Population Cycles and Habitat Fragmentation

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Collaborators

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Predator-Prey Models

Lotka-Volterra

$$\frac{dN}{dt} = rN - \alpha NP$$

$$\frac{dP}{dt} = \chi NP - \delta P$$

Rosenzweig-MacArthur

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - \frac{\alpha NP}{N + \mu}$$

$$\frac{dP}{dt} = \frac{\chi NP}{d + N} - \delta P$$

May

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - \frac{\alpha NP}{N + \mu}$$

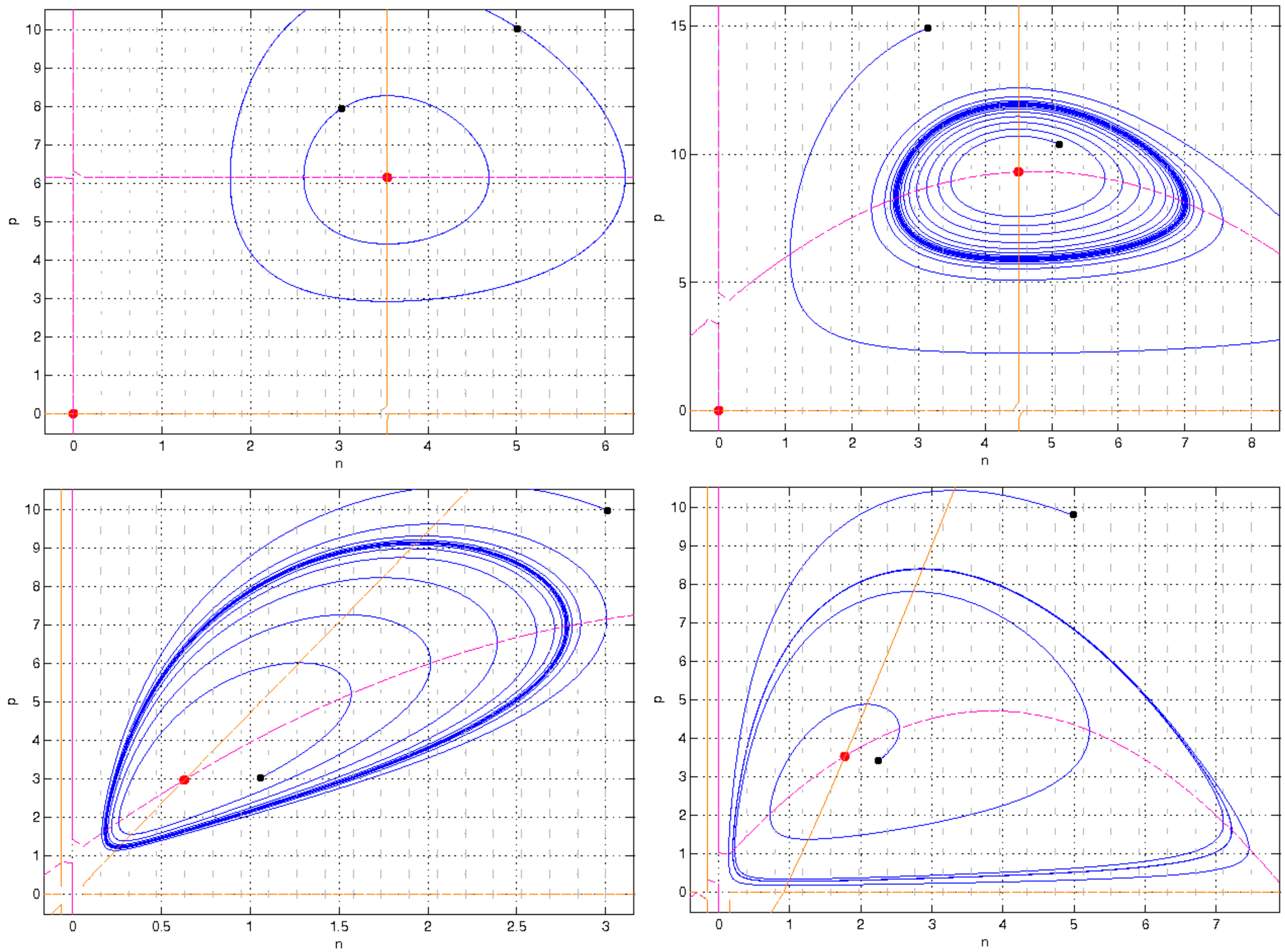
$$\frac{dP}{dt} = sP\left(1 - \frac{qP}{N}\right)$$

Variable Territory

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - \frac{\alpha NP}{N + \mu}$$

$$\frac{dP}{dt} = \frac{\chi NP}{d + N} - \delta P - \frac{sqP^2}{N}$$

Predator-Prey Phase Planes



Spatial Model

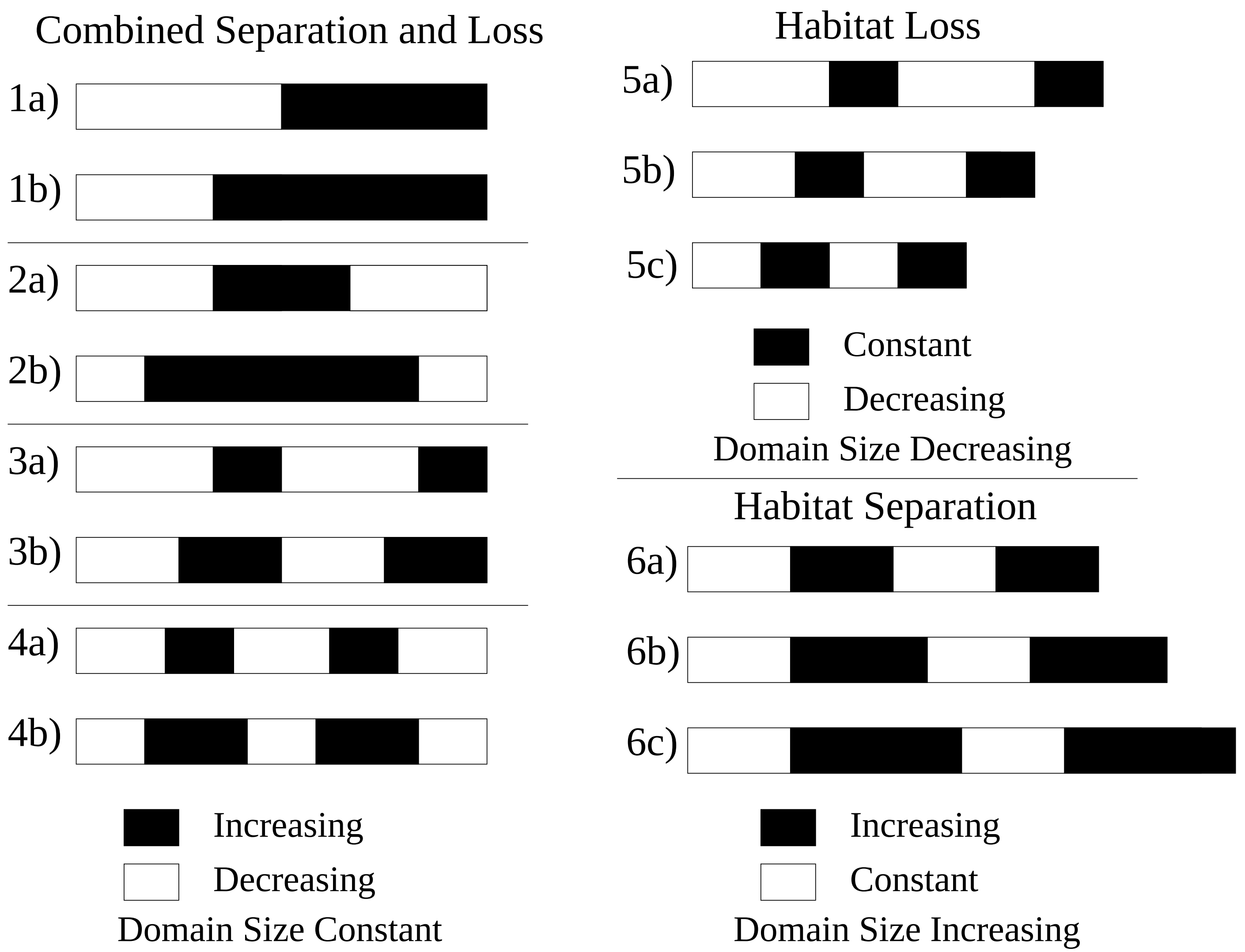
$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + f(n,p,x), \tag{1a}$$

