

# Spatial population dynamics and control



# Broad overview

- Aspects of stochasticity at a single location (with Brett Melbourne)
- Stochastic spatial spread (with Brett Melbourne)
- Control of invasive species (with Caz Taylor, Richard Hall, Julie Blackwood, Chris Costello, Rebecca Epanchin-Niell plus others for experimental/field aspects)

# New stochastic Ricker models: extinction risk could be higher

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# Extinction

- Deterministic and stochastic causes
- Demographic stochasticity \*

  - random births & deaths: within-individual scale

- Environmental stochasticity \*

  - random births & deaths: population scale

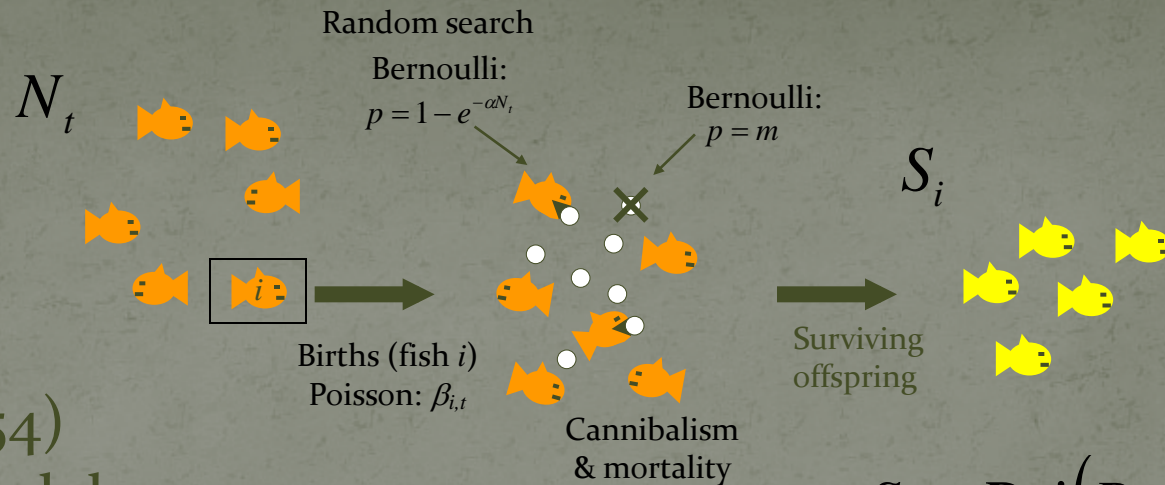
- Demographic heterogeneity

  - vital rates (birth/death): between-individual scale

- Sex ratio stochasticity

  - random: male or female?

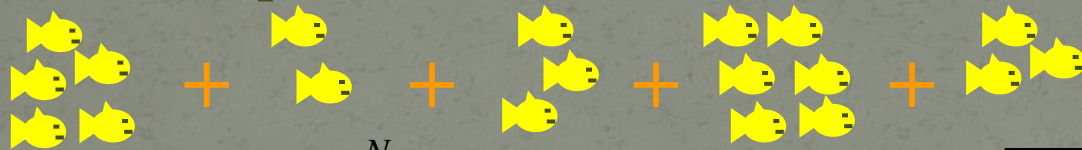
# Ricker (1954) Fisheries model



$$S_i \sim \text{Poi}(R e^{-\alpha N_t})$$

Sum up survivors over all adults

$$R = \beta(1 - m)$$

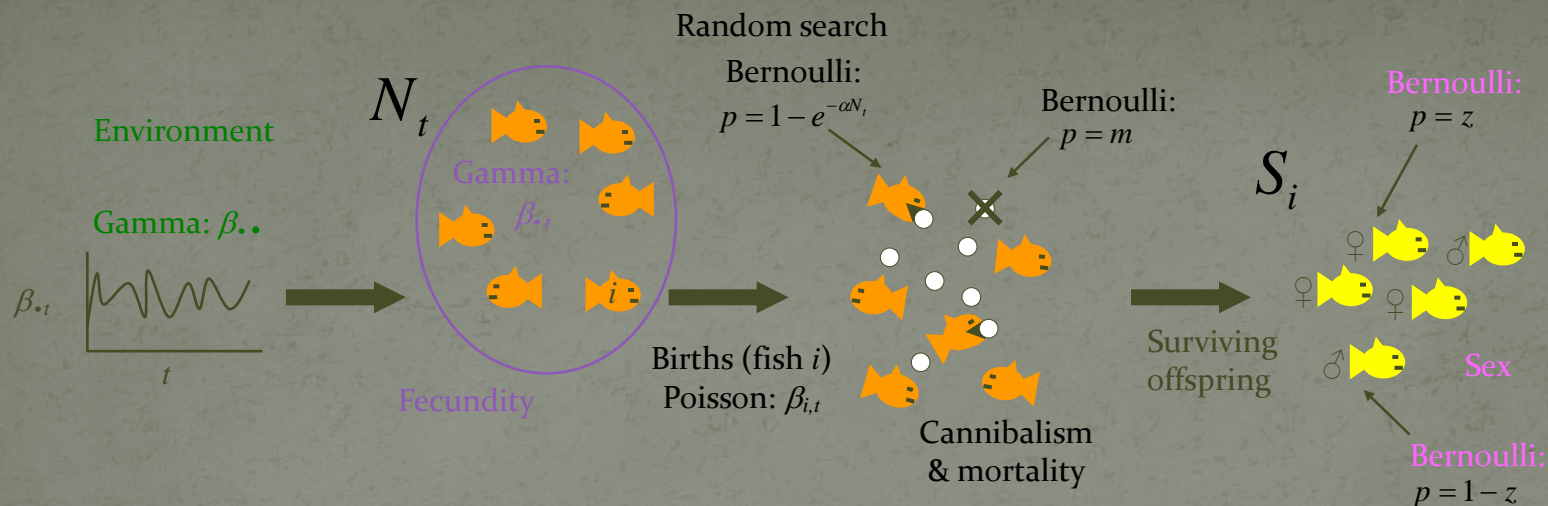


$$N_{t+1} = \sum_i^{N_t} S_i \sim \text{Poi}(N_t R e^{-\alpha N_t})$$

Poisson Ricker

(sum of Poissons is Poisson)

Model for demographic stochasticity



Demographic stochasticity	Environmental stochasticity	Demographic heterogeneity	Sex	Model	Abbr.
●				Poisson	P
●	●			Neg bin-environmental	NBe
●		●		Neg bin-demographic	NBd
●	●	●		Neg bin-gamma	NBg
●			●	Poisson-binomial	PB
●	●		●	Neg bin-binomial-environmental	NBBE
●		●	●	Neg bin-binomial-demographic	NBBd
●	●	●	●	Neg bin-binomial-gamma	NBBg

