Using Tsirelson space as the frame for HI constructions

Spiros A. Argyros
(joint work with Pavlos Motakis)

Department of Mathematics
National Technical University of Athens
Athens, Greece

2012 / Banff

The goal of this lecture is to present the construction of a new reflexive HI Banach space. This space is denoted as $\mathfrak{X}_{\text{ISP}}$ and its definition uses the method of saturation under constraints originated 20 years ago by E. Odell and Th. Schlumprecht. This method permits to use Tsirelson space as the unconditional frame and thus new features in HI spaces occur. The most significant property of the space $\mathfrak{X}_{\text{ISP}}$ is that it satisfies the hereditary Invariant Subspace Property, which means that every operator acting on every subspace of $\mathfrak{X}_{\text{ISP}}$ has a non trivial invariant subspace.

Saturated and saturated under constraints norms

We will start explaining the fundamental concepts of saturated and saturated under constraints norms. At the beginning we will present the paradigms that led to the corresponding concepts.

Saturated and saturated under constraints norms

We will start explaining the fundamental concepts of saturated and saturated under constraints norms. At the beginning we will present the paradigms that led to the corresponding concepts.

Tsirelson's norm

(B.S. Tsirelson 1972)

For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_T = \max\left\{\|x\|_0,\ \sup\{\frac{1}{2}\sum_{i=1}^{n}\|E_i x\|_T\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$. Tsirelson space is

$$T = \overline{(c_{00}, \|\cdot\|_T)}$$

- The implicit formula is due to T. Figiel and W. B. Johnson. The initial Tsirelson construction actually concerns the dual T^*.

Tsirelson's norm

(B.S. Tsirelson 1972)

For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_T = \max\left\{\|x\|_0,\ \sup\{\frac{1}{2}\sum_{i=1}^{n}\|E_i x\|_T\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.
Tsirelson space is

$$T = \overline{(c_{00}, \|\cdot\|_T)}$$

- The implicit formula is due to T. Figiel and W. B. Johnson. The initial Tsirelson construction actually concerns the dual T^*.

Tsirelson's norm

(B.S. Tsirelson 1972)

For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_T = \max\left\{\|x\|_0,\ \sup\{\frac{1}{2}\sum_{i=1}^{n}\|E_i x\|_T\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.
Tsirelson space is

$$T = \overline{(c_{00}, \|\cdot\|_T)}$$

- The implicit formula is due to T. Figiel and W. B. Johnson. The initial Tsirelson construction actually concerns the dual T^*.

Saturated norms, paradigms

Schlumprecht's norm

(Th. Schlumprecht 1992)

For $x \in c_{00}(\mathbb{N})$, $f(n) = \log_2(n+1)$, we set

$$\|x\|_S = \max\left\{\|x\|_0,\ \sup\left\{\frac{1}{f(n)}\sum_{i=1}^{n}\|E_i x\|_S\right\}\right\}$$

Where the supremum is taken over all $E_1 < \cdots < E_n$.
Schlumprecht's space is

$$S = \overline{(c_{00}, \|\cdot\|_S)}$$

Saturated norms, paradigms

Schlumprecht's norm

(Th. Schlumprecht 1992)

For $x \in c_{00}(\mathbb{N})$, $f(n) = \log_2(n+1)$, we set

$$\|x\|_S = \max\left\{\|x\|_0,\ \sup\{\frac{1}{f(n)}\sum_{i=1}^{n}\|E_i x\|_S\}\right\}$$

Where the supremum is taken over all $E_1 < \cdots < E_n$.
Schlumprecht's space is

$$S = \overline{(c_{00}, \|\cdot\|_S)}$$

Odell and Schlumprecht's norm

(E. Odell and Th. Schlumprecht 1993)

For $x \in c_{00}(\mathbb{N})$, $f(n) = \log_2(n+1)$, we set

$$\|x\|_{OS} = \max\left\{\|x\|_0,\ \sup\{\frac{1}{f(n)}\sum_{i=1}^{n}\|E_i x\|_{m_i}\}\right\}$$

Where the supremum is taken over all $(m_i, E_i)_{i=1}^{n}$ admissible and for $m \geqslant 2$, $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m}\|F_i x\|_{OS} : F_1 < \cdots < F_m\}$$

Odell - Schlumprecht space is

$$S_{OS} = \overline{(c_{00}, \|\cdot\|_{OS})}$$

Odell and Schlumprecht's norm

(E. Odell and Th. Schlumprecht 1993)

For $x \in c_{00}(\mathbb{N})$, $f(n) = \log_2(n+1)$, we set

$$\|x\|_{OS} = \max\left\{\|x\|_0,\ \sup\{\frac{1}{f(n)}\sum_{i=1}^{n}\|E_i x\|_{m_i}\}\right\}$$

Where the supremum is taken over all $(m_i, E_i)_{i=1}^{n}$ admissible and for $m \geqslant 2$, $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m}\|F_i x\|_{OS} : F_1 < \cdots < F_m\}$$

Odell - Schlumprecht space is

$$S_{OS} = \overline{(c_{00}, \|\cdot\|_{OS})}$$

Saturated under constraints norms, paradigm

- The space S_{OS} has the following remarkable property.
- Every Banach space with a 1-unconditional basis is $1+\varepsilon$ block finitely representable in every block subspace of S_{OS}.
- Three years later (1996) Odell and Schlumprecht presented the conditional version of their space.
- This is a HI space such that every Banach space with a monotone basis is $1+\varepsilon$ block finitely representable in every block subspace.

Saturated under constraints norms, paradigm

- The space S_{OS} has the following remarkable property.
- Every Banach space with a 1-unconditional basis is $1+\varepsilon$ block finitely representable in every block subspace of S_{OS}.
- Three years later (1996) Odell and Schlumprecht presented the conditional version of their space.
- This is a HI space such that every Banach space with a monotone basis is $1+\varepsilon$ block finitely representable in every block subspace.

Saturated under constraints norms, paradigm

- The space S_{OS} has the following remarkable property.
- Every Banach space with a 1-unconditional basis is $1+\varepsilon$ block finitely representable in every block subspace of S_{OS}.
- Three years later (1996) Odell and Schlumprecht presented the conditional version of their space.
- This is a HI space such that every Banach space with a monotone basis is $1+\varepsilon$ block finitely representable in every block subspace.

Saturated under constraints norms, paradigm

- The space S_{OS} has the following remarkable property.
- Every Banach space with a 1-unconditional basis is $1 + \varepsilon$ block finitely representable in every block subspace of S_{OS}.
- Three years later (1996) Odell and Schlumprecht presented the conditional version of their space.
- This is a HI space such that every Banach space with a monotone basis is $1 + \varepsilon$ block finitely representable in every block subspace.

Saturated under constraints norms, paradigm

- The space S_{OS} has the following remarkable property.
- Every Banach space with a 1-unconditional basis is $1+\varepsilon$ block finitely representable in every block subspace of S_{OS}.
- Three years later (1996) Odell and Schlumprecht presented the conditional version of their space.
- This is a HI space such that every Banach space with a monotone basis is $1+\varepsilon$ block finitely representable in every block subspace.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}$, $\{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^k \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^k$ such that $m_i \leqslant n_i$ for $i = 1, \ldots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}$, $\{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^k \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^k$ such that $m_i \leqslant n_i$ for $i = 1, \ldots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}$, $\{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^{k} \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^{k}$ such that $m_i \leqslant n_i$ for $i = 1, \ldots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}, \{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^k \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^k$ such that $m_i \leqslant n_i$ for $i = 1, \ldots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}, \{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^{k} \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^{k}$ such that $m_i \leqslant n_i$ for $i = 1, \ldots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

A family $\mathcal{F}$ of finite subsets of the naturals is said to be regular if

(i) For every $n \in \mathbb{N}, \{n\} \in \mathcal{F}$.

(ii) $\mathcal{F}$ is hereditary, i.e. if $F \in \mathcal{F}$ and $E \subset F$, then $E \in \mathcal{F}$.

(iii) $\mathcal{F}$ is spreading, i.e. if $E = \{m_i\}_{i=1}^k \in \mathcal{F}$ and $F = \{n_i\}_{i=1}^k$ such that $m_i \leqslant n_i$ for $i = 1, \dots, k$, then $F \in \mathcal{F}$.

(iv) $\mathcal{F}$ is compact, i.e. $\mathcal{F}$ does not contain an infinite strictly increasing sequence of its elements.

Concepts, regular families

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.
A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^{n} \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

Concepts, regular families

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.
A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.

A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

Concepts, regular families

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.

A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

Concepts, regular families

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.

A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

Concepts, regular families

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.

A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

The fundamental examples of regular families are

$\mathcal{A}_n = \{F \subset \mathbb{N} : \#F \leqslant n\}$

The Schreier family $\mathcal{S} = \{F \subset \mathbb{N} : \#F \leqslant \min F\}$.

For $\mathcal{F}$ a regular family, the $\mathcal{F}$-admissibility is defined.
A sequence $E_1 < \cdots < E_n$ of subsets of $\mathbb{N}$ is said to be $\mathcal{F}$-admissible, if $\{\min E_i\}_{i=1}^n \in \mathcal{F}$.

A sequence $x_1 < \cdots < x_n$ of vectors in c_{00} is $\mathcal{F}$-admissible, if $\operatorname{supp} x_1 < \cdots < \operatorname{supp} x_n$ is $\mathcal{F}$-admissible.

