M^3 irreducible closed $\mathbb{Q}HS$

State as contrapositive.

<u>Conj</u>: M is <u>not</u> an L-space $\overset{??}{\Longleftrightarrow}$ $\pi_1 M$ is LO, i.e. $\pi_1 M \hookrightarrow \mathrm{Homeo}^+(\mathbb{R})$

$\Uparrow$ $\qquad\qquad$ $\Downarrow$ $\quad$ $\cancel{\Uparrow}$

~~Drop this~~

M has a co-orient taut foliation $\xRightarrow{\text{Thurston}}$ $\pi_1 M$ acts non-triv on S^1 $\Rightarrow$ $\pi_1 M$ is virtually LO

[By Thurston and Perelman, M has a decomp into geometric pieces.]

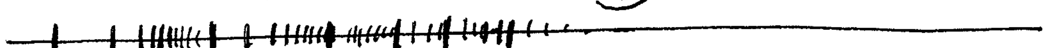
[ⓐ Conj is true for all SF (BRW 2005) and Solv mflds (BGW 2011)

ⓑ Graph mflds: For $\mathbb{Z}HS$: Clay-Lidman-Watson + Boileau-Boyer.]

<u>Hyperbolic 3-manifolds</u> [Mostow Rigidity gives strong conn between Topology $\Updownarrow$ geometry.]

$\mathcal{V} = \{\mathrm{vol}(M) \mid M \text{ hyp. of finite volume}\}$ [well-ordered closed subset of $\mathbb{R}$]

Weeks mflds $\quad S^3 \setminus \quad\quad S^3 \setminus$



0.94.. $\quad$ 2.02988... $\quad$ 3.66...

[Limit pts result from Dehn filling.]

<u>Hodgson-Weeks Census</u>: $\approx$ Volume ≤ 6.5

11,031 $\qquad$ shortest geod ≥ 0.3

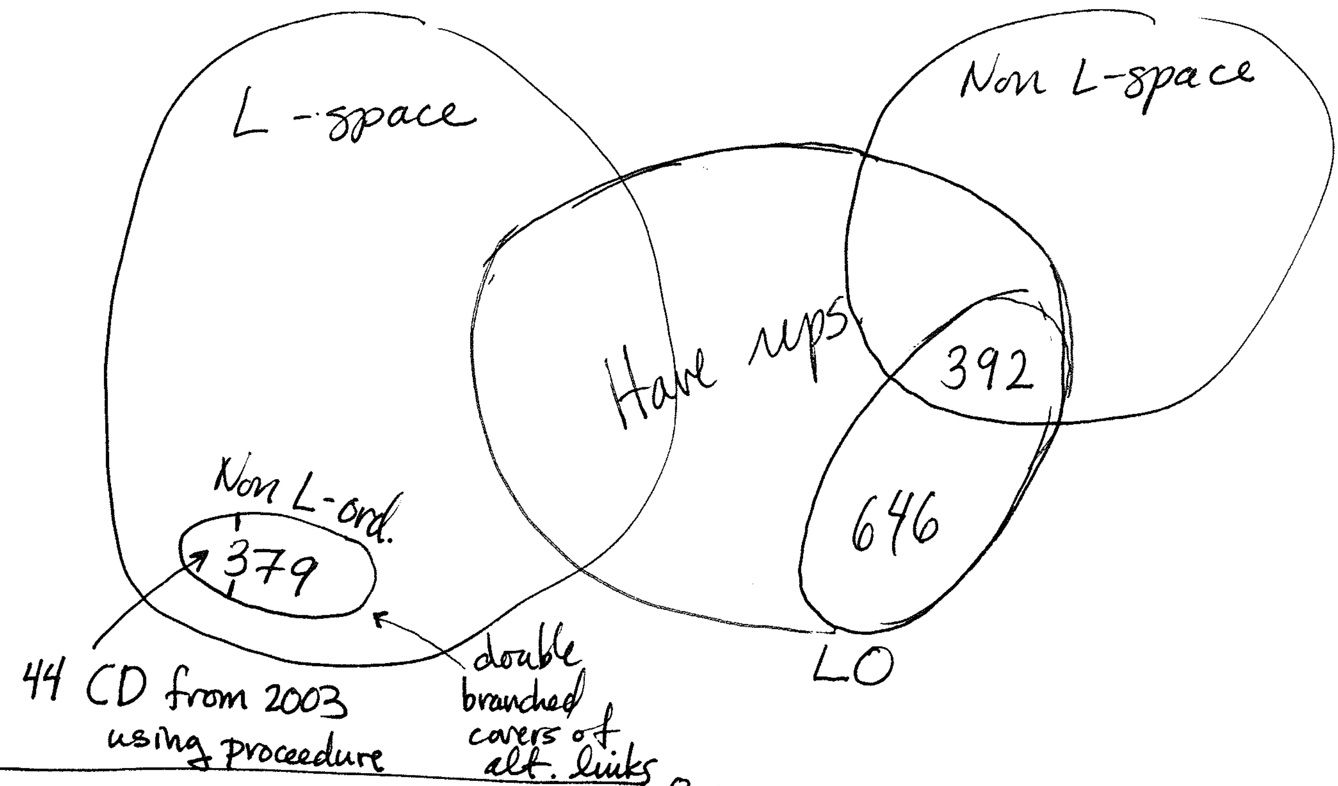
<u>Results</u>: Of the 10,903 QHSs in HW.