All models have mean:  $N_{t+1} = N_t R e^{-\alpha N_t}$

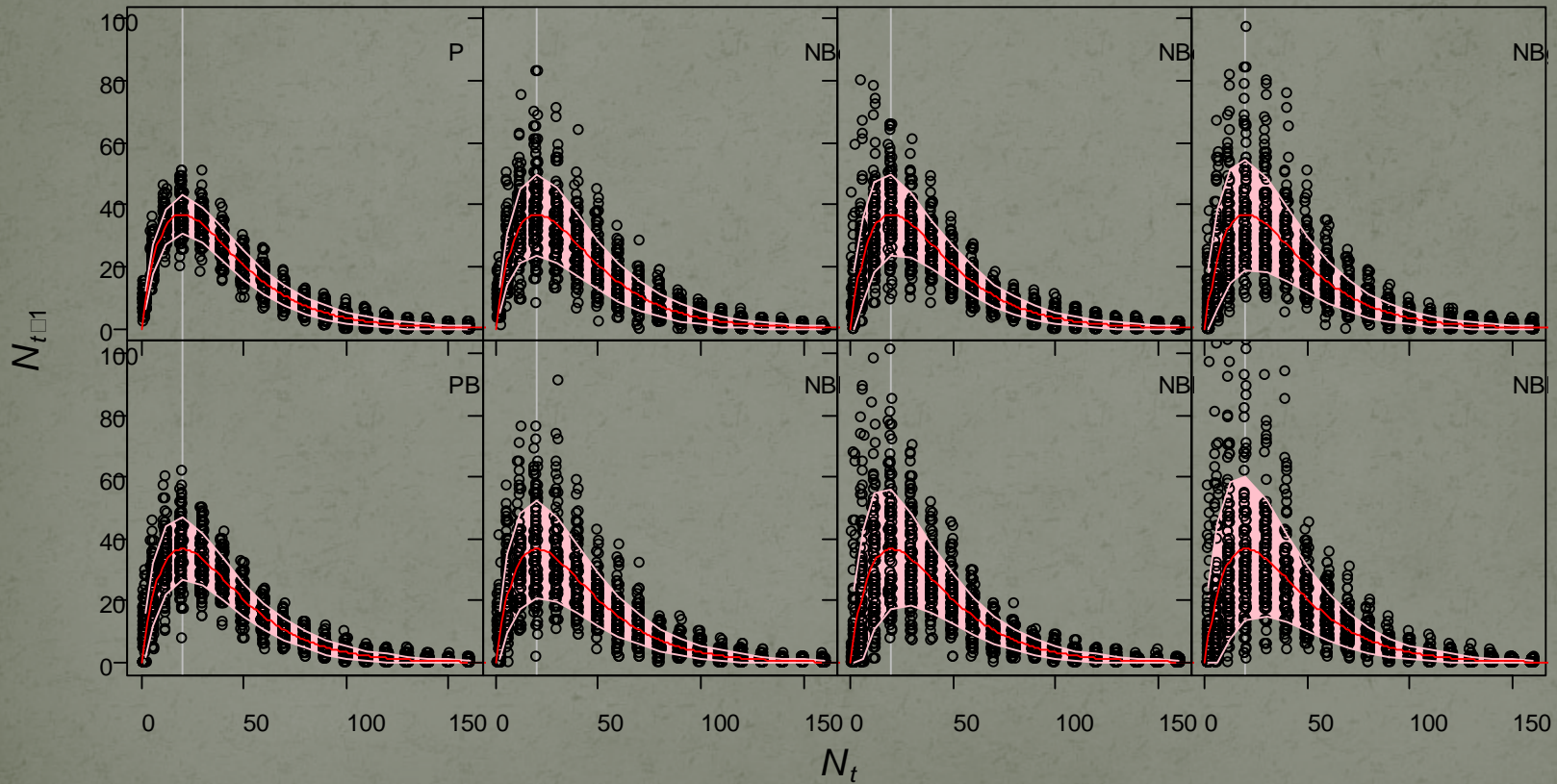
# Stochastic production functions

Demographic  
stochasticity

Environmental  
stochasticity

Demographic  
heterogeneity

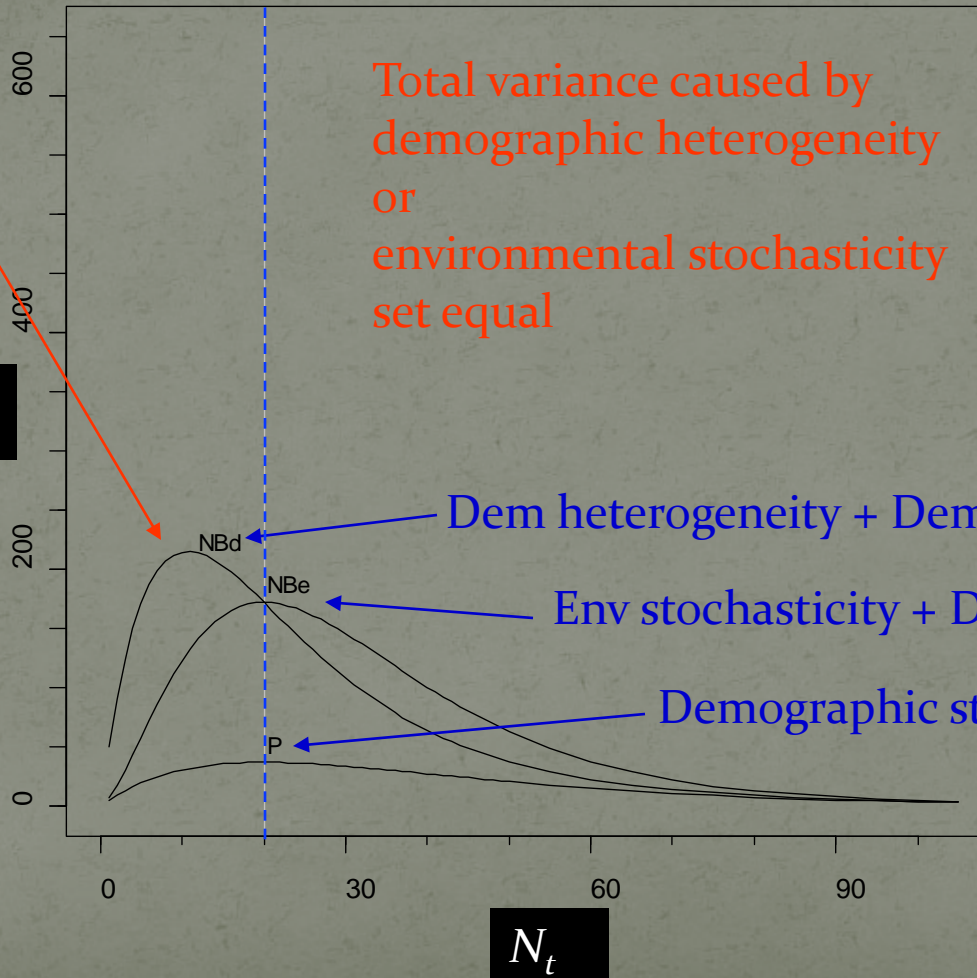
Env stoch +  
Dem het



# Variance in $N_{t+1}$

Variance is density dependent

$Var(N_{t+1})$

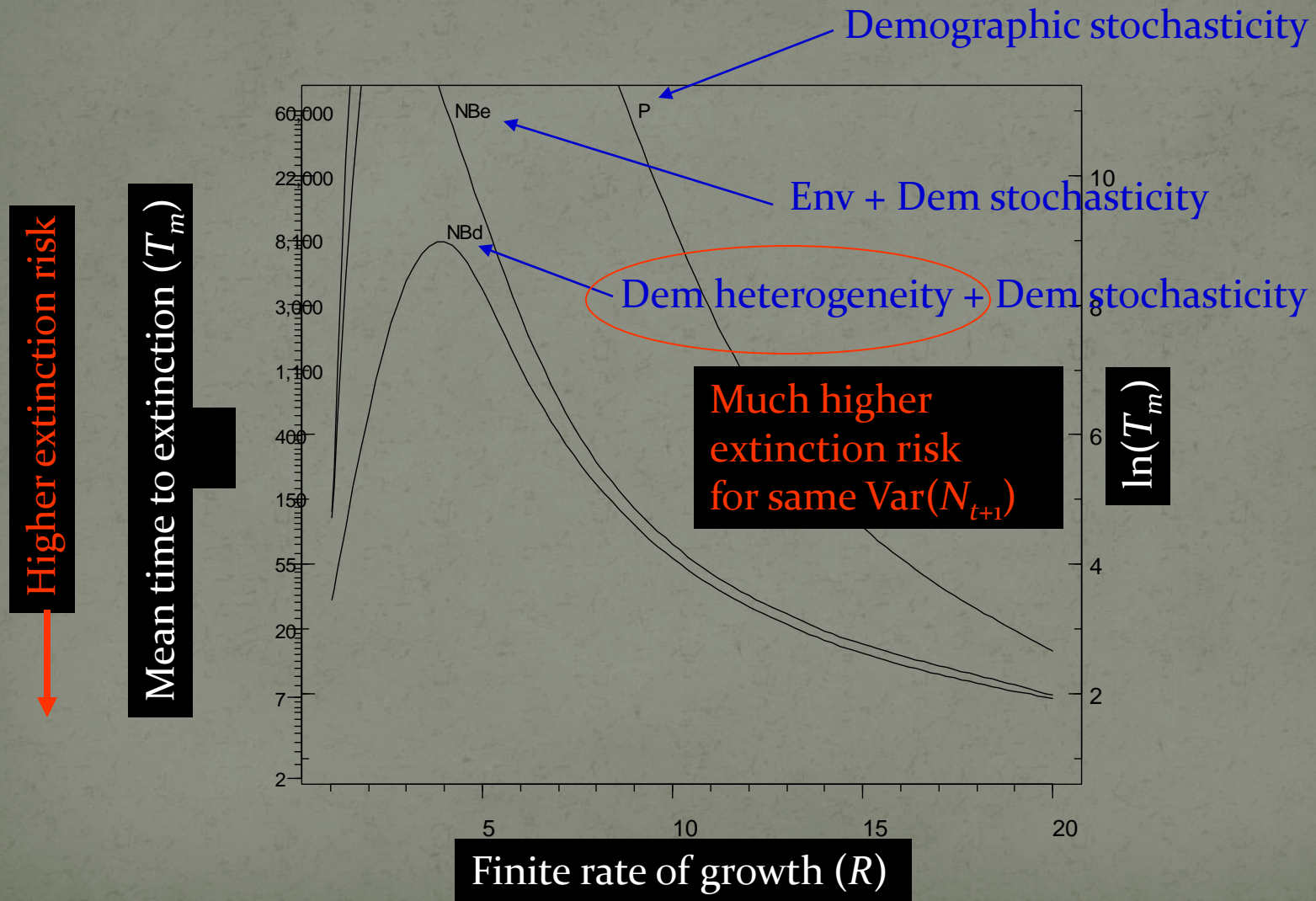




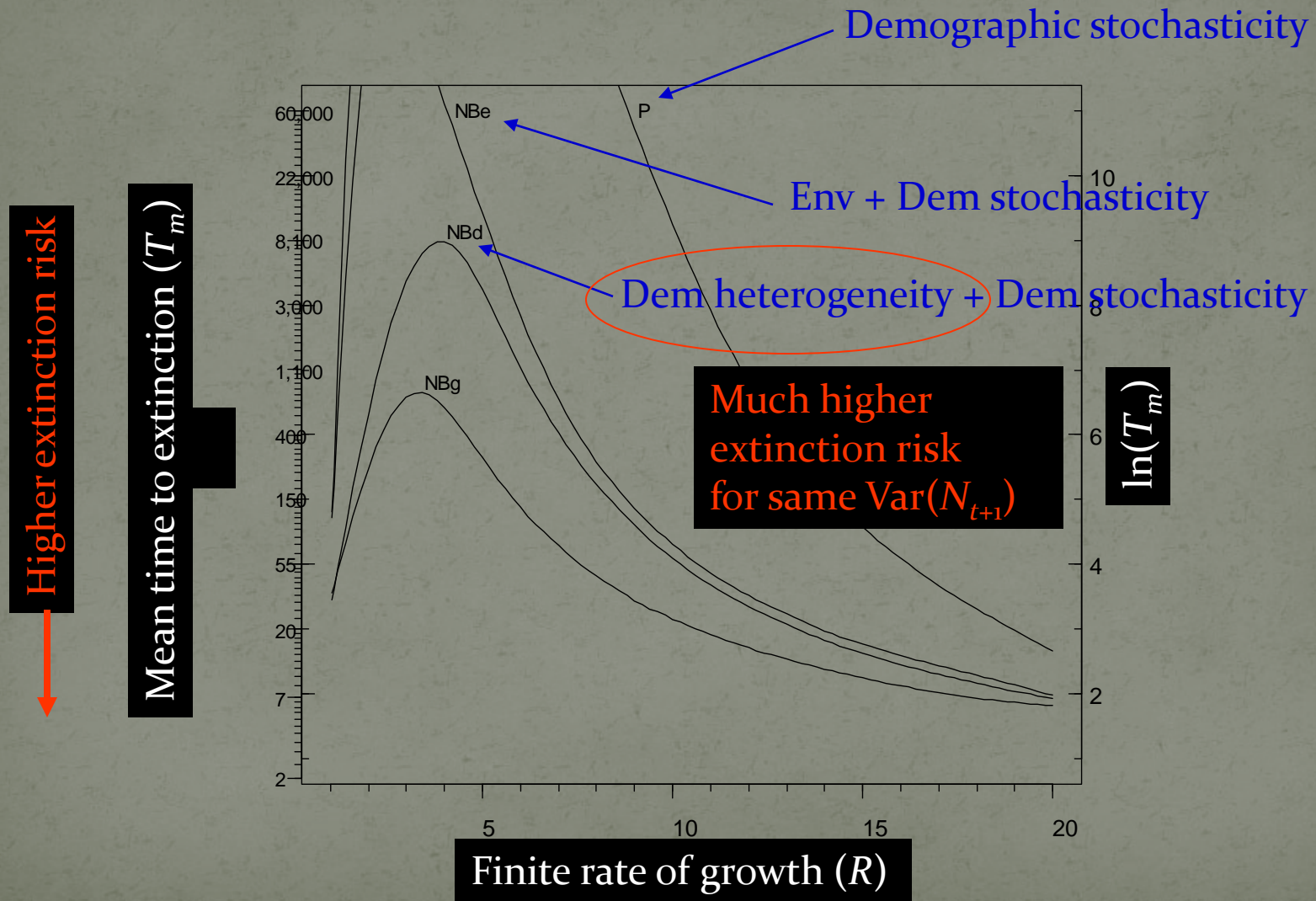
# Extinction times

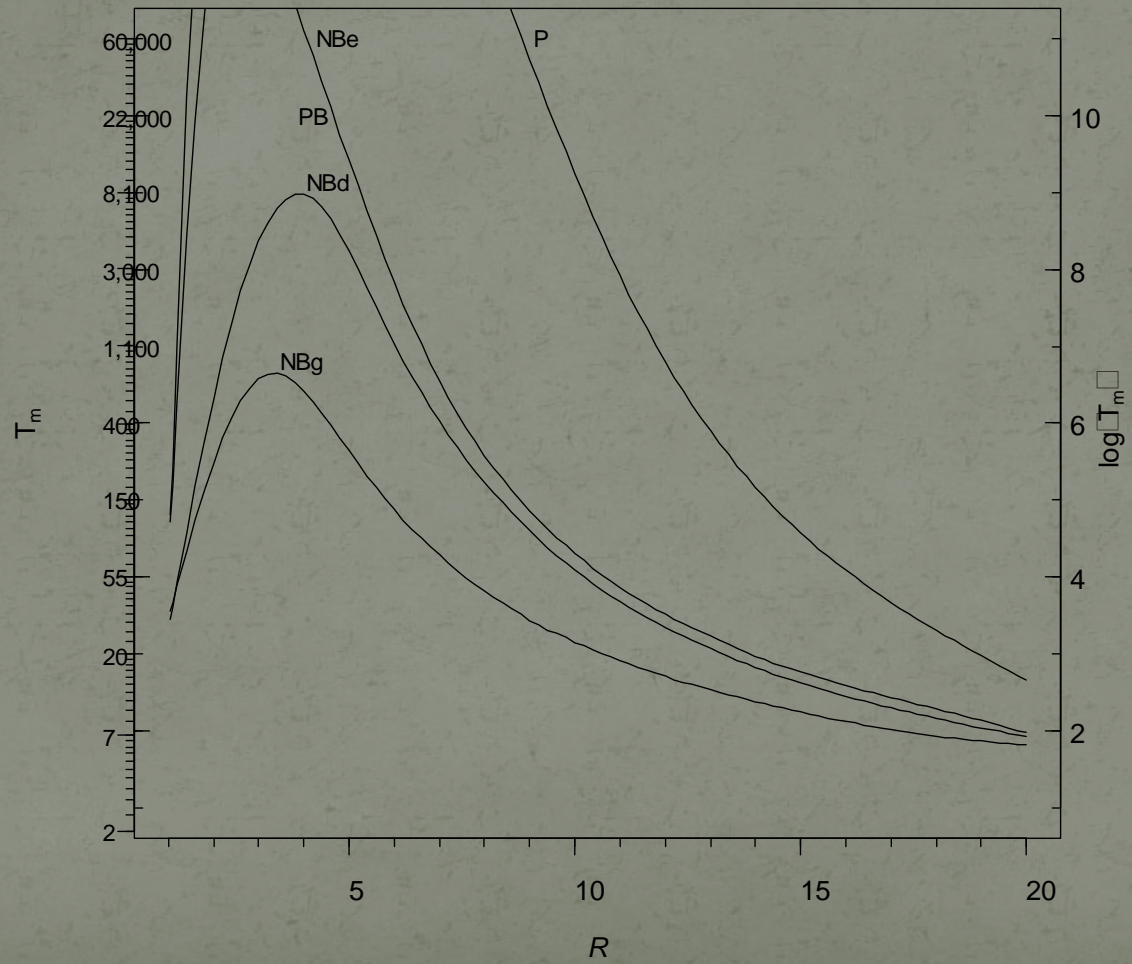
- Intrinsic mean time to extinction  
(Grimm & Wissel 2004, *Oikos* 105: 501-511)

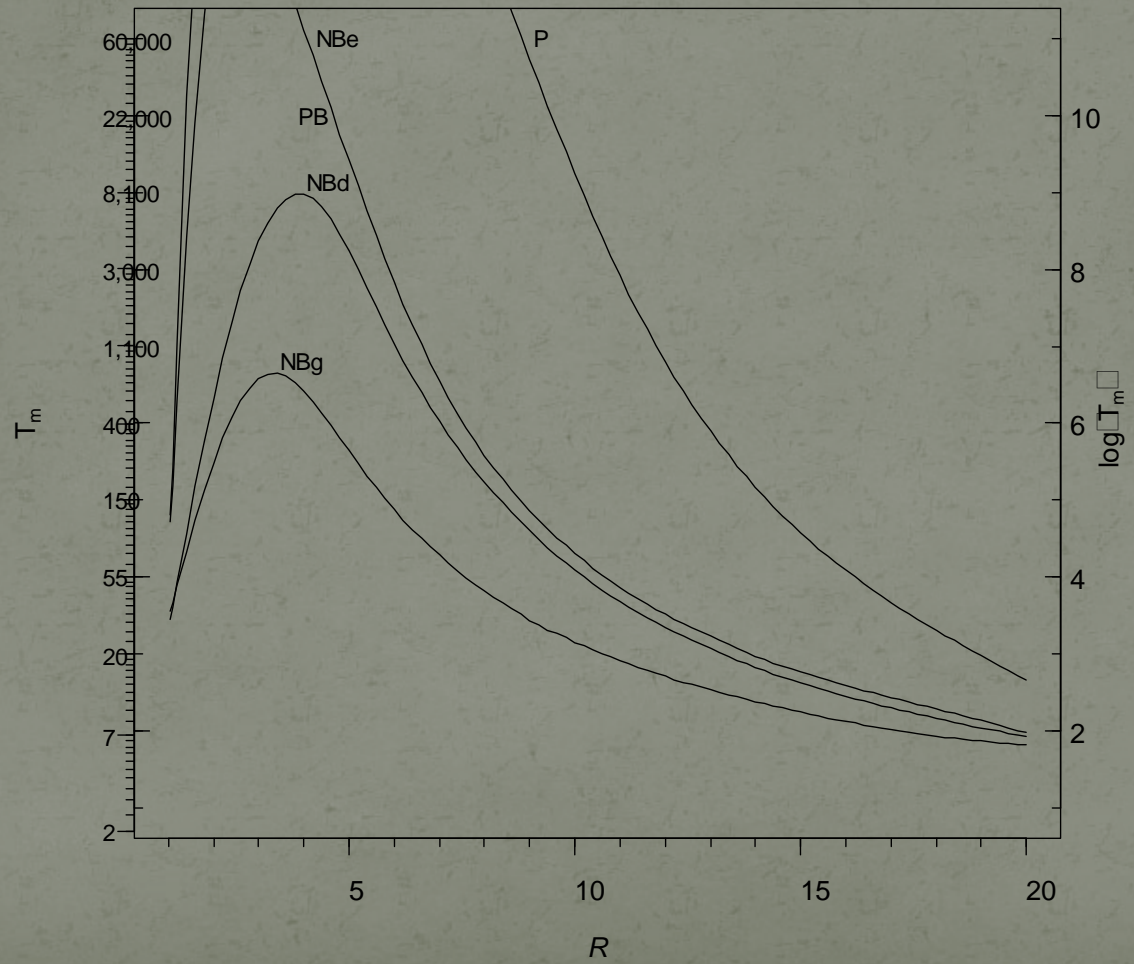
# Extinction times



# Extinction times







# Fitting models to data

## Likelihoods:

Poisson Ricker (demographic stochasticity)

$$\Pr(N_{t+1} = n_{t+1} \mid \theta, N_t = n_t) = \frac{e^{-\mu} \mu^{n_{t+1}}}{n_{t+1}!}, \quad \mu = n_t R e^{-\alpha n_t}$$

NBBg Ricker (all sources of stochasticity)

$$\int_{R_E=0}^{\infty} G(R_E) \sum_{F=0}^{n_t} \binom{n_t}{F} z^F (1-z)^{n_t-F} \binom{n_{t+1} + Fk_D - 1}{Fk_D - 1} \left( \frac{\lambda}{Fk_D + \lambda} \right)^{n_{t+1}} \left( \frac{Fk_D}{Fk_D + \lambda} \right)^{Fk_D}$$

