BIRS Workshop 12w5020
Recent Advances in Transversal and Helly-type Theorems in Geometry, Combinatorics and Topology
October 21 - October 26 of 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.
Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:30	Jürgen Eckhoff
9:30–10:00	Problem session
10:00–10:30	Coffee Break, TCPL
10:30–12 :00	Discussion and problem session
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:30–15:00	Roman Karasev
15:00–15:30	Coffee Break, TCPL
15:30–16:00	Wlodzimierz Kuperberg
16:00–17:00	Discussion
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:30	Luis Montejano
9:30–10:00	Problem session
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Discussion and problem session
11:30–13:30	Lunch
13:00–14:30	Discussion and problem session
14:30–15:00	Pablo Soberón
15:00–15:30	Coffee Break, TCPL
15:30–16:00	Ricardo Strausz
16:00–17:30	Discussion
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:30–10:00	Ferenc Fodor
10:00–10:30	Coffee Break, TCPL
10:30–11:00	Antoine Deza
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:30	Jorge Ramirez Alfonsin
9:30–10:00	Problem session
10:00–10:30	Coffee Break, TCPL
10:30–12 :00	Discussion and problem session
12:00–13:30	Lunch
13:30–14:30	Discussion and problem session
14:30–15:00	Andreas Holmsen
15:00–15:30	Coffee Break, TCPL
15:30–16:00	Alfredo Hubard
16:00–17:30	Discussion
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:30	Deborah Oliveros
9:30–10:00	Aladar Heppes
10:00–10:30	Coffee Break, TCPL
10:30–12:00	Informal Discussion
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Antoine Deza** (McMaster University)
Title: *Combinatorial, computational, and geometric approaches to the colourful simplicial depth*
Abstract: In statistics, there are several measures of the depth of a point p relative to a fixed set S of sample points in dimension d. One of the most intuitive is the simplicial depth of p introduced by Liu (1990), which is the number of simplices generated by points in S that contain p. Obtaining a lower bound for the simplicial depth is a challenging problem. Carathéodory Theorem can be restated as: The simplicial depth is at least 1 if p belongs to the convex hull of S. Bárány (1982) showed that the simplicial depth is a least a fraction of all possible simplices generated from S. Gromov (2010) improved the fraction via a topological approach. Bárány's result uses a colourful version of Carathéodory Theorem leading to the associated colourful simplicial depth. We present recent generalizations of the Colourful Carathéodory Theorem due to Arocha et al. and Homlsen et al. and our strengthening of these. We provide a new lower bound for the colourful simplicial depth improving the earlier bounds of Bárány and Matoušek and of Stephen and Thomas. Computational approaches for small dimension and the colourful linear programming feasibility problem introduced by Bárány and Onn are discussed.

Based on join works with Frédéric Meunier (ENPC Paris), Tamon Stephen (Simon Fraser), Pauline Sarrabezolles (ENPC Paris), and Feng Xie (Microsoft)

Speaker: **Jürgen Eckhoff** (Technische Universität Dortmund)
Title: *A teasing strip problem*
Abstract: The τ-strip problem consists in proving (or disproving) the following conjecture: If a finite set of points in the plane is such that every three of the points lie in some strip of width 1, then all points lie in some strip of width τ;. (Here $\tau = 1.6180...$ is the golden number.) The conjecture is more than 40 years old and, despite considerable progress, still unsolved. The talk describes a new approach, based on numerical evidence, which may help to tackle it.

Speaker: **Ferenc Fodor** (Univeristy of Szeged and University of Calgary)
Title: *Colourful and fractional (p,q)-problems.*
Abstract: In this talk we will consider colourful and fractional versions of the classical (p,q)-problem for systems of intervals in the real line. This is a preliminary report on ongoing research with I. Bárány, L. Montejano, and D. Oliveros.

Speaker: **Aladar Heppes** (Renyi Institute)
Title: *T(5) families of moderately overlapping unit discs.*
Abstract: The following extension of Danzer's theorem will be reported upon. Consider a finite family of at least five 2/3-disjoint unit discs. If any 5-tuple if the discs has a line transversal then there is a line meeting all discs. (Joint work with T. Bosztriczky and K. Brczky.

Speaker: **Andreas Holmsen** (KAIST)
Title: *On generalizations of the Erdos-Szekeres theorem.*
Abstract: The Erdos-Szekeres theorem states that every sufficiently large set of points in general position

in the plane contains a large subset wich is convexly independent. There are several results and conjectures on possible extensions to pseudo-line arrangements or convex sets. We'll present a unified viewpoint and report our progress on some of these questions. This is joint work with Michael Dobbins and Alfredo Hubard.