Concepts, regular families

For $\mathcal{F}, \mathcal{G}$ regular families, the convolution $\mathcal{F} * \mathcal{G}$ is defined:

$$\mathcal{F} * \mathcal{G} = \{E = \cup_{i=1}^{n} E_i : E_i \in \mathcal{F}, \{E_i\}_{i=1}^{n} \mathcal{G}\text{-admissible}\}$$

Using the convolution and diagonalization, the Schreier hierarchy $\mathcal{S}_\xi, \xi < \omega_1$, is defined.
(E. Odell, D. Alspach - S. A. 1986)

Concepts, regular families

For $\mathcal{F}, \mathcal{G}$ regular families, the convolution $\mathcal{F} * \mathcal{G}$ is defined:

$$\mathcal{F} * \mathcal{G} = \{E = \cup_{i=1}^{n} E_i : E_i \in \mathcal{F}, \{E_i\}_{i=1}^{n} \mathcal{G}\text{-admissible}\}$$

Using the convolution and diagonalization, the Schreier hierarchy $\mathcal{S}_\xi, \xi < \omega_1$, is defined.
(E. Odell, D. Alspach - S. A. 1986)

Concepts, regular families

For $\mathcal{F}, \mathcal{G}$ regular families, the convolution $\mathcal{F} * \mathcal{G}$ is defined:

$$\mathcal{F} * \mathcal{G} = \{E = \cup_{i=1}^{n} E_i : E_i \in \mathcal{F}, \{E_i\}_{i=1}^{n} \mathcal{G}\text{-admissible}\}$$

Using the convolution and diagonalization, the Schreier hierarchy $\mathcal{S}_\xi, \xi < \omega_1$, is defined.
(E. Odell, D. Alspach - S. A. 1986)

Concepts, regular families

For $\mathcal{F}, \mathcal{G}$ regular families, the convolution $\mathcal{F} * \mathcal{G}$ is defined:

$$\mathcal{F} * \mathcal{G} = \{E = \cup_{i=1}^{n} E_i : E_i \in \mathcal{F}, \{E_i\}_{i=1}^{n} \mathcal{G}\text{-admissible}\}$$

Using the convolution and diagonalization, the Schreier hierarchy $\mathcal{S}_\xi, \xi < \omega_1$, is defined.
(E. Odell, D. Alspach - S. A. 1986)

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

  $$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$
- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1.$

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

$$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$

- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

  $$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$

- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

  $$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$

- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

  $$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$

- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$
  $$\|x\|_W = \sup\{f(x) : f \in W\}$$
  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$
- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concept, norming sets

- A subset W of c_{00} is a norming set if

  $(e_n^*)_n \subset W$, $f \in W \Rightarrow -f \in W$ and $\|f\|_\infty \leqslant 1$.

  W is rationally convex

  W is closed under projections on intervals of $\mathbb{N}$

- For W a norming set and $x \in c_{00}$

$$\|x\|_W = \sup\{f(x) : f \in W\}$$

  and $X_W = \overline{(c_{00}, \|\cdot\|_W)}$

- The sequence $(e_n)_n$ is a bimonotone Schauder basis for the space X_W.
- Conversely, every bimonotone Schauder basis is isometrically defined by a norming set W.

Concepts, $(\theta, \mathcal{F})$ operation

For W a norming set, $0 < \theta < 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F})$ operation

if for every $\{f_i\}_{i=1}^n$ $\mathcal{F}$-admissible family in W, the functional

$$f = \theta \sum_{i=1}^{n} f_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F})$ operation

For W a norming set, $0 < \theta < 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F})$ operation

if for every $\{f_i\}_{i=1}^n$ $\mathcal{F}$-admissible family in W, the functional

$$f = \theta \sum_{i=1}^{n} f_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F})$ operation

For W a norming set, $0 < \theta < 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F})$ operation

if for every $\{f_i\}_{i=1}^n$ $\mathcal{F}$-admissible family in W, the functional

$$f = \theta \sum_{i=1}^{n} f_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F})$ operation

For W a norming set, $0 < \theta < 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F})$ operation

if for every $\{f_i\}_{i=1}^n$ $\mathcal{F}$-admissible family in W, the functional

$$f = \theta \sum_{i=1}^{n} f_i$$

belongs to W.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^n f_k$, $(f_k)_{k=1}^n \subset W_{(\theta,\mathcal{F})}$ $\mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^n f_k, (f_k)_{k=1}^n \subset W_{(\theta,\mathcal{F})}$ $\mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^{n} f_k, (f_k)_{k=1}^{n} \subset W_{(\theta,\mathcal{F})}$ $\mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^{n} f_k$, $(f_k)_{k=1}^{n} \subset W_{(\theta,\mathcal{F})}$ $\mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^{n} f_k$, $(f_k)_{k=1}^{n} \subset W_{(\theta,\mathcal{F})}$ $\mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^{n} f_k, (f_k)_{k=1}^{n} \subset W_{(\theta,\mathcal{F})} \mathcal{F}$-admissible
- a rational convex combination of the above.

Concepts, Tsirelson type norms

A Tsirelson type norming set associated to a $(\theta, \mathcal{F})$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F})}$, closed in the $(\theta, \mathcal{F})$ operation.

The minimality of $W_{(\theta,\mathcal{F})}$ yields that every $f \in W_{(\theta,\mathcal{F})}$ has one of the following forms

- $f = e_n^*$
- $f = \theta \sum_{k=1}^{n} f_k, (f_k)_{k=1}^{n} \subset W_{(\theta,\mathcal{F})} \mathcal{F}$-admissible
- a rational convex combination of the above.

Examples

The set $W_{(\frac{1}{2},\mathcal{S})}$ induces the Tsirelson norm.

For $n \geqslant 2$ and $1 < q < \infty$ the set $W_{(n^{-1/q},\mathcal{A}_n)}$ induces a Tsirelson type norm, equivalent to ℓ_p
(S. Bellenot 1986, S. A. - I. Deliyanni 1991)

- The generalized Tsirelson space with the norm induced by $W_{(\theta,\mathcal{S}_\xi)}, \xi < \omega_1$.
(S. A. 1987)

Examples

The set $W_{(\frac{1}{2},\mathcal{S})}$ induces the Tsirelson norm.

For $n \geqslant 2$ and $1 < q < \infty$ the set $W_{(n^{-1/q},\mathcal{A}_n)}$ induces a Tsirelson type norm, equivalent to ℓ_p
(S. Bellenot 1986, S. A. - I. Deliyanni 1991)

- The generalized Tsirelson space with the norm induced by $W_{(\theta,\mathcal{S}_\xi)}, \xi < \omega_1$.
(S. A. 1987)

Examples

The set $W_{(\frac{1}{2},\mathcal{S})}$ induces the Tsirelson norm.

For $n \geqslant 2$ and $1 < q < \infty$ the set $W_{(n^{-1/q},\mathcal{A}_n)}$ induces a Tsirelson type norm, equivalent to ℓ_p
(S. Bellenot 1986, S. A. - I. Deliyanni 1991)

- The generalized Tsirelson space with the norm induced by $W_{(\theta,\mathcal{S}_\xi)}, \xi < \omega_1$.
(S. A. 1987)

Examples

The set $W_{(\frac{1}{2},\mathcal{S})}$ induces the Tsirelson norm.

For $n \geqslant 2$ and $1 < q < \infty$ the set $W_{(n^{-1/q},\mathcal{A}_n)}$ induces a Tsirelson type norm, equivalent to ℓ_p
(S. Bellenot 1986, S. A. - I. Deliyanni 1991)

- The generalized Tsirelson space with the norm induced by $W_{(\theta,\mathcal{S}_\xi)}, \xi < \omega_1$.
(S. A. 1987)

Examples

The set $W_{(\frac{1}{2},\mathcal{S})}$ induces the Tsirelson norm.

For $n \geqslant 2$ and $1 < q < \infty$ the set $W_{(n^{-1/q},\mathcal{A}_n)}$ induces a Tsirelson type norm, equivalent to ℓ_p
(S. Bellenot 1986, S. A. - I. Deliyanni 1991)

- The generalized Tsirelson space with the norm induced by $W_{(\theta,\mathcal{S}_\xi)}, \xi < \omega_1$.
(S. A. 1987)

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j,\mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)},\mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j},\mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j,\mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)},\mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j},\mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j}, \mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j,\mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)},\mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j},\mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j,\mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)},\mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j},\mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, mixed Tsirelson type norms

A mixed Tsirelson type norming set associated to a $(\theta_j, \mathcal{F}_j)_j$ sequence of operations is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j), j \in \mathbb{N}$.

Examples

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n)_n}$ induces the Schlumprecht norm.

The set $W_{(\frac{1}{m_j}, \mathcal{S}_{n_j})_j}$ induces the Argyros - Deliyanni norm (1994).

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m} \sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m} \sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m}\sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max\operatorname{supp}\alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m}\sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m}\sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m} \sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

An α-average in a norming set W is an average

$$\alpha = \frac{1}{m} \sum_{i=1}^{m} f_i$$

with $m \geqslant 2$, $f_1 < \cdots < f_m$ in W

The size of α is $s(\alpha) = m$.