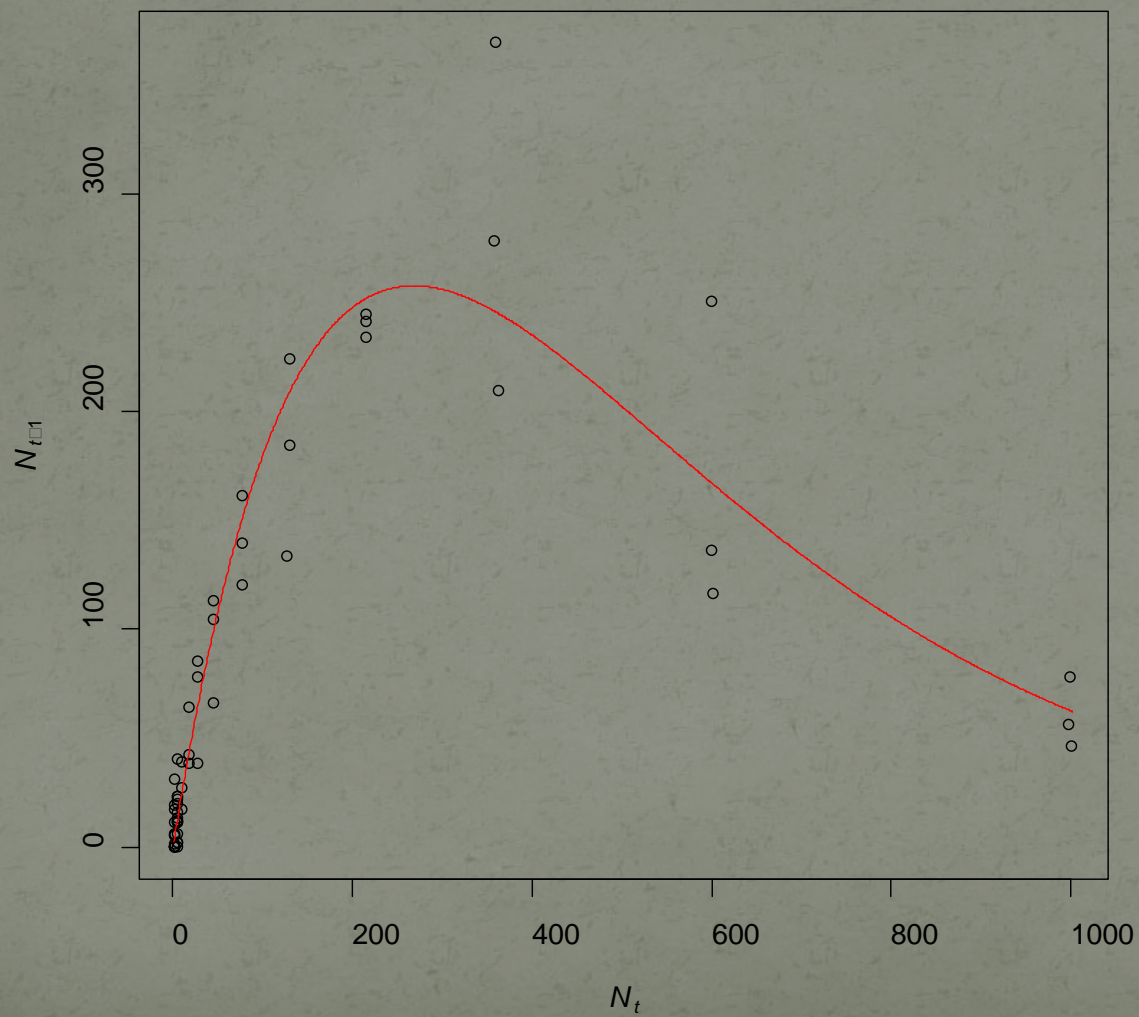
$$G(R_E) = R_E^{k_E - 1} \exp\left(-\frac{R_E k_E}{R}\right) \left(\frac{k_E}{R}\right)^{k_E} \frac{1}{\Gamma(k_E)}, \quad \lambda = F \frac{R_E}{z} e^{-\alpha n_t}$$

# Experiment

- *Tribolium castaneum* (red flour beetle)
- Same life history as Ricker's fish
- Cannibalism



# Experimental data





# Model comparison

Model	Demographic heterogeneity			Environmental stochasticity		$\Delta AIC$
	$R$	$\alpha$	$k_D$	$k_E$		
Poisson (dem stoch)	2.5	0.0036			336	
Negative binomial (dem het)	2.6	0.0037	0.15		18	
Negative binomial (env stoch)	2.7	0.0038		2.0	56	
Negative binomial-gamma	2.6	0.0037	0.26	29.2	5	
Poisson-binomial (sex)	2.7	0.0038			87	
NB-binomial (dem het)	2.6	0.0037	0.39		17	
NB-binomial (env stoch)	2.8	0.0038		13.1	10	
NB-binomial-gamma (all)	2.6	0.0037	1.15	26.6	0	

Demographic heterogeneity mistaken for environmental stochasticity

Small  $k$  value = big variance

# Conclusion

- Many species could be at much higher risk than we thought!
- ... because simpler models can wrongly conclude that environmental stochasticity dominates, whereas demographic variance has higher extinction risk (for the same variance in abundance)
- Important to include all stochasticity

Melbourne B. A. & Hastings A. (2008).

Extinction risk depends strongly on factors contributing to stochasticity.

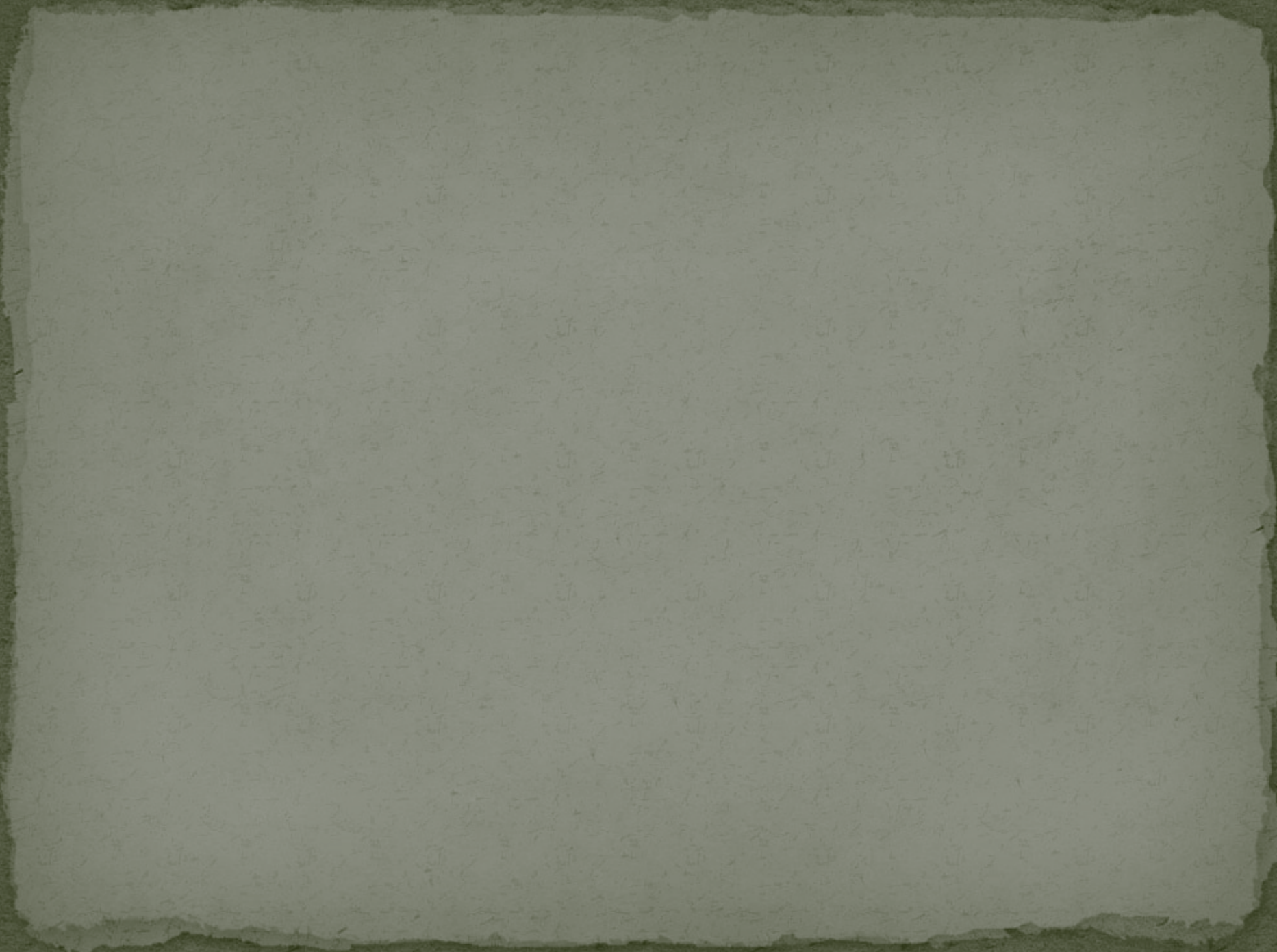
Nature 454: 100-103.

Assistants

Michelle Gibson, Dylan Hodgkiss,  
Claire Koenig, Tom McCabe, Devan  
Paulus, David Smith, Nancy Tcheou,  
Roselia Villalobos, Motoki Wu



DEB 0516150



# Stochastic dynamics of invasive spread

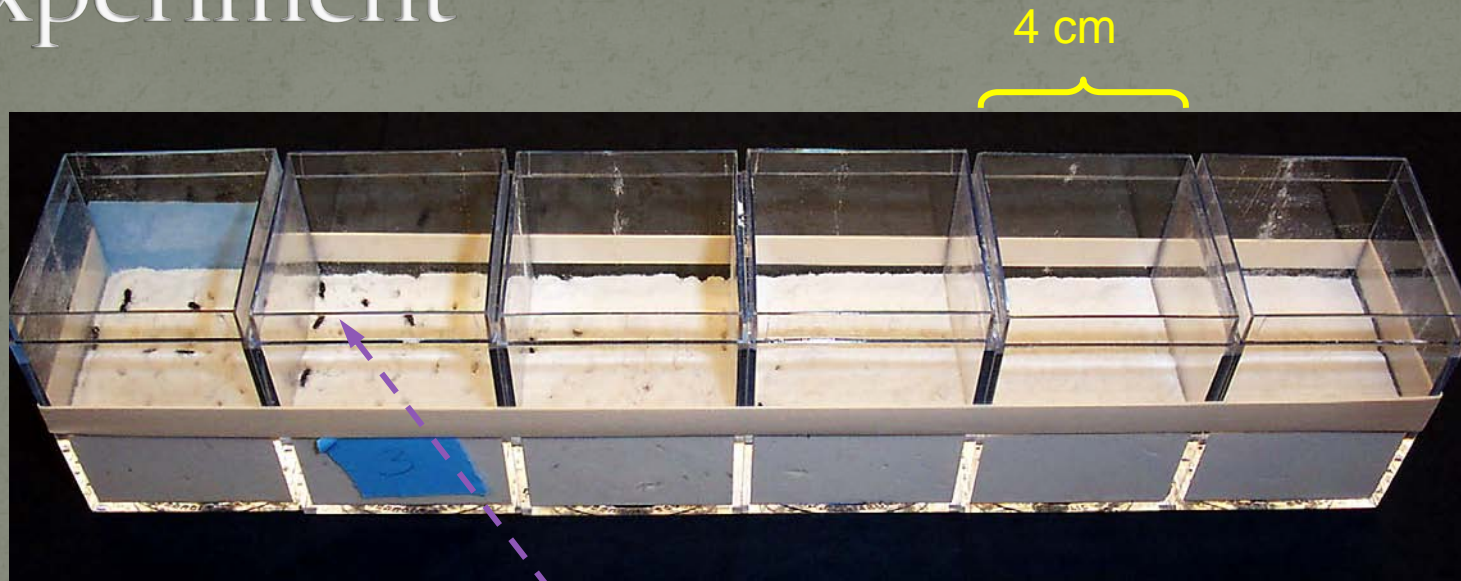
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Brett Melbourne & Alan Hastings  
University of California, Davis

# Stochastic spread

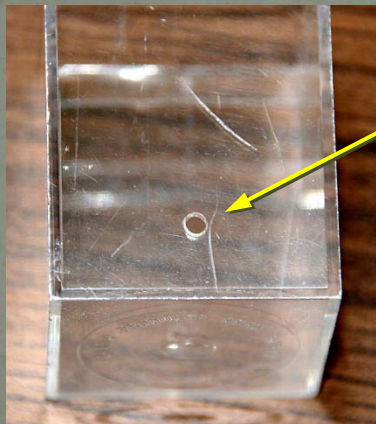
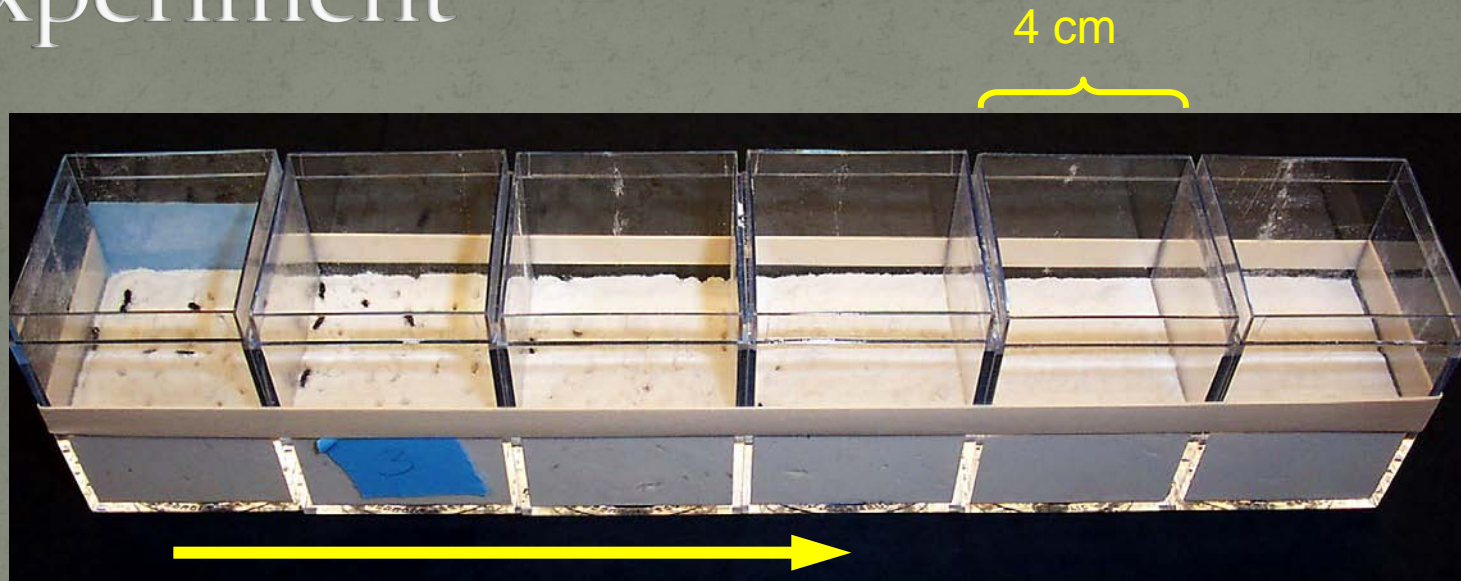
- Stochasticity (→ variance in speed)
  - Population growth & dispersal
  - Demographic, environmental, genetic
- Repeat an invasion: different
  - Nature: one realization
- Real invasions can't be repeated
  - Many times, identical conditions
  - Laboratory microcosms

