Transmit Strategies for the Gaussian Bidirectional Broadcast Channel & Latest Results

Tobias Oechtering,
KTH, Stockholm, Sweden

joint works with: Rafael Wyrembelski, Holger Boche, Clemens Schnurr, Igor Bjelaković, Eduard Jorswieck

Interactive Information Theory Workshop,
BIRS, Banff, Canada, Jan 17, 2012

Bidirectional Broadcast Channel

Restricted decode & forward bidirectional relaying

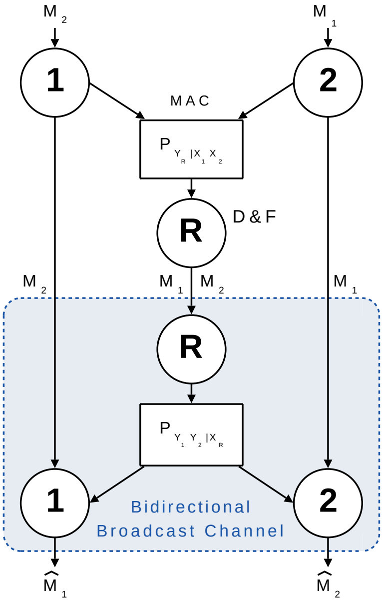
1. Phase: MAC
2. Phase: BiBC: BC with RX message cognition

BiBC capacity region

$$R_1 \leq I(X_R; Y_1)$$
$$R_2 \leq I(X_R; Y_2)$$

Practically relevant since

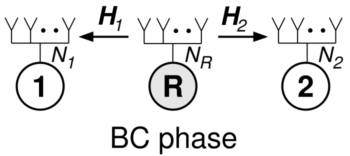
- supports modularization
- gains are easily realized

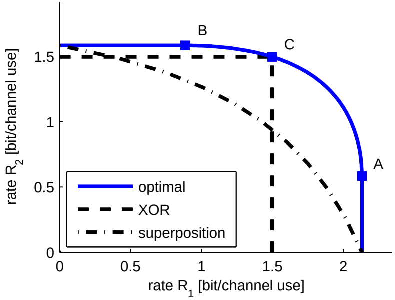


(Some) Related Literature

- Early results: Decode & forward strategies based on
  - superposition coding [Rankov et al. '05], [Oechtering et al.,'06], et al.
  - XOR operation [Larsson et al. '04], [Wu et al. '04], [Yeung '05], et. al.
  - ➠ optimal channel coding approach based on network coding idea (single information flow) found by many groups independently [Knopp '06], [Oechtering et al. '07], [Kim et al. '07], [Xie '07], [Wu '07]
- Closely related problems:
  - Common message BC (multicast) among others [Khisti '04]
  - Compound channel [Blackwell et al '59], [Wolfowitz '60], et al.
  - Slepian-Wolf coding over BC [Tuncel '06]
  - Physical-layer NC [Zhang et al. '06], [Popovski et al. '06], et al.
- Extensions: Compress or compute & forward strategies
  - [Schnurr et al '07], [Kim et al, '08], [Günduz et al.'08], [Wilson et al, '08], [Nam et al. '08], [Nazer et al. '08], [Ong et al. '10], [Lim et al. '10], et al.

Gaussian Multi-Antenna Bidirectional Relaying





Capacity Region of Bidirectional Broadcast Channel

$$C_{\mathrm{BC}} := \bigcup_{\operatorname{tr} \boldsymbol{Q} \le P,\ \boldsymbol{Q} \succeq 0} \left\{ [R_1, R_2] \in \mathbb{R}_+^2 \ :\ R_1 \le C_1(\boldsymbol{Q}), R_2 \le C_2(\boldsymbol{Q}) \right\}$$

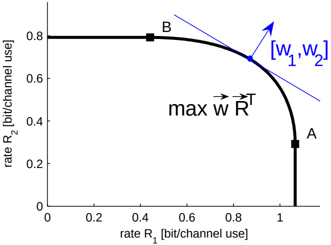
with

$$C_i(\boldsymbol{Q}) := \log\det\left(\boldsymbol{I}_{N_i} + \tfrac{1}{\sigma^2} \boldsymbol{H}_i^H \boldsymbol{Q} \boldsymbol{H}_i \right), \quad i = 1, 2.$$

