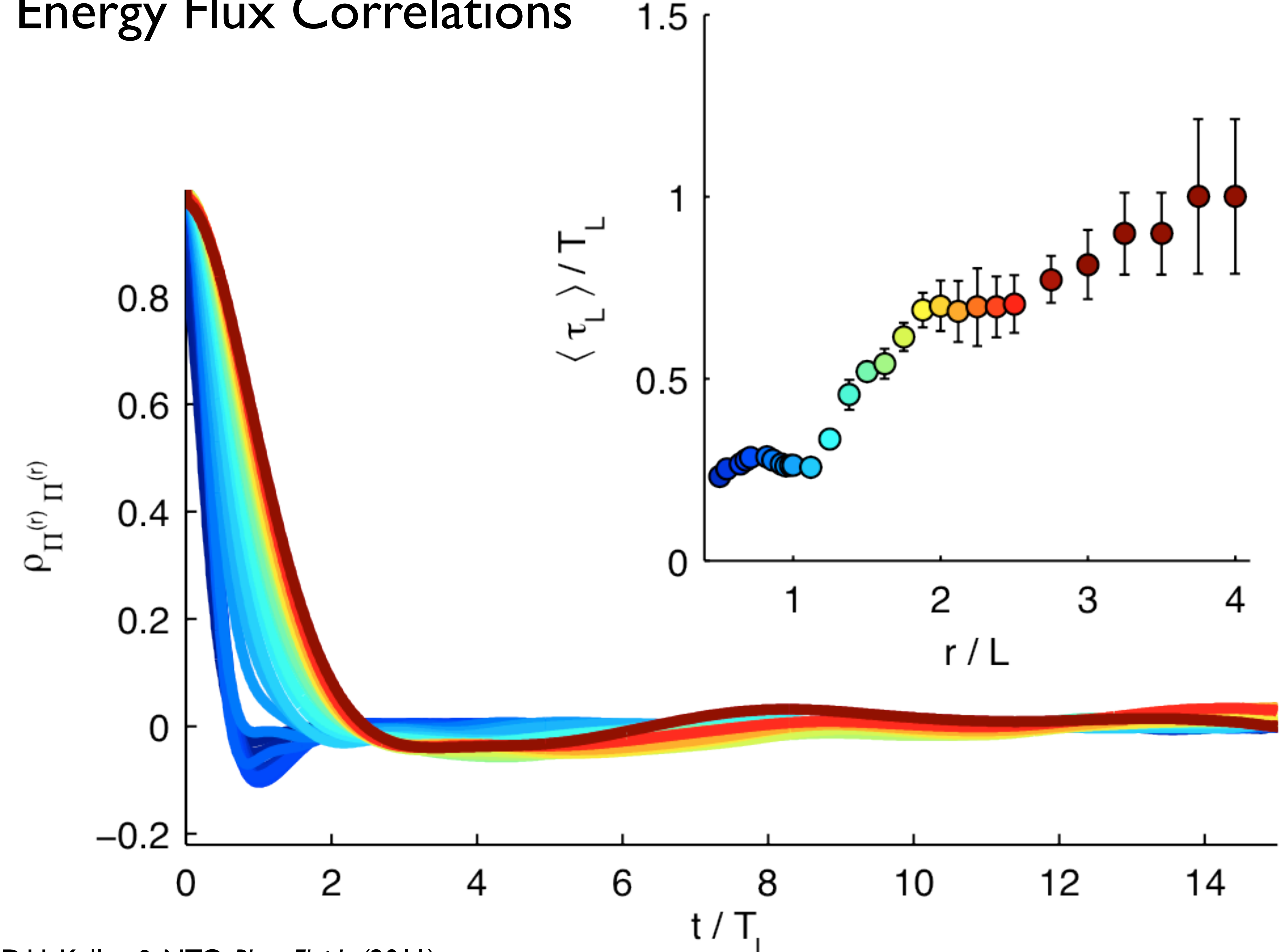
How does is change?
What are its dynamics?