# Experiment



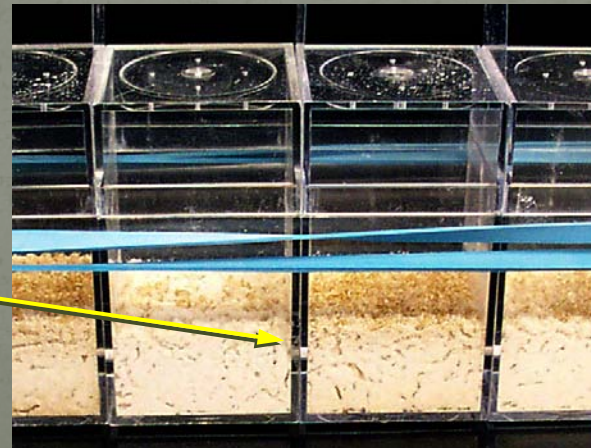
Flour beetle:  
*Tribolium*  
*castaneum*

# Experiment



Hole

Tunnel





# Lifecycle in laboratory

- Discrete time (35 day cycle)

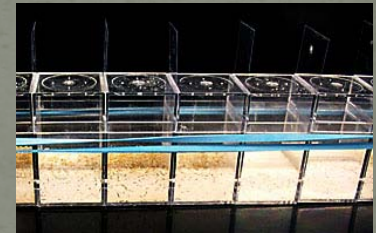
# Lifecycle in laboratory

- Discrete time (35 day cycle)
  - 1) Adults lay eggs (24 hr)
    - Fences installed; adults removed



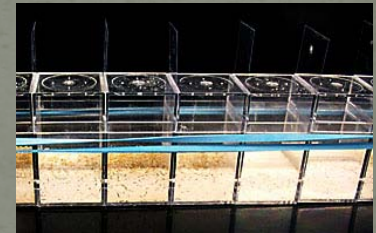
# Lifecycle in laboratory

- Discrete time (35 day cycle)
  - 1) Adults lay eggs (24 hr)
    - Fences installed; adults removed
  - 2) Larvae grow
    - Adults emerge (ca day 30)



# Lifecycle in laboratory

- Discrete time (35 day cycle)
  - 1) Adults lay eggs (24 hr)
    - Fences installed; adults removed
  - 2) Larvae grow
    - Adults emerge (ca day 30)
  - 3) Adults disperse (48 hr)
    - Census after dispersal



# Experiment

- 30 landscapes
- Constant environment
- 13 generations





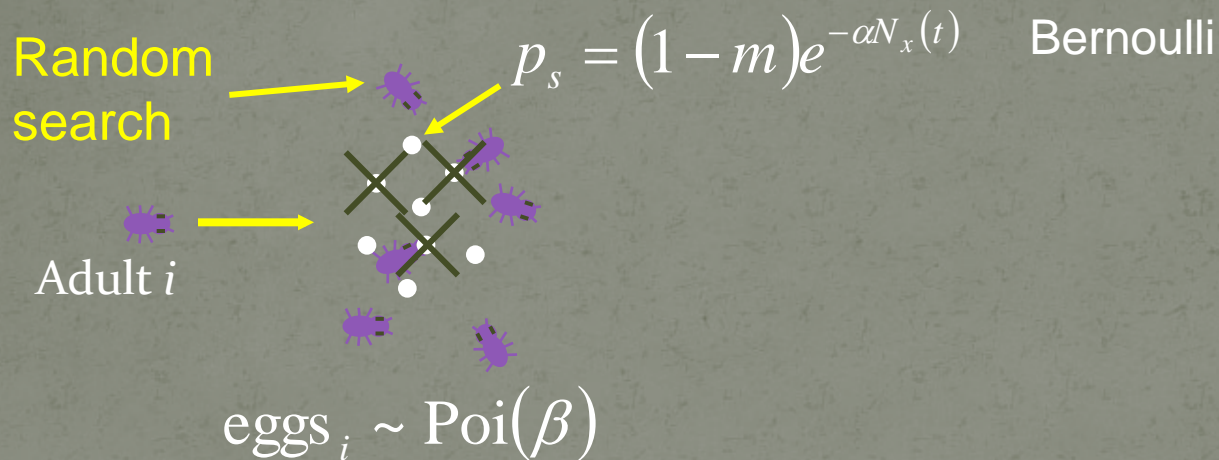
# Mechanistic stochastic models

- Individual based derivation
- Predict mean, variance, & prob dist

$$N_x(t+1) = \text{growth} + \text{migration}$$

# Growth (birth, surv) in a patch

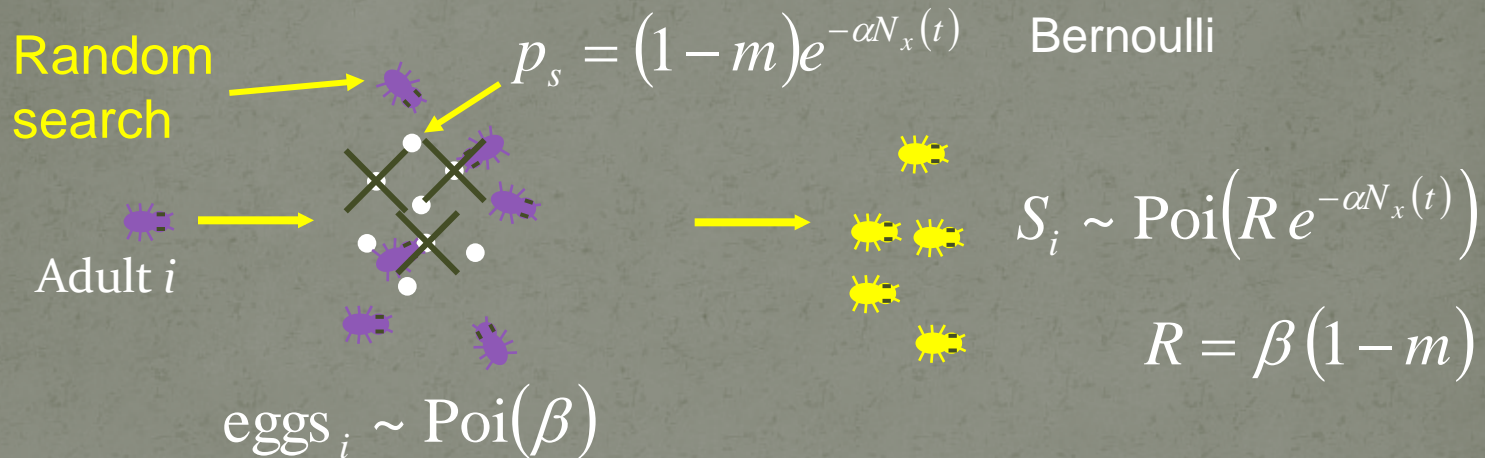
Survive cannibalism & DI mortality





# Growth (birth, surv) in a patch

Survive cannibalism & DI mortality



Patch scale:

$$N_x(t+1) = \sum_i^{N_x(t)} S_i \sim \text{Poi}(N_x(t) R e^{-\alpha N_x(t)})$$

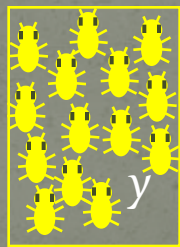
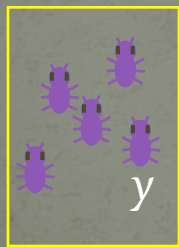
Put up survivors over all patches Ricker

# Stochastic Ricker models

Model	Dem stoch ( <i>Birth, Surv</i> )	Sex	Env stoch	Dem het
Poisson				
Neg bin				
Neg bin (Density Dep.)				
Neg bin-gamma				
Poisson-binomial				
Neg bin-binomial				
Neg bin-binomial (DD)				
Neg bin-binomial-gamma				

# Stochastic spatial model

Patch scale growth



Number surviving is  
a random variable:  
stochastic Ricker

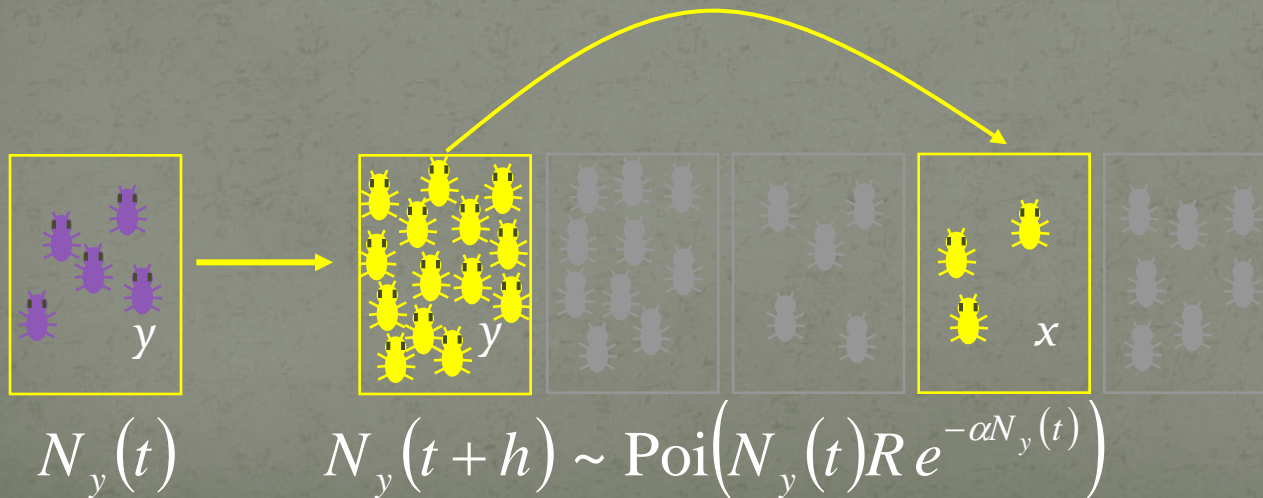
$$N_y(t)$$

$$N_y(t+h) \sim \text{Poi}\left(N_y(t) R e^{-\alpha N_y(t)}\right)$$

# Stochastic spatial model

Migration from patch  $y$  to  $x$

$$M_{y \rightarrow x} \sim \text{Poi}\left(p_{y \rightarrow x} N_y(t) R e^{-\alpha N_y(t)}\right)$$



# Stochastic spatial model

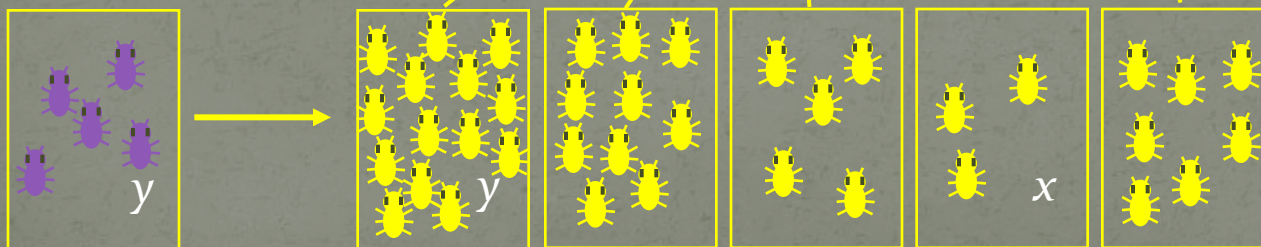
Landscape scale

$$N_x(t+1) = \sum_y M_{y \rightarrow x} \sim \text{Poi} \left( \sum_y p_{y \rightarrow x} N_y(t) R e^{-\alpha N_y(t)} \right)$$

Sum contributions from all patches

$$M_{y \rightarrow x} \sim \text{Poi} \left( p_{y \rightarrow x} N_y(t) R e^{-\alpha N_y(t)} \right)$$

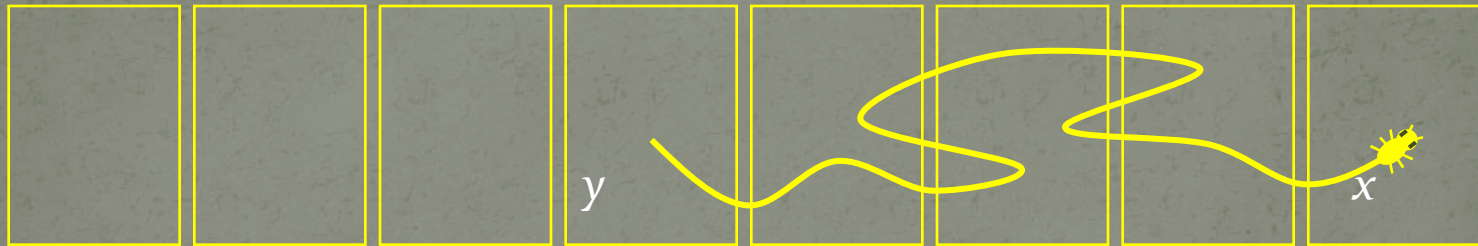
Other Ricker models work the same way



$$N_y(t)$$

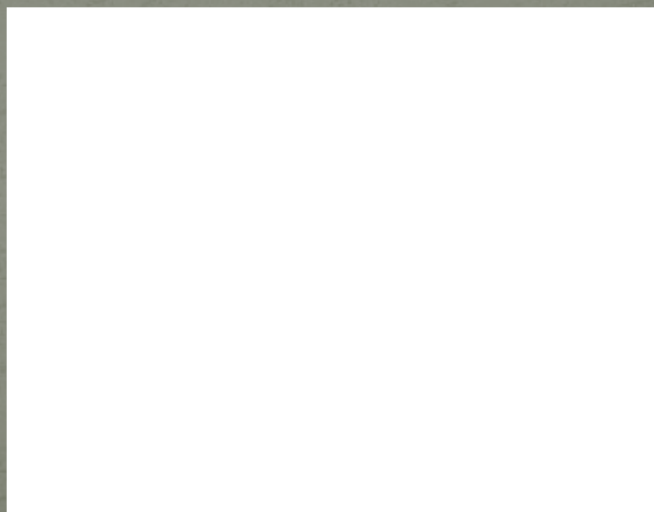
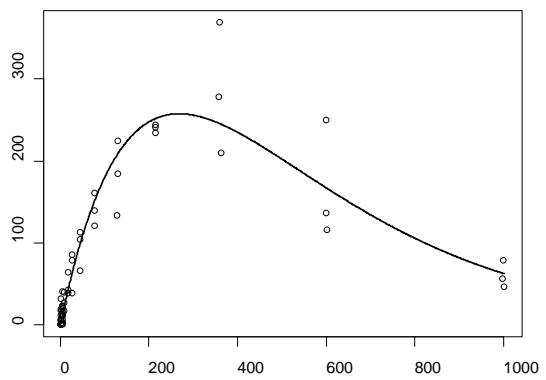
$$N_y(t+h) \sim \text{Poi} \left( N_y(t) R e^{-\alpha N_y(t)} \right)$$