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} + g(n,p), \tag{1b}$$

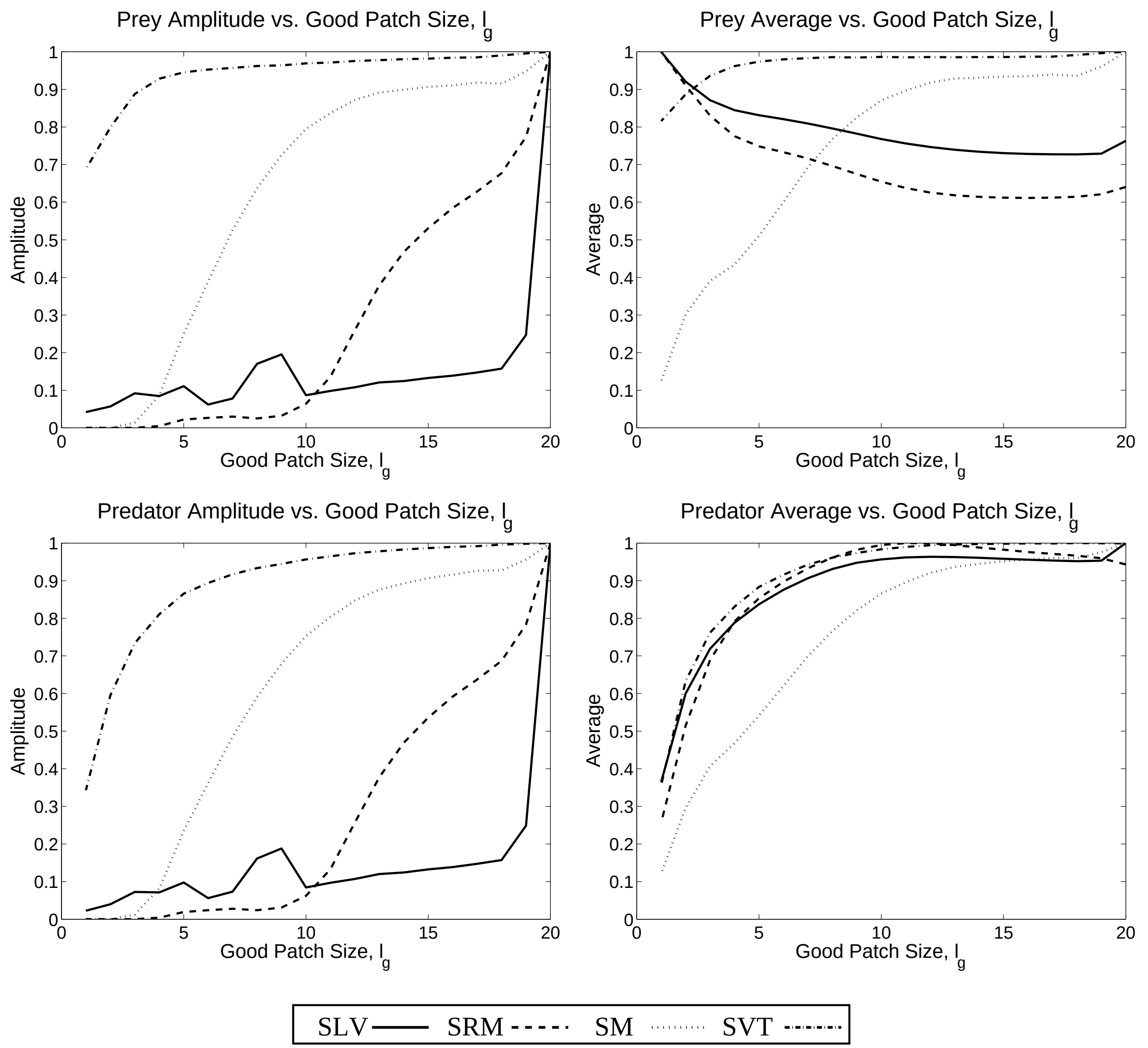
where,

$$\begin{aligned} n,\ p \quad &= \quad \text{the prey \& predator population densities,} \\ D_n,\ D_p \quad &= \quad \text{the diffusivity coefficients,} \\ f(n,p,x),\ g(n,p) \quad &= \quad \text{the reaction terms.} \end{aligned}$$