Spectral Energy Flux
Time Evolution

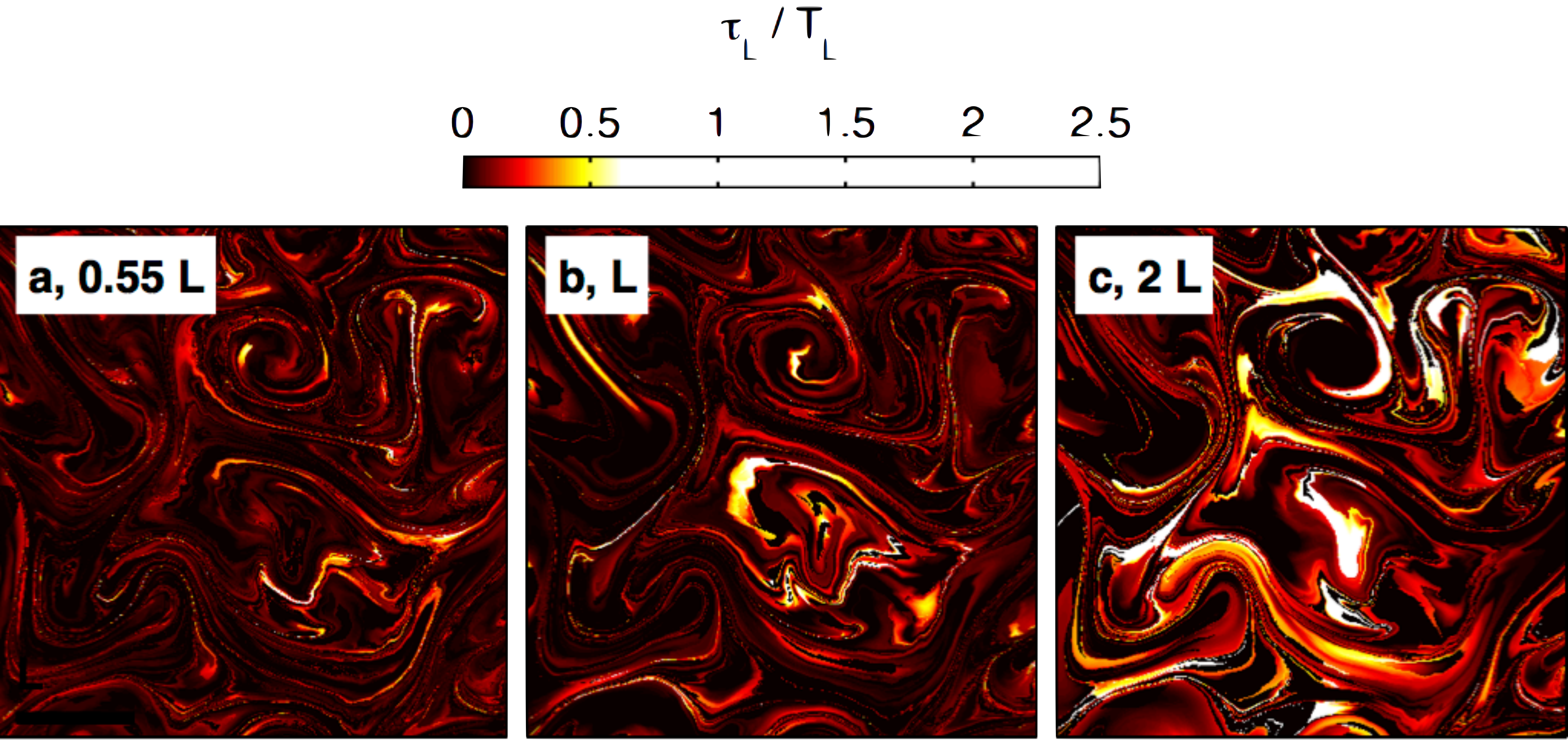


Energy Flux Correlations



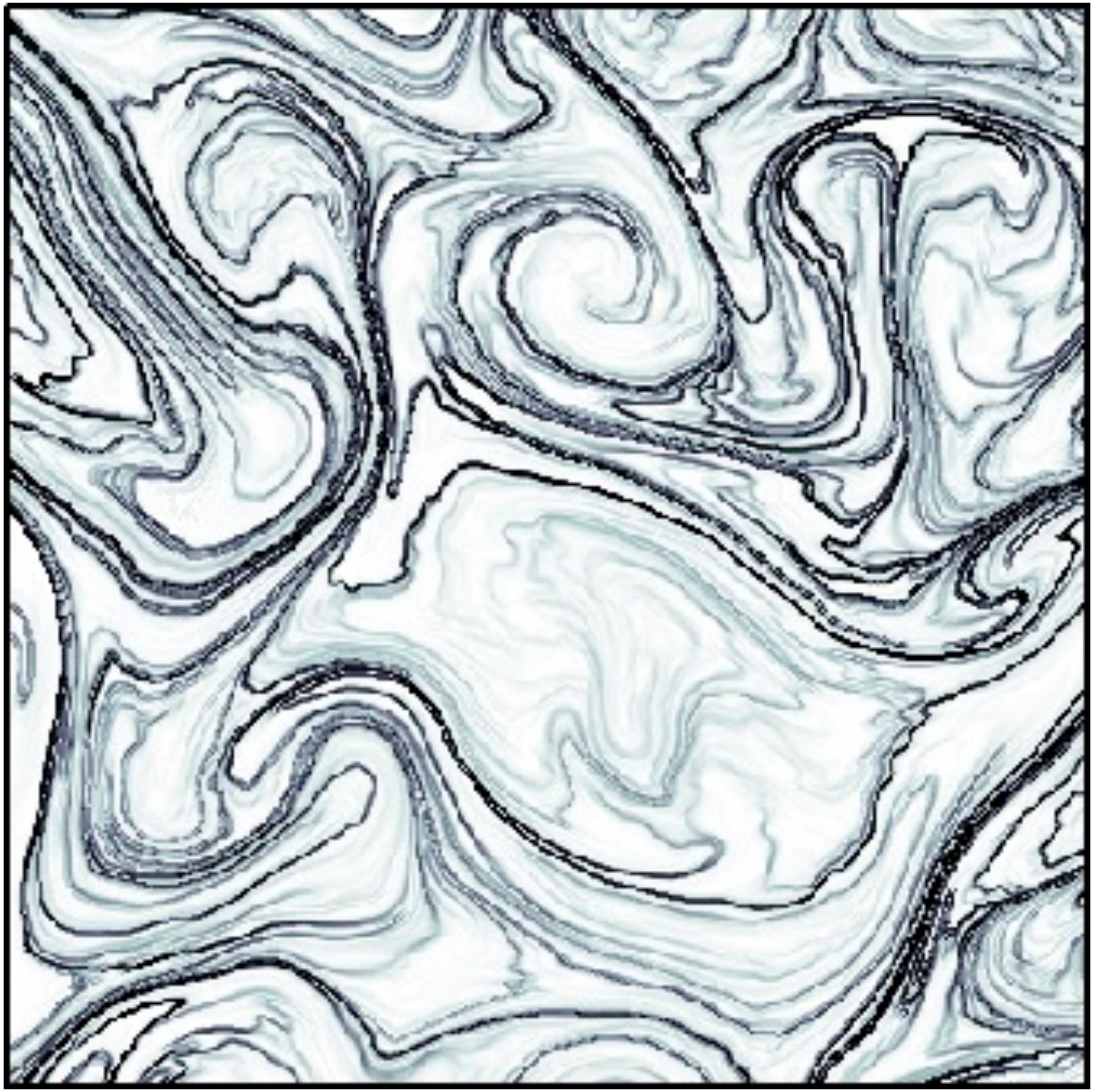
D.H. Kelley & NTO, *Phys. Fluids* (2011)

