Banff International Research Station
for Mathematical Innovation and Discovery

Workshop on Conformal and CR Geometry
July 29–August 3, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipelines Pavilion (TCPL)
*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
Evening	Socialize in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	Colin Guillarmou: On the renormalized volume
10:00–10:30	Coffee Break, TCPL
10:30–11:20	Jie Qing: Hypersurfaces in hyperbolic space and conformal metrics on domains in sphere
11:30–12:20	Pierre Albin: Compactness of relatively isospectral sets of surfaces
12:30–13:30	Lunch
14:00	Group Photo; meet in foyer of TCPL
14:15–15:00	Informal Discussions and Collaboration
15:00–15:30	Coffee Break, TCPL
15:30–16:30	Informal Discussions and Collaboration
16:30–17:20	Xianzhe Dai: Cheeger's half torsion and cone
17:30–18:20	Zhiqin Lu: On the Tian-Yau-Zelditch expansion on Riemann surfaces
18:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Michael Eastwood: Conformal and CR geometry from the parabolic viewpoint
10:00–10:30	Coffee Break, TCPL
10:30–11:20	Andreas Juhl: On the building blocks of GJMS-operators
11:30–12:20	Bent Ørsted: Extremal properties of natural functionals in conformal and CR geometry
12:30–13:30	Lunch
14:00–15:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
15:00–15:30	Coffee Break, TCPL
15:30–16:30	Informal Discussions and Collaboration
16:30–17:20	Pawel Nurowski: More explicit Fefferman-Graham metrics with G_2 holonomy
17:30–18:20	Jan Slovak: Free CR-distributions
18:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:50	Andreas Čáp: Projective compactness
10:00–10:20	Coffee Break, TCPL
10:20–11:10	Rod Gover: Conformal geometry, holography, and boundary calculus
11:20–11:50	Matthias Hammerl: Ambient and conformal holonomy
12:00–12:30	Katharina Neusser: Some complexes of differential operators
12:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Andrea Malchiodi: Recent progress on the Yamabe problem
10:00–10:30	Coffee Break, TCPL
10:30–11:20	Sagun Chanillo: Embedding CR 3-manifolds
11:30–12:20	María del Mar González: Fractional order operators in conformal geometry
12:30–13:30	Lunch
13:30–15:00	Informal Discussions and Collaboration
15:00–15:30	Coffee Break, TCPL
15:30–16:30	Informal Discussions and Collaboration
16:30–17:00	Yi Wang: Quasiconformal mappings, isoperimetric inequality and finite total Q-curvature
17:10–17:40	Antonio Ache: Asymptotics in the study of Obstruction-flat metrics
17:50–18:20	Yoshihiko Matsumoto: Asymptotics of ACH-Einstein metrics
18:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:30	Jeffrey Case: A Paneitz-type operator for CR pluriharmonic functions
9:40–10:10	Travis Willse: Highly symmetric generic 2-plane fields on 5-manifolds and Heisenberg 5-group holonomy
10:15–10:45	Coffee Break, TCPL
10:45–11:30	Informal Discussions and Collaboration
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Antonio Ache** (University of Wisconsin)
Title: *Asymptotics in the study of Obstruction-flat metrics*

Abstract: In an article published in 1983, Leon Simon developed an extremely useful approach for understanding the singular set of minimal submanifolds based on deriving asymptotics for solutions of certain nonlinear evolution equations at isolated singular points. He applied this method for proving the uniqueness of tangent cones at isolated singularities for minimal submanifolds and to the unique tangent problem for harmonic maps. A modification of Simon's original idea was recently used by Tobias Colding and William Minicozzi to prove the uniqueness of tangent cones at infinity for Ricci-flat manifolds with Euclidean Volume growth by introducing new monotonicity formulas for the Ricci curvature. In this lecture we will review some recent results and open problems that have appeared in the application of Simon's method to the understanding of the singular set of Obstruction-flat metrics (i.e., metrics whose Fefferman-Graham Obstruction tensor vanishes). These results include singularity removal theorems for Obstruction-flat metrics with constant scalar curvature.

Speaker: **Pierre Albin** (University of Illinois, Urbana-Champaign)
Title: *Compactness of relatively isospectral sets of surfaces*

Abstract: Although one can not 'hear the shape of a drum', it turns out that the set of isospectral metrics on a closed surface forms a compact set. I will discuss joint work with Clara Aldana and Frédéric Rochon regarding the corresponding statement for non-compact surfaces.