A sequence $\alpha_1 < \alpha_2 < \cdots < \alpha_n < \cdots$ is very fast growing (v.f.g.), if for $n > 1$

$$s(\alpha_n) > (\max \operatorname{supp} \alpha_{n-1})^2$$

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

For W a norming set, $0 < \theta \leqslant 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F}, \alpha)$ operation

if for every $\{\alpha_i\}_{i=1}^n$ $\mathcal{F}$-admissible and very fast growing family of α-averages in W, the functional

$$f = \theta \sum_{i=1}^{n} \alpha_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

For W a norming set, $0 < \theta \leqslant 1$ and $\mathcal{F}$ a regular family we say that *W is closed under the $(\theta, \mathcal{F}, \alpha)$ operation*

if for every $\{\alpha_i\}_{i=1}^n$ $\mathcal{F}$-admissible and very fast growing family of α-averages in W, the functional

$$f = \theta \sum_{i=1}^{n} \alpha_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

For W a norming set, $0 < \theta \leqslant 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F}, \alpha)$ operation

if for every $\{\alpha_i\}_{i=1}^n$ $\mathcal{F}$-admissible and very fast growing family of α-averages in W, the functional

$$f = \theta \sum_{i=1}^{n} \alpha_i$$

belongs to W.

Concepts, $(\theta, \mathcal{F}, \alpha)$ operations

For W a norming set, $0 < \theta \leqslant 1$ and $\mathcal{F}$ a regular family we say that W is closed under the $(\theta, \mathcal{F}, \alpha)$ operation

if for every $\{\alpha_i\}_{i=1}^n$ $\mathcal{F}$-admissible and very fast growing family of α-averages in W, the functional

$$f = \theta \sum_{i=1}^{n} \alpha_i$$

belongs to W.

Concepts, Tsirelson type norms under constraints

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

Concepts, Tsirelson type norms under constraints

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

Concepts, Tsirelson type norms under constraints

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

Concepts, Tsirelson type norms under constraints

A Tsirelson type norming set under constraints associated to a $(\theta, \mathcal{F}, \alpha)$ operation is:

The minimal norming set $W_{(\theta,\mathcal{F},\alpha)}$, closed in the $(\theta, \mathcal{F}, \alpha)$ operation.

Example

The set $W_{(1,\mathcal{S},\alpha)}$ induces an under constraints norm.

This is a reflexive space with some interesting properties.

Concepts, mixed Tsirelson type norms under constraints

A mixed Tsirelson type norming set under constraints associated to a sequence $(\theta_j, \mathcal{F}_j, \alpha)_j$ of operation is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j, \alpha)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j, \alpha), j \in \mathbb{N}$.

Example

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n, \alpha)_n}$ induces a variant of the Odell - Schlumprecht norm.

Concepts, mixed Tsirelson type norms under constraints

A mixed Tsirelson type norming set under constraints associated to a sequence $(\theta_j, \mathcal{F}_j, \alpha)_j$ of operation is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j, \alpha)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j, \alpha)$, $j \in \mathbb{N}$.

Example

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n, \alpha)_n}$ induces a variant of the Odell - Schlumprecht norm.

Concepts, mixed Tsirelson type norms under constraints

A mixed Tsirelson type norming set under constraints associated to a sequence $(\theta_j, \mathcal{F}_j, \alpha)_j$ of operation is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j, \alpha)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j, \alpha), j \in \mathbb{N}$.

Example

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n, \alpha)_n}$ induces a variant of the Odell - Schlumprecht norm.

Concepts, mixed Tsirelson type norms under constraints

A mixed Tsirelson type norming set under constraints associated to a sequence $(\theta_j, \mathcal{F}_j, \alpha)_j$ of operation is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j, \alpha)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j, \alpha), j \in \mathbb{N}$.

Example

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n, \alpha)_n}$ induces a variant of the Odell - Schlumprecht norm.

Concepts, mixed Tsirelson type norms under constraints

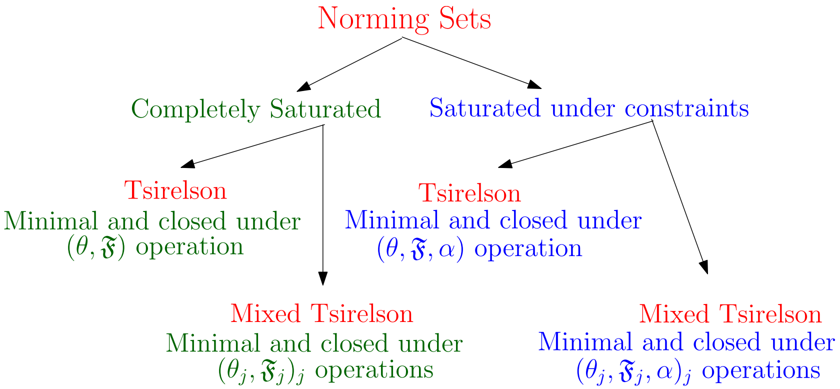
A mixed Tsirelson type norming set under constraints associated to a sequence $(\theta_j, \mathcal{F}_j, \alpha)_j$ of operation is:

The minimal norming set $W_{(\theta_j, \mathcal{F}_j, \alpha)_j}$, simultaneously closed in the operations $(\theta_j, \mathcal{F}_j, \alpha), j \in \mathbb{N}$.

Example

The set $W_{(\frac{1}{f(n)}, \mathcal{A}_n, \alpha)_n}$ induces a variant of the Odell - Schlumprecht norm.

Concepts


Norming Sets
Completely Saturated
Saturated under constraints
Tsirelson
Minimal and closed under $(\theta, \mathfrak{F})$ operation
Mixed Tsirelson
Minimal and closed under $(\theta_j, \mathfrak{F}_j)_j$ operations
Tsirelson
Minimal and closed under $(\theta, \mathfrak{F}, \alpha)$ operation
Mixed Tsirelson
Minimal and closed under $(\theta_j, \mathfrak{F}_j, \alpha)_j$ operations

The new objects

- In the case of saturated under constraints norming sets, a new class appears which lies strictly between the corresponding Tsirelson and mixed Tsirelson ones.
- It is not difficult to see that for any $(\theta, \mathcal{F})$ operation we have that
$$W_{(\theta,\mathcal{F})} = W_{(\theta^j,\mathcal{F}^j)_j}$$
Where $\mathcal{F}^j$ is the j-times convolution of the family $\mathcal{F}$.
- In the case of Tsirelson space we have that $\mathcal{S}^j = \mathcal{S}_j$, hence $W_{(\frac{1}{2^n},\mathcal{S}_n)}$, is Tsirelson's norming set.
- In the case of saturated under constraints norming sets, the set
$$W_{(\theta,\mathcal{F},\alpha)} \neq W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$$
This yields the new class $W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$, which lies strictly between the Tsirelson and mixed Tsirelson ones.

The new objects

- In the case of saturated under constraints norming sets, a new class appears which lies strictly between the corresponding Tsirelson and mixed Tsirelson ones.

- It is not difficult to see that for any $(\theta, \mathcal{F})$ operation we have that

$$W_{(\theta,\mathcal{F})} = W_{(\theta^j,\mathcal{F}^j)_j}$$

Where $\mathcal{F}^j$ is the j-times convolution of the family $\mathcal{F}$.

- In the case of Tsirelson space we have that $\mathcal{S}^j = \mathcal{S}_j$, hence $W_{(\frac{1}{2^n},\mathcal{S}_n)}$, is Tsirelson's norming set.

- In the case of saturated under constraints norming sets, the set

$$W_{(\theta,\mathcal{F},\alpha)} \neq W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$$

This yields the new class $W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$, which lies strictly between the Tsirelson and mixed Tsirelson ones.

The new objects

- In the case of saturated under constraints norming sets, a new class appears which lies strictly between the corresponding Tsirelson and mixed Tsirelson ones.
- It is not difficult to see that for any $(\theta, \mathcal{F})$ operation we have that
$$W_{(\theta,\mathcal{F})} = W_{(\theta^j,\mathcal{F}^j)_j}$$
Where $\mathcal{F}^j$ is the j-times convolution of the family $\mathcal{F}$.
- In the case of Tsirelson space we have that $\mathcal{S}^j = \mathcal{S}_j$, hence $W_{(\frac{1}{2^n},\mathcal{S}_n)}$, is Tsirelson's norming set.
- In the case of saturated under constraints norming sets, the set
$$W_{(\theta,\mathcal{F},\alpha)} \neq W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$$
This yields the new class $W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$, which lies strictly between the Tsirelson and mixed Tsirelson ones.

The new objects

- In the case of saturated under constraints norming sets, a new class appears which lies strictly between the corresponding Tsirelson and mixed Tsirelson ones.
- It is not difficult to see that for any $(\theta, \mathcal{F})$ operation we have that
$$W_{(\theta,\mathcal{F})} = W_{(\theta^j,\mathcal{F}^j)_j}$$
Where $\mathcal{F}^j$ is the j-times convolution of the family $\mathcal{F}$.
- In the case of Tsirelson space we have that $\mathcal{S}^j = \mathcal{S}_j$, hence $W_{(\frac{1}{2^n},\mathcal{S}_n)}$, is Tsirelson's norming set.
- In the case of saturated under constraints norming sets, the set
$$W_{(\theta,\mathcal{F},\alpha)} \neq W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$$
This yields the new class $W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$, which lies strictly between the Tsirelson and mixed Tsirelson ones.

