Indirect Transmitted Infectious Diseases: from Microscopic Cycles to Macroscopic Cycles

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John Snow's Ghost Map

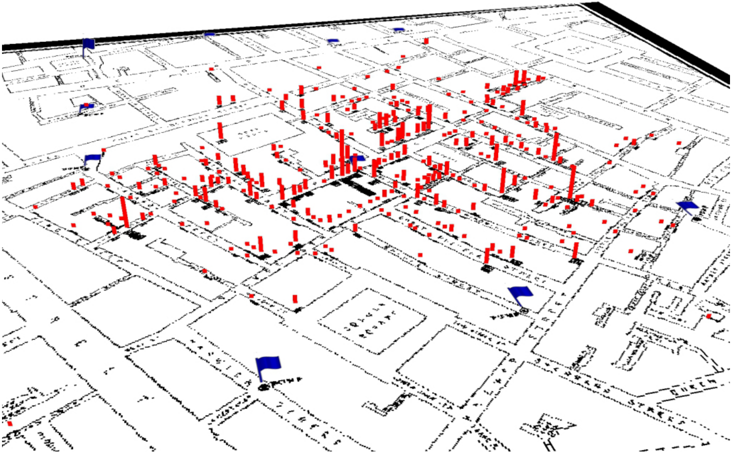


Figure: Blue flags represent pumps for drinking water and red bars represent deaths at that address

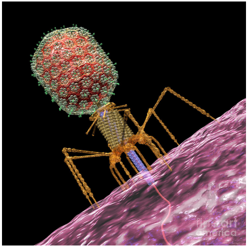
Infectious diseases

These are diseases caused by pathogenic micro organisms such as **bacteria**, virus, parasites and fungi; the disease can be spread directly or **indirectly** from one person to another.

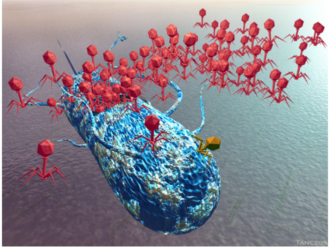
- Direct contact transmission$\rightarrow$ occurs when there is a physical contact between an infected person and a susceptible person.
- Indirect contact transmission$\rightarrow$ occurs when there is no direct human-to-human contact. Contact occurs from a reservoir to contaminated surfaces or objects, or to vectors.

- **Our focus** $\rightarrow$ bacteria $\bigoplus$ indirect transmission
- **Reservior** $\rightarrow$bacteria $\bigoplus$ bacteriophage

Bacteriophage $\rightarrow$It is a virus that prey on bacteria.



(a) phage injecting it's DNA into a bacterium

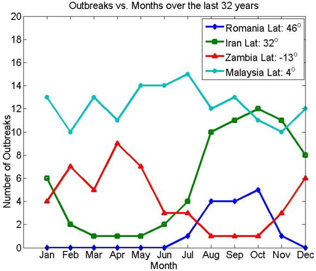


(b) bacterium producing masses of new viruses

- **Microscopic cycles** $\rightarrow$b $\bigoplus$ bacteriophage cycles
- **Macroscopic cycles** $\rightarrow$ human population cycles
- **Minimum Infection Dose (MID)-** The minimum amount of pathogen required to cause an infection in the host.

Our objectives

- Show that the cyclical outbreaks in endemic regions are driven by the cycles generated by the predator-prey relationship that exist between bacteriophage and bacteria.
- Demonstrate the importance of the relationship between the MID and the bacterial carrying capacity in relation to the existence of the cycles
- Attempt an explanation to the different nature of the outbreaks observed around the world



