Banff International Research Station

for Mathematical Innovation and Discovery

Spectral Analysis, Stability and Bifurcation in Modern Nonlinear Physical Systems (12w5073)
November 4 – 9, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

*Format of the presentations: lecture 30 min. + discussion 5 min.

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
	Lectures (Chair: Dmitry Pelinovsky)
9:00–9:35	Peter Lancaster
9:35–10:10	Michael Overton
10:10–10:20	Coffee Break, TCPL
10:20–10:55	Richard Cushman
10:55–11:30	Davide Bigoni
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
	Lectures (Chair: Phil Morrison)
14:15–14:50	Jussi Behrndt
14:50–15:10	Coffee Break, TCPL
15:10–15:45	Richard Kollar
15:45–16:20	Karsten Trunk
16:20–16:55	Dmitry Pelinovsky
16:55–17:30	Panayotis Kevrekidis
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
	Lectures (Chair: Yasuhide Fukumoto)
9:00–9:35	Michael Berry
9:35–10:10	Sergey Dobrokhotov
10:10–10:30	Coffee Break, TCPL
10:30–11:05	Setsuro Fujiié
11:30–13:30	Lunch
	Lectures (Chair: Richard Cushman)
13:30–14:05	John Maddocks
14:05–14:40	Pietro-Luciano Buono
14:40–15:15	Jeroen Lamb
15:15–15:45	Coffee Break, TCPL
15:45–16:20	Holger Waalkens
16:20–16:55	Konstantinos Efstathiou
16:55–17:30	Zensho Yoshida
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
	Lectures (Chair: Davide Bigoni)
9:00–9:35	Olivier Doaré
9:35–10:10	Oleg Kirillov
10:10–10:20	Coffee Break, TCPL
10:20–10:55	Gianne Derks
10:55–11:30	Francis Nier
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
	Lectures (Chair: Jussi Behrndt)
9:00–9:35	Edgar Knobloch
9:35–10:10	Marina Chugunova
10:10–10:20	Coffee Break, TCPL
10:20–10:55	Almut Burchard
10:55–11:30	Stephane Le Dizes
11:30–13:30	Lunch
	Lectures (Chair: Sherwin Maslowe)
13:30–14:05	Stefan Llewellyn Smith
14:05–14:40	Makoto Hirota
14:40–15:15	Yasuhide Fukumoto
15:15–15:45	Coffee Break, TCPL
15:45–16:20	Paolo Luzzatto-Fegiz
16:20–16:55	Phil Morrison
16:55–17:30	George Hagstrom
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
	Lectures (Chair: Zensho Yoshida)
9:00–9:35	Sherwin Maslowe
9:35–10:10	Katie Oliveras
10:10–10:30	Coffee Break, TCPL
10:30–11:05	Emanuele Tassi
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Spectral Analysis, Stability and Bifurcation in Modern Nonlinear Physical Systems (12w5073)
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ABSTRACTS

Speaker: **Behrndt, Jussi** (TU Graz, Austria)
Title: *Spectral theory for differential operators with indefinite weights*

Abstract: In this talk we discuss the spectral properties of a class of ordinary and partial differential operators with indefinite weight functions. These operators are not symmetric or self-adjoint with respect to a Hilbert space scalar product but they can still be viewed to be symmetric with respect to a suitably chosen Krein space inner product. However, for a detailed spectral analysis the Krein space symmetry is only of little use. In this lecture we present a general approach via decomposition and perturbation methods to obtain results on the structure of the real and non-real spectrum, as well as quantitative bounds on the non-real spectrum.

Speaker: **Berry, Michael** (University of Bristol, UK)
Title: *The exquisite sensitivity of polarization*

Abstract: Recent insights into the time-development of quantum states driven by nonhermitian matrices, and an exactly solvable model, can be applied to the evolution of optical polarization in a stratified nontransparent dielectric medium twisted cyclically along the propagation direction. The twist is chosen to encircle a degeneracy (branch-point) in the plane of parameters describing the medium. Polarization evolutions are determined analytically and illustrated as tracks on the Poincaré sphere and the stereographic plane. Even when the twist is slow, the exact evolutions differ sharply from those of the local eigenpolarizations and can display extreme sensitivity to initial conditions with the tracks exhibiting elaborate coilings and loopings that would be very interesting to explore experimentally. Underlying these dramatic violations of adiabatic intuition are the disparity of exponentials and the Stokes phenomenon of asymptotics.

Reference: Berry, M V 2011, 'Optical polarization evolution near a non-Hermitian degeneracy', J Opt. 13 115701 (15pp).

Speaker: **Bigoni, Davide** (University of Trento, Italy)
Title: *Challenging instabilities in elastic structures*

Abstract: A series of challenging structural behaviours will be presented for elastic structure evidencing:

- bifurcation and instability for tensile dead loads;
- multiple bifurcations for single-degrees-of-freedom structures;
- decreasing of buckling loads at increasing stiffness;

- recover of a stable trivial path after a buckled and unstable one;

- flutter instability as related to Coulomb friction.

Speaker: **Broer, Henk** (University of Groningen, The Netherlands)
Title: *Resonance and fractal geometry*

Abstract: A number of resonant phenomena is reviewed such as Huygens's synchronizing clocks, the tidal resonances of Moon and certain planets as well a the swing. Resonance is an interaction of various oscillations with rationally related frequencies which leads to a compatible periodic behaviour. It is conceived of in terms of parameter dependent dynamics. The resonant zones in parameter space then can consist of tongues that are arranged in a fractal pattern in which a Cantor set plays a role. In and near the Cantor set also other types of dynamics may occur, like quasi-periodic or chaotic. In the talk we discuss several examples.

Speaker: **Buono, Pietro-Luciano** (UOIT, Canada)
Title: *Stability analysis and bifurcations of the Hip-Hop orbit*

Abstract: I will discuss recent results obtained with M. Lewis, D. Offin (Queen's) and M. Kovacic (UOIT) about the Hip-Hop orbit of the Newtonian 2N-body problem. The Hip-Hop orbit (in reduced space) is a periodic solution with time-reversing and spatio-temporal symmetries and in fact, we have shown that it is a brake orbit. I will also present the analytical proof of linear instability of the Hip-Hop orbit using Maslov index methods. I will show numerical simulations of the Hip-Hop orbit as the energy is varied which exhibits a sequence of symmetry-breaking bifurcations and discuss avenues for classifying those bifurcations.

