# Models of Sparse Random Graphs and Network Algorithms

## 12w5004 – Abstracts

Feb 5–10, 2012

**David Aldous** — *Some thoughts on data compression and entropy for sparse graphs with vertex-names*

After an informal review of classic Shannon theory of entropy and data compression for random sequences, I will speculate on analogs for sparse graphs with vertex-names.

**Charles Bordenave** — *How does a uniformly sampled Markov chain behave ?*

This is joint work with P. Caputo and D. Chafai. In this talk, we will consider various probability distributions on the set of stochastic matrices with n states and on the set of Laplacian/Kirchhoff matrices on n states. They will arise naturally from the conductance model on n states with i.i.d conductances. With the help of random matrix theory, we will study the spectrum of these processes. An emphasis will be put on the case of the simple random walk on a sparse directed Erdos-Renyi graph.

**Shankar Bhamidi** — *Limited choice and randomness in evolution of networks*

The last few years have seen an explosion in network models describing the evolution of real world networks. In the context of math probability, one aspect which has seen an intense focus is the interplay between randomness and limited choice in the evolution of networks, ranging from the description of the emergence of the giant component, the new phenomenon of ”explosive percolation” and power of two choices. I will describe on going work in understanding such dynamic network models, their connections to classical constructs such as the standard multiplicative coalescent and local weak convergence of random trees.

**Nicolas Fraiman** — *Connectivity of Bluetooth graphs*

We study the connectivity of random Bluetooth graphs, these are obtained as “irrigation subgraphs” of the well-known random geometric graph model. There are two parameters that control the model: the radius r that determines the visible neighbors of each node and the number of edges c that each node is allowed to have. The randomness comes from the distribution of nodes in space and the choices of each vertex. We characterize the connectivity threshold (in c) for values of r close the critical value for connectivity in the underlying random geometric graph. This is joint work with Nicolas Broutin, Luc Devroye and Gabor Lugosi.

**Marc Lelarge** — *A new approach to the orientation of random hypergraphs*

A $h$-uniform hypergraph $H = (V, E)$ is called $(\ell, k)$-orientable if there exists an assignment of each hyperedge $e \in E$ to exactly $\ell$ of its vertices $v \in e$ such that no vertex is assigned more than $k$ hyperedges. Let $H_{n,m,h}$ be a hypergraph, drawn uniformly at random from the set of all $h$-uniform hypergraphs with $n$ vertices and $m$ edges. In this paper, we determine the threshold of the existence of a $(\ell, k)$-orientation of $H_{n,m,h}$ for $k \geq 1$ and $h > \ell \geq 1$, extending recent results motivated by applications such as cuckoo hashing or load balancing with guaranteed maximum load. Our proof combines the local weak convergence of sparse graphs and a careful analysis of a Gibbs measure on spanning subgraphs with degree constraints. It allows us to deal with a much broader class than the uniform hypergraphs.

**Pat Morin** — *Maximum interference in the highway and related models.*

Given a set D of n disks, the interference of a point p is defined as the number of disks of D that contain p. The interference of D is the maximum interference over all centers of disks in D. In this talk, we discuss upper and lower bounds on maximum interference in 1 dimension, 2 dimensions, in the worst case, and in probabilistic settings.

**Tobias Muller** — *Colouring random geometric graphs*

If we pick points $X_1, \ldots, X_n$ at random from $d$-dimensional space (i.i.d. according to some probability measure) and fix a $r > 0$, then we obtain a random geometric graph by joining points by an edge whenever their distance is $< r$. I will talk about some results on the chromatic number and the clique number of this model.

**Matthew Penrose** — *Connectivity of G(n,r,p)*

Consider a graph on n vertices placed uniformly independently at random in the unit square, in which any two vertices distant at most r apart are connected by an edge with probability p. This generalises both the classical random graph and the random geometric graph. We discuss the chances of its being disconnected without having any isolated vertices, when n is large, for various choices of the other parameters.

**Justin Salez** — *Joint distribution of distances in large random regular networks.*

We study the array of point-to-point distances in large random regular graphs equipped with exponential edge-weights. The asymptotic marginal distribution of a single entry is now well-understood, thanks to the work of Bhamidi, van der Hofstad and Hooghiemstra (2010). In this talk, we will show that the whole array, suitably recentered, converges in the weak sense to a rather simple infinite random array. Our proof consists in analyzing the invasion of the network by several mutually exclusive flows emanating from different sources and propagating simultaneously at unit rate along the edges. The result applies to both the random regular multi-graph produced by the configuration model and the uniform regular simple graph.

**Csaba Toth** — *Convex partitions.*

A convex partition is a planar straight-line graph where every bounded face is convex and the complement of the outer face is also convex. Two results are presented in this talk: (1) For every n points in the plane, there is a convex partition G such that the total edge length of G is at most O(log n/log log n) times that of a Euclidean minimum spanning tree (EMST) for the n points, and this bound is the best possible. (2) If G is a convex partition and the outer face has O(1) edges, then G contains a monotone path of at least Omega(log n /log log n) edges, and this bound is the best possible. (Joint work with Adrian Dumitrescu.)

**Joseph Yukich** — *Probabilistic Analysis of Some Geometric Networks*

We survey some techniques for establishing general limit theorems in stochastic geometry (laws of large numbers, variance asymptotics, and central limit theorems). We show how the general theorems may be applied to deduce the limit theory for various functionals of random geometric graphs, including, for example, network connectivity functionals, clique count, total edge length, and component count. The talk is based on joint work with M. Penrose, T. Schreiber, and Y. Baryshnikov.