Spatial Dependence of Integral Times

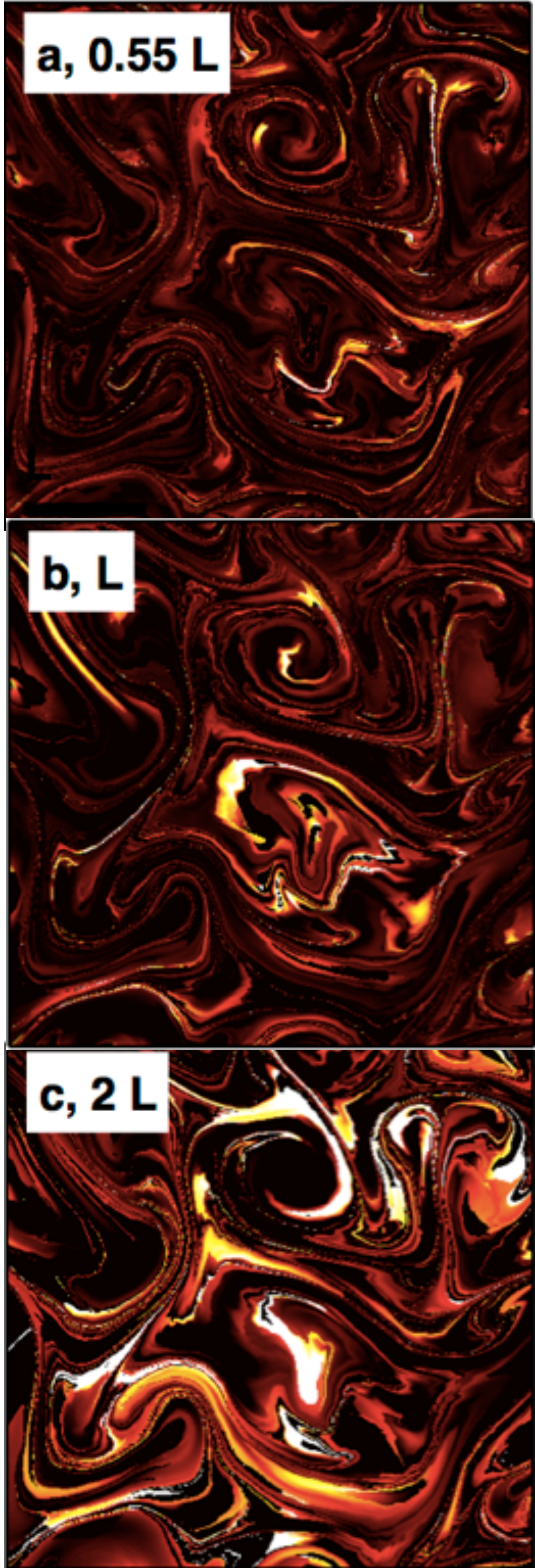


D.H. Kelley & NTO, *Phys. Fluids* (2011)