The new objects

- In the case of saturated under constraints norming sets, a new class appears which lies strictly between the corresponding Tsirelson and mixed Tsirelson ones.

- It is not difficult to see that for any $(\theta, \mathcal{F})$ operation we have that
$$W_{(\theta,\mathcal{F})} = W_{(\theta^j,\mathcal{F}^j)_j}$$
Where $\mathcal{F}^j$ is the j-times convolution of the family $\mathcal{F}$.

- In the case of Tsirelson space we have that $\mathcal{S}^j = \mathcal{S}_j$, hence $W_{(\frac{1}{2n},\mathcal{S}_n)}$, is Tsirelson's norming set.

- In the case of saturated under constraints norming sets, the set
$$W_{(\theta,\mathcal{F},\alpha)} \neq W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$$
This yields the new class $W_{(\theta^j,\mathcal{F}^j,\alpha)_j}$, which lies strictly between the Tsirelson and mixed Tsirelson ones.

Concepts

Saturated under constraints

Tsirelson

Minimal and closed under

$(\theta, \mathfrak{F}, \alpha)$ operation

Mixed Tsirelson

Minimal and closed under

$(\theta_j, \mathfrak{F}_j, \alpha)_j$ operations

The new objects

Minimal and closed under

$(\theta^j, \mathfrak{F}^j, \alpha)_j$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n} \|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1$, $k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m} \|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n}\|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1$, $k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m}\|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n} \|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1$, $k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m} \|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n}\|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1$, $k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m}\|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n}\|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1$, $k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m}\|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The space $T_{0,1}$

We will discuss the Tsirelson space under constraints with its norm induced by $W_{(1,\mathcal{S},\alpha)}$ which is also described by the following implicit formula.
For $x \in c_{00}(\mathbb{N})$ we set

$$\|x\|_{T_{0,1}} = \max\left\{\|x\|_0,\ \sup\{\sum_{i=1}^{n} \|E_i x\|_{k_i}\}\right\}$$

Where the supremum is taken over all $n \leqslant E_1 < \cdots < E_n$.

Also $k_1 \geqslant 2$ and for $i > 1, k_i > (\max E_{i-1})^2$. $\|\cdot\|_m$ is a norm on c_{00} given by

$$\|x\|_m = \sup\{\frac{1}{m}\sum_{i=1}^{m} \|F_i x\|_{T_{0,1}} : F_1 < \cdots < F_m\}$$

The spreading models of $T_{0,1}$

- The space $T_{0,1}$ is reflexive with an unconditional basis.
- Every spreading model of $T_{0,1}$ generated by a weakly null sequence is either ℓ_1 or c_0.
- Every subspace Y of $T_{0,1}$ admits both ℓ_1 and c_0 as spreading models.
- The space $T_{0,1}$ does not admit ℓ_1^2 and c_0^2 spreading models.

The spreading models of $T_{0,1}$

- The space $T_{0,1}$ is reflexive with an unconditional basis.
- Every spreading model of $T_{0,1}$ generated by a weakly null sequence is either ℓ_1 or c_0.
- Every subspace Y of $T_{0,1}$ admits both ℓ_1 and c_0 as spreading models.
- The space $T_{0,1}$ does not admit ℓ_1^2 and c_0^2 spreading models.

The spreading models of $T_{0,1}$

- The space $T_{0,1}$ is reflexive with an unconditional basis.
- Every spreading model of $T_{0,1}$ generated by a weakly null sequence is either ℓ_1 or c_0.
- Every subspace Y of $T_{0,1}$ admits both ℓ_1 and c_0 as spreading models.
- The space $T_{0,1}$ does not admit ℓ_1^2 and c_0^2 spreading models.

The spreading models of $T_{0,1}$

- The space $T_{0,1}$ is reflexive with an unconditional basis.
- Every spreading model of $T_{0,1}$ generated by a weakly null sequence is either ℓ_1 or c_0.
- Every subspace Y of $T_{0,1}$ admits both ℓ_1 and c_0 as spreading models.
- The space $T_{0,1}$ does not admit ℓ_1^2 and c_0^2 spreading models.

The spreading models of $T_{0,1}$

- The space $T_{0,1}$ is reflexive with an unconditional basis.
- Every spreading model of $T_{0,1}$ generated by a weakly null sequence is either ℓ_1 or c_0.
- Every subspace Y of $T_{0,1}$ admits both ℓ_1 and c_0 as spreading models.
- The space $T_{0,1}$ does not admit ℓ_1^2 and c_0^2 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha(\{x_n\}_n) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

  If $\alpha(\{x_n\}_n) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

  If $\alpha(\{x_n\}_n) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha(\{x_n\}_n) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

  If $\alpha(\{x_n\}_n) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

  If $\alpha(\{x_n\}_n) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha(\{x_n\}_n) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

  If $\alpha(\{x_n\}_n) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

  If $\alpha(\{x_n\}_n) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha\big(\{x_n\}_n\big) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

If $\alpha\big(\{x_n\}_n\big) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

If $\alpha\big(\{x_n\}_n\big) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha\big(\{x_n\}_n\big) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

  If $\alpha\big(\{x_n\}_n\big) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

  If $\alpha\big(\{x_n\}_n\big) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

The α-index of a block sequence $(x_n)_n$ in $T_{0,1}$ is equal to zero ($\alpha\big(\{x_n\}_n\big) = 0$), if for every v.f.g. sequence $(\alpha_k)_k$ of α-averages and every $(x_{n_k})_k$

$$\lim_k \alpha_k(x_{n_k}) = 0$$

- The α-index determines completely the spreading models generated by block sequences in $T_{0,1}$, in the following manner.
- **Proposition:** Let $(x_n)_n$ be a seminormalized block sequence in $T_{0,1}$. Then

  If $\alpha\big(\{x_n\}_n\big) \neq 0$, then $(x_n)_n$ admits ℓ_1 as a spreading model.

  If $\alpha\big(\{x_n\}_n\big) = 0$, then $(x_n)_n$ admits only c_0 spreading models.

The α-index

Assume that $\alpha(\{x_n\}_n) \neq 0$.

Then there exists $\varepsilon > 0$, $(x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k$, $f = \sum_{i=1}^{k} \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^{k} x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha(\{x_n\}_n) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

Assume that $\alpha\big(\{x_n\}_n\big) \neq 0$.

Then there exists $\varepsilon > 0$, $(x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k$, $f = \sum_{i=1}^k \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^k x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha\big(\{x_n\}_n\big) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

The α-index

Assume that $\alpha(\{x_n\}_n) \neq 0$.

Then there exists $\varepsilon > 0$, $(x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k$, $f = \sum_{i=1}^k \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^k x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha(\{x_n\}_n) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

The α-index

Assume that $\alpha(\{x_n\}_n) \neq 0$.

Then there exists $\varepsilon > 0$, $(x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k$, $f = \sum_{i=1}^k \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^k x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha(\{x_n\}_n) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

The α-index

Assume that $\alpha(\{x_n\}_n) \neq 0$.

Then there exists $\varepsilon > 0, (x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k, f = \sum_{i=1}^k \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^k x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha(\{x_n\}_n) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

The α-index

Assume that $\alpha\big(\{x_n\}_n\big) \neq 0$.

Then there exists $\varepsilon > 0, (x_\ell)_{\ell \in L}$ a subsequence of $(x_n)_n$ and $(\alpha_\ell)_{\ell \in L}$ a sequence of very fast growing α-averages with $\alpha_\ell(x_\ell) > \varepsilon$.

Then for $k \leqslant \ell_1 < \cdots < \ell_k, f = \sum_{i=1}^k \alpha_{\ell_i} \in W_{(1,\mathcal{S},\alpha)}$, hence

$$\|\sum_{i=1}^k x_{\ell_i}\|_{T_{0,1}} > \varepsilon k$$

If $\alpha\big(\{x_n\}_n\big) = 0$, then using induction on the tree complexity of the $f \in W_{(1,\mathcal{S},\alpha)}$, we prove that $(x_n)_n$ admits c_0 as a spreading model.

Strictly Singular Operators on $T_{0,1}$

- Since the space $T_{0,1}$ admits c_0 and ℓ_1 spreading models, the strictly singular operators on every subspace of it, form a non separable ideal.
- The following describes the structure of the strictly singular operators in $T_{0,1}$.
- **Theorem:** If $S : T_{0,1} \to T_{0,1}$ is strictly singular, then for every weakly null sequence $(x_n)_n$, the sequences $(x_n)_n$, $(Tx_n)_n$, do not generate the same spreading model.
- The space $T_{0,1}$ is quasi minimal.

Strictly Singular Operators on $T_{0,1}$

- Since the space $T_{0,1}$ admits c_0 and ℓ_1 spreading models, the strictly singular operators on every subspace of it, form a non separable ideal.
- The following describes the structure of the strictly singular operators in $T_{0,1}$.
- **Theorem:** If $S : T_{0,1} \to T_{0,1}$ is strictly singular, then for every weakly null sequence $(x_n)_n$, the sequences $(x_n)_n$, $(Tx_n)_n$, do not generate the same spreading model.
- The space $T_{0,1}$ is quasi minimal.