<u>L-spaces</u>: 4,584 (42%) $\quad$ <u>Non LO</u>: 379 (3%)

<u>Non L-spaces</u>: 2,742 (25%) $\quad$ <u>LO</u>: 1062 (10%)

<u>Unknown</u>: 3,578 (33%) $\quad$ <u>Unknown</u>: 9590 (87%)

Put up on board at start, at least the numbers.



<u>LO</u>: $\nexists$ an algorithm which decides if a finitely pres. group is LO. However, if G has solvable word problem, $\exists$ a procedure which, if G is non-LO provides a proof of this, and otherwise runs forever. [Build $\{g > e\} \cap B_e(r)$]

<u>Finding LO:</u>

$$\begin{tikzcd}
 & \widetilde{PSL_2\mathbb{R}} \arrow[d] \arrow[r, phantom, "\leq"] & \widetilde{Homeo^+(S^1)} \arrow[d] \arrow[r, phantom, "\leq"] & Homeo^+(\mathbb{R}) \\
\pi_1 M \arrow[r, "\rho"'] \arrow[ur, dashed] & PSL_2\mathbb{R} \arrow[r, phantom, "\lneq"] & Homeo^+(S^1 = P^1(\mathbb{R})) &
\end{tikzcd}$$

Reps ρ which lift to $\widetilde{PSL_2\mathbb{R}}$.

[Why do you have any such reps. $\pi_1 M$ is a balanced gp.]

One source:

$$\pi_1 M \leq \mathrm{Isom}^+(\mathbb{H}^3) = PSL_2\mathbb{C}$$
$$\lneq PSL_2 K \qquad K/\mathbb{Q} \text{ finite.}$$

When K has an embedding into $\mathbb{R}$, then get such a rep.

Obstruction to lifting: $e \in H^2(M;\mathbb{Z})$

Method:

- Solve Thurston gluing using PHC numerical methods.

Results:

4,375 reps for 3,361 manifolds.

Essentially these are evenly ~~dens~~ distributed

L-spaces	1244	(37%)
Non-L	891	(27%)
Unknown	1186	(35%)

Rep is about 2.5 times more likely to lift than expected.

Computing HF

[Sarkar-Wang 2006] HF is eff. computable

[L.O.T 2008–] Borded Floer Homology

[Bohua Zhan 2012] Fast prog. to compute HF.

Even more basic: Dehn filling and the exact triad

X with $\partial X =$ 

$X(\alpha) = X \cup$ Solid torus

<u>HW</u>: Dehn fillings on X which has an (ideal) $\underbrace{\text{triang. with} \leq 7 \text{ tet}}_{4587}$ and shortest geod ≥ 0.3.

An M in HW is a filling on an average of 5.3 of these X.

Also have 7,022 finite filling on these X.

Using the exact triad then gives 3,645 L-spaces in HW. Most of the rest come from Zhan's program (exp using exact triad).

How unlikely is this:

(a) Based on sizes of homology, would expect 40 L-spaces with a LO.

(b) Odds are $e^{-40} \approx 4 \times 10^{-18}$

Limitations: These are (almost) all Dehn surgeries on the minimally twisted 5-chain.