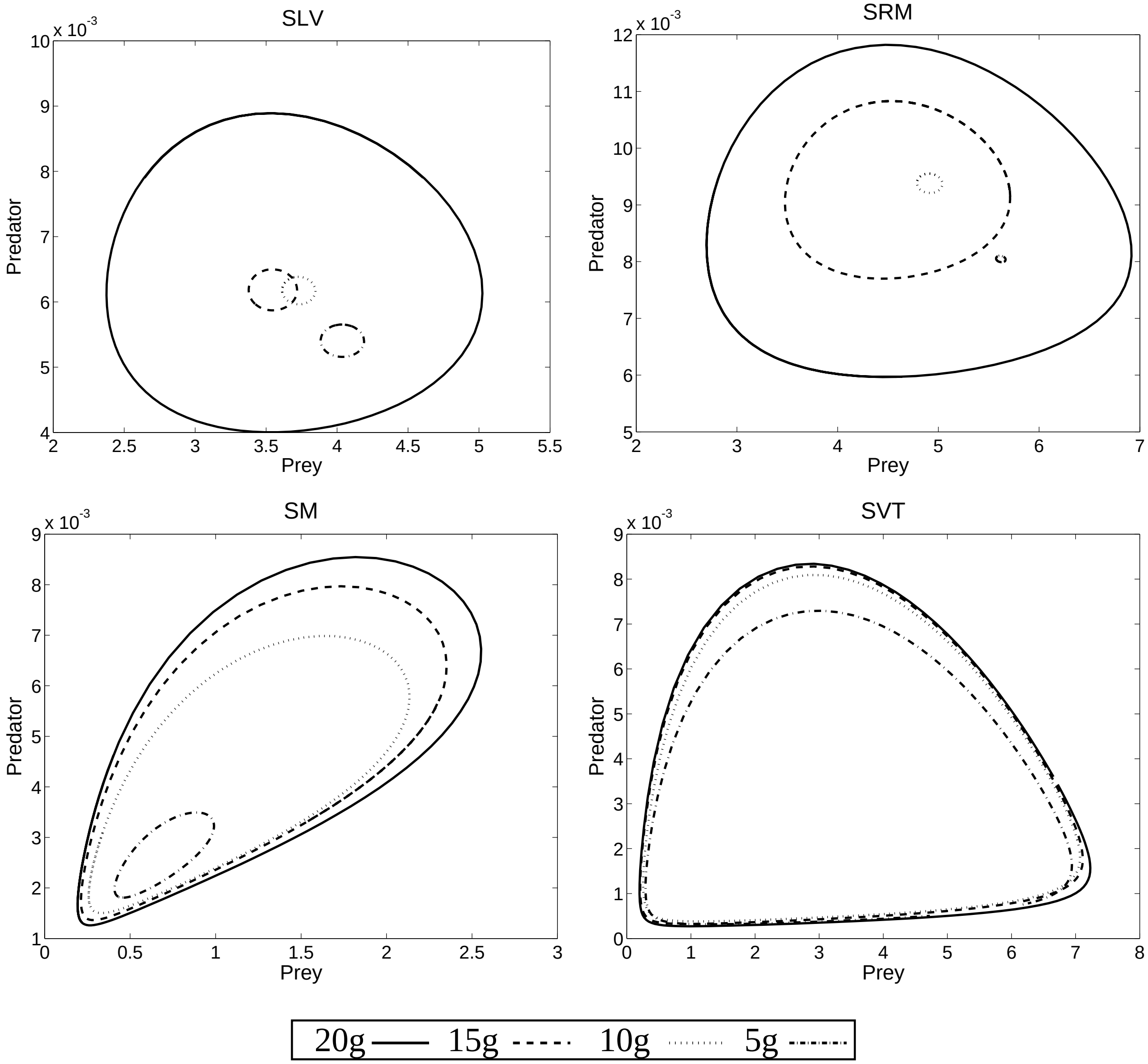
Habitat Fragmentation



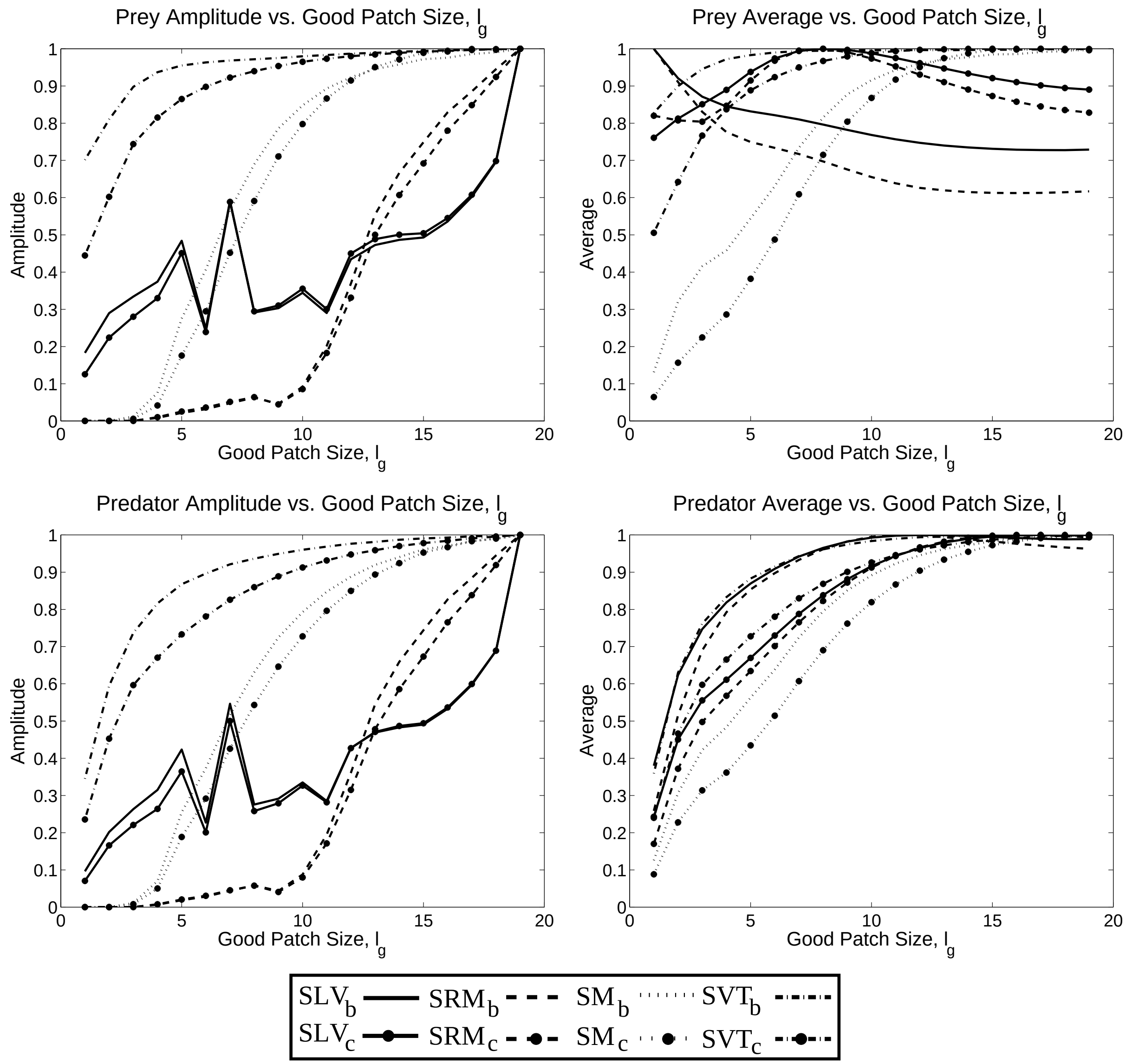
Cycles on Two-Patch Domains



Phase Planes on Two-Patch Domains

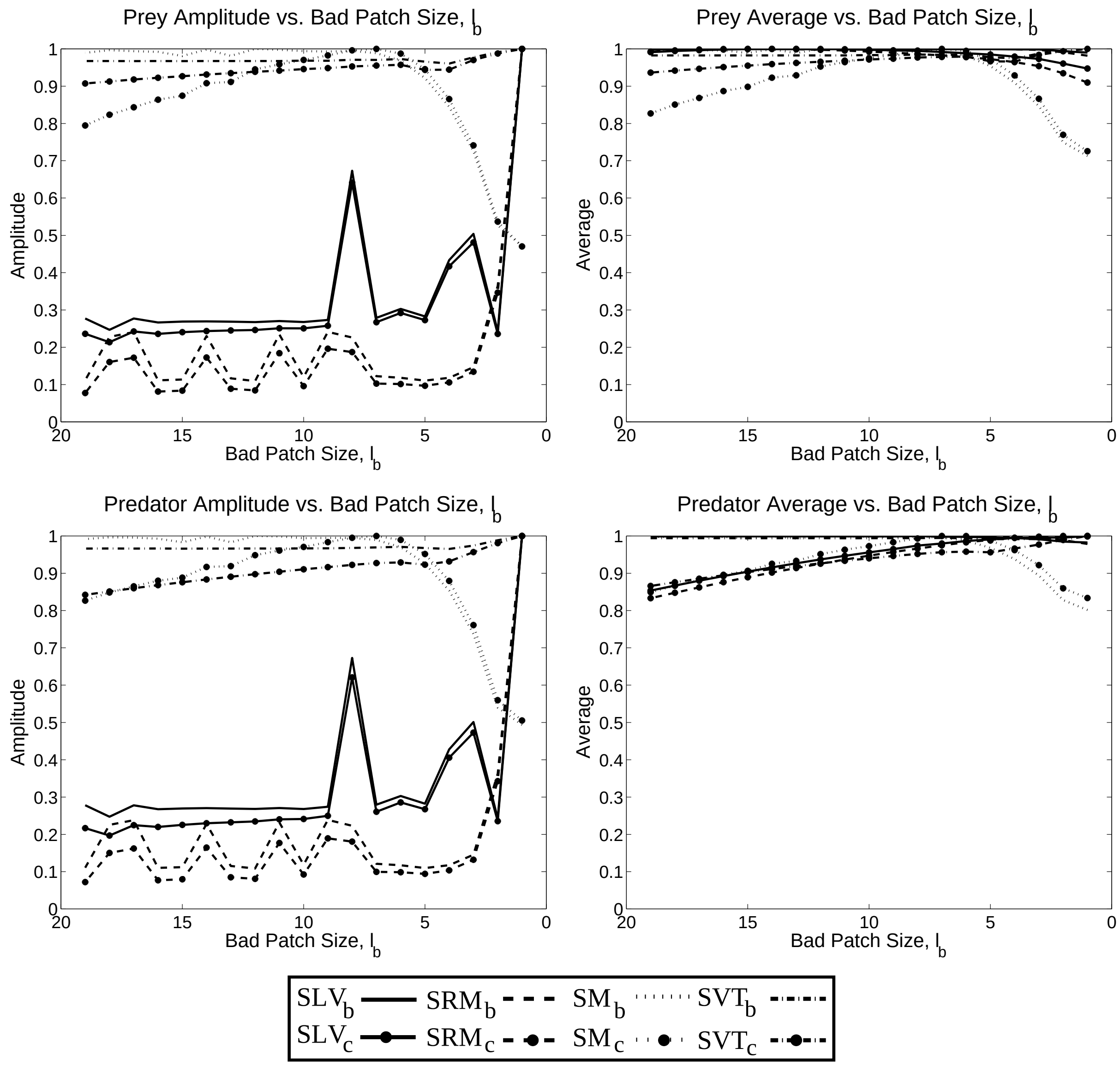


Habitat Loss - Four-Patch Domain