Speaker: **Burchard, Almut** (University of Toronto, Canada)
Title: *On computing the instability index of a non-selfadjoint differential operator associated with a thin-film equation*

Abstract: I will describe joint work with Marina Chugunova on certain non-selfadjoint fourth order differential operators that appear as linearizations of coating and rimming flows, where a thin layer of fluid coats a horizontal rotating cylinder. Our main result is that the instability index of the operator is determined by its restriction to a finite-dimensional space of trigonometric polynomials. In the proof, we use Lyapunov's method to associate the differential operator with a quadratic form, whose maximal positive subspace has dimension equal to the instability index. The quadratic form is given by a solution of Lyapunov's equation, which here takes the form of a fourth order linear PDE in two variables. Elliptic estimates for the solution of this PDE play a key role in the proof.

Speaker: **Chugunova, Marina** (University of Toronto, Canada)
Title: *Stability of regularized shock solutions in coating flows*

Abstract: We consider a model for thin liquid films in a rotating cylinder in the small surface tension limit. Using numerical computations, we show that the existence curves of regularized shock solutions on the mass-flux diagram exhibit loops. The number of loops increases and their locations move to infinity as the surface tension parameter decreases to zero. If n is the number of loops in the mass-flux diagram with $2n + 1$ solution branches, we show that $n + 1$ solution branches are stable with respect to small perturbations.

Speaker: **Cushman, Richard** (University of Calgary, Canada)
Title: *The sign exchange bifurcation in a linear Hamiltonian system*

Abstract: This talk treats in detail an example of a one parameter family of Hamiltonian systems, which exhibits an S^1-equivariant sign exchange bifurcation in its linearization about an equilibrium point.

Speaker: **Derks, Gianne** (University of Surrey, UK)
Title: *Viscosity-induced instability for Euler and averaged Euler equations in a circular domain*

Abstract: We consider the infinite time behaviour of a family of stationary solutions of Euler's equation, which can be described as constrained minima of energy on level sets of enstrophy. For free boundary conditions, this family shadows solutions of 2D Navier-Stokes equations. However, under the no-slip and under the Navier-slip boundary conditions and in a circular domain, the infinite time Navier-Stokes evolution orbit of a starting point on the family of constrained minima has order 1 distance to the family, however small the viscosity is. The viscosity in the Navier-Stokes equations is a singular perturbation for Euler's equation and one might suspect that the viscosity-induced instability is related to this singularity. This is not the case: we show that the same phenomenon can be observed for the averaged Euler equations and second grade fluids with Navier-slip boundary conditions in a circular domain.

Speaker: **Doaré, Olivier** (ENSTA-Paristech, UME, Palaiseau, France)
Title: *Influence of dissipation on local and global instabilities*

Abstract: In many mechanical or physical systems, damping has a stabilizing but also a destabilizing effect. In some ranges of parameters, these systems possess a stable equilibrium without damping, which becomes an unstable equilibrium when a small amount of damping is added. In mechanics, such systems are for instance the double pendulum subjected to a follower force studied by Ziegler [9], the Beck's column addressed by Sugiyama and Langthjem [8] or the cantilevered fluid-conveying pipe [6]. In fact, most gyroscopic systems present this feature [5]. Stability analysis of continuous, but finite length systems such as Beck's columns or fluid-conveying pipes are referred to as global, since it depends on both the properties of the medium and its boundary conditions.

In some media of infinite length, waves that are neutral in absence of dissipation become temporally amplified when damping terms are added in the wave equation. The concept of wave energy, introduced in plasma physics [7], represents a considerable value to the discussion of this effect. The energy of a wave is defined as the work done on the system to generate the neutral wave from $t = -\infty$ to $t = 0$. Consequently, a wave is of negative energy if its establishment lowers the total energy of the system. It was then found that negative energy waves are destabilized by addition of damping. Studies considering waves propagating in an infinite medium are referred to as *local*. Additionally to plasma physics, negative energy waves have been studied in mechanics, in the context of compliant panels interaction with inviscid flows [1], the instabilities of the surface between two non-miscible fluids [2].

The aim of the work presented here is to address the relationship between local and global instabilities with respect to destabilisation by damping. The fluttering fluid-conveying pipe and the plate in axial flow are considered as systems representative of many other systems in physics. It is found that the presence of gyroscopic terms in the wave equation is necessary to have negative energy waves in the system and that the existence of negative energy waves is a necessary condition to observe destabilisation by dissipation in the finite length system. Some simple local criteria, based on characteristic length of rigidity and damping forces are used to develop simple criteria that predicts global instability [3]. Finally, some recent work on energy harvesting using fluttering piezoelectric flexible plates will be considered in the particular context of destabilisation by damping [4].

References

1. T. B. Benjamin. The threefold classification of unstable disturbances in flexible surfaces bounding inviscid flows. *Journal of Fluid Mechanics*, 16(3):436-450, 1963.

2. R. A. Cairns. The role of negative energy waves in some instabilities of parallel flows. *Journal of Fluid Mechanics*, 92:1-14, 1979.

3. O. Doaré. Dissipation effect on local and global stability of fluid-conveying pipes. *Journal of Sound and Vibration*, 329(1):72-83, 2010.

4. O. Doaré and S. Michelin. Piezoelectric coupling in energy-harvesting fluttering flexible plates: linear stability analysis and conversion efficiency. *Journal of Fluids and Structures*, 27(8):1357-1375, 2011.

5. O. N. Kirillov and F. Verhulst. Paradoxes of dissipation-induced destabilization or who opened Whitney's umbrella? *Z. Angew. Math. Mech.*, 90(6): 462-488, 2010.

6. I. Lottati and A. Kornecki. The effect of an elastic foundation and of dissipative forces on the stability of fluid-conveying pipes. *Journal of Sound and Vibration*, 2(109):327-338, 1986.

7. T. H. Stix. The Theory of Plasma Waves. 1962.

8. Y. Sugiyama and M. A. Langthjem. Physical mechanism of the destabilizing effect of damping in continuous non-conservative dissipative systems. *International Journal of Non-Linear Mechanics*, 42(1):132-145, 2007.

