## BIRS 2012 – Nils Bruin: 5. Workshop Problems – continued

**N7.** Consider the genus 1 curve

$$C\colon y^2 = 2x^4 - 17.$$

Writing $\theta = \sqrt[4]{17/2}$ and $L = \mathbb{Q}(\theta)$, we can consider the map

$$\begin{array}{rccc} \gamma\colon & C(\mathbb{Q}) & \to & L^\times/L^{\times 2}\mathbb{Q}^\times \\ & (x,y) & \mapsto & x-\theta \end{array}$$

which plays the same role we have seen before. Check that any $\delta \in L^\times$ representing an element in the image of $\gamma$ would have to have $N(\delta) \in 2\mathbb{Q}^{times2}$. Verify that such $\delta$ do not exist.

For added satisfaction, check that $C$ does have points everywhere locally.

**N8.** Magma has a command `TwoCoverDescent` that implements the computation of fake 2-Selmer sets of hyperelliptic curves. Read its documentation and explain its computations for the curve

$$C\colon y^2 = -x^6 + 2x^5 + 3x^4 - x^3 + x^2 + x - 3$$

You might want to run `SetVerbose("Selmer",4);` to see some of the work it is doing.

**N9.** Determine the rational points on

$$C\colon y^2 = (x^2+3)(x^4 - 18x^2 + 9).$$