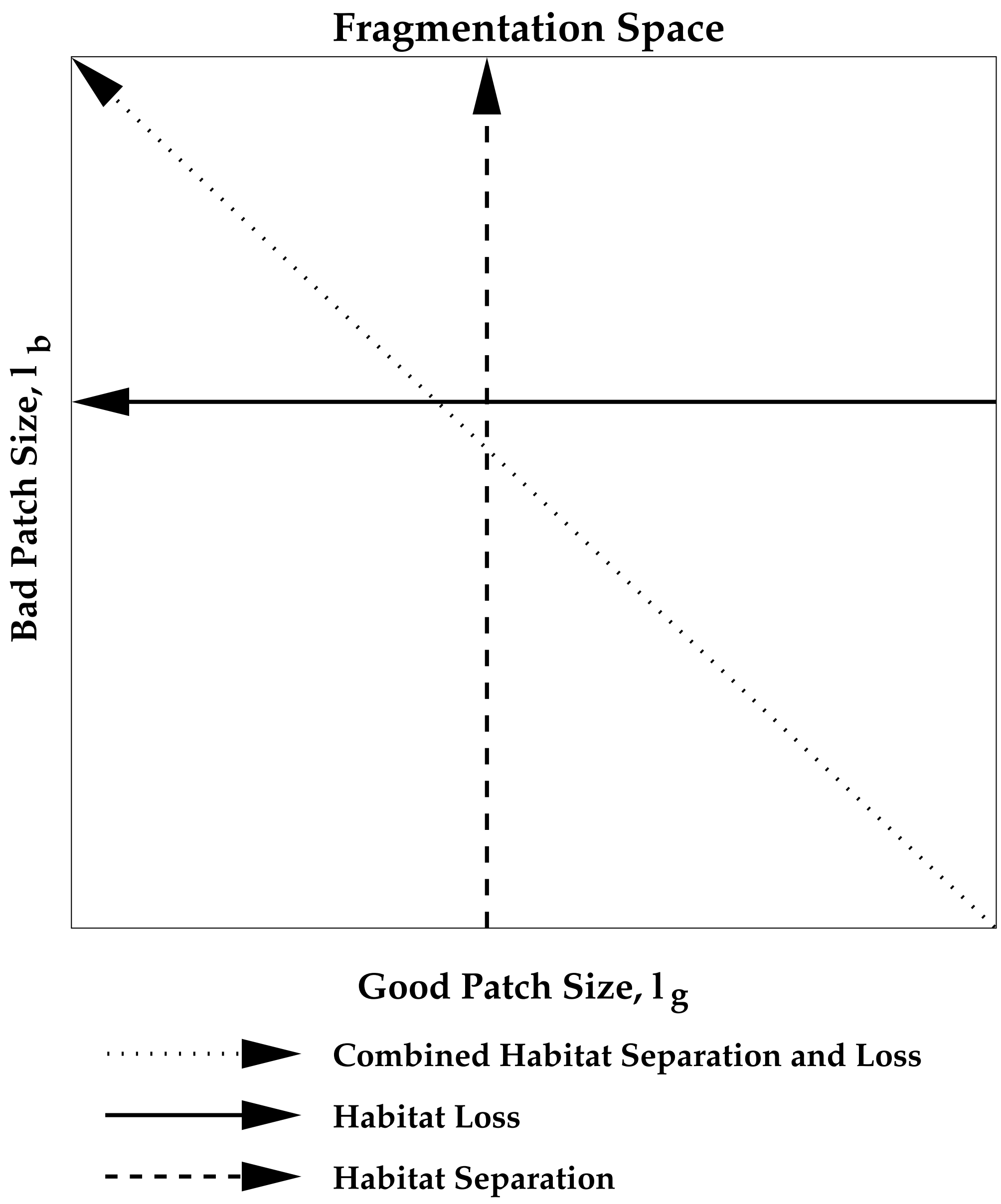
Domain: $n_g = n_b = 2$, $l_b = 12$, and $1 \leq l_g \leq 19$.

Habitat Separation - Four-Patch Domain

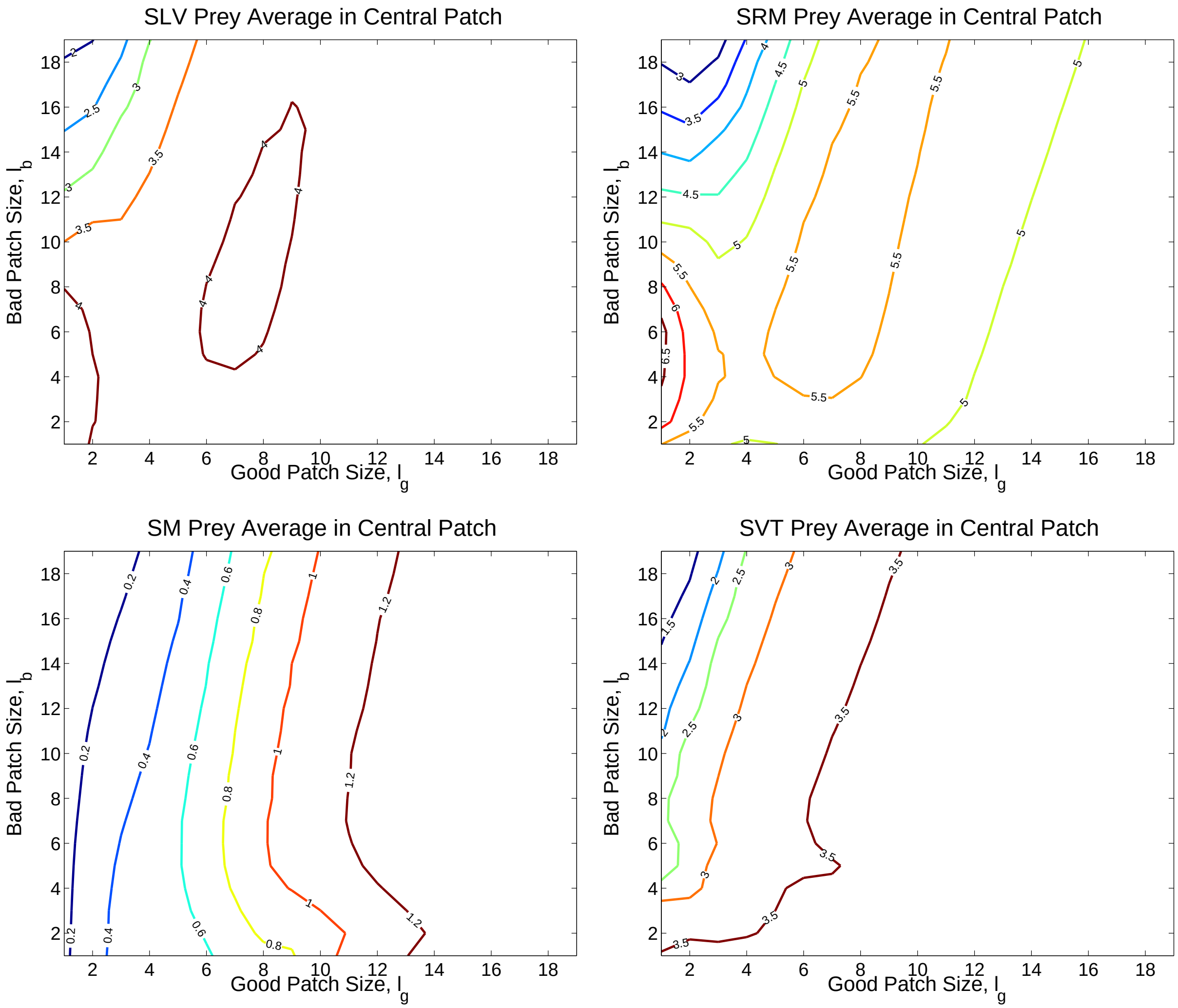


Domain: $n_g = n_b = 2$, $l_b = 12$, and $1 \leq l_g \leq 19$.