Strictly Singular Operators on $T_{0,1}$

- Since the space $T_{0,1}$ admits c_0 and ℓ_1 spreading models, the strictly singular operators on every subspace of it, form a non separable ideal.
- The following describes the structure of the strictly singular operators in $T_{0,1}$.
- **Theorem:** If $S : T_{0,1} \to T_{0,1}$ is strictly singular, then for every weakly null sequence $(x_n)_n$, the sequences $(x_n)_n$, $(Tx_n)_n$, do not generate the same spreading model.
- The space $T_{0,1}$ is quasi minimal.

Strictly Singular Operators on $T_{0,1}$

- Since the space $T_{0,1}$ admits c_0 and ℓ_1 spreading models, the strictly singular operators on every subspace of it, form a non separable ideal.
- The following describes the structure of the strictly singular operators in $T_{0,1}$.
- **Theorem:** If $S : T_{0,1} \to T_{0,1}$ is strictly singular, then for every weakly null sequence $(x_n)_n$, the sequences $(x_n)_n$, $(Tx_n)_n$, do not generate the same spreading model.
- The space $T_{0,1}$ is quasi minimal.

Strictly Singular Operators on $T_{0,1}$

- If S is a strictly singular operator and $(x_n)_n$ generates a c_0 spreading model, the above yields that $(Sx_n)_n$ is a norm null sequence. Also if $(x_n)_n$ generates an ℓ_1 spreading model, then $(Sx_n)_n$ is either norm null, or admits only c_0 spreading models.
- As consequence, for every S, T strictly singular operators, ST is compact.

Strictly Singular Operators on $T_{0,1}$

- If S is a strictly singular operator and $(x_n)_n$ generates a c_0 spreading model, the above yields that $(Sx_n)_n$ is a norm null sequence. Also if $(x_n)_n$ generates an ℓ_1 spreading model, then $(Sx_n)_n$ is either norm null, or admits only c_0 spreading models.
- As consequence, for every S, T strictly singular operators, ST is compact.

- If S is a strictly singular operator and $(x_n)_n$ generates a c_0 spreading model, the above yields that $(Sx_n)_n$ is a norm null sequence. Also if $(x_n)_n$ generates an ℓ_1 spreading model, then $(Sx_n)_n$ is either norm null, or admits only c_0 spreading models.
- As consequence, for every S, T strictly singular operators, ST is compact.

The spaces $T^n_{0,1}$

The space $T_{0,1}$ belongs to a sequence of spaces sharing similar properties described by the following.

Theorem (S. A., K. Beanland, P. Motakis)

For every $n \in \mathbb{N}$ there exists a reflexive Banach space $T^n_{0,1}$ with a 1-unconditional basis, such that every Y subspace of $T^n_{0,1}$ satisfies the following properties.

(i) *For every $S_1, \ldots, S_{n+1}$ strictly singular operators on Y, the composition $S_1 \cdots S_{n+1}$ is a compact operator.*

(ii) *There exist $S_1, \ldots S_n$ strictly singular operators, such that $S_1 \cdots S_n$ is not compact.*

- The norm on $T^n_{0,1}$ is induced by the norming set $W_{(1,\mathcal{S}_n,\alpha)}$

The spaces $T^n_{0,1}$

The space $T_{0,1}$ belongs to a sequence of spaces sharing similar properties described by the following.

Theorem (S. A., K. Beanland, P. Motakis)

For every $n \in \mathbb{N}$ there exists a reflexive Banach space $T^n_{0,1}$ with a 1-unconditional basis, such that every Y subspace of $T^n_{0,1}$ satisfies the following properties.

(i) *For every $S_1, \ldots, S_{n+1}$ strictly singular operators on Y, the composition $S_1 \cdots S_{n+1}$ is a compact operator.*

(ii) *There exist $S_1, \ldots S_n$ strictly singular operators, such that $S_1 \cdots S_n$ is not compact.*

- The norm on $T^n_{0,1}$ is induced by the norming set $W_{(1,\mathcal{S}_n,\alpha)}$

A problem

- Since every subspace of $T^n_{0,1}$ admits c_0 and ℓ_1 as spreading models, the space $T^n_{0,1}$ does not contain an asymptotic ℓ_p subspace. Hence, by a theorem of N. Tomczak-Jaegermann and V. Milman, it does not contain a boundedly distortable subspace.
- **Problem:** Is every $T^n_{0,1}$ arbitrarily distortable? If yes, does there exist an asymptotic biorthogonal system determining the distortion?

A problem

- Since every subspace of $T^n_{0,1}$ admits c_0 and ℓ_1 as spreading models, the space $T^n_{0,1}$ does not contain an asymptotic ℓ_p subspace. Hence, by a theorem of N. Tomczak-Jaegermann and V. Milman, it does not contain a boundedly distortable subspace.
- **Problem:** Is every $T^n_{0,1}$ arbitrarily distortable? If yes, does there exist an asymptotic biorthogonal system determining the distortion?

A problem

- Since every subspace of $T^n_{0,1}$ admits c_0 and ℓ_1 as spreading models, the space $T^n_{0,1}$ does not contain an asymptotic ℓ_p subspace. Hence, by a theorem of N. Tomczak-Jaegermann and V. Milman, it does not contain a boundedly distortable subspace.
- **Problem:** Is every $T^n_{0,1}$ arbitrarily distortable? If yes, does there exist an asymptotic biorthogonal system determining the distortion?

The space $\mathfrak{X}_{\text{ISP}}$

Theorem (S. A., P. Motakis) There exists a reflexive space $\mathfrak{X}_{\text{ISP}}$ with a Schauder basis $\{e_n\}_n$ satisfying the following properties.

(i) The space $\mathfrak{X}_{\text{ISP}}$ is hereditarily indecomposable.

(ii) Every seminormalized weakly null sequence $\{x_n\}_n$ has a subsequence generating either ℓ_1 or c_0 as a spreading model. Moreover every infinite dimensional subspace Y of $\mathfrak{X}_{\text{ISP}}$ admits both ℓ_1 and c_0 as spreading models.

(iii) For every Y infinite dimensional closed subspace of $\mathfrak{X}_{\text{ISP}}$ and every $T \in \mathcal{L}(Y, \mathfrak{X}_{\text{ISP}})$, $T = \lambda I_{Y,\mathfrak{X}_{\text{ISP}}} + S$ with S strictly singular.

The space $\mathfrak{X}_{\text{ISP}}$

Theorem (S. A., P. Motakis) There exists a reflexive space $\mathfrak{X}_{\text{ISP}}$ with a Schauder basis $\{e_n\}_n$ satisfying the following properties.

(i) The space $\mathfrak{X}_{\text{ISP}}$ is hereditarily indecomposable.

(ii) Every seminormalized weakly null sequence $\{x_n\}_n$ has a subsequence generating either ℓ_1 or c_0 as a spreading model. Moreover every infinite dimensional subspace Y of $\mathfrak{X}_{\text{ISP}}$ admits both ℓ_1 and c_0 as spreading models.

(iii) For every Y infinite dimensional closed subspace of $\mathfrak{X}_{\text{ISP}}$ and every $T \in \mathcal{L}(Y, \mathfrak{X}_{\text{ISP}})$, $T = \lambda I_{Y,\mathfrak{X}_{\text{ISP}}} + S$ with S strictly singular.

Theorem (S. A., P. Motakis) There exists a reflexive space $\mathfrak{X}_{_{\text{ISP}}}$ with a Schauder basis $\{e_n\}_n$ satisfying the following properties.

(i) The space $\mathfrak{X}_{_{\text{ISP}}}$ is hereditarily indecomposable.

(ii) Every seminormalized weakly null sequence $\{x_n\}_n$ has a subsequence generating either ℓ_1 or c_0 as a spreading model. Moreover every infinite dimensional subspace Y of $\mathfrak{X}_{_{\text{ISP}}}$ admits both ℓ_1 and c_0 as spreading models.

(iii) For every Y infinite dimensional closed subspace of $\mathfrak{X}_{_{\text{ISP}}}$ and every $T \in \mathcal{L}(Y, \mathfrak{X}_{_{\text{ISP}}})$, $T = \lambda I_{Y,\mathfrak{X}_{\text{ISP}}} + S$ with S strictly singular.

The space $\mathfrak{X}_{\text{ISP}}$

Theorem (S. A., P. Motakis) There exists a reflexive space $\mathfrak{X}_{\text{ISP}}$ with a Schauder basis $\{e_n\}_n$ satisfying the following properties.

(i) The space $\mathfrak{X}_{\text{ISP}}$ is hereditarily indecomposable.

(ii) Every seminormalized weakly null sequence $\{x_n\}_n$ has a subsequence generating either ℓ_1 or c_0 as a spreading model. Moreover every infinite dimensional subspace Y of $\mathfrak{X}_{\text{ISP}}$ admits both ℓ_1 and c_0 as spreading models.

(iii) For every Y infinite dimensional closed subspace of $\mathfrak{X}_{\text{ISP}}$ and every $T \in \mathcal{L}(Y, \mathfrak{X}_{\text{ISP}})$, $T = \lambda I_{Y,\mathfrak{X}_{\text{ISP}}} + S$ with S strictly singular.

The space $\mathfrak{X}_{\text{ISP}}$

Theorem (S. A., P. Motakis) There exists a reflexive space $\mathfrak{X}_{\text{ISP}}$ with a Schauder basis $\{e_n\}_n$ satisfying the following properties.

(i) The space $\mathfrak{X}_{\text{ISP}}$ is hereditarily indecomposable.