9. H. Ziegler. Die stabilitätskrieterien der elastomechanik. *Ing.-Arch.*, 20:49-56, 1952.

Speaker: **Dobrokhotov, Sergey** (Ishlinski Institute for Problems in Mechanics of Russian Academy of Sciences and Moscow Institute of Physics and Technology, Russia)
Title: *Tunneling, librations and normal forms in quantum double well with magnetic field*

Abstract: A problem for a Schrödinger-particle placed into a symmetric one-dimensional double-well potential is considered in many textbooks on Quantum mechanics. It is well known that the level splitting in some cases can be computed semiclassically taken into account the presence of the small parameter h. A multi-dimensional analogue of the problem has been intensively studied by V. Maslov, A. Poljakov, E. M. Harrel, B. Helfer, J. Sjöstrand, B. Simon etc. for lowest energy levels. The main result of this study is so-called splitting formula $\triangle E = A\exp(-\frac{J}{h})$ with the phase J based on a certain classical trajectories known as instanton. The constructive but complicated formula of the amplitude A also connected with the instanton was given in the papers by S. Yu. Dobrokhotov, V. Kolokoltsov and V. Maslov. In this talk we show that the final splitting formula takes more natural and simple form if one changes the instanton by so-called libration (unstable closed trajectories) and use the normal forms coming from classical mechanics (J. Brüning, S. Yu. Dobrokhotov, E. S. Semenov, "Unstable closed trajectories, librations and splitting of the lowest eigenvalues in quantum double well problem," *Regular and Chaotic Dynamics*, **11**, 2, (2006), 167-180).

Also we consider the following question: what happens with the splitting in the presence of magnetic field? It turns out to be very nontrivial. The derivation of explicit formulas for the splitting of eigenvalues is based on a passage from rapidly oscillating WKB-functions $e^{\frac{iS(x)}{\hbar}}$ to rapidly decaying functions $e^{\frac{-S(x)}{\hbar}}$. This passage changes the corresponding classical Hamiltonian $H = p^2/2 + V(x)$ to the "tunneling" Hamiltonian $H = -p^2/2 + V(x)$, which allows one to use the theory of the classical Hamiltonian systems for the description of tunnelling effects. This idea does not work if magnetic field is present: the "tunneling" Hamiltonian becomes a complex function, so one cannot use classical Hamiltonian systems anymore. We found that in 2-D case using the partial Fourier transform and mixed momentum-position coordinates one can reduce the quantum double-well problem with magnetic field to the standard quantum double-well problem and to study the splitting in this situation also.

This work was done together with J.Brüning and R.V.Nekrasov and was supported by the DFG-RAN project 436 RUS 113/990/0-1 and by the RFBR grants nos. 11-01-00973 and 11-01-12058

Speaker: **Efstathiou, Konstantinos** (University of Groningen, The Netherlands)
Title: *Uncovering fractional monodromy*

Abstract: The uncovering of the role of monodromy in integrable Hamiltonian fibrations has been one of the major advances in the study of integrable Hamiltonian systems in the past few decades: on one hand monodromy turned out to be the most fundamental obstruction to the existence of global action-angle coordinates while, on the other hand, it provided the correct classical analogue for the interpretation of the structure of quantum joint spectra. Fractional monodromy is a generalization of the concept of monodromy: instead of restricting our attention to the toric part of the fibration we extend our scope to also consider singular fibres. In this paper we analyze fractional monodromy for n_1:$(-n_2)$ resonant Hamiltonian systems with n_1, n_2 coprime natural numbers. We consider, in particular, systems that for $n_1, n_2 > 1$ contain one-parameter families of singular fibres which are 'curled tori'. We simplify the geometry of the fibration by passing to an appropriate branched covering. In the branched covering the curled tori and their neighborhood become untwisted thus simplifying the geometry of the fibration: we essentially obtain the same type of generalized monodromy independently of n_1, n_2. Fractional monodromy is then recovered by pushing the results obtained in the branched covering back to the original system.

Speaker: **Fujiié, Setsuro** (Ritsumeikan University, Japan)
Title: *Semi-classical resonances associated with unstable equilibria*

Abstract: We consider the semi-classical Schrödinger operator $P := -h^2\Delta + V(x)$ in $\mathbb{R}^n$, where h is a small positive (semi-classical) parameter and $V(x)$ is a real-valued smooth potential decaying at infinity.

When regarded as operator on L^2, P is a self adjoint operator and positive energies belong to the essential spectrum. However, if the classical dynamics for the corresponding classical Hamiltonian $p(x, \xi) := |\xi|^2 + V(x)$ has *trapped* trajectories on $p^{-1}(z_0)$ for a real positive energy z_0, it is expected that there exist the so-called *resonances* close to z_0 in the lower half complex plane.

Resonances are defined as complex eigenvalues of P with *outgoing* condition at infinity, or equivalently as poles of the resolvent $(P - z)^{-1}$, which can be meromorphically continued from $\mathbb{C}_+$ to $\mathbb{C}_-$ as operator from $L^2_{\text{comp}}(\mathbb{R}^n)$ to $L^2_{\text{loc}}(\mathbb{R}^n)$. The imaginary part of resonances, called *width*, means the reciprocal of the exponential decay rate of the corresponding states for the evolution as time tends to $+\infty$. The most typical potential which creates resonances is the so-called *well in an island*, and the resonances near the bottom of the well, which is a stable equilibrium, have exponentially small width with respect to the semi-classical parameter h ([4], [3]).

We are interested in the width of resonances associated with an unstable equilibrium of the potential. If $x = x_0$ is an unstable equilibrium, i.e. a local maximum of the potential, then the point $(x, \xi) = (x_0, 0)$ in the phase space is a hyperbolic fixed point of the Hamilton vector field, and it is itself a trapped trajectory. Hence resonances may appear near the energy $E_0 := V(x_0)$.

Contrary to the case of a stable equilibrium, the trap by an unstable equilibrium is much weaker, and, as consequence, the resonance width should be large. In fact, the width of resonances is known to be $\delta_0 h$, where δ_0 is a constant independent of h ([5], [1], [2]) if the trapped set consists only of this hyperbolic fixed point, in other words, if the unstable equilibrium is the unique global maximum of the potential.

Here we develop the above result, and allow the existence of other higher bumps of potential. These bumps create *homoclinic* trajectories converging to the hyperbolic fixed point as time tends to $+\infty$ and $-\infty$. Assuming that the trapped set at the energy level E_0 consists of the hyperbolic fixed point and these homoclinic trajectories, we give lower bounds for the resonance width.

First, the resonance width is bounded from below by $\delta_1 h$ with smaller constant δ_1 than δ_0 if the trap is weak in the following sense: either (i) the unstable equilibrium is anisotropic and the homoclinic trajetctories are tangent to the direction of the smallest curvature at the fixed point, or (ii) the outgoing and incoming stable manifolds associated with the fixed point intersect transversally along the homoclinic trajectories.