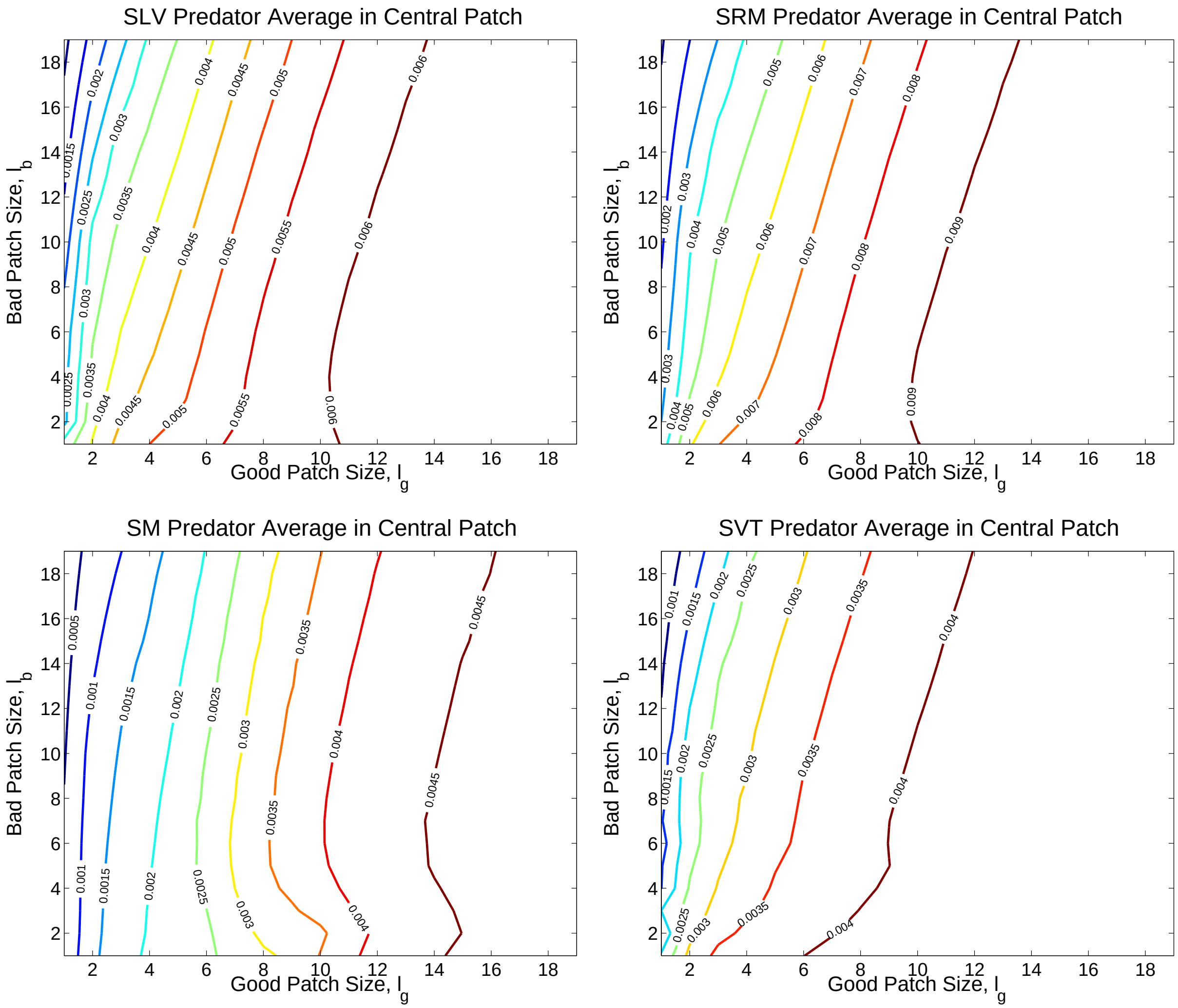
Fragmentation Space



Central Good Patch, Prey Average



Central Good Patch, Predator Average



Conclusions

- **General:**
  - Habitat Loss & Combined Loss - similar
  - Good patch size is more important than bad patch size
- **Ecological Implications (Combined Loss):**
  - Cycle amplitude decreases with fragmentation
  - Thresholds may be present

Model Simplification - Prey

Consider the system linearized around the trivial steady state.

$$\frac{\partial n}{\partial t} = rn + D_n \frac{\partial^2 n}{\partial x^2}$$

where

$$0 \le x \le L, \qquad n(0,t) = n(L,t) = 0.$$

The solution is

$$n(x,t) = \sum_{m=1}^{\infty} A_m e^{[r - D_n (m\pi/L)^2]t} \sin(\frac{m\pi x}{L}),$$

with

$$\lambda_m = r - D_n \left(\frac{m\pi}{L}\right)^2.$$

Model Simplification - Prey

The leading eigenvalue is

$$\lambda_1 = r - D_n \left(\frac{\pi}{L}\right)^2$$

and so the leading order term in the solution is

$$A_1 e^{[r - D_n (\pi/L)^2]t}.$$

This quantity is the exact solution to the ordinary differential equation

$$\frac{dn}{dt} = \left(r - D_n (\frac{\pi}{L})^2\right) n.$$

Thus we can determine persistence in the spatial model using λ_1.

Model Simplification - Predator

Linearizing about $(n, p) = (n^*, 0)$ where n^* =some constant.

$$\frac{\partial p}{\partial t} = \chi c n^* p - \delta p + D_p \frac{\partial^2 n}{\partial x^2},$$

where

$$0 \leq x \leq L, \qquad n(0,t) = n(L,t) = 0.$$

The leading order dynamics are governed by

$$\frac{dp}{dt} = \left(\chi c n^* - \delta - D_p (\frac{\pi}{L})^2\right) p.$$

Spatially Implicit Growth and Death Rates

Define:

$$
\begin{aligned}
r_L =& r - D_n \left(\frac{\pi}{L}\right)^2, \\
\delta_L =& \delta + D_p \left(\frac{\pi}{L}\right)^2, \\
s_L =& s - D_p \left(\frac{\pi}{L}\right)^2.
\end{aligned}
$$

Rewrite the growth and death rates using the eigenvalues above.

