# Alberta Number Theory Days

# May 10-12, 2013

# Schedule

All talks will take place in room 201 of the TransCanada Pipelines Pavilion.

**Saturday, May 11**

09:00 *Welcoming remarks*

09:10 **Charles Doran**
*Hypergeometric Equations from Geometry*

10:10 **Richard Guy**
*Some Unsolved Problems*

10:35 *Coffee (TransCanada Pipelines Pavilion)*

11:00 **Colin Weir**
*The $a$-numbers of Suzuki curves*

11:30 **Soroosh Yazdani**
*Level lowering, Mordell-Weil sieve, and local Szpiro conjecture*

12:20 *Lunch*

14:10 **Noam Elkies**
*How many elliptic curves can have the same prime conductor?*

15:00 *Coffee (TransCanada Pipelines Pavilion)*

15:30 **Adam Felix**
*Various generalizations of Artin's conjecture for primitive roots*

16:30 **Michael Jacobson**
*Relation Generation in Quadratic Number and Function Fields*

**Sunday, May 12**

09:00 **Clifton Cunningham**
*$L$-packets and abelian varieties*

9:50 *Coffee (Corbett Hall)*

10:20 **David Roe**
*Geometrizing Characters of Tori*

11:20 **Amir Akbary**
*Limiting Distributions of the Error Terms*

Banff International Research Station for Mathematical Innovation and Discovery

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## Speakers, Titles, and Abstracts

| *Saturday, May 11* | *morning* |
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### 9:00–9:10 AM *Welcoming remarks*

### 9:10–10:00 AM

**Charles Doran** (University of Alberta)
*Hypergeometric Equations from Geometry*

We consider families of elliptic curves, K3 surfaces, and Calabi-Yau threefolds over the thrice punctured sphere whose Picard-Fuchs ODEs are of hypergeometric type. Given a hypergeometric function, mirror symmetry suggests how to find such geometric families in most cases. It poses a puzzle, however, for the "14th case" variation of Hodge structure for Calabi-Yau threefolds. We describe two constructions of a family of K3 surface fibered Calabi-Yau threefolds realizing the 14th case VHS, the second which brings us back full circle to families of elliptic curves over the $J$-line.

### 10:10–10:35 AM

**Richard Guy** (University of Calgary)
*Some Unsolved Problems*

### 10:35–10:55 AM *Coffee* (TransCanada Pipeline Pavilion)

### 11:00–11:25 AM

**Colin Weir** (University of Calgary)
*The $a$-numbers of Suzuki curves*

For $l$ a prime different from $p$, the $l$-torsion structure of Jacobians of curves in characteristic $p$ is well understood. The $p$-torsion structure however, is much more mysterious. We will investigate the structure of the $p$-torsion group scheme of the Jacobians of Suzuki curves; a very interesting family of maximal curves in characteristic 2. We will compute the $a$-numbers of these curves, an invariant of $p$-torsion group schemes, and discuss recent progress towards computing their isomorphism types. This is work in progress with Rachel Pries and Beth Malmskog.

### 11:30–12:20 PM

**Soroosh Yazdani** (University of Lethbridge)
*Level lowering, Mordell-Weil sieve, and local Szpiro conjecture*

In this talk, we investigate the local Szpiro conjecture for certain families of elliptic curves. In particular, we show that for certain primes $p$, if $E$ is a semistable elliptic curve with minimal discriminant of the form $p^r M^{6l}$ with $l > 2$, then $r < 6$.

## 14:10-15:00 PM

**Noam Elkies** (Harvard University)
*How many elliptic curves can have the same prime conductor?*

The question in the title is open; it is not even known whether the number is bounded as the prime varies. We explain why the question is natural and outline various approaches. Finally we exhibit the largest count known (nearly twice the previous record) and describe how it was found.

## 15:00–15:25 PM *Coffee* (TransCanada Pipeline Pavilion)

## 15:30–16:20 PM

**Adam Felix** (University of Lethbridge)
*Various generalizations of Artin's conjecture for primitive roots*

A primitive root modulo a prime $p$ is an integer which generates the group of non-zero residues modulo $p$. For primes $p$, we can always find a primitive root modulo $p$. In 1927, Artin conjectured that a density for the set of primes for which a fixed integer is a primitive root modulo $p$ exists. Hooley showed that this is true upon the generalized Riemann hypothesis. Hooley's proof will be reviewed and various generalizations will be discussed.

## 16:30–17:20 PM

**Michael Jacobson** (University of Calgary)
*Relation Generation in Quadratic Number and Function Fields*

Relations in the ideal class group of quadratic number and function fields are useful in a variety of computations, including computing invariants, solving the discrete logarithm problem, and computing isogenies and endomorphism rings of elliptic curves. In this talk, we discuss on-going work on improving relation generation using sieving. In the number field case we discuss challenges and recent efforts to apply sieving to computing endomorphism rings and isogenies. In the function field case, we present recent algorithmic improvements that yield substantial practical improvements for discrete logarithm computation in high-genus quadratic function fields defined over characteristic two finite fields.

This is joint work with J.-F. Biasse, Andreas Stein, and Wilke Trei

## 9:00–9:50 AM

**Clifton Cunningham** (University of Calgary)
*$L$-packets and abelian varieties*

Let $E/Q$ be an elliptic curve without complex multiplication. A theorem of Elkies states that $E$ has infinitely many primes of supersingular reduction; this is equivalent to the infinitude of a certain $L$-packet of automorphic representations of $SL(2)$. I will explain this equivalence and discuss the analogue for abelian varieties of higher dimension, paying special attention to the case of abelian surfaces. Joint work with Jeffrey Achter.

## 9:50–10:15 AM *Coffee* (Corbett Hall)

## 10:20–11:10 AM

**David Roe** (University of Calgary)
*Geometrizing Characters of Tori*

The passage from functions to sheaves has proven a valuable tool in the geometric Langlands program. In this talk I'll describe a "geometric avatar" for the group of characters of $T(K)$, where $T$ is an algebraic torus over a local field $K$. I will then give some potential applications to the classical Langlands correspondence. This is joint work with Clifton Cunningham.

## 11:20–12:10 PM

**Amir Akbary** (University of Lethbridge)
*Limiting Distributions of the Error Terms*

We describe the concept of limiting distribution through some number theoretical examples. We also review a classical result of Wintner from 1935 on limiting distribution of the error term of the prime number theorem and report on our recent work (joint with Ng and Shahabi) on generalizations of this result.