(ii) Every seminormalized weakly null sequence $\{x_n\}_n$ has a subsequence generating either ℓ_1 or c_0 as a spreading model. Moreover every infinite dimensional subspace Y of $\mathfrak{X}_{\text{ISP}}$ admits both ℓ_1 and c_0 as spreading models.

(iii) For every Y infinite dimensional closed subspace of $\mathfrak{X}_{\text{ISP}}$ and every $T \in \mathcal{L}(Y, \mathfrak{X}_{\text{ISP}})$, $T = \lambda I_{Y,\mathfrak{X}_{\text{ISP}}} + S$ with S strictly singular.

The space $\mathfrak{X}_{\text{ISP}}$

(iv) For every Y infinite dimensional subspace of $\mathfrak{X}_{\text{ISP}}$ the ideal $\mathcal{S}(Y)$ of the strictly singular operators is non separable.

(v) For every Y subspace of $\mathfrak{X}_{\text{ISP}}$ and every Q, S, T in $\mathcal{S}(Y)$ the operator QST is compact. Hence for every $T \in \mathcal{S}(Y)$ either $T^3 = 0$ or T commutes with a non zero compact operator.

(vi) For every Y infinite dimensional closed subspace of X and every $T \in \mathcal{L}(Y)$, T admits a non-trivial closed invariant subspace. In particular every $T \neq \lambda I_Y$, for $\lambda \in \mathbb{R}$ admits a non-trivial hyperinvariant subspace.

The space $\mathfrak{X}_{\text{ISP}}$

(iv) For every Y infinite dimensional subspace of $\mathfrak{X}_{\text{ISP}}$ the ideal $\mathcal{S}(Y)$ of the strictly singular operators is non separable.

(v) For every Y subspace of $\mathfrak{X}_{\text{ISP}}$ and every Q, S, T in $\mathcal{S}(Y)$ the operator QST is compact. Hence for every $T \in \mathcal{S}(Y)$ either $T^3 = 0$ or T commutes with a non zero compact operator.

(vi) For every Y infinite dimensional closed subspace of X and every $T \in \mathcal{L}(Y)$, T admits a non-trivial closed invariant subspace. In particular every $T \neq \lambda I_Y$, for $\lambda \in \mathbb{R}$ admits a non-trivial hyperinvariant subspace.

The space $\mathfrak{X}_{\text{ISP}}$

(iv) For every Y infinite dimensional subspace of $\mathfrak{X}_{\text{ISP}}$ the ideal $\mathcal{S}(Y)$ of the strictly singular operators is non separable.

(v) For every Y subspace of $\mathfrak{X}_{\text{ISP}}$ and every Q, S, T in $\mathcal{S}(Y)$ the operator QST is compact. Hence for every $T \in \mathcal{S}(Y)$ either $T^3 = 0$ or T commutes with a non zero compact operator.

(vi) For every Y infinite dimensional closed subspace of X and every $T \in \mathcal{L}(Y)$, T admits a non-trivial closed invariant subspace. In particular every $T \neq \lambda I_Y$, for $\lambda \in \mathbb{R}$ admits a non-trivial hyperinvariant subspace.

(iv) For every Y infinite dimensional subspace of $\mathfrak{X}_{\text{ISP}}$ the ideal $\mathcal{S}(Y)$ of the strictly singular operators is non separable.

(v) For every Y subspace of $\mathfrak{X}_{\text{ISP}}$ and every Q, S, T in $\mathcal{S}(Y)$ the operator QST is compact. Hence for every $T \in \mathcal{S}(Y)$ either $T^3 = 0$ or T commutes with a non zero compact operator.

(vi) For every Y infinite dimensional closed subspace of X and every $T \in \mathcal{L}(Y)$, T admits a non-trivial closed invariant subspace. In particular every $T \neq \lambda I_Y$, for $\lambda \in \mathbb{R}$ admits a non-trivial hyperinvariant subspace.

The norming set W_{ISP}

The norm on the space $\mathfrak{X}_{\text{ISP}}$ is induced by a norming set W_{ISP} which is the minimal set satisfying the following properties.

(Type I_α functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operations. If f is of type I_α and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operation, then the weight of f is $w(f) = n$.

(Type II functionals) The set W_{ISP} includes all $E\phi$, with E an interval of the naturals and $\phi = \frac{1}{2}\sum_{k=1}^{n} f_k$, where $f_1 < \cdots < f_n$ is an $\mathcal{S}$-admissible special family of type I_α special functionals.
(A special family satisfies the property, that for $k > 1$, $w(f_k)$ determines uniquely the sequence $\{f_i\}_{i=1}^{k-1}$.)

For $E\phi$ type II functional, the weights of $E\phi$ are $\hat{w}(\phi) = \{w(f_k) : E \cap \operatorname{supp} f_k \neq \varnothing\}$.

The norming set W_{ISP}

The norm on the space $\mathfrak{X}_{\text{ISP}}$ is induced by a norming set W_{ISP} which is the minimal set satisfying the following properties.

(Type I$_\alpha$ functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operations. If f is of type I$_\alpha$ and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operation, then the weight of f is $w(f) = n$.

(Type II functionals) The set W_{ISP} includes all $E\phi$, with E an interval of the naturals and $\phi = \frac{1}{2}\sum_{k=1}^{n} f_k$, where $f_1 < \cdots < f_n$ is an $\mathcal{S}$-admissible special family of type I$_\alpha$ special functionals.
(A special family satisfies the property, that for $k > 1$, $w(f_k)$ determines uniquely the sequence $\{f_i\}_{i=1}^{k-1}$.)

For $E\phi$ type II functional, the weights of $E\phi$ are $\hat{w}(\phi) = \{w(f_k) : E \cap \operatorname{supp} f_k \neq \varnothing\}$.

The norming set W_{ISP}

The norm on the space $\mathfrak{X}_{\text{ISP}}$ is induced by a norming set W_{ISP} which is the minimal set satisfying the following properties.

(Type I_α functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operations. If f is of type I_α and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operation, then the weight of f is $w(f) = n$.

(Type II functionals) The set W_{ISP} includes all $E\phi$, with E an interval of the naturals and $\phi = \frac{1}{2}\sum_{k=1}^{n} f_k$, where $f_1 < \cdots < f_n$ is an $\mathcal{S}$-admissible special family of type I_α special functionals.
(A special family satisfies the property, that for $k > 1$, $w(f_k)$ determines uniquely the sequence $\{f_i\}_{i=1}^{k-1}$.)

For $E\phi$ type II functional, the weights of $E\phi$ are $\hat{w}(\phi) = \{w(f_k) : E \cap \operatorname{supp} f_k \neq \varnothing\}$.

The norming set W_{ISP}

The norm on the space $\mathfrak{X}_{\text{ISP}}$ is induced by a norming set W_{ISP} which is the minimal set satisfying the following properties.

(Type I_α functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operations. If f is of type I_α and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operation, then the weight of f is $w(f) = n$.

(Type II functionals) The set W_{ISP} includes all $E\phi$, with E an interval of the naturals and $\phi = \frac{1}{2}\sum_{k=1}^{n} f_k$, where $f_1 < \cdots < f_n$ is an $\mathcal{S}$-admissible special family of type I_α special functionals.
(A special family satisfies the property, that for $k > 1$, $w(f_k)$ determines uniquely the sequence $\{f_i\}_{i=1}^{k-1}$.)

For $E\phi$ type II functional, the weights of $E\phi$ are $\hat{w}(\phi) = \{w(f_k) : E \cap \text{supp}\, f_k \neq \varnothing\}$.

The norming set W_{ISP}

The norm on the space $\mathfrak{X}_{\text{ISP}}$ is induced by a norming set W_{ISP} which is the minimal set satisfying the following properties.

(Type I_α functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operations. If f is of type I_α and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \alpha)$ operation, then the weight of f is $w(f) = n$.

(Type II functionals) The set W_{ISP} includes all $E\phi$, with E an interval of the naturals and $\phi = \frac{1}{2}\sum_{k=1}^{n} f_k$, where $f_1 < \cdots < f_n$ is an $\mathcal{S}$-admissible special family of type I_α special functionals.
(A special family satisfies the property, that for $k > 1$, $w(f_k)$ determines uniquely the sequence $\{f_i\}_{i=1}^{k-1}$.)

For $E\phi$ type II functional, the weights of $E\phi$ are $\hat{w}(\phi) = \{w(f_k) : E \cap \operatorname{supp} f_k \neq \varnothing\}$.

The norming set W_{ISP}

(β-averages) A β-average is an average $\beta = \frac{1}{n}\sum_{k=1}^{n} E_k\phi_k$, where $E_k\phi_k$ are of type II with pairwise disjoint weights. The size $s(\beta)$ and very fast growing sequences $(\beta_k)_k$ are defined in the same manner as for α averages.

(Type I_β functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operations. If f is of type I_β and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operation, then the weight of f is $w(f) = n$.

Since $W_{(\frac{1}{2},\mathcal{S})}$ is the same with $W_{(\frac{1}{2^n},\mathcal{S}_n)_n}$, we have that W_{ISP} is a subset of $W_{(\frac{1}{2},\mathcal{S})}$.