Spatially Implicit Models

LV

$$\frac{dn}{dt} = r_L n - cnp,$$
$$\frac{dp}{dt} = \chi cnp - \delta_L p.$$

RM

$$\frac{dn}{dt} = \left(r_L - \frac{n}{k}\right) n - \frac{cnp}{d+n},$$
$$\frac{dp}{dt} = \frac{\chi cnp}{d+n} - \delta_L p.$$

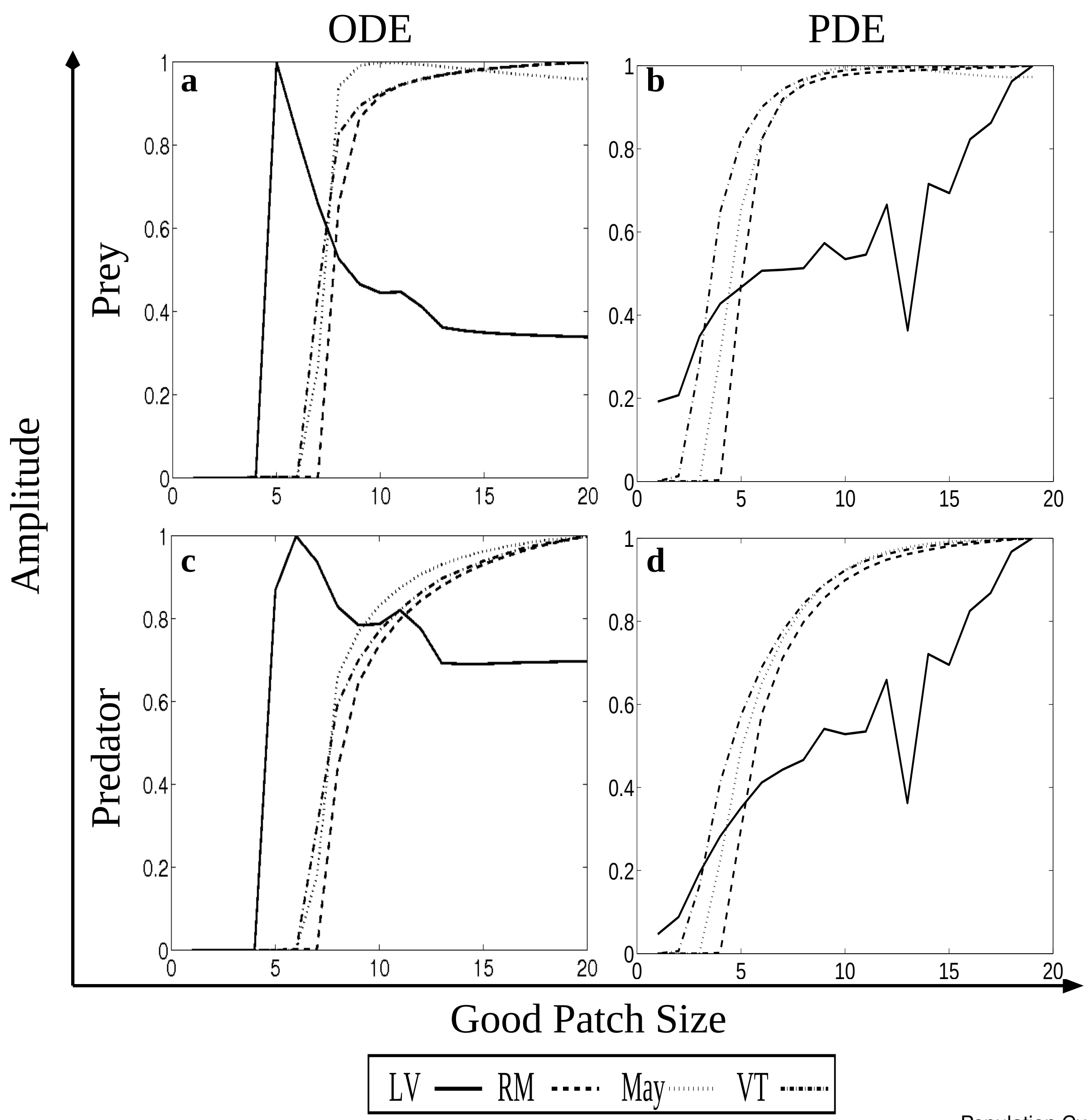
May

$$\frac{dn}{dt} = \left(r_L - \frac{n}{k}\right) n - \frac{cnp}{d+n},$$
$$\frac{dp}{dt} = p\left(s_L - \frac{qp}{n}\right).$$

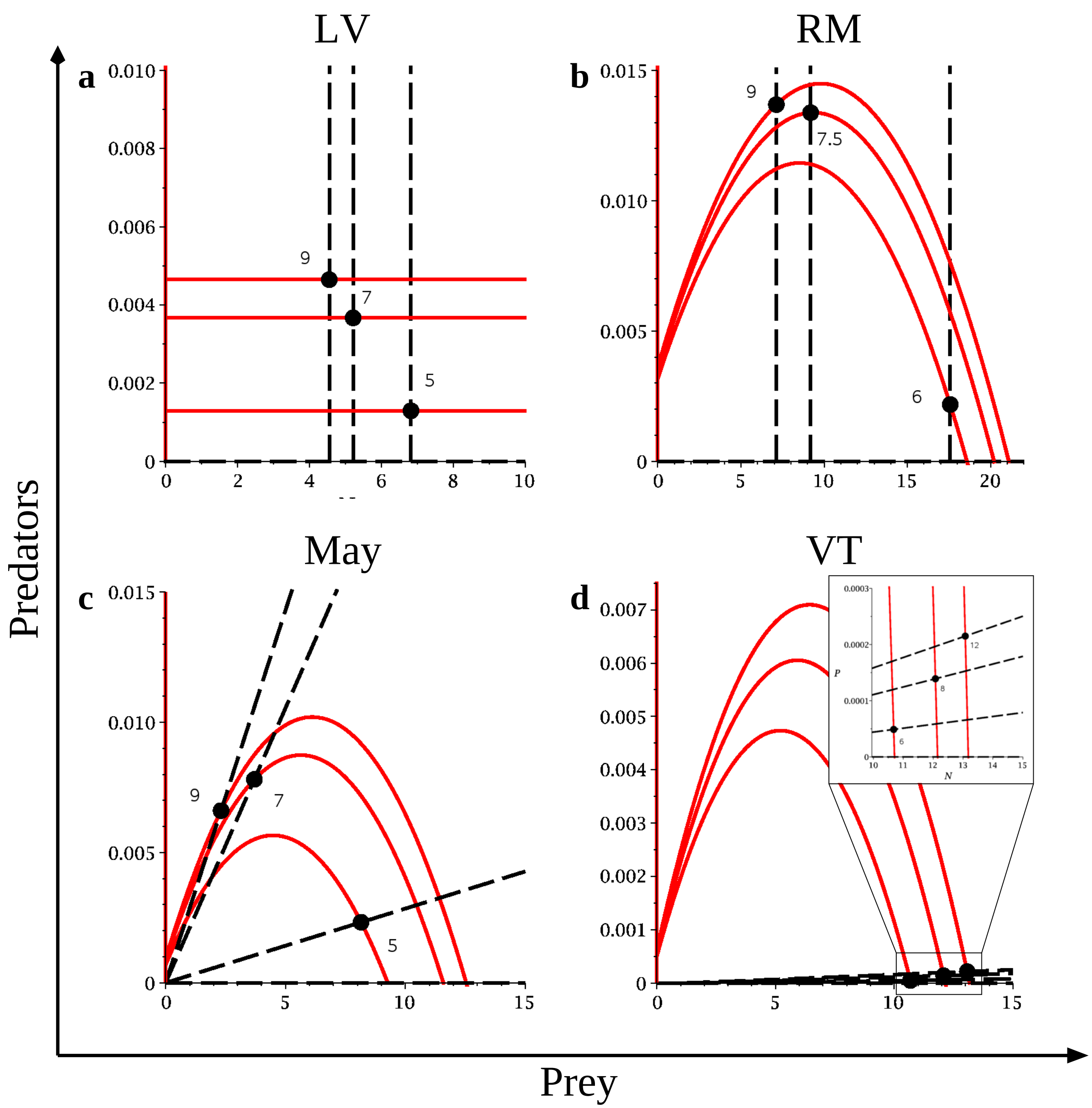
VT

$$\frac{dn}{dt} = \left(r_L - \frac{n}{k}\right) n - \frac{cnp}{d+n},$$
$$\frac{dp}{dt} = \frac{\chi cnp}{d+n} - \delta_L p - \frac{sqp^2}{n}.$$