$p_{y \rightarrow x}$  (individuals)



- Poisson diffusion
  - individuals have same  $D$
- Poisson-gamma diffusion
  - individuals have different  $D$
  - longer tail



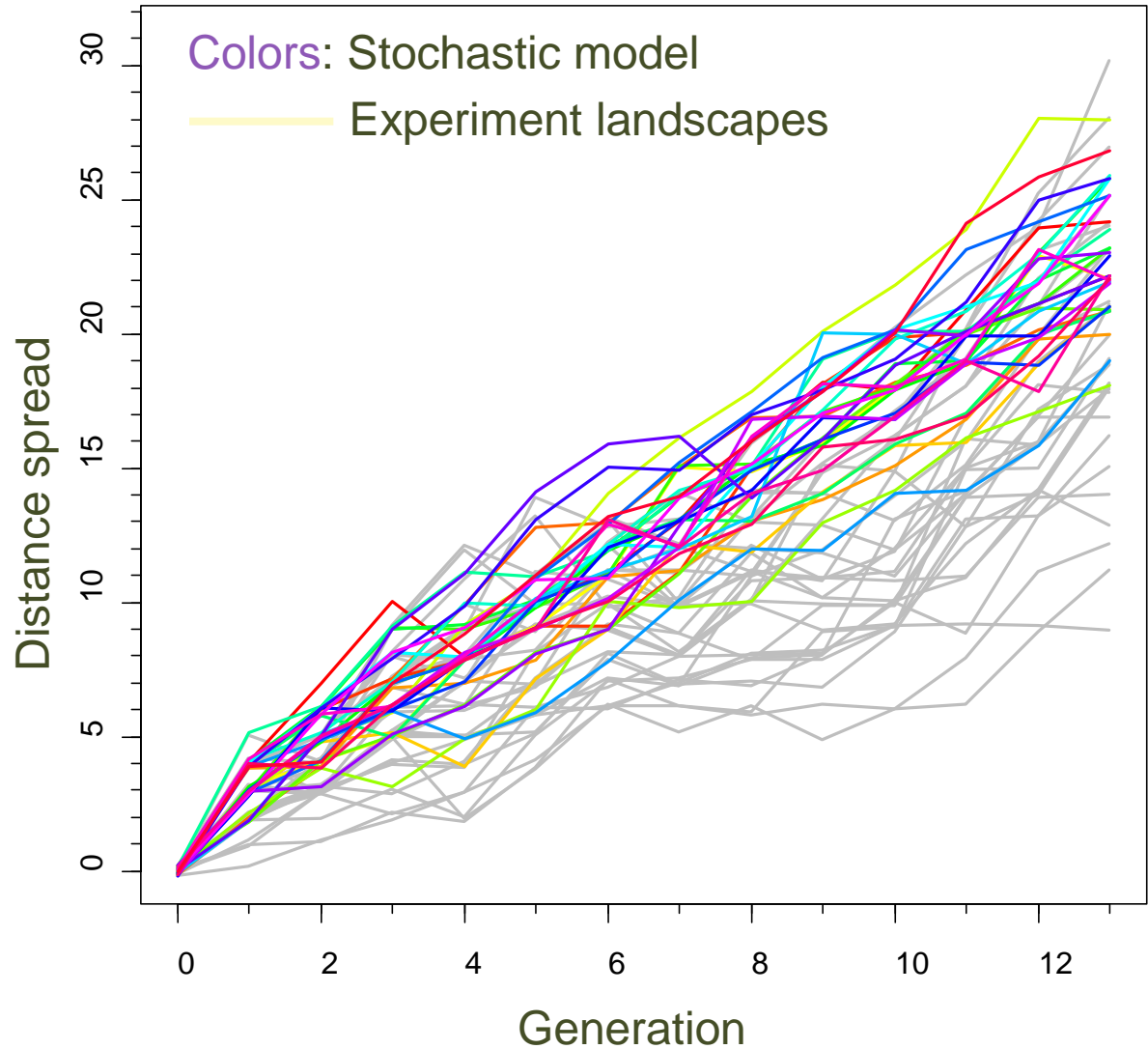




# Variance in spread rates

## Stochastic model

Dem stoch  
Sex  
Env stoch  
Dem het  
Pois diffusion



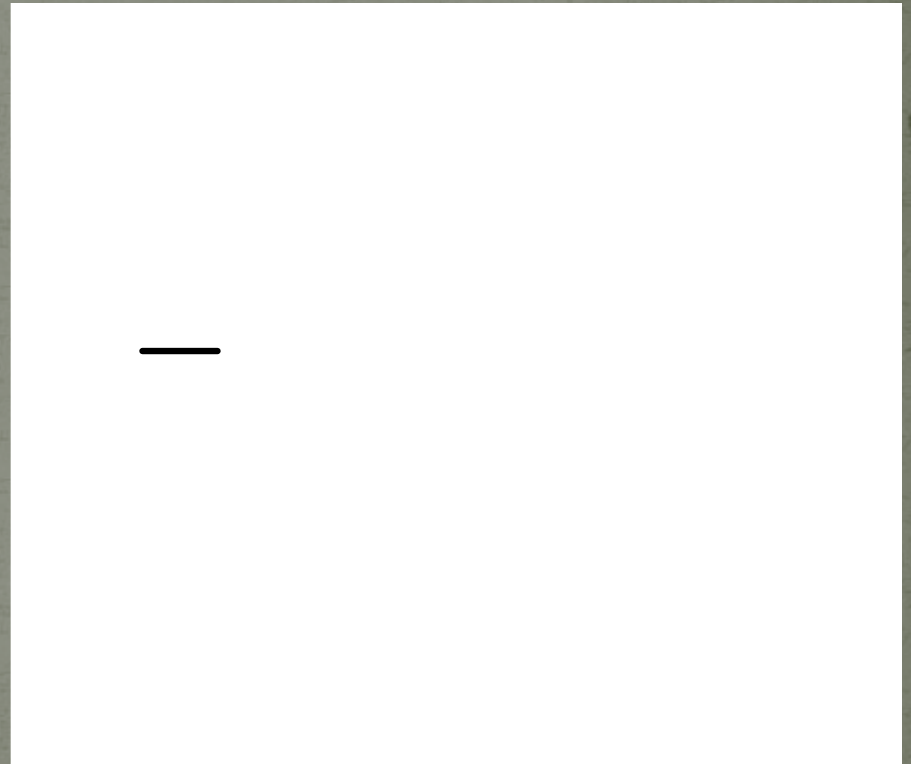
# Founder effects?

Landscapes started  
with 20 individuals

Stochastic spatial model fit

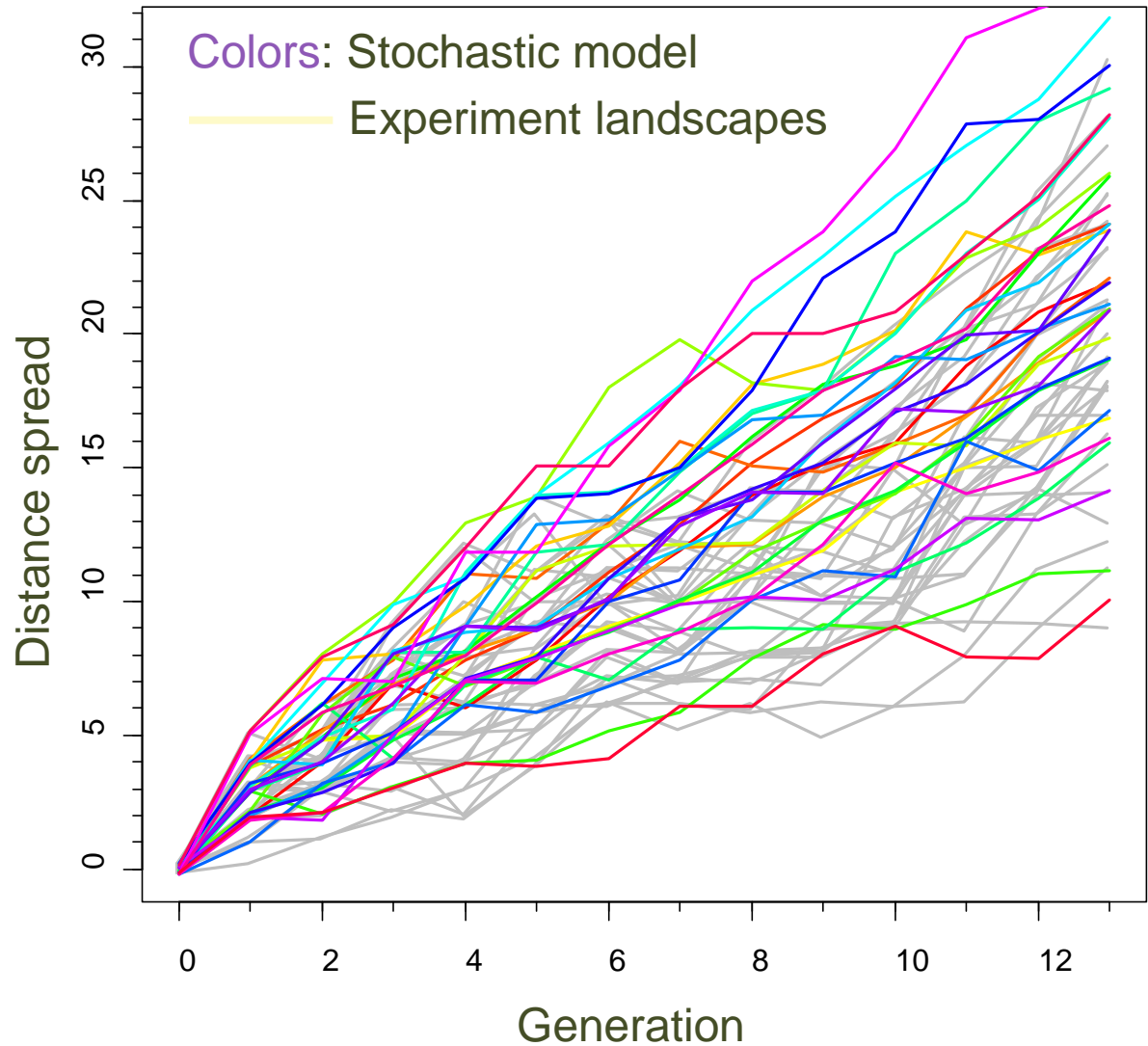
$R$ ,  $\alpha$ ,  $D$  common

$R$ ,  $\alpha$ ,  $D$  unique

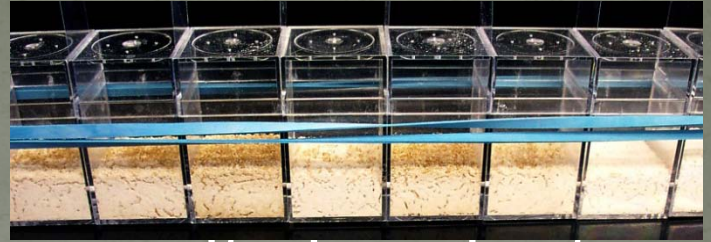


Variance  
in spread  
rates

Stochastic  
process  
+  
founder  
effects



# Conclusion



- Variance in spread rates between multiple realizations very high
- Not entirely explained by stochastic population processes
- Founder effects seem to be important – test experimentally

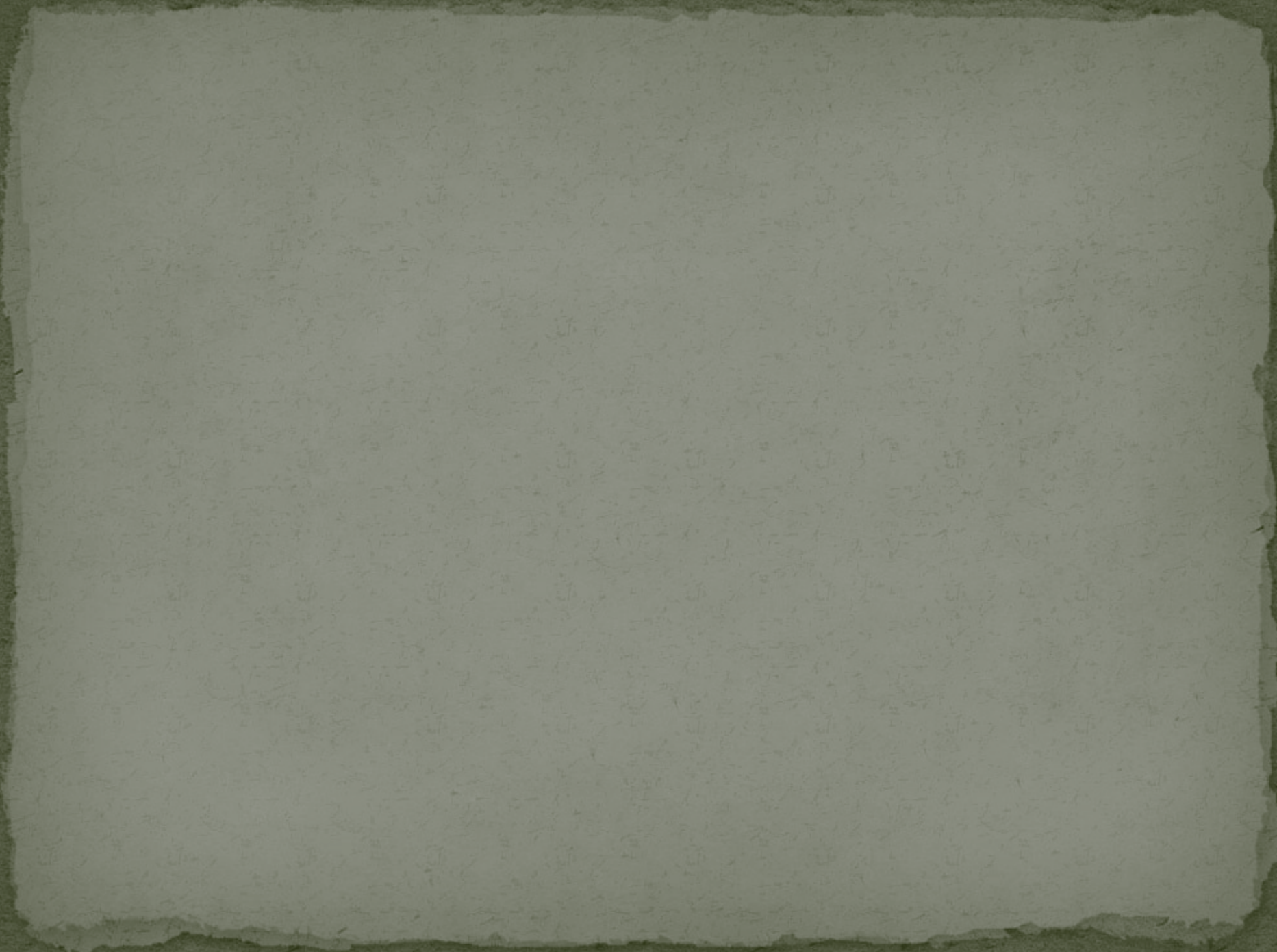
# Acknowledgments



## Assistants:

Claire Koenig  
David Smith  
Roselia Villalobos  
Motoki Wu.