Transmit Covariance Optimization Problem

$$\underset{\mathrm{tr}\,\boldsymbol{Q}\leq P,\,\boldsymbol{Q}\succeq 0}{\arg\max} \sum_{i=1}^{2} w_i \log\det\left(\boldsymbol{I}_{N_i} + \frac{1}{\sigma^2}\boldsymbol{H}_i^H \boldsymbol{Q} \boldsymbol{H}_i\right)$$

➠ **Let's study this opt. problem!**



1. Optimal Transmit Strategies for the MISO case
2. Optimal Transmit Strategies for the MIMO case
3. Latest Results and Conclusion

First Study – MISO Case $N_1 = N_2 = 1$

MISO optimization problem

$$Q_{\text{opt}}(\boldsymbol{w}) = \underset{\operatorname{tr} Q \leq P,\, Q \geq 0}{\arg\max} \sum_{i=1}^{2} w_i \log\left(1 + \frac{1}{\sigma^2} \boldsymbol{h}_i^H Q \boldsymbol{h}_i\right)$$

- Outline:
  - Subspace optimality and orthogonal channels
  - Single-beam optimality and its consequences
  - Optimal beamforming vector
- Results are published in [Trans SP '09].

First Observations

Proposition: Subspace optimality

An optimal transmit strategy transmits only into the subspace spanned by the channels, otherwise transmit power can be reduced while achieving the same rates.

➡ Optimal transmit strategy Q_{opt} has always $\operatorname{rank}(Q_{\text{opt}}) \leq 2$!

Proposition: Orthogonal channels

For orthogonal channels any rate pair can be achieved with a single-beam as well as with a two-beam strategy.

➡ For orthogonal channels the capacity region can be also achieved using the superposition encoding strategy.

Optimality of the Single-Beam Strategy

Theorem: **Single-beam optimality**

For the MISO case we can always find an optimal single-beam transmit strategy $(\mathrm{rank}(Q_{\mathrm{opt}}) = 1)$.

Proof outline:

- *Orthogonal channels:* Optimality follows immediately from previous propositions.
- *Non-orthogonal channels:* Any rank-two transmit strategy contradicts with the Karush-Kuhn-Tucker conditions so that the optimal transmit strategy has to have rank one. □

➠ **Optimal strategy is to perform a single beam onto the subspace spanned by the channels!**

Consequences of Single-Beam Optimality

For the *bidirectional broadcast channel* ...

Signal Processing

... the relay forms a single beam instead of individual beams for each user as for the classical MISO broadcast.

- Correlated channels will be beneficial (result not shown).

Channel Coding

... it is sufficient to use an one-dimensional Gaussian codebook instead of a codebook with a dimension equal to the number of transmit antennas.

- Reduction of coding complexity!

Optimal Beamforming Vector

Theorem: Property of Optimal Beamforming Vector

$$\boldsymbol{Q} := P\boldsymbol{q}\boldsymbol{q}^H, \qquad \boldsymbol{q} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2, \qquad \boldsymbol{h}_i = |\boldsymbol{h}_i|\boldsymbol{u}_i,\ a_i \in \mathbb{C},$$

then

$$\arg(a_1) - arg(a_2) = \varphi \qquad |\rho|e^{i\varphi} = \boldsymbol{u}_1^H\boldsymbol{u}_2$$

- Normalized beamforming vector

$$\boldsymbol{q}(t) = \frac{t\boldsymbol{u}_1 + (1-t)e^{-j\varphi}\boldsymbol{u}_2}{||t\boldsymbol{u}_1 + (1-t)e^{-j\varphi}\boldsymbol{u}_2||}, \qquad t \in [0,1]$$

- $[R_1(t), R_2(t)]$ with $R_i(t) := \log(1 + \frac{P}{\sigma^2}|\boldsymbol{h}_k^H\boldsymbol{q}(t)|^2)$, $t \in [0,1]$ parametrizes the curved section of the capacity region!
- Egalitarian solution easily calculated from $R_1(t_{eg}) = R_2(t_{eg})$.