Second, in the case where neither (i) nor (ii) holds, the resonance width becomes smaller, but estimated from below by $\delta_2 h = |\log h|$ under some quantitative condition on the geometry of the homoclonic trajectories.

We prove these results in the following way: Let u be a resonant state, i.e. an outgoing solution corresponding to a resonance. Consider it microlocally in the phase space, and continue it along the set of homoclinic trajectories. We show that if the width of the resonance was smaller than the above lower bound (i.e. $\delta_1 h$ in the first case and $\delta_2 h = |\log h|$ in the second case), then the amplitude of the resonant state would be smaller after a tour, which is a contradiction.

For the continuation of microlocal solutions, we use two theories. Along the homoclinic trajectories where the Hamilton vector field is regular, we use the standard WKB theory of Maslov, and for the continuation beyond the hyperbolic fixed point, which is essential for our problem, we use a formula in [2], which gives the microlocal semiclassical behavior of u on the outgoing stable manifold in terms of that on the incoming stable manifold.

This is a joint work with Jean-Francois Bony (Univ. Bordeaux I), Thierry Ramond (Univ. Paris Sud) and Maher Zerzeri (Univ. Paris Nord).

References

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2. Bony, J.-F., Fujiié, S., Ramond, T., Zerzeri, M. : Journal of Functional Analysis, 252-1,1 (2007), pp. 68125.

3. Fujiié, S., Lahamar-Benbernou, A., Martinez, A. : J. Math. Soc. Japan Vol.63, No.1 (2011) pp.1-78.

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5. Sjöstrand, J. : Springer Lecture Notes in Math., 1256 (1987), pp. 402429.

Speaker: **Fukumoto, Yasuhide** (Institute of Mathematics for Industry, Kyushu University, Japan)
Hirota, Makoto (Japan Atomic Energy Agency, Japan)
Mie, Youichi (Sumitomo Rubber Industries, Ltd. Japan)

Title: *Lagrangian and Eulerian hybrid method for symmetric breaking bifurcation of a rotating flow*

Abstract: A steady Euler flow of an inviscid incompressible fluid is characterized as an extremum of the total kinetic energy (=the Hamiltonian) with respect to perturbations constrained to an isovortical sheet (=coadjoint orbits). We exploit the criticality in the Hamiltonian to calculate the energy of three-dimensional waves on a steady vortical flow, and, as a by-product, to calculate the mean flow, induced by nonlinear interaction of waves with themselves.

We apply these formulas to the linear and weakly nonlinear stability of a rotating flow confined in a cylinder of elliptic cross-section. The linear instability, parametric resonance between a pair of Kelvin waves, is known as the Moore-Saffman-Tsai-Widnall (MSTW) instability. The linear stability characteristics is well captured from the viewpoint of Krein's theory of Hamiltonian spectra. Furthermore, with the mean flow induced by the Kelvin waves, a hybrid method of combining the Eulerian and the Lagrangian approaches is developed to deduce the amplitude equations to third order.

Speaker: **Hagstrom, George** (CIMS, New York University, USA)
Morrison, Phil (Physics Department, University of Texas, USA)

Title: *On the Continuum Hamiltonian Hopf Bifurcation II*

Abstract: The subject of this talk is the mathematical theory of the *continuum Hamiltonian Hopf* (CHH) bifurcation, defined as the bifurcation of modes with non-zero real part from the imaginary axis

and the continuous spectrum in a linear Hamiltonian system. Physical examples of such bifurcations are given in part I [1]. We consider linear Hamiltonian systems formulated as differential equations on Banach space: $\dot{f} = JHf$, J and H corresponding to the symplectic form and Hamiltonian. The goal of the theory is to determine criteria for the stability of the spectrum of JH under small perturbations. In the canonical case, for bounded H and with perturbations measured by the operator norm, bifurcations only occur if regions of the spectrum corresponding to positive and negative Krein signature are adjacent [2]. The emphasis here is on noncanonical Hamiltonian systems. In this case, J is singular and it is necessary to consider perturbations to J as well as to H. If the linear system comes from the linearization of a nonlinear equation about an equilibrium, shifting to a neighboring equilibrium leads to changes in both J and H. The choice of the norm for perturbations and the type of perturbations that are allowed have large effects on the resulting theory. Dynamically accessible perturbations, which are defined as perturbations that conserve the Casimir invariants and thus preserve the structure of J, play a crucial role in this theory. We present these ideas in detail for the specific example of the Vlasov-Poisson system linearized about a homogeneous equilibrium $f_0(v)$ [3]. We define the family of perturbations of the linearized Vlasov operator through perturbations of f_0. We prove that for each f_0 there is an arbitrarily small $\delta f_0'$ in the Sobolev space $W^{1,1}(\mathbb{R})$ such that $f_0 + \delta f_0$ is unstable. Dynamically accessible perturbations of f_0 are compositions with area preserving rearrangements. Under these perturbations f_0 will always be stable if the continuous spectrum is only of positive signature, where the signature of the continuous spectrum is defined as in previous work [4]. If there is a signature change, then there is a rearrangement of f_0 that is unstable and arbitrarily close to f_0 with f_0' in $W^{1,1}$. This result is analogous to Krein's theorem for the continuous spectrum. We discuss extensions of these results to general noncanonical Hamiltonian systems, in particular, the consequences and the physical meaning of different choices of the set of perturbations.

1. "On the continuum Hamiltonian-Hopf bifurcation I," by P.J. Morrison.

2. Ju.L. Daleckii and M.G. Krein,"Stability of Solutions of Differential Equations in Banach Space."

3. G.I. Hagstrom and P.J. Morrison, Trans. Theory and Stat. Phys. 39, 466 (2011).

4. P.J. Morrison, Trans. Theory and Stat. Phys. 29, 397 (2000).

Speaker: **Hirota, Makoto** (Japan Atomic Energy Agency, Japan)
Title: *Lagrangian approach to weakly and strongly nonlinear stability analyses of fluid models*

Abstract: Stability problems of various fluid models (PDEs) are widely addressed in the studies of complex fluids, geophysical fluids, astrophysical and laboratory plasmas and so on. If the model is physically well-posed, it is promising to find its Hamiltonian structure and Casimir invariants in the dissipationless limit, which can yield a priori estimates for Lyapunov stability. More detailed stability analysis is often facilitated by restoring the Lagrangian description of fluid, especially when there are many Lagrangian invariants (i.e., frozen-in fields). The Lagrangian viewpoint is advantageous in that it enjoys an effective use of variational principle. For example, in linear stability analysis, the variational principle renders the eigenvalue problem being composed of Hermitian and anti-Hermitian operators. The concepts of action-angle variables and adiabatic invariance can be formulated for not only discrete spectrum but also continuous one. By invoking the Lie series expansion, weakly nonlinear analysis is also performed systematically, and the normal forms for mode-mode couplings are extracted by least algebraic manipulations. Even for strongly nonlinear problem such as explosive instability, the variational principle enables us to infer its mechanism in a heuristic manner. Our recent advancements in this Lagrangian approach will be overviewed.