The norming set W_{ISP}

(β-averages) A β-average is an average $\beta = \frac{1}{n}\sum_{k=1}^{n} E_k\phi_k$, where $E_k\phi_k$ are of type II with pairwise disjoint weights. The size $s(\beta)$ and very fast growing sequences $(\beta_k)_k$ are defined in the same manner as for α averages.

(Type I_β functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operations. If f is of type I_β and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operation, then the weight of f is $w(f) = n$.

Since $W_{(\frac{1}{2}, \mathcal{S})}$ is the same with $W_{(\frac{1}{2^n}, \mathcal{S}_n)_n}$, we have that W_{ISP} is a subset of $W_{(\frac{1}{2}, \mathcal{S})}$.

The norming set W_{ISP}

(β-averages) A β-average is an average $\beta = \frac{1}{n}\sum_{k=1}^{n} E_k\phi_k$, where $E_k\phi_k$ are of type II with pairwise disjoint weights. The size $s(\beta)$ and very fast growing sequences $(\beta_k)_k$ are defined in the same manner as for α averages.

(Type I_β functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operations. If f is of type I_β and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operation, then the weight of f is $w(f) = n$.

Since $W_{(\frac{1}{2},\mathcal{S})}$ is the same with $W_{(\frac{1}{2^n},\mathcal{S}_n)_n}$, we have that W_{ISP} is a subset of $W_{(\frac{1}{2},\mathcal{S})}$.

The norming set W_{ISP}

(β-averages) A β-average is an average $\beta = \frac{1}{n}\sum_{k=1}^{n} E_k\phi_k$, where $E_k\phi_k$ are of type II with pairwise disjoint weights. The size $s(\beta)$ and very fast growing sequences $(\beta_k)_k$ are defined in the same manner as for α averages.

(Type I_β functionals) The set W_{ISP} is closed in the $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operations. If f is of type I_β and is the result of $(\frac{1}{2^n}, \mathcal{S}_n, \beta)$ operation, then the weight of f is $w(f) = n$.

Since $W_{(\frac{1}{2}, \mathcal{S})}$ is the same with $W_{(\frac{1}{2^n}, \mathcal{S}_n)_n}$, we have that W_{ISP} is a subset of $W_{(\frac{1}{2}, \mathcal{S})}$.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

In every HI construction one could find seminormalized weakly null sequences $(x_n)_n$ with the property, for every strictly singular operator S, $(Sx_n)_n$ is a norm null sequence. In the case of $\mathfrak{X}_{\text{ISP}}$ space, the sequences $(x_n)_n$ generating c_0 spreading models have the aforementioned property, thus it is critical to determine sequences generating c_0 spreading models. There exists a criterion which is based on α and β indices.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

In every HI construction one could find seminormalized weakly null sequences $(x_n)_n$ with the property, for every strictly singular operator S, $(Sx_n)_n$ is a norm null sequence. In the case of $\mathfrak{X}_{\text{ISP}}$ space, the sequences $(x_n)_n$ generating c_0 spreading models have the aforementioned property, thus it is critical to determine sequences generating c_0 spreading models. There exists a criterion which is based on α and β indices.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in W_{ISP} and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that
  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write
  $$\alpha(\{x_k\}_k) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in W_{ISP} and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that
  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write
  $$\alpha(\{x_k\}_k) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in W_{ISP} and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that
  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write
  $$\alpha(\{x_k\}_k) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in W_{ISP} and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that

  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write

  $$\alpha(\{x_k\}_k) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in W_{ISP} and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that
  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write
  $$\alpha\big(\{x_k\}_k\big) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{_\text{ISP}}$

- (α index) Let $\{x_k\}_k$ be a block sequence in $\mathfrak{X}_{_\text{ISP}}$ that satisfies the following.

  For any n, for any very fast growing sequence $\{\alpha_q\}_q$ of α-averages in $W_{_\text{ISP}}$ and for any $\{F_k\}_k$ increasing sequence of subsets of the naturals, such that $\{\alpha_q\}_{q\in F_k}$ is $\mathcal{S}_n$-admissible, the following holds.

  For any $\{x_{n_k}\}_k$ subsequence of $\{x_k\}_k$, we have that
  $$\lim_k \sum_{q\in F_k} |\alpha_q(x_{n_k})| = 0.$$

  Then we say that the α-index of $\{x_k\}_k$ is zero and write
  $$\alpha\big(\{x_k\}_k\big) = 0.$$

- The β index is similarly defined.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

c_0 spreading models in $\mathfrak{X}_{\text{ISP}}$

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

The α, β indices provide the following criterion for sequences generating c_0 spreading models.

If $(x_n)_n$ is a seminormalized block sequence in $\mathfrak{X}_{\text{ISP}}$, then the following are equivalent.

- $(x_n)_n$ admits only c_0 as a spreading model.
- The indices α and β on $(x_n)_n$ are equal to zero.

It is not difficult to see that if either α or β index is not equal to zero, then $(x_n)_n$ contains a subsequence generating ℓ_1 as a spreading model.

Special convex combinations

- (((n,ε) basic special convex combinations) A convex combination $\sum_{k\in F} c_k e_k$ is a (n,ε) b.s.c.c. if

(i) the set F belongs to $\mathcal{S}_n$

(ii) for any $G \in \mathcal{S}_{n-1}, G \subset F$, we have that $\sum_{k\in G} c_k < \varepsilon$.

- ((n,ε) special convex combinations) Let $x_1 < \cdots < x_m$ be vectors in c_{00} and $\psi(k) = \min \operatorname{supp} x_k$, for $k = 1,\ldots,m$. Then $x = \sum_{k=1}^m c_k x_k$ is said to be a (n,ε) s.c.c., if $\sum_{k=1}^m c_k e_{\psi(k)}$ is a (n,ε) b.s.c.c.

Special convex combinations

- (((n,ε) basic special convex combinations) A convex combination $\sum_{k\in F} c_k e_k$ is a (n,ε) b.s.c.c. if

(i) the set F belongs to $\mathcal{S}_n$

(ii) for any $G \in \mathcal{S}_{n-1}, G \subset F$, we have that $\sum_{k\in G} c_k < \varepsilon$.

- (((n,ε) special convex combinations) Let $x_1 < \cdots < x_m$ be vectors in c_{00} and $\psi(k) = \min \operatorname{supp} x_k$, for $k = 1,\ldots,m$. Then $x = \sum_{k=1}^m c_k x_k$ is said to be a (n,ε) s.c.c., if $\sum_{k=1}^m c_k e_{\psi(k)}$ is a (n,ε) b.s.c.c.

Special convex combinations

- (((n, ε) basic special convex combinations) A convex combination $\sum_{k \in F} c_k e_k$ is a (n, ε) b.s.c.c. if

(i) the set F belongs to $\mathcal{S}_n$

(ii) for any $G \in \mathcal{S}_{n-1}, G \subset F$, we have that $\sum_{k \in G} c_k < \varepsilon$.

- (((n, ε) special convex combinations) Let $x_1 < \cdots < x_m$ be vectors in c_{00} and $\psi(k) = \min \operatorname{supp} x_k$, for $k = 1, \ldots, m$. Then $x = \sum_{k=1}^{m} c_k x_k$ is said to be a (n, ε) s.c.c., if $\sum_{k=1}^{m} c_k e_{\psi(k)}$ is a (n, ε) b.s.c.c.

Special convex combinations

- (((n,ε) basic special convex combinations) A convex combination $\sum_{k\in F} c_k e_k$ is a (n,ε) b.s.c.c. if

(i) the set F belongs to $\mathcal{S}_n$

(ii) for any $G \in \mathcal{S}_{n-1}, G \subset F$, we have that $\sum_{k\in G} c_k < \varepsilon$.

- (((n,ε) special convex combinations) Let $x_1 < \cdots < x_m$ be vectors in c_{00} and $\psi(k) = \min \operatorname{supp} x_k$, for $k = 1, \ldots, m$. Then $x = \sum_{k=1}^m c_k x_k$ is said to be a (n,ε) s.c.c., if $\sum_{k=1}^m c_k e_{\psi(k)}$ is a (n,ε) b.s.c.c.

Special convex combinations

- (((n,ε) basic special convex combinations) A convex combination $\sum_{k\in F} c_k e_k$ is a (n,ε) b.s.c.c. if

(i) the set F belongs to $\mathcal{S}_n$

(ii) for any $G \in \mathcal{S}_{n-1}, G \subset F$, we have that $\sum_{k\in G} c_k < \varepsilon$.

- ((n,ε) special convex combinations) Let $x_1 < \cdots < x_m$ be vectors in c_{00} and $\psi(k) = \min \operatorname{supp} x_k$, for $k = 1, \ldots, m$. Then $x = \sum_{k=1}^m c_k x_k$ is said to be a (n,ε) s.c.c., if $\sum_{k=1}^m c_k e_{\psi(k)}$ is a (n,ε) b.s.c.c.

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n, ε) special convex combination $\sum_{i\in F} c_i x_i$, with $\{x_i\}_{i\in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i\in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n, ε) s.c.c. $\sum_{k\in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k\in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n, ε) special convex combination $\sum_{i \in F} c_i x_i$, with $\{x_i\}_{i \in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i \in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n, ε) s.c.c. $\sum_{k \in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k \in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n, ε) special convex combination $\sum_{i \in F} c_i x_i$, with $\{x_i\}_{i \in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i \in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n, ε) s.c.c. $\sum_{k \in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k \in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n,ε) special convex combination $\sum_{i\in F} c_i x_i$, with $\{x_i\}_{i\in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i\in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n,ε) s.c.c. $\sum_{k\in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k\in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n,ε) special convex combination $\sum_{i\in F} c_i x_i$, with $\{x_i\}_{i\in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i\in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n,ε) s.c.c. $\sum_{k\in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k\in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

The basic inequality

The great advantage of using Tsirelson space as the unconditional frame for the space $\mathfrak{X}_{\text{ISP}}$ is the following inequality.