NSF Biological Invasions IGERT  
DGE 0114432  
NSF DEB 0516150



# Problem

- *Spartina alterniflora*
- Native to eastern US (and Gulf)
- Invasive in western U.S.
- 2 sites
  - S.F. Bay – replacing native
  - Willapa Bay – invading bare ground

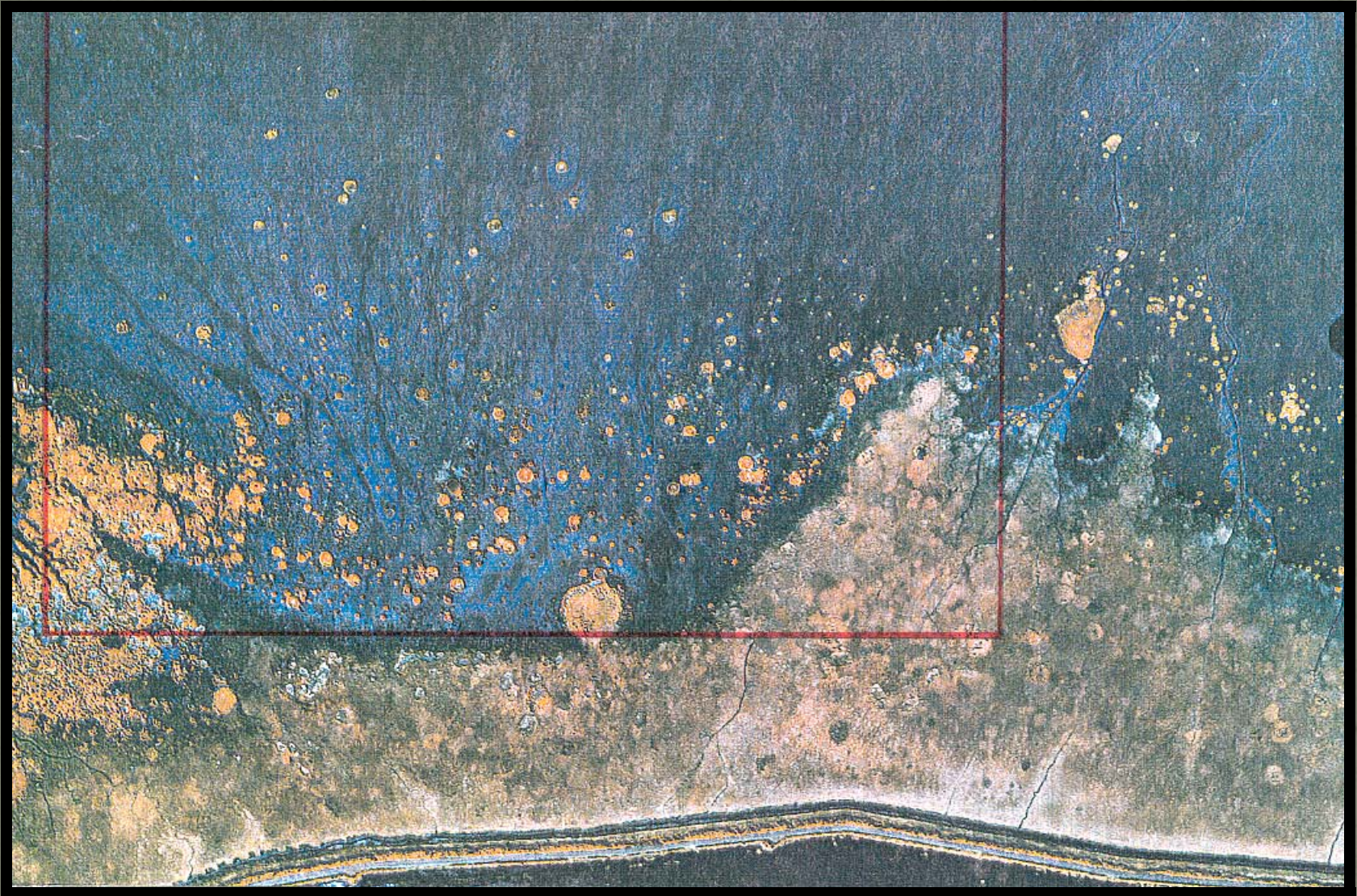




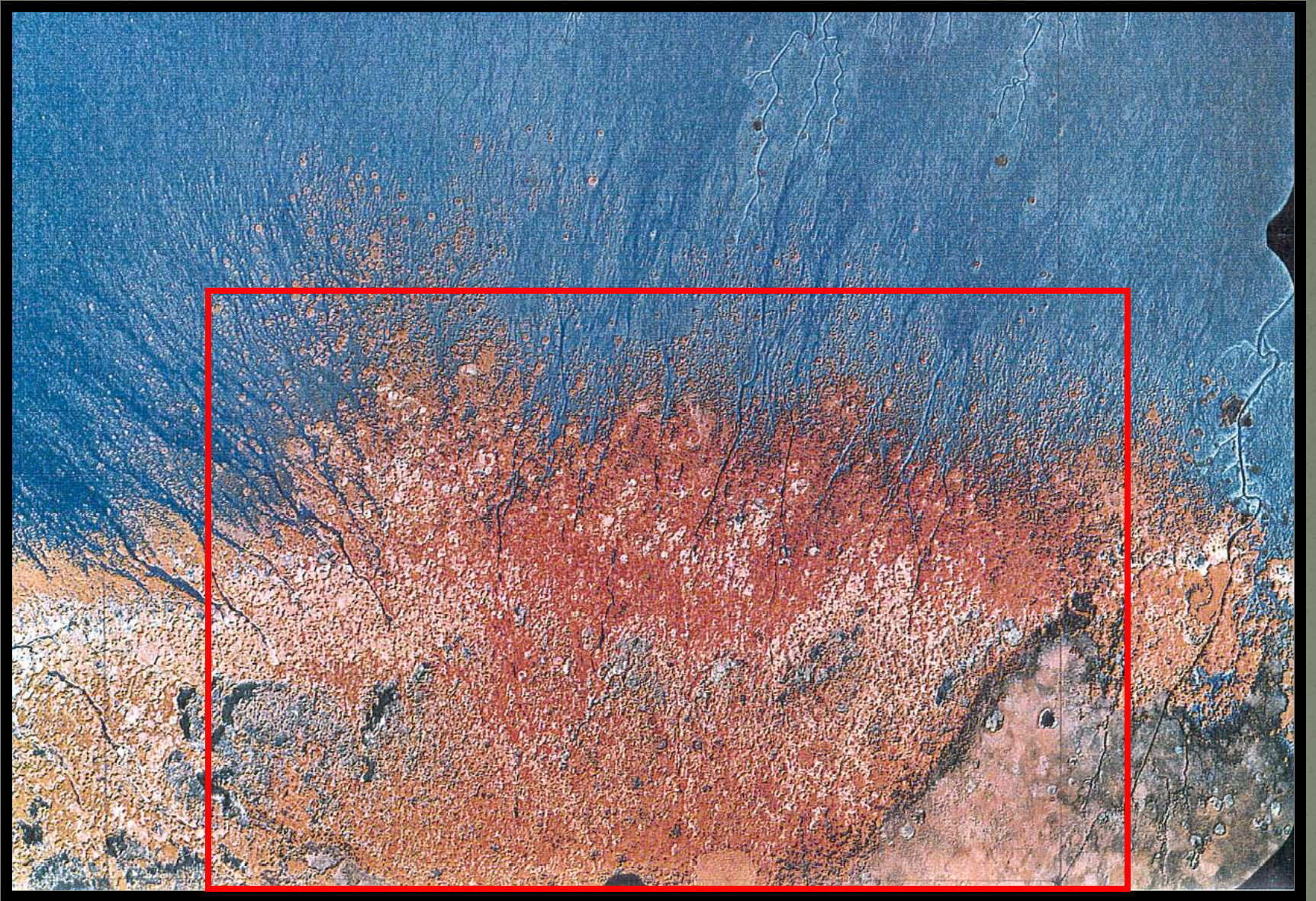


## Willapa National Wildlife Refuge

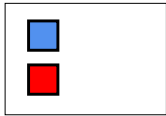
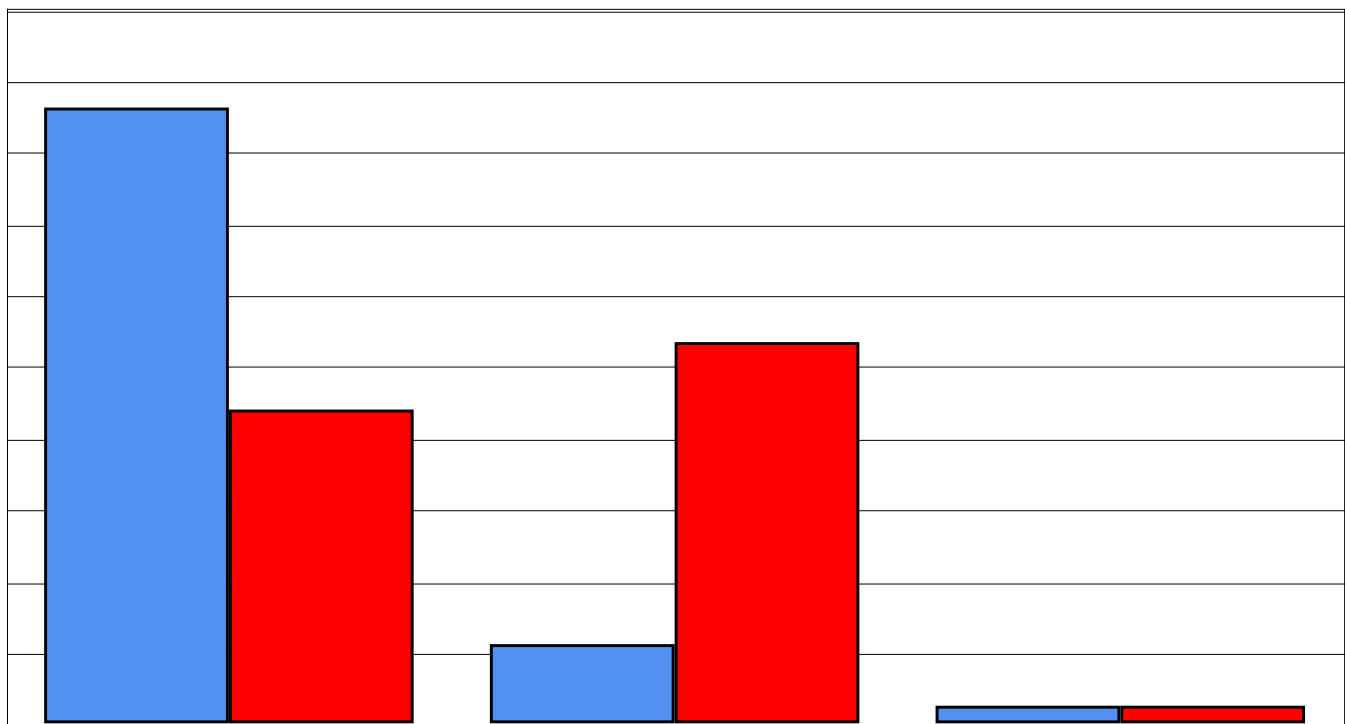