Speaker: **Kevrekidis, Panayotis** (Department of Mathematics and Statistics, University of Massachusetts, USA)
Title: *Existence, Stability and Dynamics of Some Single- and Multi-Component Solitary Waves: From Theory to Experiments*

Abstract: In this talk, we will present an overview of recent theoretical, numerical and experimental work concerning the static, stability, bifurcation and dynamic properties of coherent structures that can emerge in one- and higher-dimensional settings within Bose-Einstein condensates at the coldest temperatures in the universe (i.e., at the nanoKelvin scale). We will discuss how this ultracold quantum mechanical setting can be approximated at a mean-field level by a deterministic PDE of the nonlinear Schrodinger type and what the fundamental nonlinear waves of the latter are, such as dark solitons and vortices. Then, we will try to go to a further layer of simplified description via nonlinear ODEs encompassing the dynamics of the waves within the traps that confine them, and the interactions between them. Finally, we will attempt to compare the analytical and numerical implementation of these reduced descriptions to recent experimental results and speculate towards a number of interesting future directions within this field.

Speaker: **Kirillov, Oleg** (Helmholtz-Zentrum Dresden-Rossendorf, Germany)
Title: *Ziegler-Bottema dissipation-induced instability and related topics*

Abstract: Exactly 60 years ago Ziegler [1] observed (I) that viscous dissipation can move pure imaginary eigenvalues of a Lyapunov stable time-reversible non-conservative mechanical system (Ziegler's pendulum loaded by a follower force) to the right half of the complex plane and (II) that the threshold of asymptotic stability generically does not converge to the threshold of the Lyapunov stability of the non-damped system when dissipation coefficient tends to zero. In 1956 Bottema [2] related the structurally unstable situation (II) to the Whitney umbrella singularity [3] of the stability boundary. I will show the examples of Hamiltonian, reversible and $\mathcal{PT}$-symmetric systems of physics and mechanics with the similar effects of dissipation-induced instabilities and non-commuting limits of vanishing dissipation. I will discuss the relation of these effects to the multiple non-derogatory eigenvalues occurring both on the stability boundary and inside the domain of asymptotic stability, show the connection to the spectral abscissa minimization [4] and in the Hamiltonian case will demonstrate that a suitable combination of damping and nonconservative positional forces can destabilize the eigenvalues with both positive and negative Krein (symplectic) signature of the unperturbed system [5-7].

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Speaker: **Knobloch, Edgar** (Department of Physics, University of California at Berkeley, USA)
Title: *The magnetorotational instability and its saturation*

Abstract: The magnetorotational instability is a magnetic field induced instability of differential rotation that is likely to be of fundamental importance in astrophysics because of its angular momentum transport properties. In this talk I will review some of the essential properties of this instability, both in the dissipationless regime and in the dissipative regime, emphasizing the role played by magnetic cross-helicity in determining the nature of this instability. Applications to transport of angular momentum require an understanding of the amplitude of the instability. Its evolution is complex, however, because it involves three radically different timescales: the rotation frequency, the inverse Alfven travel time and the dissipation rate. I will describe an asymptotically reduced model that sheds light on the equilibration process both in an intermediate, nominally dissipationless regime, and in the ultimate regime where dissipation takes over, showing how phase mixing can saturate Maxwell and Reynolds stresses even when the instability is still evolving.

Speaker: **Kollar, Richard** (Comenius University, Slovakia)
Title: *Graphical Krein Signature and its Applications*

Abstract: We present a couple of applications of a simple graphical interpretation of the Krein signature well-known in the spectral theory of polynomial operator pencils. First, we show a simple generalization of the Evans function, the Evans-Krein function, that allows the calculation of Krein signatures in a way that is easy to incorporate into existing Evans function evaluation codes at virtually no additional computational cost. The graphical Krein signature also enables us to give elegant proofs of index theorems for linearized Hamiltonians in the finite dimensional setting: a general result implying as a corollary Vakhitov-Kolokolov criterion (or Grillakis-Shatah-Strauss criterion) generalized to problems with arbitrary kernels, and a count of real eigenvalues for linearized Hamiltonian systems in canonical form. Finally we demonstrate how the graphical approach can be used to derive new types of criteria prohibiting Hamiltonian-Hopf bifurcations under collisions of two eigenvalues of opposite signature. This is a joint work with Peter Miller (U Michigan).

Speaker: **Lamb, Jeroen** (Imperial College, London, UK) Title: *Additive noise does not destroy a pitchfork bifurcation*

Abstract: It is well-known from [CF98: Crauel and Flandoli, "Additive noise destroys a pitchfork bifurcation", Journal of Dynamics and Differential Equations 10 Nr. 2 (1998), 259-274] that adding noise to a system with a deterministic pitchfork bifurcation yields a unique random attracting fixed point with negative Lyapunov exponent for all parameters. Based on this observation, [CF98] concludes that the deterministic bifurcation is destroyed by the additive noise.

However, we show that there is qualitative change in the random dynamics at the bifurcation point in the sense that after the bifurcation, the Lyapunov exponent cannot be observed almost surely in finite time. We associate this bifurcation with a breakdown of both uniform attraction and equivalence under uniformly continuous topological conjugacies, and with non-hyperbolicity of the dichotomy spectrum at the bifurcation point.

This is joint work with Mark Callaway, Doan Thai Son, and Martin Rasmussen (all at Imperial College London).

Speaker: **Lancaster, Peter** (University of Calgary, Canada)
Title: *Linear Algebra meets Mechanics in Gyroscopic Systems*

Abstract: A general stability criterion associated with the names of Thomson, Tait and, more recently, Krechetnikov will be discussed. A precise formulation will be presented in the language of linear algebra.