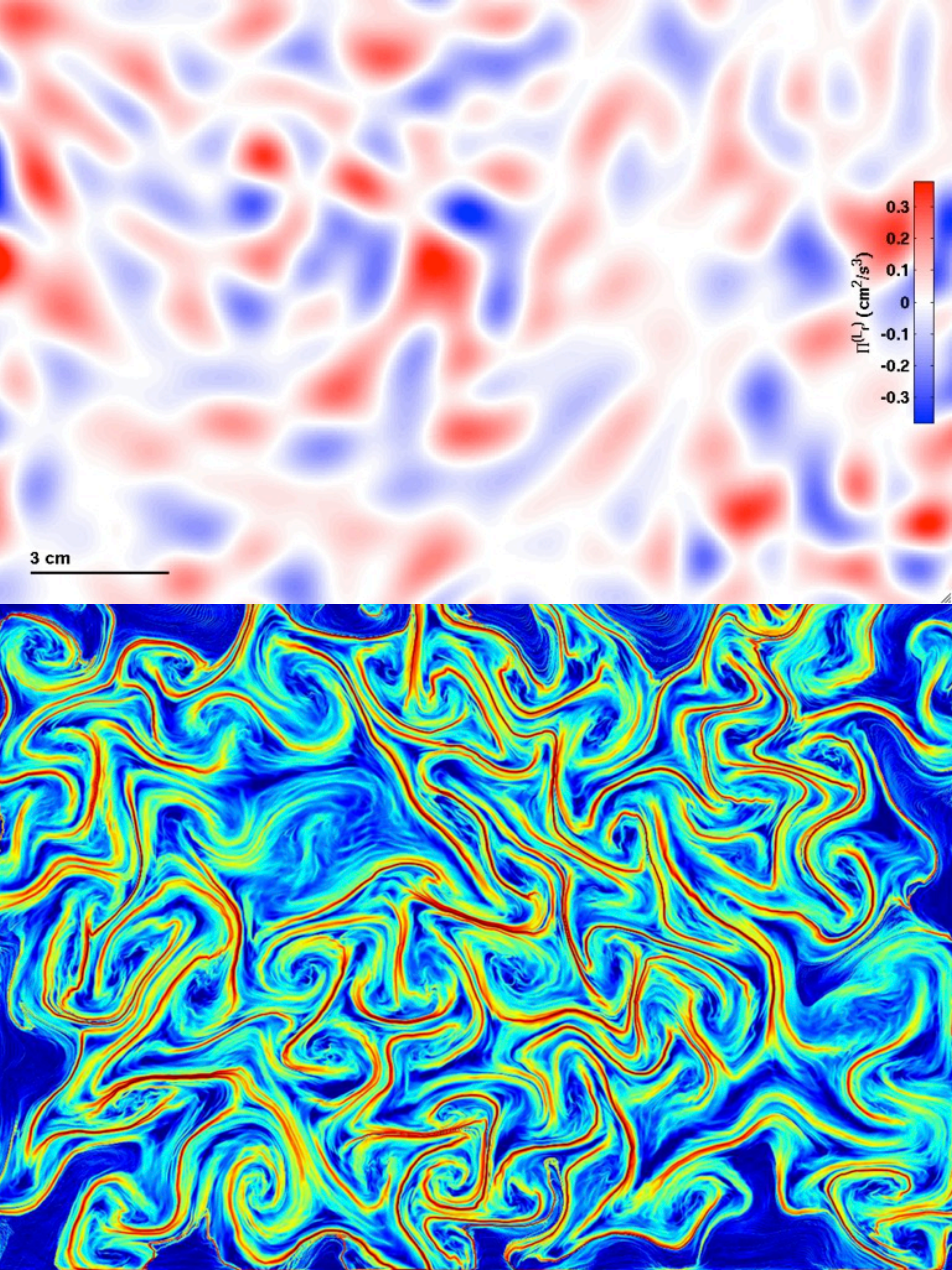
LCS?