Extended Study – MIMO Case

MIMO optimization problem

$$Q_{\text{opt}}(w) = \underset{\operatorname{tr} Q \leq P,\, Q \geq 0}{\arg\max} \sum_{i=1}^{2} w_i \log\det\left(I_{N_i} + \frac{1}{\sigma^2} H_i^H Q H_i\right)$$

- Outline:
  - Subspace optimality and 'Orthogonal' channels
  - Karush-Kuhn-Tucker conditions – Unsymmetric Riccati equation
  - Special case: Full rank transmission
  - Special case: Parallel channels
- Results are published in [Trans Com '09].

First Observations

Proposition: Subspace optimality

An optimal transmit strategy transmits only into the vector space spanned by the set of column vectors of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$.

Proposition: 'Orthogonal channels'

$\boldsymbol{P}_i$ projector onto the vector space spanned by the set of column vectors of $\boldsymbol{H}_i$, $i = 1, 2$.

Any rate pair achievable with $\boldsymbol{Q}$ can be achieved with equivalent transmit strategies $\hat{\boldsymbol{Q}}$ with rank $\hat{r}$ satisfying

$$\max\{r_1, r_2\} \leq \hat{r} \leq \min\{r_1 + r_2, N_R\}, \qquad r_i := \operatorname{rank}(\boldsymbol{P}_i \boldsymbol{Q} \boldsymbol{P}_i).$$

- In general an optimal solution $\boldsymbol{Q}_{\text{opt}}$ will be not unique!
  - ➡ Makes analysis of the general optimization problem difficult.

Special Case: Invertible Channels

Lagrangian

$$L(\boldsymbol{Q}, \mu, \boldsymbol{\Psi}) = -\sum_{i=1}^{2} w_i C_i(\boldsymbol{Q}) - \mu\big(P - \operatorname{tr} \boldsymbol{Q}\big) - \operatorname{tr} \boldsymbol{Q}\boldsymbol{\Psi}$$

Karush-Kuhn-Tucker conditions

$$\sum_{i=1}^{2} w_i \boldsymbol{H}_i(\sigma^2 \boldsymbol{I}_{N_i} + \boldsymbol{H}_i^H \boldsymbol{Q} \boldsymbol{H}_i)^{-1} \boldsymbol{H}_i^H = \mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi} \tag{1}$$

$$\begin{aligned} \boldsymbol{Q} &\succeq 0, & P &\geq \operatorname{tr} \boldsymbol{Q}, \\ \boldsymbol{\Psi} &\succeq 0, & \mu &\geq 0, \\ \operatorname{tr} \boldsymbol{Q}\boldsymbol{\Psi} &= 0, & \mu\big(P - \operatorname{tr} \boldsymbol{Q}\big) &= 0 \end{aligned}$$

- If $\boldsymbol{H}_i^{-1}$ exists, then (1) can be expressed as...

Special Case: Invertible Channels

Lagrangian

$$L(\boldsymbol{Q}, \mu, \boldsymbol{\Psi}) = -\sum_{i=1}^{2} w_i C_i(\boldsymbol{Q}) - \mu\Big(P - \operatorname{tr} \boldsymbol{Q}\Big) - \operatorname{tr} \boldsymbol{Q}\boldsymbol{\Psi}$$

Karush-Kuhn-Tucker conditions

$$\sum_{i=1}^{2} w_i(\sigma^2 \boldsymbol{H}_i^{-H}\boldsymbol{H}_i^{-1} + \boldsymbol{Q})^{-1} = \mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi}$$

$$\begin{aligned} \boldsymbol{Q} &\succeq 0, & P &\geq \operatorname{tr} \boldsymbol{Q}, \\ \boldsymbol{\Psi} &\succeq 0, & \mu &\geq 0, \\ \operatorname{tr} \boldsymbol{Q}\boldsymbol{\Psi} &= 0, & \mu\Big(P - \operatorname{tr} \boldsymbol{Q}\Big) &= 0 \end{aligned}$$

- with substitutions $\boldsymbol{A}_i := \sigma^2 \boldsymbol{H}_i^{-H}\boldsymbol{H}_i^{-1}$ and $\boldsymbol{B} := \mu \boldsymbol{I}_{N_R} - \boldsymbol{\Psi}$ ...