Aerial photos courtesy of Washington State DNR



Aerial photos courtesy of Washington State DNR



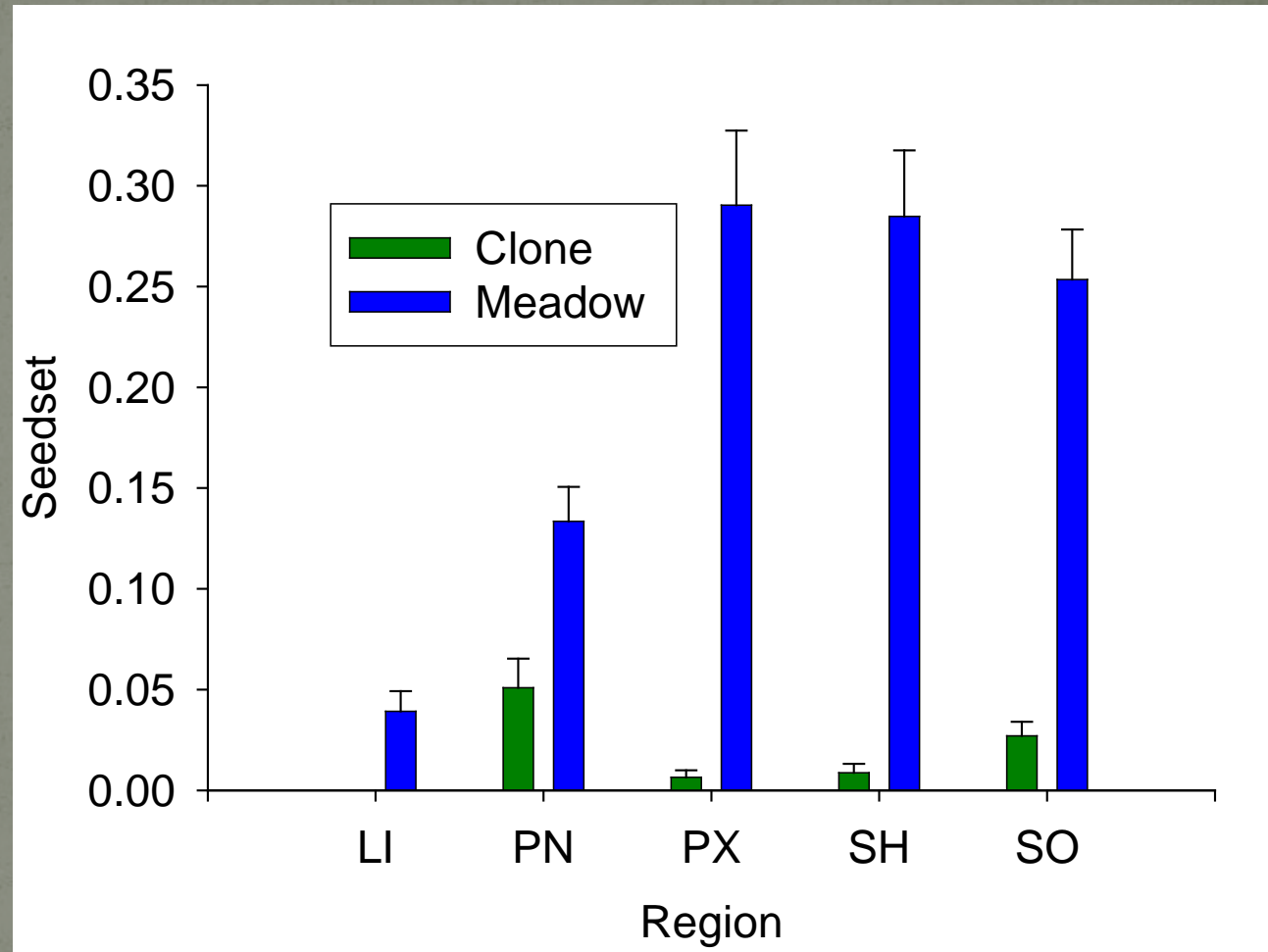


# Models

- Spatially-explicit Stochastic Simulation
  - Consequences of an Allee effect
  - Compare to analytic model to justify use of latter in designing control strategies
- Analytical Non Spatial Model
  - Finding Optimal Control Strategies

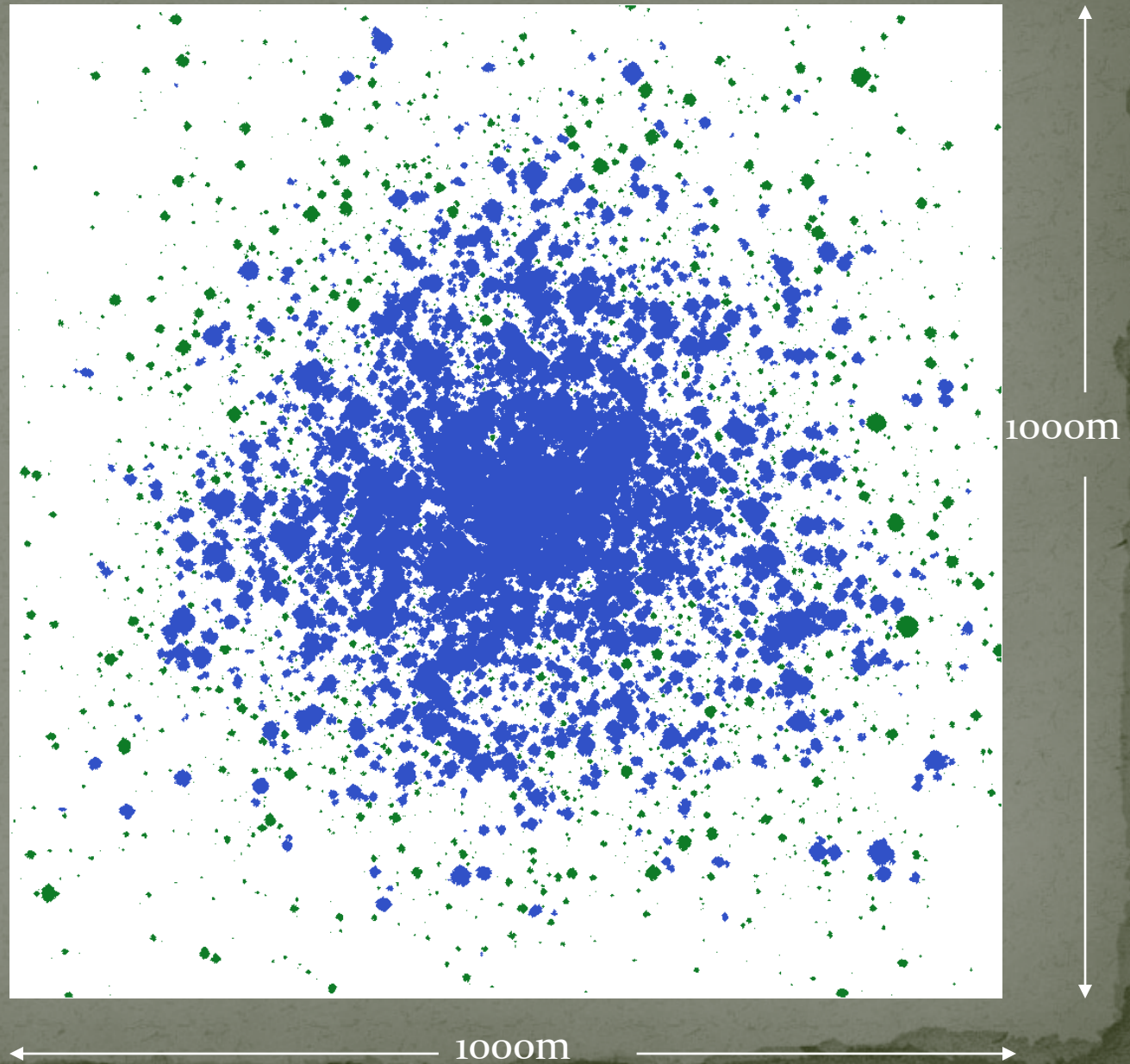
# Field Results: Allee effect

Low density  
plants set <  
10 X the  
seed



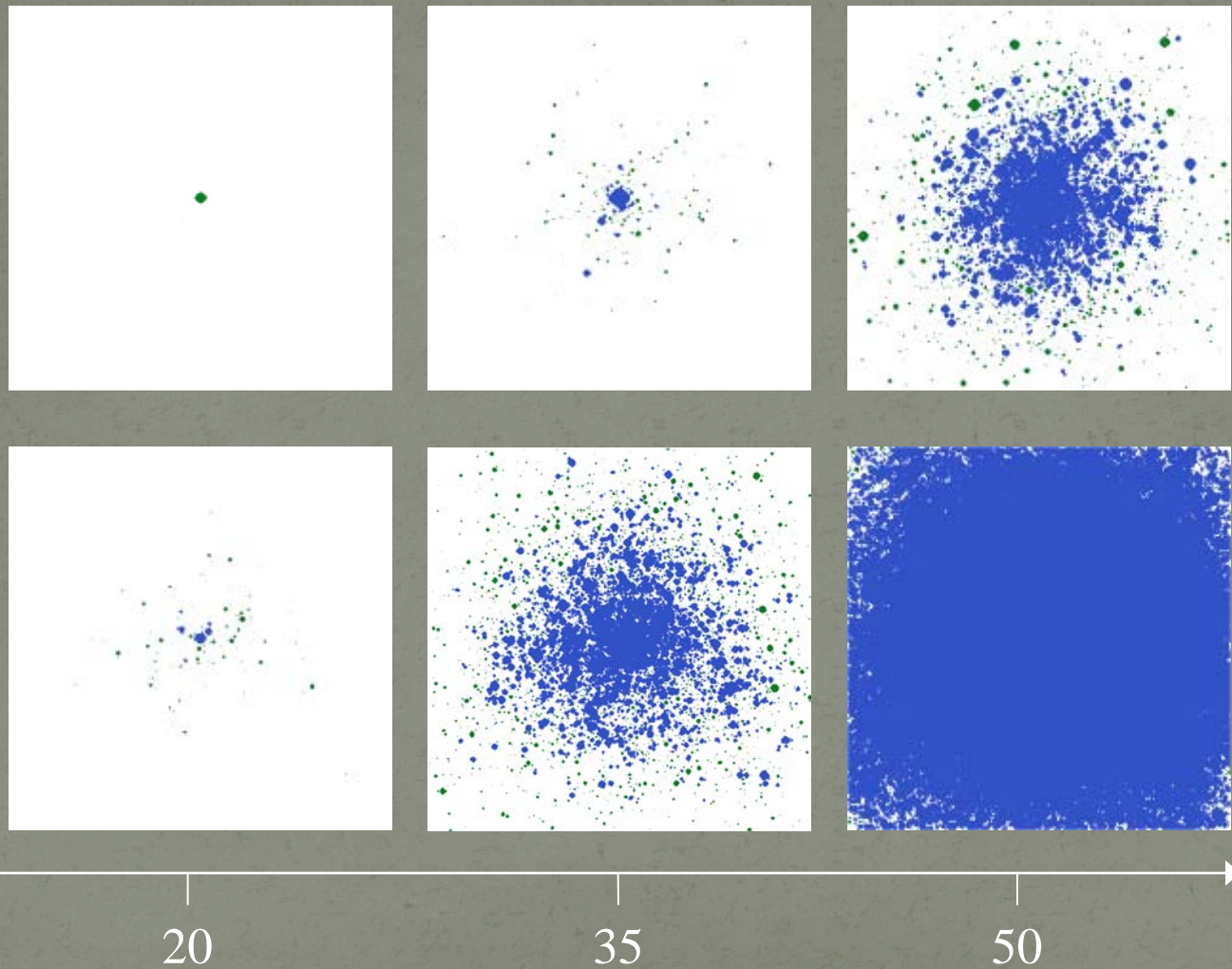
# Spatially-explicit simulation model

- One square km
- Parameterized from
  - GIS maps (Civille)
  - Field data (Davis, Taylor, Civille, Grevstad)
- Run for 100 years, time step 1 year
- Clones have low seed production
- Meadows have high seed production





# Allee Effect Slows Invasion



# Analytical Non-spatial Model

Seedling Area

$$+1 = +$$

Clone Area

$$+1 = + (1 - \eta)$$

Meadow Area

$$+1 = \eta +$$

Parameters are dependent on density and numbers of individuals.

: FECUNDITY OF CLONES

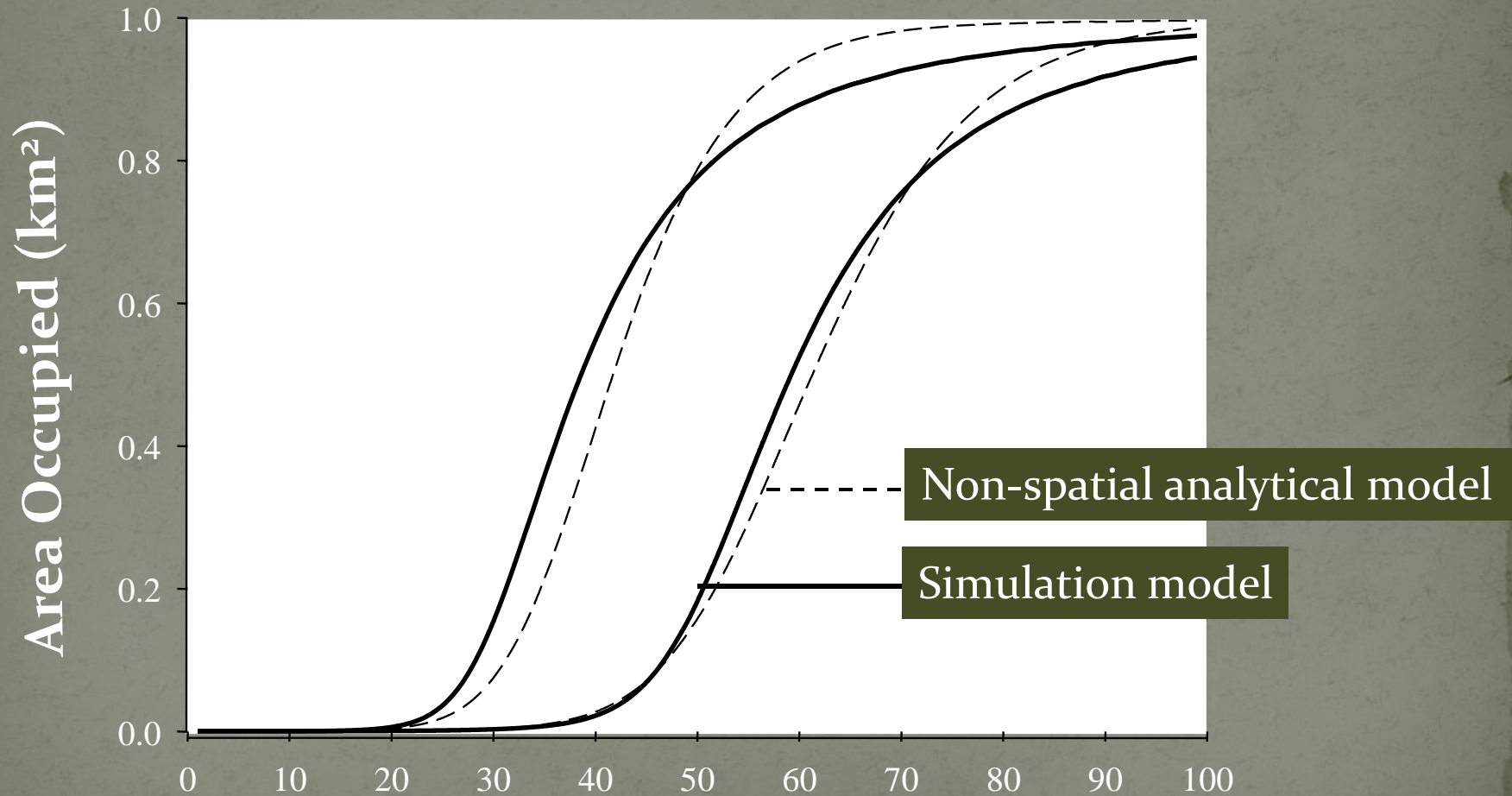
: FECUNDITY OF MEADOWS

: GROWTH RATE OF CLONES

: GROWTH RATE OF MEADOWS

$\eta$  : MERGE RATE OF CLONES INTO MEADOWS

# Analytical model predicts same dynamics as simulation model



# Control of *Spartina*



2003 Control



# Control questions

- How much *Spartina* needs to be removed every year to eradicate invasion within 10 years
- Is it better to prioritize removal of *fast growing* but *low seed producing clones* or is it better to prioritize removal of *slow-growing* but *high seed producing meadows*?