Speaker: **Alfredo Hubard** (École Normale Superieure)
Title: *Topology and geometry of realization spaces by families of convex bodies.*
Abstract: We say that two families of convex bodies have the same combinatorial type if there is a selfhomeomorphism of the cylinder $S^{d-1}xR$ that maps the graphs of the support functions of one family to the the graphs of the support functions of the other one. We metrize the space of families of convex bodies with the Hausdorff metric. This talk will be about results on the topology and geometry of all families with a fixed combinatorial type.

Speaker: **Roman Karasev** (Dept. of Mathematics, Moscow Institute of Physics and Technology)
Title: *Projective center point and Tverberg theorems*
Abstract: We present projective versions of the center point theorem and Tverberg's theorem, interpolating between the original and the so-called "dual" center point and Tverberg theorems. Furthermore we give a common generalization of these and many other known (transversal, constraint, dual, and colorful) Tverberg type results in a single theorem, as well as some essentially new results about partitioning measures in projective space.

We focus on two classical topics in discrete geometry: the center point theorem from Neumann and Rado and Tverberg's theorem. Many deep generalizations of these classical results have been made in the last three decades, starting from the topological generalization by Bárány, Shlosman, and Szűcz. A good review on this topic and numerous references are given in Matoušek's book. After this book was published, new achievements were made by Hell, Engström and Engström–Norén, K., and Blagojević–M.–Ziegler , establishing "constrained", "dual", and "optimal colorful" Tverberg type theorems.

The discrete center point theorem states the following: For any finite set $X \subset \mathbb{R}^d$ there exists a *center point* $c \in \mathbb{R}^d$ such that any closed half-space $H \ni c$ contains at least $\left\lceil \frac{|X|}{d+1} \right\rceil$ points of X. In K. (2008) a dual center point theorem and a dual Tverberg theorem for families of hyperplanes were proved. The dual center point theorem states: For any family of n hyperplanes in general position in $\mathbb{R}^d$ there exists a point c such that any ray starting at c intersects at least $\left\lceil \frac{n}{d+1} \right\rceil$ hyperplanes.

Here the use of the adjective "dual" is rather descriptive, it does not refer to projective duality. Thus it is interesting to dualize it once more projectively and compare it with the original center point theorem.
Definition: Any two distinct hyperplanes H_1 and H_2 partition $\mathbb{R}P^d$ into two pieces. In this paper, we always consider the pieces as being closed. If H_1 and H_2 coincide then we consider $H_1 = H_2$ as one piece and the whole $\mathbb{R}P^d$ as the other.

The projective dual of the "dual center point theorem" reads: Assume that X is a family of n points in $\mathbb{R}P^d$ and $c \in \mathbb{R}P^d$ is another point such that the family $X \cup c$ is in general position. Then there exists a hyperplane $W \subseteq \mathbb{R}P^d$ such that together with any hyperplane $H_1 \ni c$ it partitions $\mathbb{R}P^d$ into two parts each containing at least $\left\lceil \frac{n}{d+1} \right\rceil$ points of X.

From the proof of this theorem we can assure that W does not contain c; however if we omit the general position assumption then the theorem remains true by a compactness argument but W may happen to contain c.

Now we are going to interpolate between the original center point theorem and the latter "dual to dual" version (they appear as special cases when V is the hyperplane at infinity or when V is a point):

Theorem 1 [Projective center point theorem]:
Suppose that $V \subset \mathbb{R}P^d$ is a projective subspace of dimension v and X is a finite point set with $|X| = n$. Put

$$r = \left\lceil \frac{n}{(d-v)(v+1)+1} \right\rceil.$$

Then there exists a projective subspace $W \subset \mathbb{R}P^d$ of dimension $d-v-1$ such that any pair of hyperplanes $H_1 \supseteq V$ and $H_2 \supseteq W$ partitions $\mathbb{R}P^d$ into two parts each containing at least r points of X.

If we require the general position assumption, that no r points of X together with V are contained in a hyperplane, then W may be chosen disjoint from V.