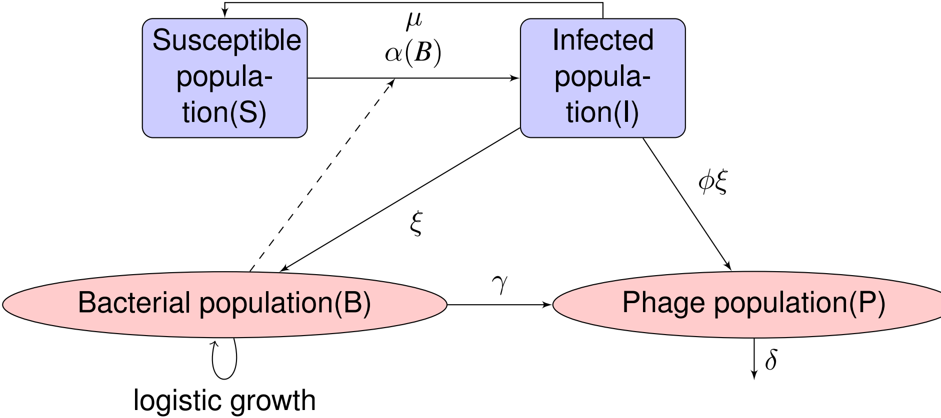
(Emch, et al., 2008)

Human population

- No immunity
- Divide human population into two classes: susceptible and infected class
- Constant population

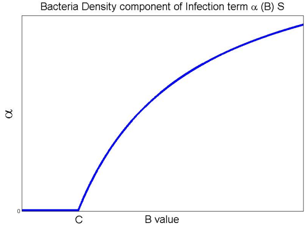
Bacteria-phage system $\rightarrow$ predator-prey relationship

Block diagram



- δ =phage death rate, (0.5-7.9) visions day^{-1}
- ξ= pathogen shed rate (10-100) cell liter^{-1} day^{-1}
- μ = human recovery rate (0.1) day^{-1}

Incidence term



$$\alpha(B) = \begin{cases} 0, & B < c; \\ \frac{a(B-c)}{(B-c)+H}, & B \geq c. \end{cases}$$

- K= pathogen carrying capacity (10^6) cell liter^{-1}
- H= half saturation pathogen density, $(10^6 - 10^8)$ cell litter^{-1}
- a= maximum rate of infection, (0.1) day^{-1}
- c= MID, (10^6) cell liter^{-1}

Predation term

$$\gamma \frac{BP}{K_1 + B}$$

- $\gamma =$ phage absorption rate (-) liter virion^{-1} day^{-1}
- $\beta =$phage burst size,(80-100) visions day^{-1}
- $K_1 =$ half saturation bacteria predation density, (-) cell

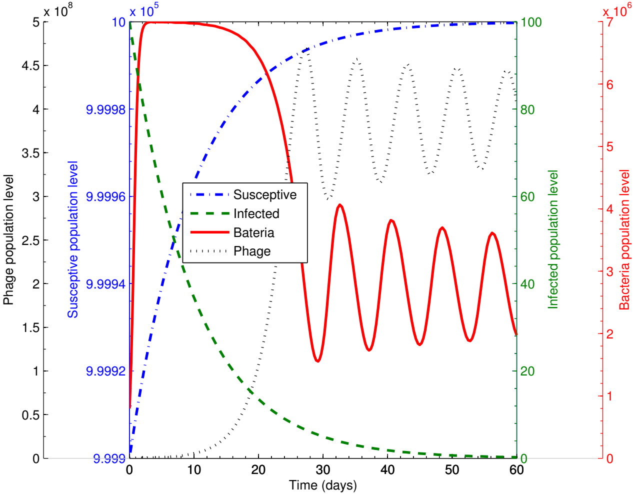
(Jensen, et al., 2006)

System of equations

$$
\begin{aligned}
\frac{dS}{dt} &= -\alpha(B)S + \mu I, \\
\frac{dI}{dt} &= \alpha(B)S - \mu I, \\
\frac{dB}{dt} &= rB\left(1 - \frac{B}{K}\right) - \gamma\frac{B}{K_1 + B}P + \xi I, \\
\frac{dP}{dt} &= \beta\gamma\frac{B}{K_1 + B}P - \delta P + \phi\xi I \\
N &= S + I
\end{aligned}
$$

