# North West Functional Analysis Seminar (BIRS 12w2163)

Organizers:

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#### **1** Overview of the Workshop

The Northwest Functional Analysis Seminar (NWFAS) is a bi-annual regional scientific meeting of academic researchers and graduate students in functional analysis. Participants are drawn from universities in Western Canada and the American Northwest. The two main goals of the meeting are to enable researchers from a large geographical area to stay in touch with developments in the general field but outside their respective areas of specialization, and to provide a forum for junior researchers (junior faculty, postdoctoral fellows, and graduate students) to present their results to a wider audience and to form contacts with other functional analysts within the region. The majority of talks are given by junior researchers, with a few presentations by senior researchers on topics currently attracting high levels of interest.

This two-day workshop is the fifth edition of the seminar, following very successful earlier meetings held in 2003, 2005, 2007 and 2009. In addition to presenting a strong scientific program, the workshop provides the only venue by which the region's researchers in functional analysis come into contact regularly. The NWFAS also has formative role in developing the profiles of early-career researchers. The session is organized around a dozen forty-minute talks, allowing time for informal discussion.

A PIMS Collaborative Research Group in Operator Algebras and Noncommutative Geometry led by researchers in Victoria, Edmonton, and Regina wrapped up its activities by the spring of 2012, and this session was an opportunity to discuss the results of the CRG.

# 2 Overview of the Field

Functional analysis is a very broad field, encompassing the analysis of general families of functions on spaces (topological, or measurable) and the algebras of operators that act on those functions. It is the basis for foundational results in a wide variety of fields of mathematics and its applications, including the existence and uniqueness of solutions to differential equations, the analysis of dynamical systems, harmonic analysis, mathematical physics, quantum mechanics, numerical methods, among others. Hilbert spaces, Banach spaces, Fourier spaces, group algebras, Banach algebras, C\*-algebras, von Neumann algebras, are some of the main objects of study.

Topics of interest include the embedding and approximation properties for families of functions on topological or measurable spaces and groups, properties of individual and algebras of (linear) operators on these families of functions, classification and identification problems for these algebras, quantum extensions of classical notions in functional analysis.

### **3** Recent Developments and Open Problems

As a broad area, there are many open problems in functional analysis and its sub-disciplines. Some are very specific – for instance, when does a function come about as a restriction of some differentiable function (the Whitney problem), or when does a linear operator have an invariant subspace (the invariant subspace problem). Some are very general – for instance, what is a useful classification scheme for C\*-algebras (e.g. the Elliott program). Some bring together diverse areas into some commonality – for instance, how do the invariants of a dynamical system correlate with the invariants of similar C\*-algebras that encode the dynamics. A long standing problem has been to generalize the beautiful results of Fourier analysis on locally compact abelian groups to a wide class of groups and their quantum analogues.

# **4 Presentation Highlights**

The highlight of the presentations was the wide diversity of talks delivered and the success of the young researchers to show their stuff. This meeting brought together researchers in harmonic analysis (who study the functional analysis of groups) with researchers in C\*-algebras, others in quantum physics, and more in dynamical systems and even tilings. For the first time with the North West Functional Analysis Seminar, we were able to post videos of the talks online, through the BIRS web server, which allowed these talks to reach a much wider audience. The speakers were very accommodating in allowing their presentations to be recorded and broadcast.

### 5 Scientific Progress Made

The progress made here was in the communication between the diverse groups, helping to identify what expertise there is within Western Canada to collaborate on some of these challenges. For instance, Dr. Gilad Gour, a quantum physicist at the University of Calgary, gave his very first talk to a functional analysis crowd, and discovered that many of his problems in QM can be formulated, and perhaps solved, in the framework of C\*-algebra, and in particular in the language of completely positive maps that represent quantum channels and other constructions in QM. He went on to reformulate his talk for a national mathematical meeting in the summer, based on his exposure to the audience at NWFAS. Dr. Antoine Julien gave an excellent presentation on substitution tilings, a geometric construction that leads to aperiodic structures such as Penrose tiles, which it turns out results in a spectral triple that is a important notion in linear operator theory. A large group of presentations focused on the harmonic analysis of groups: compactifications, factorization, amenability, trace formulas, among other. A very interesting presentation by a postdoctoral fellow, David Alonso-Gutierrez focused on the convex geometry of high dimensional bodies, which has important connections to the properties of linear operators in high, and infinite, dimensional linear spaces.

# **6** Outcome of the Meeting

Some of the discussions at the meeting have led to ongoing collaborations between research teams – for instance, many of the questions coming up in quantum communications and quantum cryptography are nicely phrased in the language of states, positive operators, completely positive maps, in C\*-algebras. The Quantum group of physicists at Calgary that Dr. Gour works with is now interacting with the C\*-algebraists to extend this language into mathematical tools that can attack some of these problems.

The communications also made clear the strengths of these groups of researchers in Western Canada – the strong groups in harmonic analysis and geometry at Alberta and Saskatchewan, quantum research at Alberta and Calgary, dynamics at Victoria, are some of the examples.

We left encouraged with the revealed strengths and looking forward to further interactions.