Speaker: **Le Dizes, Stephane** (IRPHE, CNRS & Aix-Marseille University, France)
Title: *Waves on vortices; Landau damping versus radiative growth*

Abstract: In the first part of the talk, I will analyse the characteristics of the linear waves living on a vortex in an incompressible inviscid homogeneous fluid. I will show that a large axial wavenumber asymptotic analysis can be used to provide information on their spatial structure and dispersion relation. The stabilizing role of critical point singularities will be discussed and analysed in this framework. Asymptotic results will be illustrated and compared to numerical results for a family of vortices ranging from the Rankine vortex (disk of uniform vorticity) to the Lamb-Oseen vortex (gaussian vorticity profile). In the second part of the talk, I will consider the waves on similar vortices but in a fluid uniformly stratified in the direction of the vortex axis. I will show that stratification is a source of instability. Using the large axial wavenumber asymptotic analysis, I will show that the instability mechanism is associated with the radiative character of the waves. Connections with similar instability in shallow water or in a compressible fluid will be made. Experimental evidence of the radiative instability will be also provided.

Speaker: **Llewellyn Smith, Stefan** (University of California at San Diego, USA)
Title: *The continuous spectrum in the Moore-Saffman-Tsai-Widnall instability*

Abstract: High-Reynolds number flows are dominated by vortical structures. The instability of coherent vortex structures is hence of scientific importance, but also has real-world applications, as in the aircraft wake problems which affects flight operations near airports. Vortex filaments are unstable to a number of instabilities: the long wavelength Crow instability, the short wavelength Moore-Saffman-Tsai- Widnall (MSTW) instability and the ultra-short wavelength elliptical instability. The MSTW instability concerns a vortex in strain and was first examined by Moore and Saffman in a general context but with asymptotically small strain. Most of the actual studies since have concentrated on the case of a piecewise continuous profile of vorticity (a Rankine vortex) which supports discrete normal modes. We consider (i) the more general case of non-infinitesimal strain using exact solutions of the Euler equations called hollow vortices and (ii) smooth vorticity profiles by looking at an initial-value problem.

Speaker: **Luzzatto-Fegiz, Paolo** (Physical Oceanography, Woods Hole Oceanographic Institution, USA)
Williamson, Charles (Sibley School of Mechanical and Aerospace Engineering, Cornell University, USA)
Title: *Resonant instability in two-dimensional vortex arrays*

Abstract: While resonant (Hamiltonian Hopf) bifurcations are common in fluid dynamics, it appears that such instabilities have not yet been observed for one or two vortices, in two-dimensions. Intrigued by this fact, we examine conditions for the development of a Hamiltonian Hopf instability in vortex arrays. By building on the theory of Krein signatures for Hamiltonian systems, and considering constraints owing to impulse conservation, we show that a resonant instability (developing through coalescence of two eigenvalues) cannot occur for one or two vortices. We illustrate this deduction by examining available linear stability results for one or two vortices. Our work indicates that a resonant instability may, however, occur for three or more vortices. For these more complex flows, we propose a simple model, based on an elliptical vortex representation, to detect the onset of a resonant instability. We provide an example in support of our theory by examining three co-rotating vortices, for which we also perform a linear stability analysis. The stability boundary in our model is in good agreement with the full stability calculation. In addition, we show that eigenmodes associated with an overall rotation or an overall displacement of the vortices always have eigenvalues equal to zero and $\pm i\Omega$, respectively, where Ω is the angular velocity of the array. These results, for overall rotation and displacement modes, can also be used to immediately check the accuracy of a detailed stability calculation.

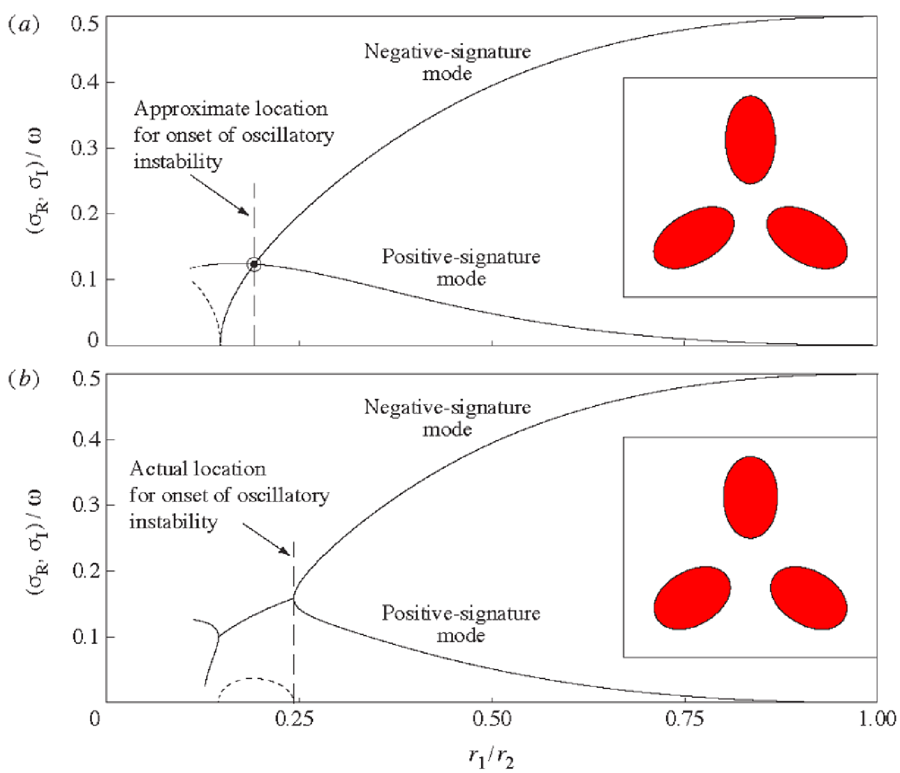

Figure 1: Selected eigenvalues for three co-rotating vortices, as a function of vortex separation distance r_1/r_2. (a) is the prediction for the development of the first resonance from the elliptical model. (b) shows results from an accurate linear stability analysis. The vortex shapes corresponding to the onset of instability are shown in the insets. Dashed and solid lines denote real (unstable) and imaginary (stable) parts, respectively. (See also Luzzatto-Fegiz & Williamson, Proc. Roy. Soc. A 2011.)

Speaker: **Maddocks, John** (EPFL, Swiss Federal Institute of Technology, Switzerland)
Title: *Parameter continuation, semi-classical path integrals, and looping probability of DNA*

Abstract: In families of isoperimetrically constrained variational principles the signs of the eigenvalues of the Hessian of the energy with respect to the Lagrange multipliers enters in to the second order necessary conditions. I will explain how the computation of looping probability for an elastic polymer, such as DNA, can be cast in terms of path integrals where the leading order approximation involves an isoperimetric variational principle, and the first, or semi-classical, correction involves the determinant of the same Hessian of the energy with respect to the Lagrange multipliers.