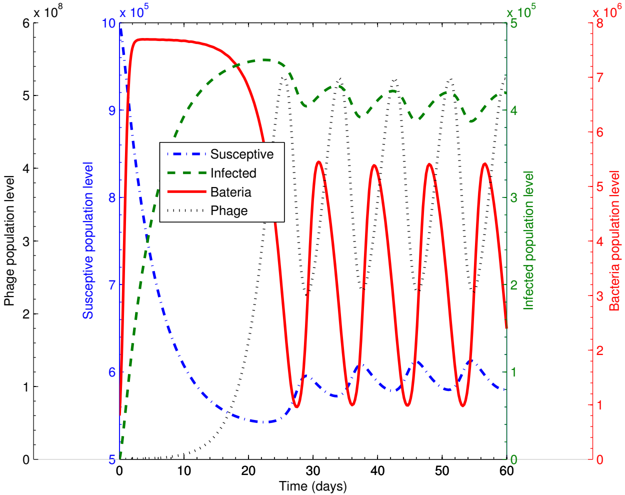
Case 1a: No shedding with $K \leq c$

Only microscopic cycles are observed



Case 1b: No shedding with $K \geq c$

B microscopic and macroscopic cycles are observed

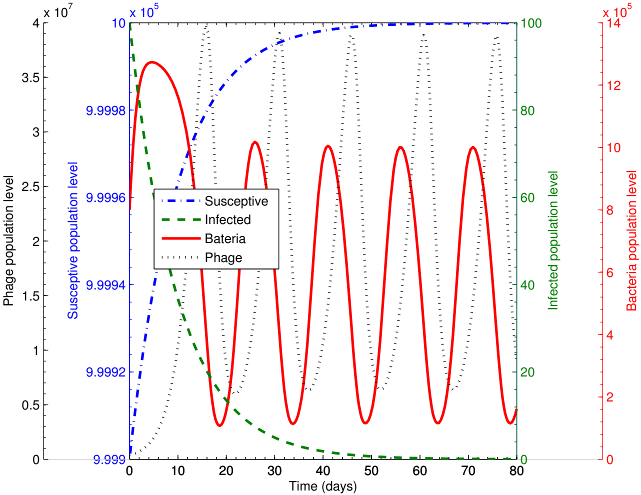


Observations

- Microscopic cycles always exist and the infected class does not have much control over them
- The infected class is only affected when the bacterial level goes above the MID

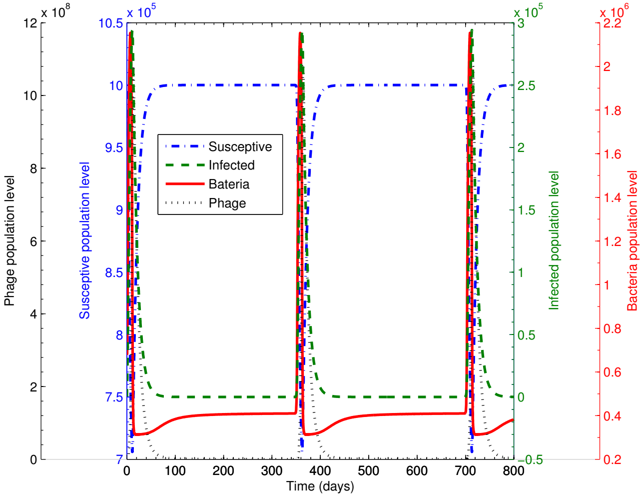
Case 2a: Shedding with $K \leq c$

Only microscopic cycles are observed



Case 2b: No shedding with $K \geq c$

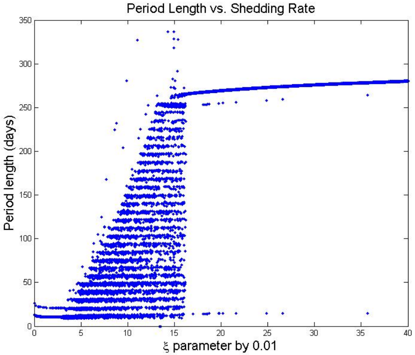
Both microscopic and macroscopic cycles are observed