FTLE Field

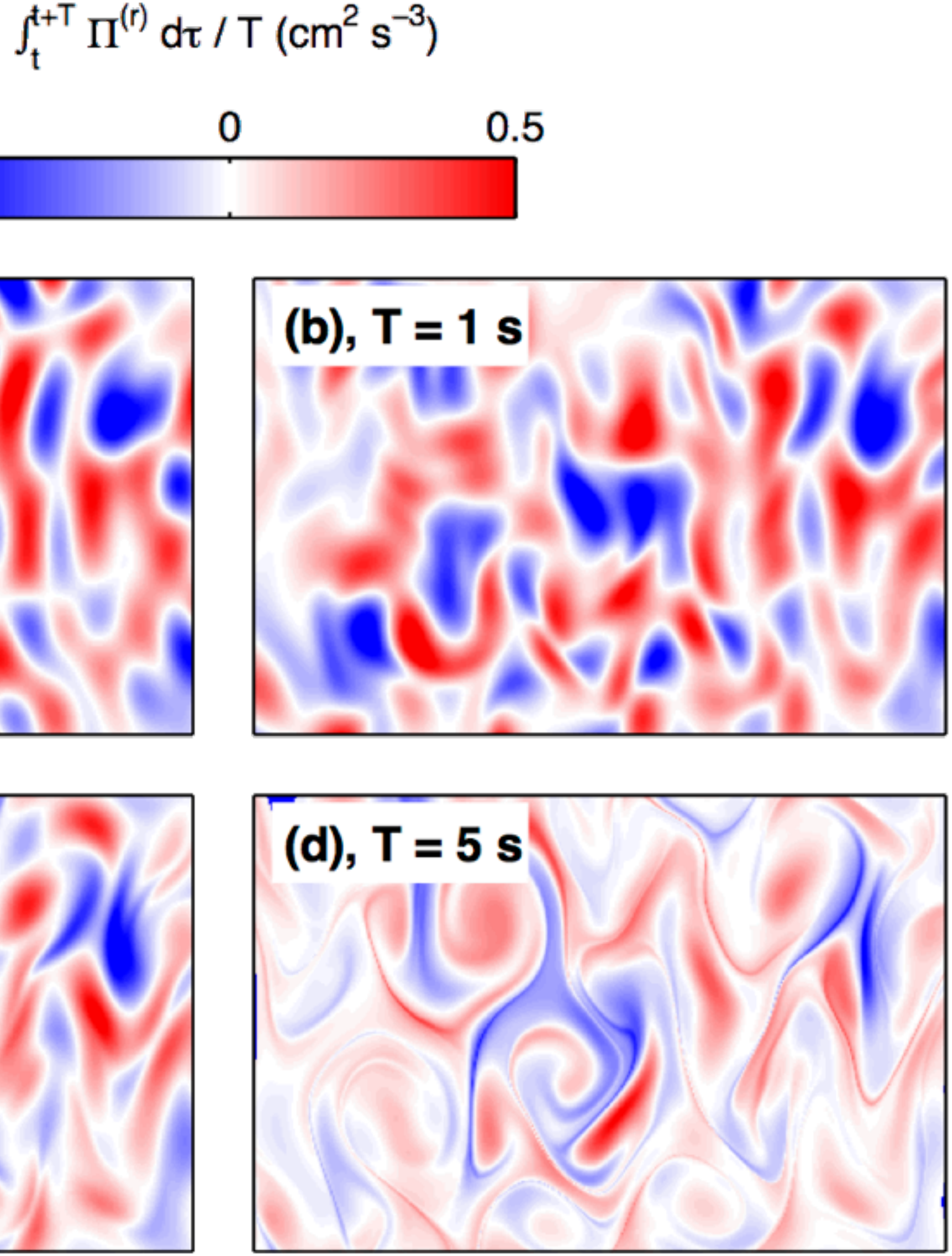


D.H. Kelley & NTO, *Phys. Fluids* (2011)



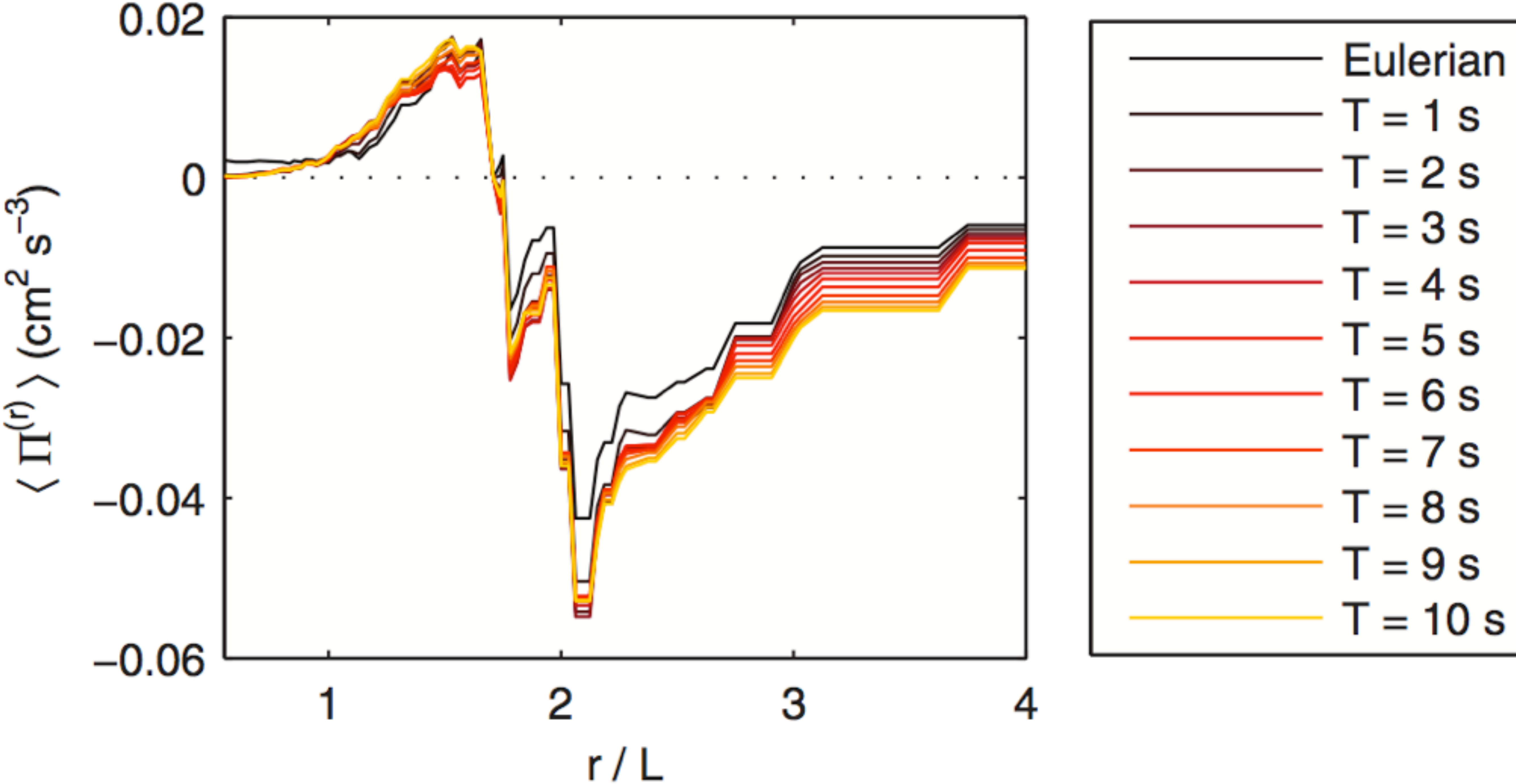
3 cm
$\Pi^{(L_r)}$ (cm^2/s^3)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3