Special Case: Invertible Channels

Lagrangian

$$L(\boldsymbol{Q}, \mu, \boldsymbol{\Psi}) = -\sum_{i=1}^{2} w_i C_i(\boldsymbol{Q}) - \mu\big(P - \operatorname{tr}\boldsymbol{Q}\big) - \operatorname{tr}\boldsymbol{Q}\boldsymbol{\Psi}$$

Karush-Kuhn-Tucker conditions

$$w_1(\boldsymbol{A}_1 + \boldsymbol{Q})^{-1} + w_2(\boldsymbol{A}_2 + \boldsymbol{Q})^{-1} = \boldsymbol{B}$$

$$\boldsymbol{Q} \succeq 0, \qquad P \geq \operatorname{tr}\boldsymbol{Q},$$

$$\boldsymbol{\Psi} \succeq 0, \qquad \mu \geq 0,$$

$$\operatorname{tr}\boldsymbol{Q}\boldsymbol{\Psi} = 0, \qquad \mu\big(P - \operatorname{tr}\boldsymbol{Q}\big) = 0$$

- multiplication with $(\boldsymbol{A}_1 + \boldsymbol{Q})$ and $(\boldsymbol{A}_2 + \boldsymbol{Q})$ we get...

Special Case: Invertible Channels

Lagrangian

$$L(\mathbf{Q}, \mu, \mathbf{\Psi}) = -\sum_{i=1}^{2} w_i C_i(\mathbf{Q}) - \mu\Big(P - \operatorname{tr} \mathbf{Q}\Big) - \operatorname{tr} \mathbf{Q}\mathbf{\Psi}$$

Karush-Kuhn-Tucker conditions

$$\mathbf{Q}\mathbf{B}\mathbf{Q} + \mathbf{Q}\mathbf{B}\mathbf{A}_2 + \mathbf{A}_1\mathbf{B}\mathbf{Q} - \mathbf{Q} = w_1\mathbf{A}_2 + w_2\mathbf{A}_1 - \mathbf{A}_1\mathbf{B}\mathbf{A}_2,$$

$$\begin{aligned} \mathbf{Q} &\geq 0, & P &\geq \operatorname{tr} \mathbf{Q}, \\ \mathbf{\Psi} &\geq 0, & \mu &\geq 0, \\ \operatorname{tr} \mathbf{Q}\mathbf{\Psi} &= 0, & \mu\Big(P - \operatorname{tr} \mathbf{Q}\Big) &= 0 \end{aligned}$$

- ... a quadratic matrix equation (also known as unsymmetric Riccati equation). A solution method exists, but further analytical results are not available so far (we do not know).

Special Case: Invertible Channels

Lagrangian

$$L(\mathbf{Q}, \mu, \mathbf{\Psi}) = -\sum_{i=1}^{2} w_i C_i(\mathbf{Q}) - \mu\Big(P - \operatorname{tr}\mathbf{Q}\Big) - \operatorname{tr}\mathbf{Q}\mathbf{\Psi}$$

Karush-Kuhn-Tucker conditions

$$w_1(\mathbf{A}_1 + \mathbf{Q})^{-1} + w_2(\mathbf{A}_2 + \mathbf{Q})^{-1} = \mathbf{B} \tag{2}$$

$$\mathbf{Q} \succeq 0, \qquad P \geq \operatorname{tr}\mathbf{Q},$$

$$\mathbf{\Psi} \succeq 0, \qquad \mu \geq 0,$$

$$\operatorname{tr}\mathbf{Q}\mathbf{\Psi} = 0, \qquad \mu\Big(P - \operatorname{tr}\mathbf{Q}\Big) = 0$$

- Notice, at this step we can multiply $(\mathbf{A}_1 + \mathbf{Q})$ and $(\mathbf{A}_2 + \mathbf{Q})$ from the left **or** from the right!