Comparison of ODE & PDE Solutions



Nullclines



Fragmentation with Edge Behaviour

Edge Behaviour

$$D_1 \frac{\partial u_1}{\partial x} = D_2 \frac{\partial u_2}{\partial x},$$
$$u_1 = \frac{\alpha}{1-\alpha} \frac{D_2}{D_1} u_2,$$

where,

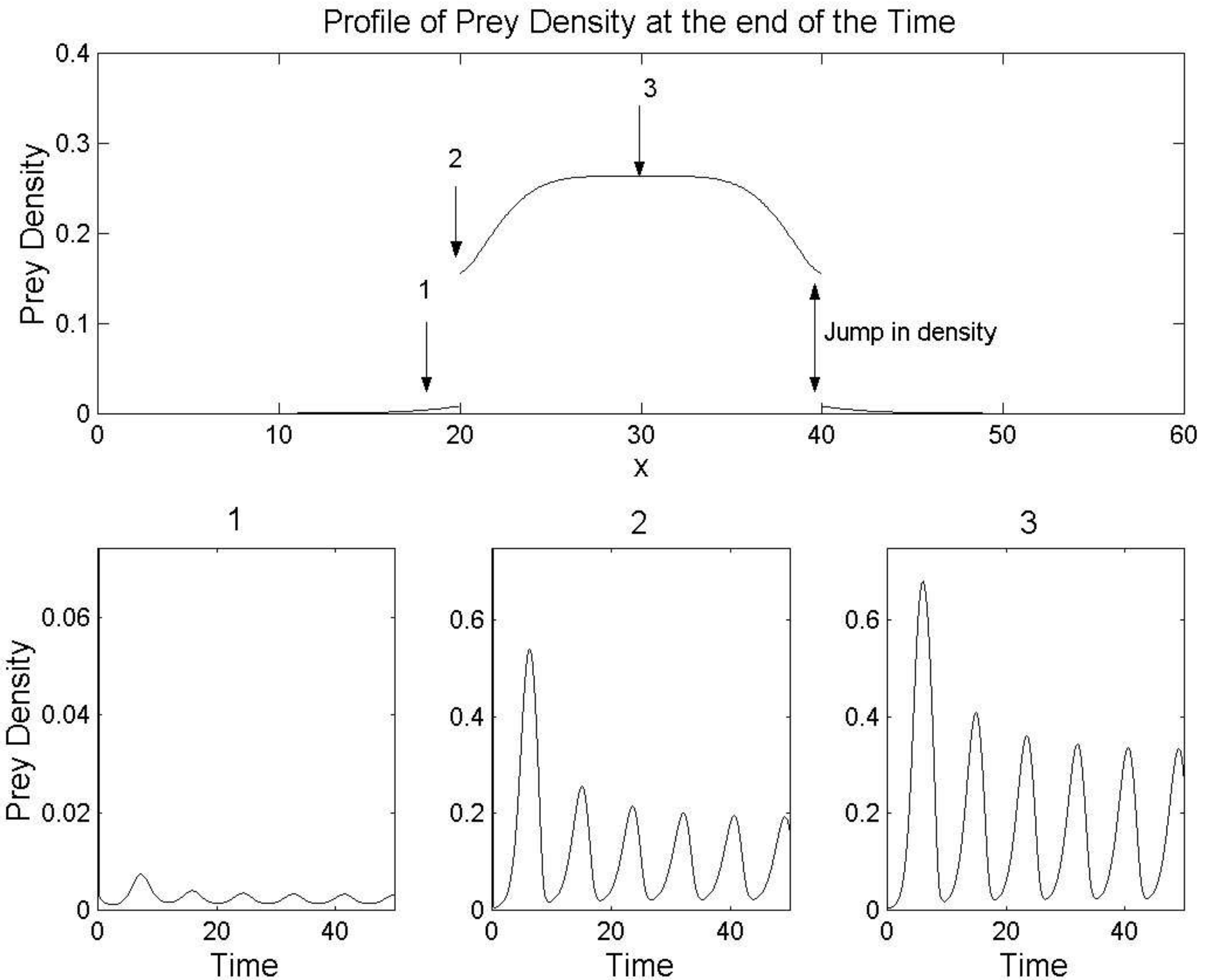
$D_1,\ D_2$=the diffusivity coefficients in patches 1 and 2,

$u_1,\ u_2$ =the population densities at the edge of patches 1 and 2.