Lagrangian Flux



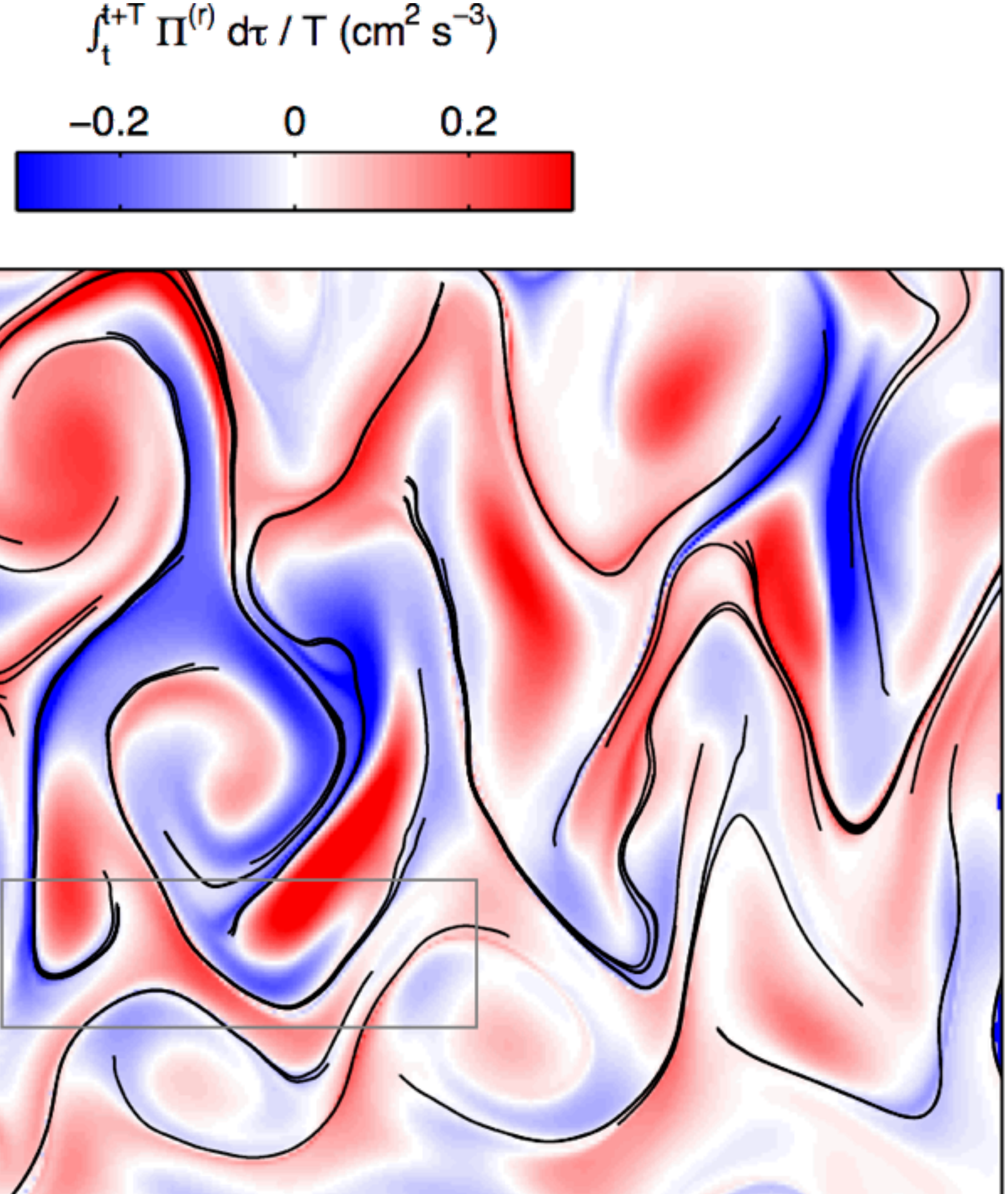
D.H. Kelley, M.R. Allshouse, & NTO, *Phys. Rev. E* (2013)