Speaker: **Maslowe, Sherwin** (Dept. of Mathematics, McGill University, Canada)
Title: *Evolution Equations for Finite Amplitude Waves in Parallel Shear Flows*

Abstract: The evolution of a wave packet propagating on a shear flow with velocity profile $\bar{u}(y)$ can be investigated by employing an amplitude $A(X,T)$ that varies slowly in space and time. In weakly nonlinear hydrodynamic stability problems, the PDE satisfied by A is often the Ginzburg-Landau equation. Whether or not this is the case depends on the dynamics of the critical layer, a thin layer centered on the point y_c, where $\bar{u} = c$, the perturbation phase speed. To determine the coefficients of terms in the amplitude equation, one must sometimes evaluate singular integrals. In such cases, the outcome will depend on whether viscous or nonlinear effects are dominant in the critical layer. When the amplitude satisfies the Ginzburg-Landau equation, the critical layer is relatively passive in that its primary role is to determine the coefficients of the dispersive and nonlinear terms. Recently, however, other formulations have been developed leading to an integro-differential equation governing the amplitude evolution. In such cases, it is the critical layer that dictates the form of the evolution equation. One finite amplitude approach that will be discussed features dispersive effects that are prevalent in the critical layer rather than viscosity. This approach is most appropriate in many geophysical shear flows, because the Reynolds numbers there are typically very large.

Speaker: **Morrison, Phil** (Physics Department, University of Texas, USA)
Hagstrom, George (CIMS, New York University, USA)
Title: *On the Continuum Hamiltonian Hopf bifurcation I*

Abstract: In Hamiltonian systems, the Hamiltonian Hopf (HH) bifurcation occurs when two pairs of stable eigenvalues $\pm i\omega_R^{1,2}(\eta)$ collide at some parameter value, η_0, and bifurcate to the quartet $\pm\omega_R \pm i\omega_I$. According to the Krein-Moser theorem, this bifurcation can only happen if the colliding eigenvalue pairs have opposite signature, which can be determined by evaluating the energy on the eigenfunction [1]. Such a transition to instability (overstability) is seen in the discrete spectrum of PDEs that describe many physical systems, such as the fluid plasma two-stream instability, top-hat distribution description of Jean's instability, and the contour dynamics description of shear flow or Kelvin-Helmholtz instability [2].

The continuum Hamiltonian Hopf (CHH) bifurcation is a similar, but mathematically more challenging, bifurcation that occurs in Hamiltonian PDEs with a continuous spectrum. Examples include the Vlasov equation, Euler's fluid equation, MHD, etc. To understand this bifurcation it is necessary to first attach a signature to the continuous spectrum [3] of Hamiltonian PDEs, a nontrivial task since eigenfunctions of the continuous spectrum are non-normalizable. Having the signature, a version of Krein-Moser theorem is possible provided an appropriate definition of structural stability and parameter variation are given [4]. Thus, in the CHH bifurcation, the continuous spectrum plays the role of one of the eigenvalue pairs of the HH bifurcation.

In this talk I will review the HH bifurcation in the PDE context, give examples, and describe how the CHH bifurcation appears in a variety of physical systems. (Rigorous aspects of CHH will be described in a companion talk [5]). I will also argue that the single-wave model [6] is a nonlinear normal form that captures the long time asymptotics of this bifurcation in a class of PDEs.

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Speaker: **Nier, Francis** (Université de Rennes, France)
Title: *Accurate estimates for the exponential decay of semigroups with non-self-adjoint generators: phenomena, examples, known results and applications*

Abstract: When one handles semigroups $(e^{-tL})_{t\geq 0}$ in a Hilbert space with non self-adjoint generators L with $\mathbb{R}\mathrm{e}\ L \geq 0$, three natural objects govern in different ways the exponential decay w.r.t $t \geq 0$ of $\|e^{-tL}\|$:

- the numerical range of L,
- the resolvent norm $\|(L - i\lambda)^{-1}\|$ along $+i\mathbb{R}$,
- the spectrum of L.

Distinguishing the corresponding effects becomes crucial when $L = L^{\varepsilon}$ depends on a small parameter with respect to which the exponential decay has to be analyzed.
Parameter dependent non self-adjoint operators occur naturally in some models or can be obtained after introducing artificially self-adjoint deformations of self-adjoint generators. I will first recall a simple result about estimating both C_{ε} and α_{ε} in

$$\|e^{-tL_{\varepsilon}}\| \leq C_{\varepsilon} e^{-\alpha_{\varepsilon} t},$$

in term of the numerical range, the resolvent norm and the spectrum. Then this discussion will be illustrated by various examples studied with collaborators during the recent years and related with linearized fluid mechanics equations, Fokker-Planck equations and quantum shape resonances.

Speaker: **Oliveras, Katie** (Seattle University, Mathematics Department, USA)
Deconinck, Bernard (University of Washington, Department of Applied Mathematics, USA)
Title: *Stability of stationary periodic solutions to the Euler equations*

Abstract: Euler's equations describe the dynamics of gravity waves on the surface of an ideal fluid with arbitrary depth. In this talk, I discuss the stability of one- and two- dimensional traveling wave solutions for the full set of Eulers equations via a generalization of a non-local formulation of the water wave problem due to Ablowitz, Fokas and Musslimani. Transforming the non-local formulation into a traveling coordinate frame, we obtain a new scalar equation for the stationary solutions using the original physical variables. Using this new equation, we develop a numerical scheme to determine traveling wave solutions by exploiting the bifurcation structure of the non-trivial periodic solutions. Next, we determine numerically the spectral stability for the periodic traveling wave solution by extending Fourier-Floquet analysis to apply to the non-local problem. We can generate the full spectra for all traveling wave solutions. In addition to recovering

well-known results such as the Benjamin-Feir instability for one-dimensional traveling waves in deep water, we confirm the presence of high-frequency instabilities for shallow water waves.