Ovaskainen & Cornell (2003) J. Applied Probability

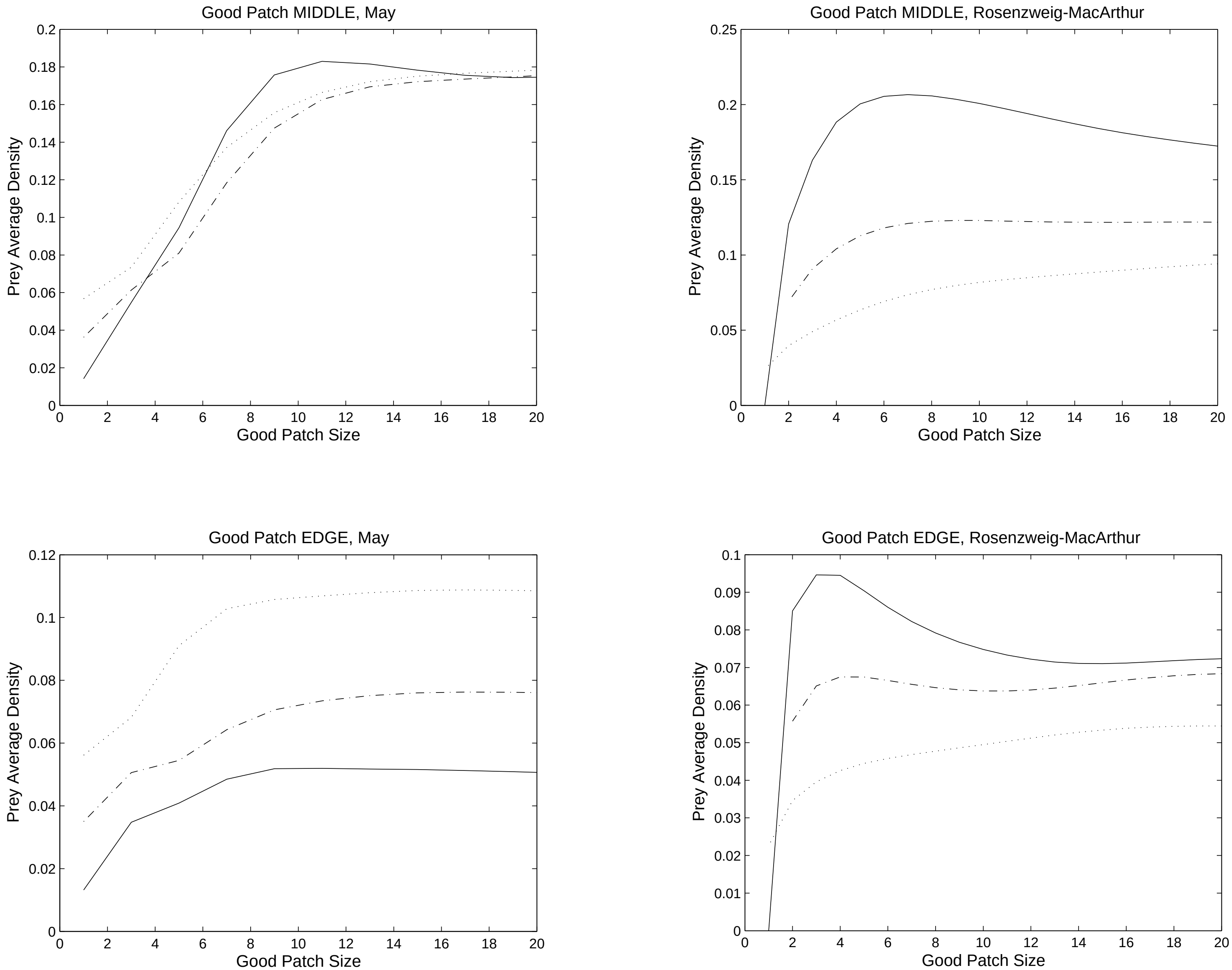
Maciel & Lutscher (2013) Am Nat

Prey Only

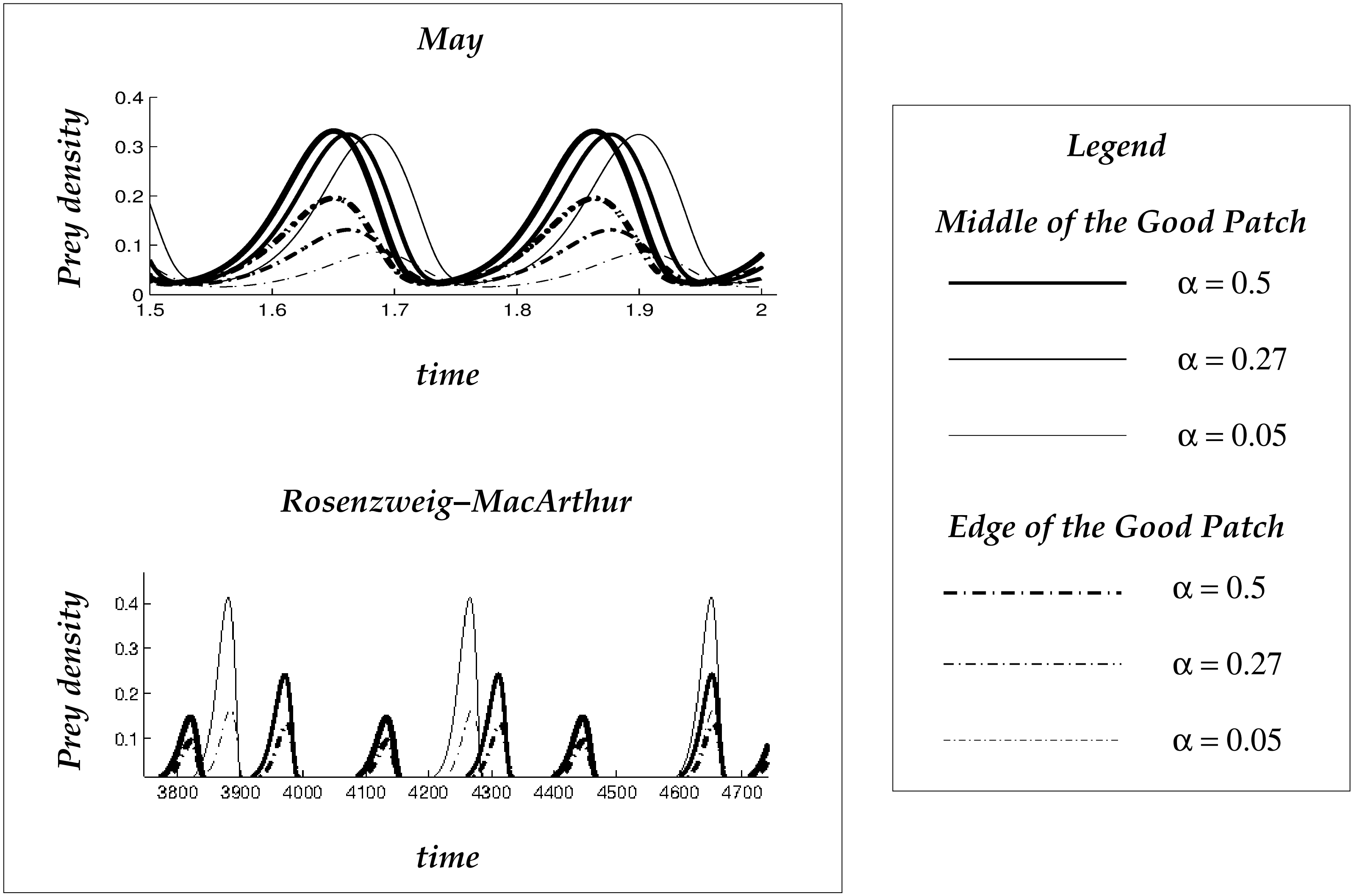


Profile of Prey Density at the end of the Time
Prey Density
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
X
1
2
3
Jump in density
1
2
3
Prey Density
0.06
0.04
0.02
0.6
0.4
0.2
Time

Effect of Good Patch Size



Effect of Patch Preference

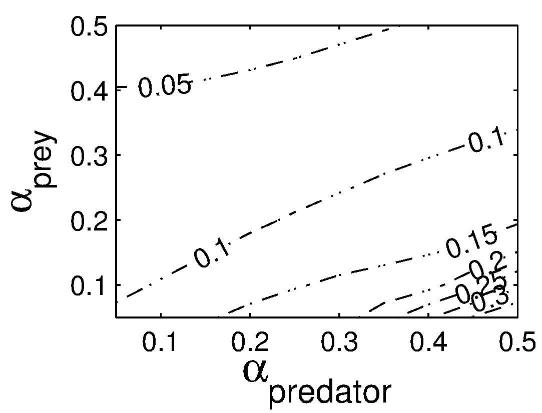


Different Patch Preference

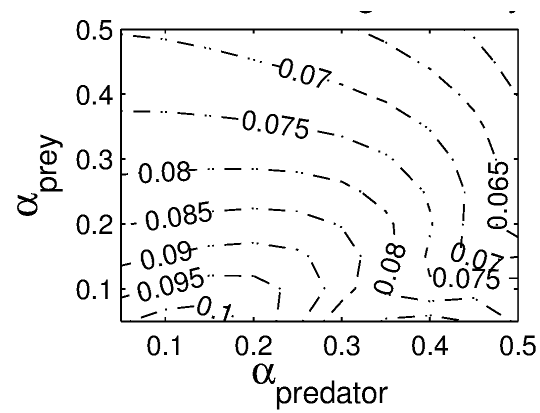
Good Patch EDGE

May Model

Prey Average Density

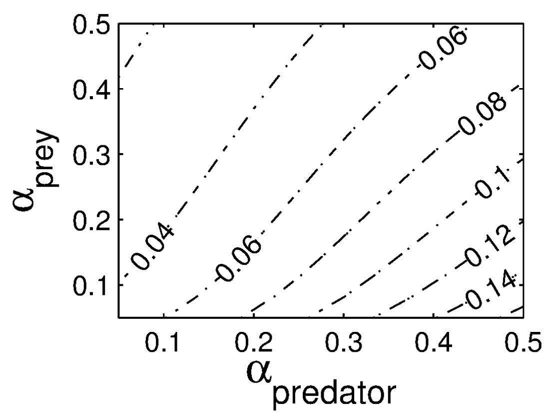


Predator Average Density

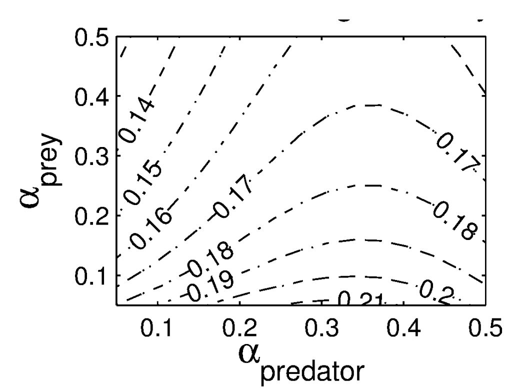


Rosenzweig–MacArthur Model

Prey Average Density



Predator Average Density



Snowshoe hare

Oh my gosh, I've got to run -
Lynx is here to spoil the fun.
My feet are built to tread the snow,
But so are hers, you ought to know.
Oh no, she's gaining - a spurt of speed -
Am I to be her bunny feed?
Zig, zag, dash - I'm past her,
Today *my* furry feet are faster!
I never ever want to lose
My lucky little running shoes.

from Lucky Hares and Itchy Bears, (1996) by Susan Ewing, Alaska Northwest Books, Portland, OR.