Stable Averages



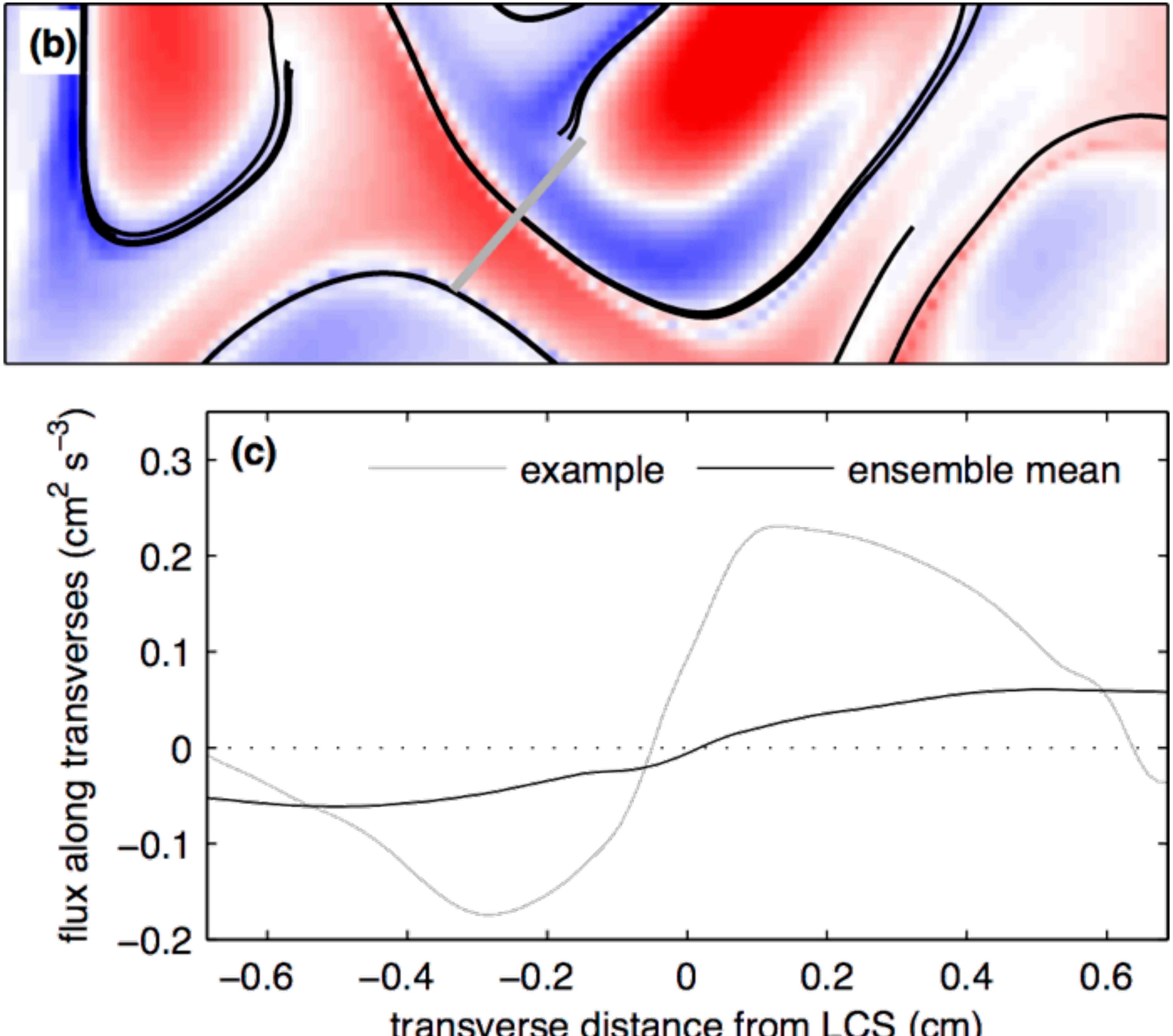
D.H. Kelley, M.R. Allshouse, & NTO, *Phys. Rev. E* (2013)

LCS Overlay



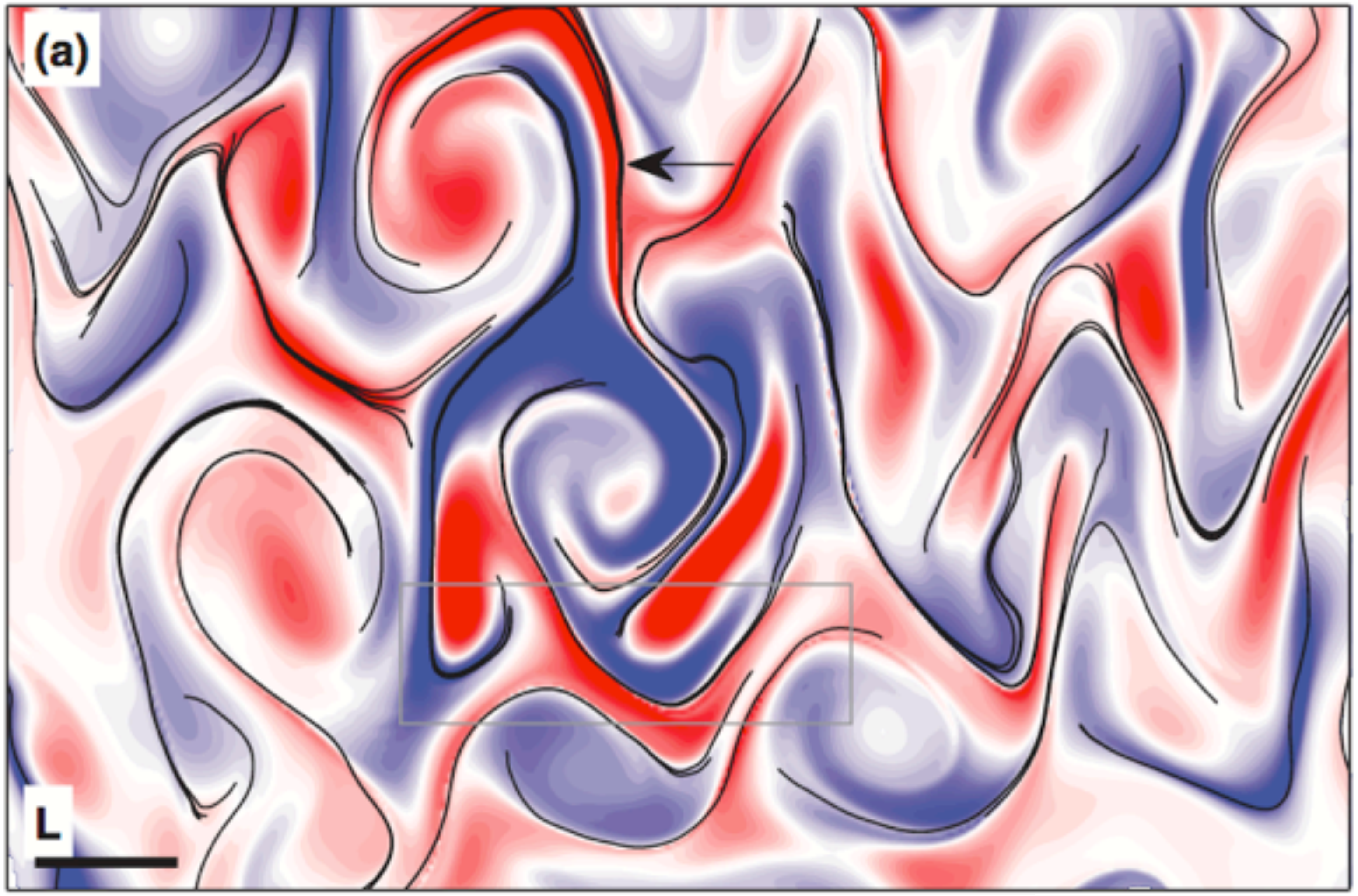
D.H. Kelley, M.R. Allshouse, & NTO, *Phys. Rev. E* (2013)