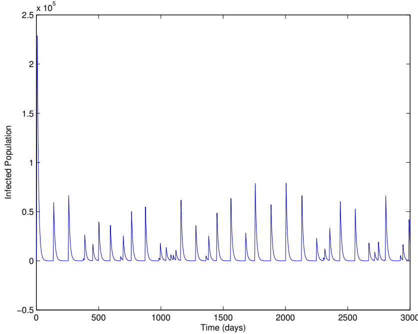
Observations

- Unlike the previous case, the period of these cycles if approximately 1 year, which corresponds to annual outbreaks observed in some endemic areas
- The cycles exist at low level and only enter human population when the bacterial levels increased passed the MID
- The bacterial population peak before the infected human population

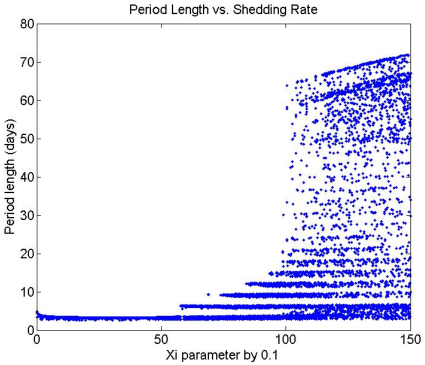
chaos



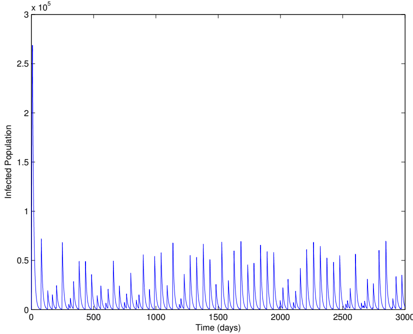
(a) shedding rate vs period with r=1



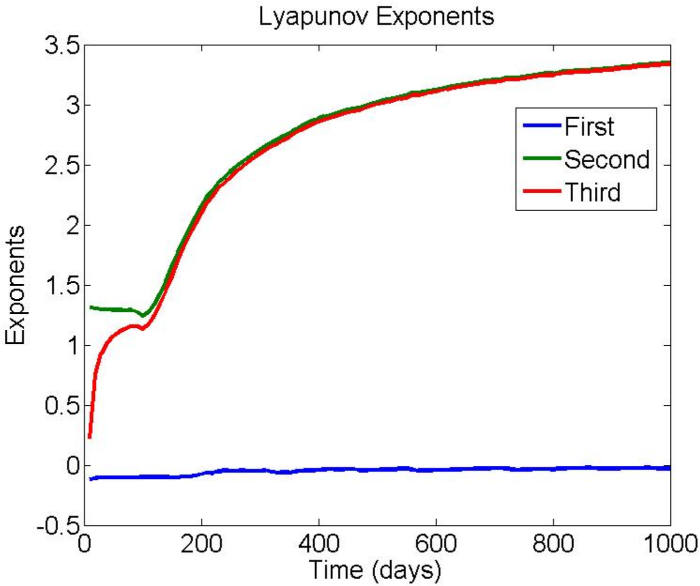
(b) trajectory for r=1 and $\xi = 11$



(a) shedding rate vs period with r=5



(b) tracjectory for r=5 and $\xi = 53$


Lyapunov Exponents
Exponents
Time (days)
First
Second
Third
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
200
400
600
800
1000

Conclusion

- As the cycles in the bacteria-phage system exist in the in the absence of human contribution to the bacterial and phage levels , and because the bacterial cycles peak before the human cycles when they exist in both systems, we can conclude that it is the microscopic cycles that are driving the macroscopic cycles
- If the phage level could be enhanced in some way to keep the bacteria below c, then only microscopic cycles will always exist
- The existence of chaotic behaviour explains the lack of clear periodicity in some endemic areas and the unpredictable nature of outbreaks in countries near the equator

Future work

- Explicitly include the role of infection derived immunity through the use of a recovered class
- Determine the exact conditions for the existence of limit cycles
- Determine the relation between the amplitude and the period of the cycles to other parameters in the system
- Include alternate forms of phage predation

Appreciation

- I would like to express my sincere gratitude to my supervisor Dr. Hao Wang for his constant support, guidance and motivation

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Thank you for LIstening