For a (n, ε) special convex combination $\sum_{i \in F} c_i x_i$, with $\{x_i\}_{i \in F}$ a finite normalized block sequence, we have that

$$\|\sum_{i \in F} c_i x_i\|_{\text{ISP}} \leqslant \frac{6}{2^n} + 12\varepsilon$$

The above yields the following. For every $(x_k)_k$ normalized block sequence such that either α or β index is not zero, there exists $\delta > 0$ such that

- For every n and for every $\frac{1}{2^{2n}} > \varepsilon > 0$ there exist (n, ε) s.c.c. $\sum_{k \in F} c_k x_k$ with

$$\delta < \|2^n \sum_{k \in F} c_k x_k\|_{\text{ISP}} \leqslant 7$$

Determining c_0 spreading models

We are ready to see how starting with an arbitrary normalized block sequence $(x_n)_n$, in at most two steps, a further block sequence can be chosen, generating a c_0 spreading model.

- If α and β indices of $(x_n)_n$ are zero, we are done. Otherwise, there exists a further block sequence $(y_k)_k$ with each y_k a $(k, \frac{1}{2^{2k}})$ s.c.c. such that $z_k = 2^k y_k$ is a seminormalized block sequence.
- It is shown the α index of (z_k) is equal to zero. If the β index of (z_k) is equal to zero, then we are done.
- Otherwise repeating the previous procedure to the sequence $(z_k)_k$, we arrive at a sequence $(w_k)_k$, for which both α and β indices are zero.

Determining c_0 spreading models

We are ready to see how starting with an arbitrary normalized block sequence $(x_n)_n$, in at most two steps, a further block sequence can be chosen, generating a c_0 spreading model.

- If α and β indices of $(x_n)_n$ are zero, we are done. Otherwise, there exists a further block sequence $(y_k)_k$ with each y_k a $(k, \frac{1}{2^{2k}})$ s.c.c. such that $z_k = 2^k y_k$ is a seminormalized block sequence.
- It is shown the α index of (z_k) is equal to zero. If the β index of (z_k) is equal to zero, then we are done.
- Otherwise repeating the previous procedure to the sequence $(z_k)_k$, we arrive at a sequence $(w_k)_k$, for which both α and β indices are zero.

Determining c_0 spreading models

We are ready to see how starting with an arbitrary normalized block sequence $(x_n)_n$, in at most two steps, a further block sequence can be chosen, generating a c_0 spreading model.

- If α and β indices of $(x_n)_n$ are zero, we are done. Otherwise, there exists a further block sequence $(y_k)_k$ with each y_k a $(k, \frac{1}{2^{2k}})$ s.c.c. such that $z_k = 2^k y_k$ is a seminormalized block sequence.
- It is shown the α index of (z_k) is equal to zero. If the β index of (z_k) is equal to zero, then we are done.
- Otherwise repeating the previous procedure to the sequence $(z_k)_k$, we arrive at a sequence $(w_k)_k$, for which both α and β indices are zero.

Determining c_0 spreading models

We are ready to see how starting with an arbitrary normalized block sequence $(x_n)_n$, in at most two steps, a further block sequence can be chosen, generating a c_0 spreading model.

- If α and β indices of $(x_n)_n$ are zero, we are done. Otherwise, there exists a further block sequence $(y_k)_k$ with each y_k a $(k, \frac{1}{2^{2k}})$ s.c.c. such that $z_k = 2^k y_k$ is a seminormalized block sequence.
- It is shown the α index of (z_k) is equal to zero. If the β index of (z_k) is equal to zero, then we are done.
- Otherwise repeating the previous procedure to the sequence $(z_k)_k$, we arrive at a sequence $(w_k)_k$, for which both α and β indices are zero.

Determining c_0 spreading models

We are ready to see how starting with an arbitrary normalized block sequence $(x_n)_n$, in at most two steps, a further block sequence can be chosen, generating a c_0 spreading model.

- If α and β indices of $(x_n)_n$ are zero, we are done. Otherwise, there exists a further block sequence $(y_k)_k$ with each y_k a $(k, \frac{1}{2^{2k}})$ s.c.c. such that $z_k = 2^k y_k$ is a seminormalized block sequence.
- It is shown the α index of (z_k) is equal to zero. If the β index of (z_k) is equal to zero, then we are done.
- Otherwise repeating the previous procedure to the sequence $(z_k)_k$, we arrive at a sequence $(w_k)_k$, for which both α and β indices are zero.

Strictly singular operators

- The structure of the space $\mathfrak{X}_{_{\text{ISP}}}$, permits the easy construction of strictly singular and non-compact operators. More precisely, the following holds.
- **Proposition:** Let $(x_n)_n$ and $(y_n)_n$ be seminormalized block sequences in $\mathfrak{X}_{_{\text{ISP}}}$, such that $(x_n)_n$ generates an ℓ_1 spreading model and $(y_n)_n$ generates a c_0 spreading model. Then there exists $L \subset \mathbb{N}$ and S a strictly singular operator in $\mathcal{L}(\mathfrak{X}_{_{\text{ISP}}})$, such that $Sx_n = y_n$, for all $n \in L$.
- On the other hand, the composition of every three strictly singular operators is a compact one.
- The previous two steps which we need to arrive to a c_0 spreading model is the reason for the necessity of the composition of three strictly singular operators in order obtain a compact one.

Strictly singular operators

- The structure of the space $\mathfrak{X}_{_{\rm ISP}}$, permits the easy construction of strictly singular and non-compact operators. More precisely, the following holds.
- **Proposition:** Let $(x_n)_n$ and $(y_n)_n$ be seminormalized block sequences in $\mathfrak{X}_{_{\rm ISP}}$, such that $(x_n)_n$ generates an ℓ_1 spreading model and $(y_n)_n$ generates a c_0 spreading model. Then there exists $L \subset \mathbb{N}$ and S a strictly singular operator in $\mathcal{L}(\mathfrak{X}_{_{\rm ISP}})$, such that $Sx_n = y_n$, for all $n \in L$.
- On the other hand, the composition of every three strictly singular operators is a compact one.
- The previous two steps which we need to arrive to a c_0 spreading model is the reason for the necessity of the composition of three strictly singular operators in order obtain a compact one.

Strictly singular operators

- The structure of the space $\mathfrak{X}_{\text{ISP}}$, permits the easy construction of strictly singular and non-compact operators. More precisely, the following holds.
- **Proposition:** Let $(x_n)_n$ and $(y_n)_n$ be seminormalized block sequences in $\mathfrak{X}_{\text{ISP}}$, such that $(x_n)_n$ generates an ℓ_1 spreading model and $(y_n)_n$ generates a c_0 spreading model. Then there exists $L \subset \mathbb{N}$ and S a strictly singular operator in $\mathcal{L}(\mathfrak{X}_{\text{ISP}})$, such that $Sx_n = y_n$, for all $n \in L$.
- On the other hand, the composition of every three strictly singular operators is a compact one.
- The previous two steps which we need to arrive to a c_0 spreading model is the reason for the necessity of the composition of three strictly singular operators in order obtain a compact one.

Strictly singular operators

- The structure of the space $\mathfrak{X}_{\text{ISP}}$, permits the easy construction of strictly singular and non-compact operators. More precisely, the following holds.
- **Proposition:** Let $(x_n)_n$ and $(y_n)_n$ be seminormalized block sequences in $\mathfrak{X}_{\text{ISP}}$, such that $(x_n)_n$ generates an ℓ_1 spreading model and $(y_n)_n$ generates a c_0 spreading model. Then there exists $L \subset \mathbb{N}$ and S a strictly singular operator in $\mathcal{L}(\mathfrak{X}_{\text{ISP}})$, such that $Sx_n = y_n$, for all $n \in L$.
- On the other hand, the composition of every three strictly singular operators is a compact one.
- The previous two steps which we need to arrive to a c_0 spreading model is the reason for the necessity of the composition of three strictly singular operators in order obtain a compact one.

Strictly singular operators

- The structure of the space $\mathfrak{X}_{_{\rm ISP}}$, permits the easy construction of strictly singular and non-compact operators. More precisely, the following holds.
- **Proposition:** Let $(x_n)_n$ and $(y_n)_n$ be seminormalized block sequences in $\mathfrak{X}_{_{\rm ISP}}$, such that $(x_n)_n$ generates an ℓ_1 spreading model and $(y_n)_n$ generates a c_0 spreading model. Then there exists $L \subset \mathbb{N}$ and S a strictly singular operator in $\mathcal{L}(\mathfrak{X}_{_{\rm ISP}})$, such that $Sx_n = y_n$, for all $n \in L$.
- On the other hand, the composition of every three strictly singular operators is a compact one.
- The previous two steps which we need to arrive to a c_0 spreading model is the reason for the necessity of the composition of three strictly singular operators in order obtain a compact one.

Thank you!