Special Case: ... and Full Rank Transmission

Full Rank Transmission and Invertible Channels

Assume: $\boldsymbol{H}_i^{-1}$ exists & the optimal covariance matrix has $\operatorname{rank} \boldsymbol{Q} = N$

➠ $\boldsymbol{\Psi} = \boldsymbol{0}$ and therefore $\boldsymbol{B} = \mu \boldsymbol{I}_N$

- Interchanging multiplications from the left and the right ...

$$w_1(\boldsymbol{A}_2 + \boldsymbol{Q}) + w_2(\boldsymbol{A}_1 + \boldsymbol{Q}) = \mu(\boldsymbol{A}_1 + \boldsymbol{Q})(\boldsymbol{A}_2 + \boldsymbol{Q}) = \mu(\boldsymbol{A}_2 + \boldsymbol{Q})(\boldsymbol{A}_1 + \boldsymbol{Q})$$

shows that matrices $(\boldsymbol{A}_1 + \boldsymbol{Q})$ and $(\boldsymbol{A}_2 + \boldsymbol{Q})$ commute!

Special Case: ... and Full Rank Transmission

Full Rank Transmission and Invertible Channels

Assume: $\boldsymbol{H}_i^{-1}$ exists & the optimal covariance matrix has $\operatorname{rank} \boldsymbol{Q} = N$

➠ $\boldsymbol{\Psi} = \boldsymbol{0}$ and therefore $\boldsymbol{B} = \mu \boldsymbol{I}_N$

- Interchanging multiplications from the left and the right ...

$$w_1(\boldsymbol{A}_2 + \boldsymbol{Q}) + w_2(\boldsymbol{A}_1 + \boldsymbol{Q}) = \mu(\boldsymbol{A}_1 + \boldsymbol{Q})(\boldsymbol{A}_2 + \boldsymbol{Q}) = \mu(\boldsymbol{A}_2 + \boldsymbol{Q})(\boldsymbol{A}_1 + \boldsymbol{Q})$$

shows that matrices $(\boldsymbol{A}_1 + \boldsymbol{Q})$ and $(\boldsymbol{A}_2 + \boldsymbol{Q})$ commute!

➠ Both have the same eigenspace, i.e., $(\boldsymbol{A}_i + \boldsymbol{Q}) = \boldsymbol{U}\boldsymbol{\Sigma}_i\boldsymbol{U}^H, i = 1, 2,$ which can be computed from

$$(\boldsymbol{A}_2 + \boldsymbol{Q}) - (\boldsymbol{A}_1 + \boldsymbol{Q}) = \underbrace{\boldsymbol{A}_2 - \boldsymbol{A}_1}_{=\sigma^2\left((\boldsymbol{H}_2\boldsymbol{H}_2^H)^{-1} - (\boldsymbol{H}_1\boldsymbol{H}_1^H)^{-1}\right)} = \boldsymbol{U}(\boldsymbol{\Sigma}_2 - \boldsymbol{\Sigma}_1)\boldsymbol{U}^H$$

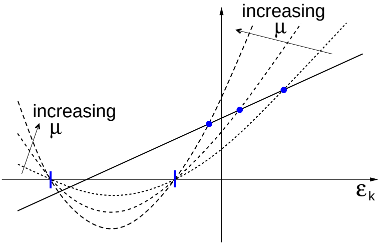
Optimal Eigenvalues

Proposition: Quadratic Equation Condition

$\boldsymbol{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_N]$ diagonalizes matrix equation

$$\underbrace{w_1(\delta_{2,k} + \epsilon_k) + w_2(\delta_{1,k} + \epsilon_k)}_{\text{linear in } \epsilon_k} = \underbrace{\mu(\delta_{1,k} + \epsilon_k)(\delta_{2,k} + \epsilon_k)}_{\text{quadratic in } \epsilon_k}$$

with $\delta_{i,k} = \boldsymbol{u}_k^H \boldsymbol{A}_i \boldsymbol{u}_k > 0$ and $\epsilon_k = \boldsymbol{u}_k^H \boldsymbol{Q} \boldsymbol{u}_k > 0$, $k = 1, 2, \ldots, N$.