Speaker: **Andreas Čáp** (University of Vienna)
Title: *Projective Compactness*

Abstract: My talk reports on joint work in progress with A. Rod Gover (University of Auckland). Projective compactness for an affine connection is a notion analogous to conformal compactness of a Riemannian metric. One considers a smooth manifold $\bar{M}$ with boundary ∂M and interior M. Basically, an affine connection on M is said to be projectively compact if its projective class smoothly extends to the boundary. It turns out, that there is a natural parameter $\alpha \in (0, 2]$ controlling this extension, so one arrives at the notion of projective compactness of order α.

We analyze implications of projective compactness related to completeness of the connection and show that for connections admitting a parallel volume form, the order of projective compactness precisely captures the asymptotic behavior of the volume when going towards the boundary. More precisely, a certain power of the volume density is a defining density for the boundary. Via the Levi Civita connection, one can also define projective compactness of a pseudo–Riemannian metric. Here the main case of interest is projective compactness of order two, and we give a sufficient condition on the asymptotic form of the metric to ensure this property. If the order of projective compactness is one or two, then the natural defining density for the boundary obtained above is in the domain of a first BGG operator. Hence there are the natural geometric conditions that the defining density is a solution or, more restrictively, a normal solution

of this operator. We analyze several consequences of these conditions obtaining relations to reductions of projective holonomy and to Einstein metrics.

Speaker: **Jeffrey Case** (Princeton University)
Title: *A Paneitz-type operator for CR pluriharmonic functions*
Abstract: We introduce a fourth order CR invariant operator on pluriharmonic functions in three dimensional CR manifolds which generalizes the operator discovered by Branson, Fontana and Morpurgo. For a distinguished class of contact forms, this operator also admits a natural "Q-curvature." In this talk, I will describe these objects and their relation to some problems in CR geometry. This is joint work with Paul Yang.

Speaker: **Sagun Chanillo** (Rutgers University)
Title: *Embedding CR 3-manifolds*
Abstract: This is joint work with Hung-Lin Chiu and Paul Yang.

    We show that under non-negativity assumptions on the CR Paneitz operator and positivity assumptions on the CR-Yamabe constant, one can globally embed CR 3-manifolds. We also consider the converse question as to when embedded CR 3-manifolds have non-negative Paneitz operators.

Speaker: **Xianzhe Dai** (University of California, Santa Barbara)
Title: *Cheeger's half torsion and cone*
Abstract: The analytic torsion is a special combination of determinants introduced by Ray-Singer as an analytic analog of the Reidemeister torsion. By the Ray-Singer conjecture, established independently by Cheeger and Müller, it is a topological invariant. For even dimensional manifolds however, it becomes trivial. Cheeger's half torsion remedies the situation, but it is no longer a topological invariant. Instead it is a conformal invariant. We show how the cone will lead us to some half torsion like quantities and study their conformal invariance.

Speaker: **Michael Eastwood** (Australian National University)
Title: *Conformal and CR geometry from the parabolic viewpoint*
Abstract: In recent years, the unification of various classical differential structures has been achieved by means of a common viewpoint known as parabolic differential geometry. Conformal and CR geometry are the two prime examples of parabolic differential geometry. I shall explain the parabolic viewpoint, especially with reference to conformal and CR geometry. Then I'll indicate why it is helpful and speculate on where it might be leading us next.

Speaker: **María del Mar González** (Universitat Politècnica de Catalunya)
Title: *Fractional order operators in conformal geometry*
Abstract: In the talk we will have a closer look at the relations between scattering operators of asymptotically hyperbolic metrics and Dirichlet-to-Neumann operators for uniformly degenerate elliptic boundary value problems. Joint with J. Qing, we consider fractional Yamabe problems, that include the boundary Yamabe problem formulated by Escobar. We observe an interesting Hopf type maximum principle together with interplays between analysis of weighted trace Sobolev inequalities and the conformal structure of the underlying manifolds, which extend the phenomena displayed in the classic Yamabe problem. In the last part of the talk, I will introduce an ongoing project on the complex/CR version.