The ordinary center point theorem is usually stated for measures, which follows from the discrete version by an approximation argument. Here is the corresponding version:

Theorem 2 [Projective center point theorem for measures]:
Suppose that $V \subset \mathbb{R}P^d$ is a projective subspace of dimension v and μ is a probability measure on $\mathbb{R}P^d$. Then there exists a projective subspace $W \subset \mathbb{R}P^d$ of dimension $d-v-1$ such that any pair of hyperplanes $H_1 \supseteq V$ and $H_2 \supseteq W$ partitions $\mathbb{R}P^d$ into two parts P_1 and P_2 so that

$$\mu(P_1), \mu(P_2) \geq \frac{1}{(d-v)(v+1)+1}.$$

We also interpolate between Tverberg's theorem and its dual. We generalize further and state a very general theorem incorporating almost all that we know about (dual, transversal, constrained, colorful) Tverberg type theorems. In particular this implies a projective center transversal theorem, which generalizes Theorem 2.

Exchanging quantors can possibly lead to other interesting theorems. As an instance of this we prove another projective Tverberg theorem and a transversal generalization.

For more information see the Arxiv preprint 1207.2204.

Speaker: **Wlodzimierz Kuperberg** (Auburn University)
Title: *Variations on the Hadwiger theme*
Abstract: The (original) *Hadwiger number* of a convex body K is the maximum number of mutually non-overlapping translates of K, each touching K. Without losing the affine invariant nature of the Hadwiger number, several generalizations and modifications are proposed. One can replace translates of K with its t-homothetic images with a positive or negative t or a combination of both types. One can even consider a variation with $t \to \infty$ or $t \to -\infty$ (in a certain sense) or a combination of both. Another possibility is to consider mutually non-overlapping t-homothetic images of K *contained in* K and touching the boundary of K from inside—a dual counterpart to the previous notions (here we should assume $0 < |t| < 1$). Some examples will be shown in which the optimal arrangements are tight.

Speaker: **Luis Montejano** (Instituto de Matemáticas UNAM)
Title: *A new topological Helly theorem*
Abstract: We prove the following new topological Helly Theorem. For that for a topological space X with the property that $H_i(U) = 0$ for $i > d$ and every open subset U of X, a finite family of open sets in X has nonempty intersection if for any subfamily of size$j, 1 < j < d+1$, the $(d-j)$-dimensional reduced homology group of its intersection is zero, where $H_{-1}(A) = 0$ if and only if A is not empty.

The fact that this result is non-expensive -in the sense that it does requiere the sets to be simple- from the homotopy pont of view (we only requiere its (d-1) homology group to be zero allow us to obtain new results concerning transversal affine planes to families of convex sets.

Speaker: **Deborah Oliveros** (Instituto de Matemáticas, UNAM)
Title: *About piercing numbers of families of planes, lines and intervals.*
Abstract: In this talk we present some ideas and bounds for the (p, q) problem and for piercing numbers of some families of affine hyperplanes, lines and intervals following the spirit of Erdös-Gallai. This is joint work with M. Huicochea, J. Jernimo and L. Montejano.

Speaker: **Jorge Ramirez Alfonsin** (Université Montpellier 2)
Title: *On transversals to Tetrahedra*

Abstract: Let $m(k,d,\lambda)$be themaximum positive integer$n$ suchthateveryset of$n$points (not necessarilyin generalposition) in $\mathbb{R}^d$hasthepropertythattheconvexhullsofall k-sets have a transversal $(d-\lambda)$-plane. It is conjectured that $m(k,d,\lambda) = d - \lambda + k + \lceil \frac{k}{\lambda} \rceil - 1$

In this talk we shall discuss recent progress toward the validity of this conjecture in the case when $k = 4$. This is a join work with J. Arocha, J. Bracho and L. Montejano.

Speaker: **Pablo Soberón** (UCL)
Title: *Equal coefficients in coloured Tverberg partitions*
Abstract: We analise a variant of the coloured Tverberg partitions where the convex hulls of the colourful sets are required to intersect using the same coefficients. We give a theorem of this kind with an optimal number of colour classes and points, and extend it to intersections with tolerance.

Speaker: **Ricardo Strausz** (Instituto de Matémticas, UNAM)
Title: *A generalisation of Tverberg's theorem*
Abstract: In this lecture the following generalisation of Tverberg's theorem is presented: every set of (t+1)(k-1)(d+1)+1 points in the euclidian d-space admits a k-Tverberg partition with tolerance t. That is, there is a k-Tverberg partition such that, whenever t points are removed from the configuration, the partition of the remaining points is still intersecting. This is a joint work with Pablo Soberon and answers positively a conjecture of Natalia Garcia-Colin.