- **Solution:** Intersection between line and parabola. (negative sol. can be excluded)
- Choose μ such that $\sum_{k=1}^{N} \epsilon_k = P$

Optimal Transmit Covariance Matrix

Optimal Transmit Covariance Matrix

The optimal transmit covariance for the case of invertible channels and a full rank transmission is given by

$$\boldsymbol{Q} = \boldsymbol{U}\text{diag}\left[\delta_{i,1} + \epsilon_1, \delta_{i,2} + \epsilon_2, \ldots, \delta_{i,N} + \epsilon_N\right] \boldsymbol{U}^H - \boldsymbol{A}_i, \quad i = 1, 2,$$

which can be completely calculated by the previous procedure.

- Possible extension to case where $\boldsymbol{Q}^{-1}$, $(\boldsymbol{H}_1\boldsymbol{H}_1^H)^{-1}$, and $(\boldsymbol{H}_2\boldsymbol{H}_2^H)^{-1}$ exist (Sherman-Morrison-Woodbury formula).

Optimal Transmit Covariance Matrix

Optimal Transmit Covariance Matrix

The optimal transmit covariance for the case of invertible channels and a full rank transmission is given by

$$Q = U \mathrm{diag}\left[\delta_{i,1} + \epsilon_1, \delta_{i,2} + \epsilon_2, \ldots, \delta_{i,N} + \epsilon_N\right] U^H - A_i, \quad i = 1, 2,$$

which can be completely calculated by the previous procedure.

- Possible extension to case where Q^{-1}, $(H_1 H_1^H)^{-1}$, and $(H_2 H_2^H)^{-1}$ exist (Sherman-Morrison-Woodbury formula).

Open Problem: Generalization

Non-full rank transmission & channels with different subspaces

Problem: $\Psi \neq 0$

Potential difficulty: Optimal solution may be not unique!

Complete Solution for Special Case: Parallel Channels

Definition: Parallel Channels (e.g. OFDM)

$$\boldsymbol{H}_1\boldsymbol{H}_1^H = \boldsymbol{W}\boldsymbol{S}_1\boldsymbol{W}^H \qquad \boldsymbol{H}_2\boldsymbol{H}_2^H = \boldsymbol{W}\boldsymbol{S}_2\boldsymbol{W}^H$$

with $\boldsymbol{S}_i = \mathrm{diag}(s_{i,1}, s_{i,2}, \ldots, s_{i,N}) \succeq \boldsymbol{0}$, $i = 1, 2$, and $\boldsymbol{W}$ unitary.

- Optimal eigenvectors
  - Hadamard Inequality: $\boldsymbol{Q} = \boldsymbol{W}\boldsymbol{\Sigma}_Q\boldsymbol{W}^H$, $\boldsymbol{\Sigma}_Q = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$
- Optimal eigenvalues
  - Weighted rate sum maximization:

$$R_\Sigma(\boldsymbol{w}) = \max_{\boldsymbol{\lambda}} \sum_{k=1}^{N}\sum_{i=1}^{2} w_i \log(1 + \tfrac{1}{\sigma^2} s_{i,k}\lambda_k) \text{ s.t. } ||\boldsymbol{\lambda}||_1 \leq P,\ \lambda_k \geq 0$$

- Previous procedure solves even non-full rank case.

Details

BiBC Under Channel Uncertainty

- Channel uncertainty is a ubiquitous phenomenon in practical systmes
  - Assume that it is only known that the exact channel realization belongs to a pre-specified set of channels $\mathcal{S}$
  - We need universal strategies that work for all realizations simultaneously

Theorem: *Capacity Region of Compound BiBC [TCom'10]*

$$R_1 \leq \inf_{s \in \mathcal{S}} I(X_R; Y_{1,s}) \quad \text{and} \quad R_2 \leq \inf_{s \in \mathcal{S}} I(X_R; Y_{2,s})$$

$Y_{i,s}$ channel output at node i for channel realization $s \in \mathcal{S}$.