Speaker: **Rod Gover** (University of Auckland)
Title: *Conformal geometry, holography, and boundary calculus*
Abstract: The hyperbolic ball provides a concrete setting for identifying the isometry group of $\mathbb{H}^{n+1}$ with the conformal group of S^n, and so a geometric foundation for Poisson transforms linking $G = SO(n+1,1)$ representations of $G = SO(n+1,1)$, as induced by its maximal parabolic, to those induced by its maximal compact subgroup. Generalising "curved analogues" of this picture, such as Poincare-Einstein metrics,

form the basis of many new developments in mathematics and physics. I will describe a natural calculus for these (and more general) conformally compact structures. The mathematics surrounding this leads to the identification and construction of natural linear boundary problems. The same tools enable the asymptotics of these problems to be solved to all orders. The calculus yields directly generalisations of the GJMS operators and the Q-curvature. The problems considered form a class of problems of interest in the conjectural AdS/CFT correspondence of quantum gravity. This is joint work with Andrew Waldron and Emanuele Latini.

Speaker: **Colin Guillarmou** (École Normale Supérieure, Paris)
Title: *On the renormalized volume*
Abstract:

Speaker: **Matthias Hammerl** (University of Vienna)
Title: *Ambient and conformal holonomy*
Abstract: I am going to present a recently developed method to compare the holonomy of the Fefferman-Graham ambient metric of a conformal structure with the tractor holonomy of the same structure. This is joint work with A. Cap (Univ. of Vienna), A. R. Gover (Univ. Auckland) and R. Graham (Univ. Washington).

Speaker: **Andreas Juhl** (Uppsala University)
Title: *On the building blocks of GJMS-operators*
Abstract: It has been shown recently that the conformally invariant powers of the Laplacian which are known as the GJMS-operators are linear combinations of compositions of certain geometric second-order differential operators. These operators (building blocks) have additional interesting properties. We give a simple formula which describes the relation of the building blocks for a given metric g and the conformal compactification of a Poincaré-Einstein metric associated to g. We also discuss the relation of this formula to the structure of residue families.

Speaker: **Zhiqin Lu** (University of California, Irvine)
Title: *On the Tian-Yau-Zelditch expansion on Riemann surfaces*
Abstract: It is well-known that the convergence of the TYZ expansion depends on the geometry of the manifold. In particular, the convergence heavily depends on the injectivity radius. In this talk, we give a preliminary report on the TYZ expansions on a degeneration family of compact Riemann surfaces. This is joint work with Chiung-ju Liu.

Speaker: **Andrea Malchiodi** (International School for Advanced Studies (SISSA))
Title: *Recent progress on the Yamabe problem*
Abstract: The Yamabe problem has drawn attention since its formulation in 1960. While its solvability was known since 1984, due to the work of T. Aubin and R. Schoen, more recently new proofs were given and the compactness question was basically settled. I will review some results in this direction, as well as the deep relation the Yamabe problem has with the Positive Mass Theorem in General Relativity, giving some counterpart in the CR case.

Speaker: **Yoshihiko Matsumoto** (University of Tokyo)
Title: *Asymptotics of ACH-Einstein metrics*
Abstract: We discuss asymptotically complex hyperbolic Einstein metrics, the CR counterpart of well-studied asymptotically hyperbolic Einstein metrics. Just as in the AH case, the Einstein constraint condition makes the CR structure on the boundary at infinity determine the expansion of the metric up to some finite order. An important aspect regarding ACH metrics is that the boundary CR structures are just "partially integrable" in general, and it is surprising that a local obstruction to the existence of smooth solutions to the Einstein equation can occur only for non-integrable structures. As an application, we can

construct CR invariant powers of the sub-Laplacian and Q-curvature for partially integrable CR manifolds via the Graham-Zworski approach. Then the total integral of the Q-curvature is generically nonzero.

Speaker: **Katharina Neusser** (Australian National University)
Title: *Some complexes of differential operators*
Abstract: We shall present a method for constructing complexes of invariant differential operators on manifolds endowed with various geometric structures. The geometric structures will mainly be certain bracket generating distributions. For these structures the constructed complexes provide fine resolutions of the sheaf of locally constant functions and so can serve as an alternative to the de Rham complex. In the case of parabolic geometries we recover the BGG complexes associated to the trivial representation. Joint work with Robert Bryant, Michael Eastwood and Rod Gover.

Speaker: **Pawel Nurowski** (University of Warsaw)
Title: *More explicit Fefferman-Graham metrics with G_2 holonomy*
Abstract: This talk is a report on a joint work with Ian Anderson.

A Monge equation $z' = F(x, y, y', y'', z)$, with a real function F such that its second derivative with respect to y'' is nonvanishing, uniquely defines a conformal structure $[g_F]$ of metrics. These metrics have signature $(3, 2)$ and live on a 5-dimensional manifold parametrized by (x, y, y', y'', z).