Speaker: **Overton, Michael** (Courant Institute of Mathematical Sciences, NYU, USA)
Title: *Optimization of Polynomial Roots, Eigenvalues and Pseudospectra*

Abstract: The root radius and root abscissa of a monic polynomial are respectively the maximum modulus and the maximum real part of its roots; both these functions are nonconvex and are non-Lipschitz near polynomials with multiple roots. We begin the talk by giving constructive methods for efficiently minimizing these nonconvex functions in the case that there is just one affine constraint on the polynomial's coefficients. We then turn to the spectral radius and spectral abscissa functions of a matrix, which are analogously defined in terms of eigenvalues. We explain how to use nonsmooth optimization methods to find local minimizers of these quantities for parameterized matrices and how to use nonsmooth analysis to study local optimality conditions for these nonconvex, non-Lipschitz functions. Finally, the pseudospectral radius and abscissa of a matrix A are respectively the maximum modulus or maximum real part of elements of its pseudospectrum (the union of eigenvalues of all matrices within a specified distance of A). These functions are also nonconvex but, it turns out, locally Lipschitz, although the pseudospectrum itself is not a Lipschitz set-valued map. We briefly discuss a new method to compute these quantities efficiently for a large sparse matrix A. We discuss applications from control and from Markov chain Monte Carlo as examples throughout the talk. Coauthors of relevant papers include Vincent Blondel, Jim Burke, Kranthi Gade, Sara Grundel, Mert Gurbuzbalaban, Nicola Guglielmi, Adrian Lewis and Alexandre Megretski.

Speaker: **Pelinovsky, Dmitry** (McMaster University, Canada)
Title: *On transverse stability of discrete line solitons*

Abstract: We obtain the sharp criterion of transverse stability and instability of line solitons in the discrete nonlinear Schrodinger (dNLS) equation on a square two-dimensional lattice near the anti-continuum limit. The fundamental (single-site) line soliton is proved to be transversely stable (unstable) when it bifurcates from the hyperbolic (elliptic) point of the dispersion surface. The results hold for both focusing and defocusing dNLS equation via a staggering transformation. We also consider the one-dimensional dNLS equation with the continuous diffraction term and prove that the fundamental line soliton is transversely unstable in both cases when it bifurcates from the hyperbolic and elliptic points of the dispersion surface. In the former case, the instability is caused by the resonance between eigenvalues of negative energy (Krein signature) and the continuous spectrum of positive energy. Analytical results are illustrated numerically.

Speaker: **Tassi, Emanuele** (Centre de Physique Theorique, CNRS, Marseille, France)
Title: *Negative energy modes in some models for plasma physics*

Abstract: An important issue for the stability properties of continuous media, such as plasmas or fluids, is that of negative energy modes. These are spectrally stable modes, possessing negative energy. Their identification is important, because negative energy modes can be destabilized by small perturbations, induced, for instance, by dissipation. A general and effective framework for the study of negative energy modes is the Hamiltonian one. Indeed, the knowledge of the Hamiltonian structure of a system, allows to unambiguously identify the presence of negative energy modes, through the reduction of the Hamiltonian for the linearized system to its normal form. In this contribution I will present two examples of Hamiltonian (in particular Lie-Poisson) systems of interest for plasma physics, that possess negative energy modes, when linearized about homogeneous equilibria. The two models describe the phenomena of magnetic reconnection and of electron temperature gradient driven turbulence, respectively. Both systems exhibit Krein bifurcations when negative energy modes merge with positive energy modes for critical values of the wavelength of the perturbations.

Speaker: **Trunk, Carsten** (Technical University of Ilmenau, Germany)
Title: *On $\mathcal{PT}$ symmetric operators in Krein spaces*

Abstract: We consider so-called $\mathcal{PT}$ symmetric operators in the Krein space $(L_2(\mathbb{R}), [\cdot,\cdot])$, where $[\cdot,\cdot]$ is given via the fundamental symmetry $\mathcal{P}f(x) = f(-x)$. The action of the anti-linear operator $\mathcal{T}$ on a function of a real spatial variable x is defined by $\mathcal{T}f(x) = \overline{f(x)}$.

An operator A is said to be $\mathcal{PT}$-symmetric if it commutes with $\mathcal{PT}$.

In the last decade a generalization of the harmonic oscillator using a complex deformation was investigated. This operator is defined via the differential expression

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon > 0.$$

We will start our investigations with the discussion of some simple cases (like ϵ even) and we will concentrate on a description of self-adjoint, $\mathcal{P}$-selfadjoint and $\mathcal{PT}$-symmetric operators related to such a differential expression and their spectral properties. The talk is based on joint works with T. Ya. Azizov (Voronezh).

Speaker: **Waalkens, Holger** (University of Groningen, The Netherlands)
Title: *Scattering monodromy in the two-center problem*

Abstract: We have earlier shown that the Euler-Jacobi problem of the motion of a particle in the field of two fixed attracting centers has monodromy, i.e. there are topological obstructions to the global construction of action-angle variables in this integrable system. This study concerned the bound motions where the (regular part of the) phase space is foliated by Liouville-Arnold tori. The obstructions are caused by isolated singular fibers which give rise to twists in the Liouville-Arnold torus bundle such that globally action-angle variables cannot be uniquely defined. For the corresponding quantum system, this implies the nonuniqueness of quantum numbers. We now show that for the unbound motions in the two-center problem, there are similar obstructions which cause what has been coined scattering monodromy in the context of planar central scattering where scattering monodromy implies, e.g., the nonuniqueness of quantum phase shifts.

Speaker: **Yoshida, Zensho** (University of Tokyo, Japan)
Title: *Singular Casimir elements and their roles in fluid/plasma dynamics*

Abstract: Bifurcation of equilibrium points in fluids or plasmas is studied using the notion of Casimir foliation that occurs in a noncanonical Hamiltonian formulation of an ideal fluid or plasma. The nonlinearity of the system makes the Poisson operator inhomogeneous on phase space (the function space of state variables), resulting in a nontrivial center of the Poisson algebra; the center elements are called Casimirs. Orbits are constrained on level-sets of Casimirs, i.e. Casimir leaves. Even if a Hamiltonian is simple (typically a fluid/plasma Hamiltonian is just the "norm" of phase space, unlike bumpy Hamiltonians modeling strongly coupled systems), energy contours on a Casimir leaf may have considerably complicated shapes. Invoking a simple model of plasma, we show that the equilibrium points on Casimir leaves bifurcate as Casimir parameters change. We may compare the energies of bifurcated equilibrium points to estimate the stability. In ideal dynamics, however, a higher-energy state may sustain stably by other Casimir constraints; in fact "resonant singularities" generate infinite number of "singular Casimir elements" which foliate the phase space and separate different equilibrium points. A singular perturbation (introduced by finite dissipation) destroys the Casimir leaves, removing the topological constraint and allowing the state vector to move towards lower-energy state in unconstrained phase space. We propose an extended Hamiltonian mechanical representation of such an instability caused by a singular perturbation.