- Results are extended to arbitrary varying channels.

Robust Transmit Strategies for the MISO case

- CSI uncertainty: $\boldsymbol{h}_{i,0}$ nominal (known) channel, [Loyka et al.'08]

$$y_i = (\boldsymbol{h}_{i,0} + \boldsymbol{d}_i)\boldsymbol{x} + n_i, \qquad i = 1,2$$

perturbation $\boldsymbol{d}_i \in \mathcal{D}_i$

$$\mathcal{D}_i := \{\boldsymbol{d}_i : \sigma_1(\boldsymbol{d}_i) = \|\boldsymbol{d}_i\| \leq \epsilon_i\}$$

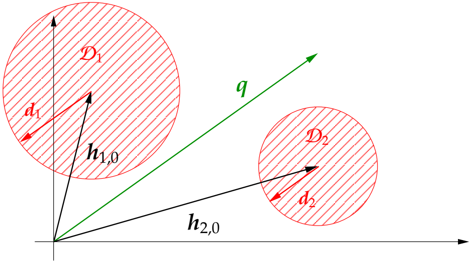
Optimal Robust Transmit Strategy

- Worst case capacity region of the MISO BiBC under channel uncertainty $\mathcal{D}_i$ is given by set of rate pairs

$$R_1 \leq \log\left(1 + \tfrac{1}{\sigma^2}(|\boldsymbol{h}_{1,0}^H \boldsymbol{q}| - \epsilon_1)^2\right), \quad R_2 \leq \log\left(1 + \tfrac{1}{\sigma^2}(|\boldsymbol{h}_{2,0}^H \boldsymbol{q}| - \epsilon_2)^2\right)$$

for some transmit strategy $\boldsymbol{Q} = \boldsymbol{q}\boldsymbol{q}^H$ with $tr(\boldsymbol{Q}) \leq P$.

Worst-Case Perturbation



- Worst-case perturbations can explicitly be characterized as

$$\boldsymbol{d}_i(\boldsymbol{q}) = -\epsilon_i \, e^{-j\varphi_i} \, \boldsymbol{u}_{\boldsymbol{q}} \quad \text{with } \boldsymbol{u}_{\boldsymbol{q}} = \boldsymbol{q}/\|\boldsymbol{q}\| \text{ and } \varphi_i = arg(\boldsymbol{h}_{i,0}^H \boldsymbol{u}_{\boldsymbol{q}})$$

➠ Worst-case $\boldsymbol{d}_i$ are anti-parallel to transmit strategy $\boldsymbol{q}$

- Some extensions to MIMO case possible.

Conclusion

Bidirectional broadcast channel is an appealing problem which

- is practically relevant
  - modularization, realizes some network coding gains,
- is closely related to multicast, P2P channel,
- allows derivation of closed form results in the Gaussian case

MISO: single-beam optimaity manifests single information flow view, favors correlated channels

MIMO: closed-form procedure to find full rank solution
$\rightarrow$ extension open,

- has many interesting extensions, e.g.,
  - compound and arbitrary varying channel versions
    $\rightarrow$ robust transmit strategies.

Thank you for your attention!