The Fefferman-Graham ambient metric g_A associated with the conformal structures $[g_F]$, generically, has holonomy reduced from $SO(4, 3)$ to the full split real form of the exceptional group G_2.

For some choices of the function F the Fefferman-Graham ambient metric g_A of the corresponding conformal structures $[g_F]$ can be written, in a suitably chosen coordinate system, in terms of elementary functions of the coordinates only.

In this talk I will explain why for the known choices of F this is possible.

In addition, I will show other ansatzes for F, for which g_A will have full split G_2 holonomy and be expressible in terms of elementary functions of the coordinates.

The examples I will present will explain what is the most general ansatz for F to get g_A's with these properties.

Speaker: **Bent Ørsted** (Aarhus University, Denmark)
Title: *Extremal properties of natural functionals in conformal and CR geometry*
Abstract: Many interesting natural functionals in conformal and CR geometry have been found, some with links to recent results in spectral geometry and holographic principles from field theories. The functionals may be thought of as certain functions on the space of metrics, and we shall give examples of stationary points and local extrema; in particular for conformal and CR spheres we shall treat determinants of natural differential operators using methods from representation theory. Finally we shall discuss the prospects for extensions to other parabolic geometries. This lecture in based in part on joint work with N. M. Møller.

Speaker: **Jie Qing** (University of California, Santa Cruz)
Title: *Hypersurfaces in hyperbolic space and conformal metrics on domains in sphere*
Abstract: In this talk I will introduce a global correspondence between properly immersed horospherically convex hypersurfaces in hyperbolic space and complete conformal metrics on subdomains in the boundary at infinity of hyperbolic space. I will discuss when a horospherically convex hypersurface is proper and when the hyperbolic Gauss map is injective. These are expected to be useful to the understandings of both elliptic problems of Weingarten hypersurfaces in hyperbolic space and elliptic problems of complete conformal metrics on subdomains in sphere.

Speaker: **Jan Slovák** (Masaryk University)
Title: *Free CR-distributions*
Abstract: The lecture will be based on my long cooperation with Gerd Schmalz. The free CR-distributions are inherited on generic n^2-codimensional submanifolds in C^{n+n^2}. In the lowest dimensional case $n = 1$,

this coincides with the hypersurface case of Cartan and Poincaré, but it displays surprising properties for higher dimensions. In particular, this is also an instance of well understood parabolic geometry and there is the Fefferman-like construction of a natural circle bundle, which mimics the hypersuface case. The geometry on the circle bundle is modelled on skew-Hermitian matrices in the same way as the conformal 4-dimensional case. The construction of basic invariants of these geometries is obtained using a blend of the classical exterior calculus and the direct knowledge of the relevant cohomological data known for the parabolic geometry in question.

Speaker: **Yi Wang** (Stanford University)
Title: *Quasiconformal mappings, isoperimetric inequality and finite total Q-curvature*
Abstract: In this talk, we are going to prove the isoperimetric inequality on the noncompact conformally flat four manifold with totally finite Q-curvature and $\frac{1}{4\pi^2} \int_M Q(x) dv_M(x) < 1$. To do this, we will look at the construction of the quasiconformal mapping with suitable quantity of Jacobian. The affirmative answer to the existence of such mapping would imply the bilipschitz parametrization of the manifold as well as the isoperimetric inequality. Moreover, if the Q-curvature is non-negative, the conformal factor relates to A_1 weight.

Speaker: **Travis Willse** (Australian National University)
Title: *Highly symmetric generic 2-plane fields on 5-manifolds and Heisenberg 5-group holonomy*
Abstract: Nurowski used Cartan's solution of the local equivalence problem for generic 2-plane fields on 5-manifolds to show that any such plane field induces a canonical signature-$(2,3)$ conformal structure on the underlying manifold. Exploiting a local normal form given by Cartan, we consider a natural infinite-dimensional class of highly symmetric 2-plane fields (which includes the submaximally symmetric plane fields) and show that the conformal structures they induce enjoy additional geometric structures. We produce explicit (signature-$(3,4)$) Fefferman-Graham ambient metrics for these conformal structures and show that the local holonomy of these metrics and the local normal conformal holonomy of the underlying structures are equal to the Heisenberg 5-group; this holonomy group acts indecomposably but not irreducibly.