LCS Divide Dynamically Distinct Regions



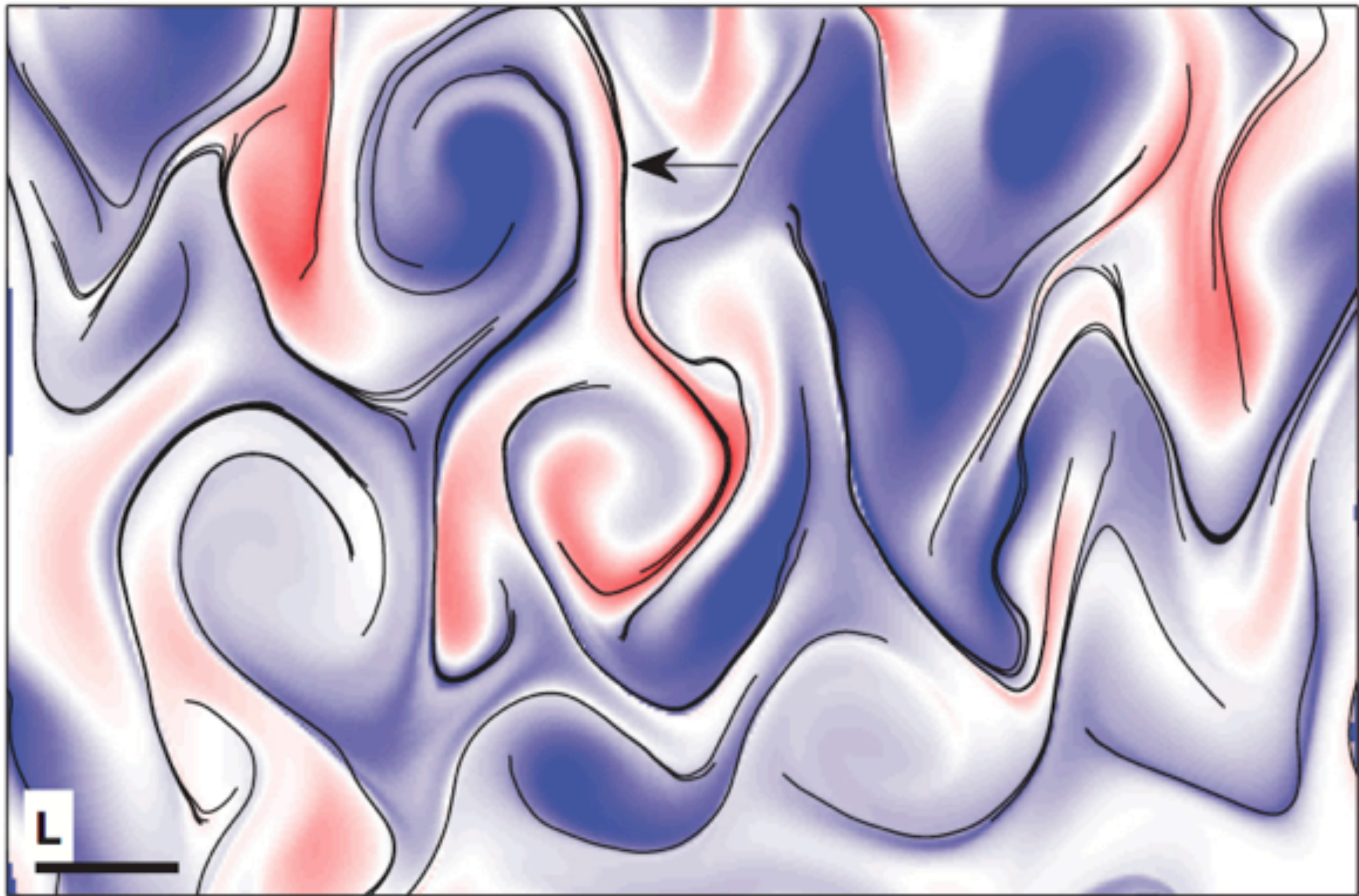
D.H. Kelley, M.R. Allshouse, & NTO, *Phys. Rev. E* (2013)

Different Scales?



r = 1.75 L_f

r = 2.09 L_f



Summary

Spectral transport couples to spatial transport

LCS tend to separate dynamically distinct regions

Can this be put on a solid mathematical foundation?

How can we incorporate dynamics into definitions of structure?

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