Own Related References I

- C. Schnurr, T. J. Oechtering, and S. Stańczak,
  On Coding for the Broadcast Phase in the Two-Way Relay Channel,
  Proc. 41st Annual Conf. on Inf. Sciences and Systems, Mar. 2007.
- T. J. Oechtering, C. Schnurr, I. Bjelaković, and H. Boche,
  Broadcast Capacity Region of Two-Phase Bidirectional Relaying,
  IEEE Transactions on Information Theory, vol. 54, no. 1, 2008, pp. 454-458.
- R. F. Wyrembelski, T. J. Oechtering, I. Bjelaković, C. Schnurr, and H. Boche
  Capacity of Gaussian MIMO Bidirectional Broadcast Channels
  Proc. IEEE Int. Symp. on Inf. Theory, July 2008, pp. 584–588
- T. J. Oechtering, R. F. Wyrembelski, and H. Boche
  Multi-Antenna Bidirectional Broadcast Channels - Optimal Transmit Strategies
  IEEE Trans. on Signal Proc., May 2009, pp. 1948–1958.
- T. J. Oechtering, E. A. Jorswieck, R. F. Wyrembelski, and H. Boche
  On the Optimal Transmission for the MIMO Bidirectional Broadcast Channel
  IEEE Trans. on Com., vol. 57, Dec. 2009, pp. 3817-3826.

Own Related References II

R. F. Wyrembelski, I. Bjelaković, T. J. Oechtering, and H. Boche
On the Capacity of Bidirectional Broadcast Channels under Channel Uncertainty
Proc. IEEE Int. Conf. Commun., pp. 1-5, June 2009.

R. F. Wyrembelski, I. Bjelaković, T. J. Oechtering, and H. Boche
Optimal Coding Strategies for Bidirectional Broadcast Channels under Channel Uncertainty
IEEE Trans. Commun., vol. 58, no. 10, pp. 2984-2994, Oct. 2010.

Optimal eigenvalues

- Lagrangian: $L(\boldsymbol{\lambda}, \mu, \boldsymbol{\nu}) = \mu(P - \|\boldsymbol{\lambda}\|_1) + \sum_{k=1}^{N} \left[\nu_k \lambda_k - \sum_{i=1}^{2} w_i \log(1 + \frac{1}{\sigma^2} s_{i,k} \lambda_k) \right]$,

- Optimal λ_k follows from the Karush-Kuhn-Tucker conditions

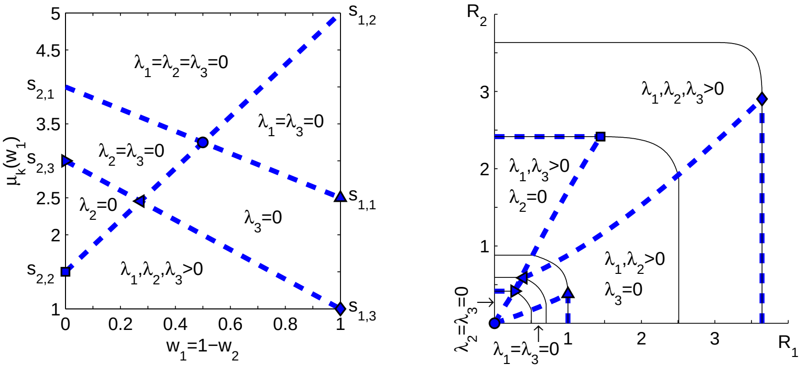
$$w_1 s_{1,k}(\sigma^2 + s_{2,k}\lambda_k) + w_2 s_{2,k}(\sigma^2 + s_{1,k}\lambda_k) = (\mu - \nu_k)(\sigma^2 + s_{1,k}\lambda_k)(\sigma^2 + s_{2,k}\lambda_k)$$

  - Quadratic equation in λ_k; negative solution can be excluded
  - μ has to be chosen so that power constraint is fulfilled
  - ν_k controls positivity condition of λ_k

The case $\nu_k = \lambda_k = 0$ defines a thresholds $\mu_k(\boldsymbol{w})$ where mode k is activated, i.e., for larger $P \Rightarrow$ smaller μ, we will have $\lambda_k > 0$,

$$\mu_k(\boldsymbol{w}) = \sigma^{-2}(w_1 s_{1,k} + w_2 s_{2,k}).$$

Modes Areas of Parallel Channels with $N_1 = N_2 = N_R = 3$



- Eigenvalue $s_{i,k}$ corresponds to the k-th eigenvector of $\boldsymbol{H}_i\boldsymbol{H}_i^H$

➠ The activated modes, $\lambda_k > 0$, change with the weights. At the weights corresponding to '∘' beamforming is never optimal.

Back