# Control Strategy


- $T_t < \text{MAX}$  = Total area removed in year t
- $0 \leq X_t \leq 1$  = fraction of  $T_t$  that was meadows
- $0 \leq (1 - X_t) \leq 1$  = fraction of  $T_t$  that was clones

$$+1 = + (1 - \eta) - (1 - )$$


$$+1 = \eta + -$$

# Control Objectives

1. Eradicate invasion in one square km region within 10 years
2. Minimize Cost X Risk of colonizing other sites



Total area removed  
in 10 years

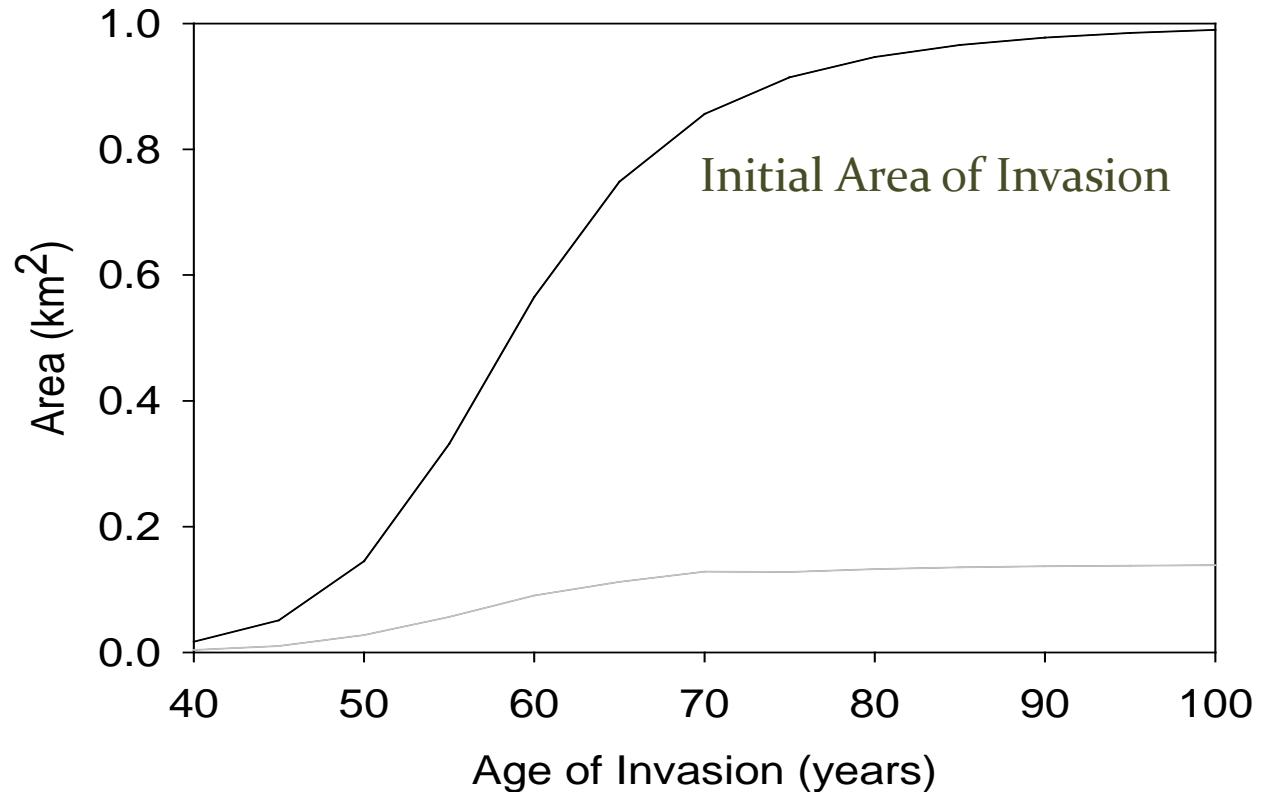


Total seed production during 10  
years



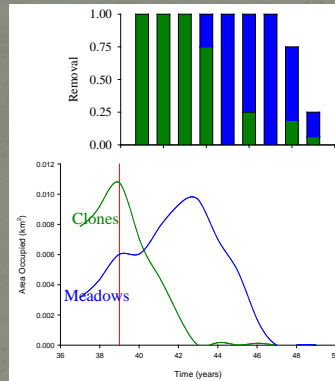
# Minimum Removal needed to Eradicate within 10 years

Equivalent of 15-20% of initial invasion has to be removed annually

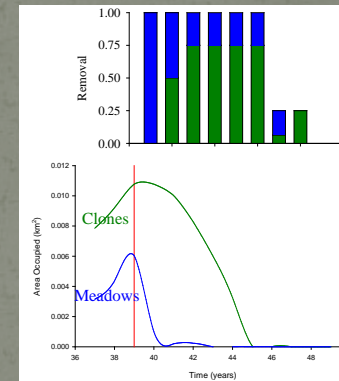


# Optimal Control Strategies

## Low Budget *Clones First*

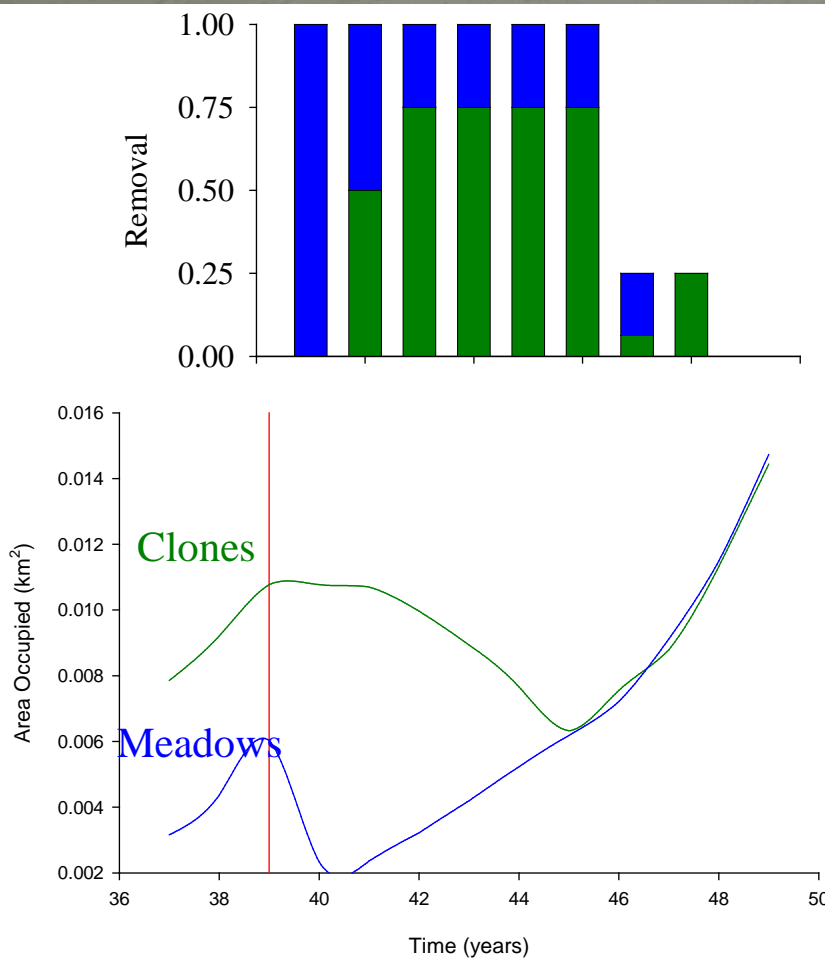


## High Budget *Meadows First*

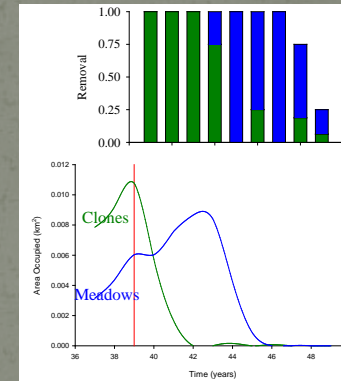


# Switch Control Strategies

## Low Budget *Meadow First*

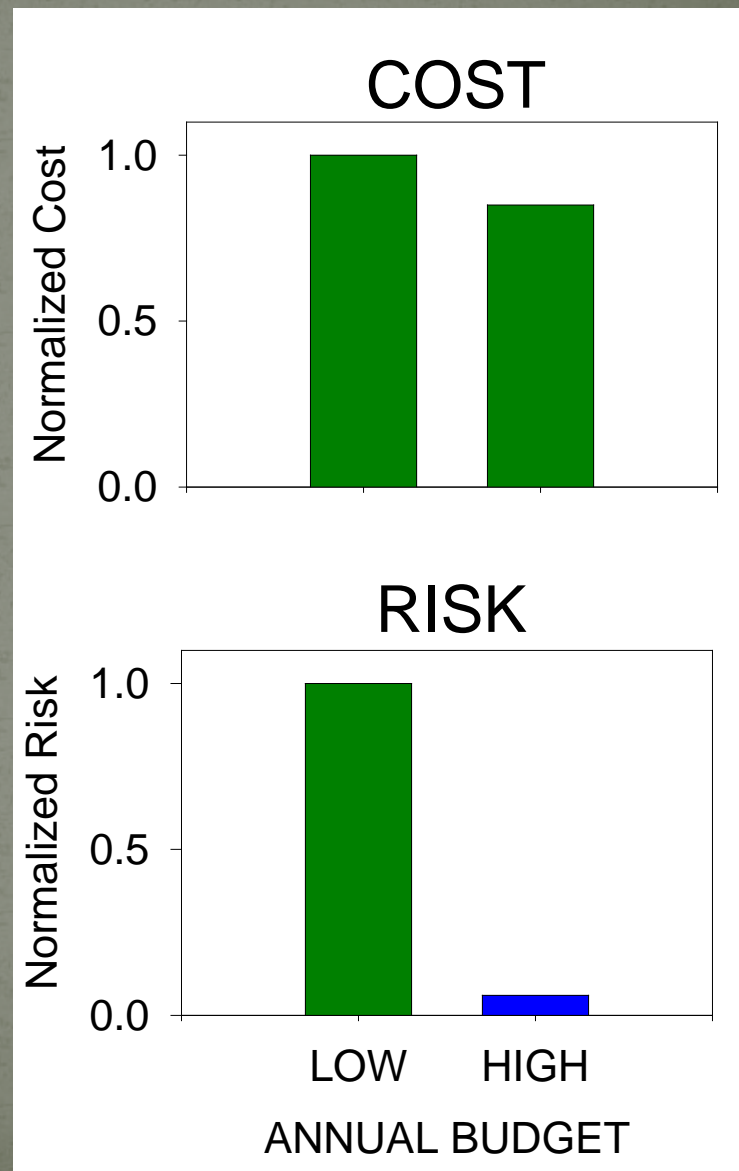


## High Budget *Clones First*



# Summary

	Low Budget	High Budget
Minimize Cost Only	Clones First	Clones First
Minimize Risk Only	Clones First	Meadows First
Minimize Cost and Risk	Clones First	Meadows First
No Allee Effect	Clones First	Clones First



# Linear control model

- Density independent
- Three classes – seedlings, juveniles and adults
  - Express model in terms of area occupied
- If the model were nonlinear this would become a dynamic programming problem –
  - Difficult numerical problem – cannot really get a solution
  - So, can we simplify in this case?

(Hastings, Hall and Taylor, TPB in press)

$$N_{t+1} = LN_t,$$

$$N_{t+1} = L(N_t - H_{t+1}).$$

$$N_T = L^T N_0 - \sum_{i=1}^T L^{T+1-i} H_i,$$

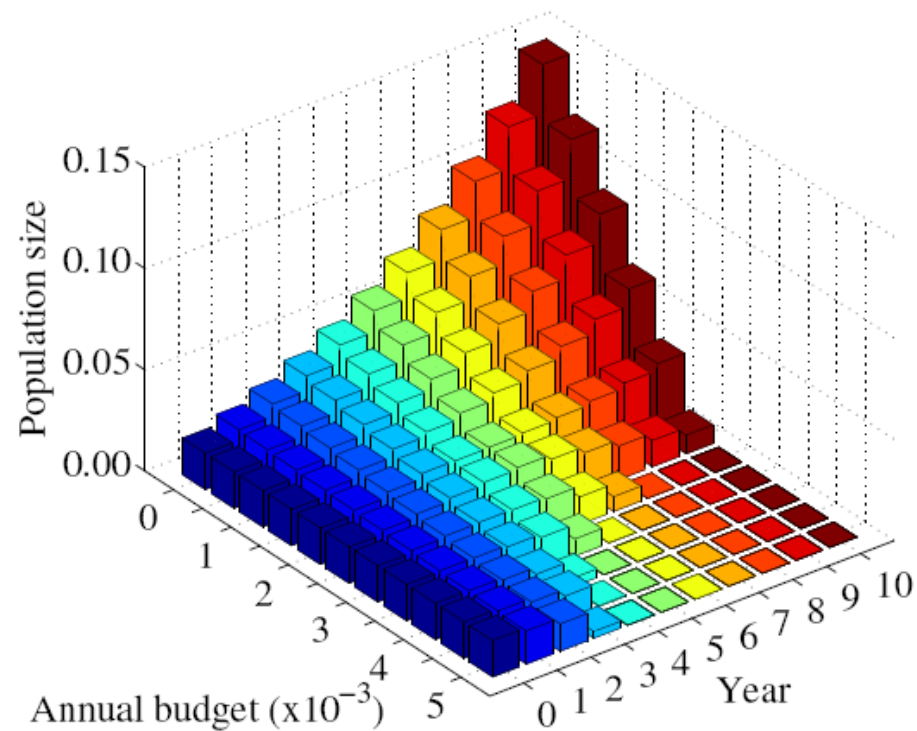
Population = size without control – contribution of removed

# What classes should be removed?

- One year ahead?
  - The class that contributes the most area (normalized by 'cost') should be removed first
- “Infinitely” far ahead?
  - The class that has the highest reproductive values (normalized by 'cost') should be removed first
- Therefore do intermediate case, finite time horizon, which becomes a linear programming problem (from previous slide)

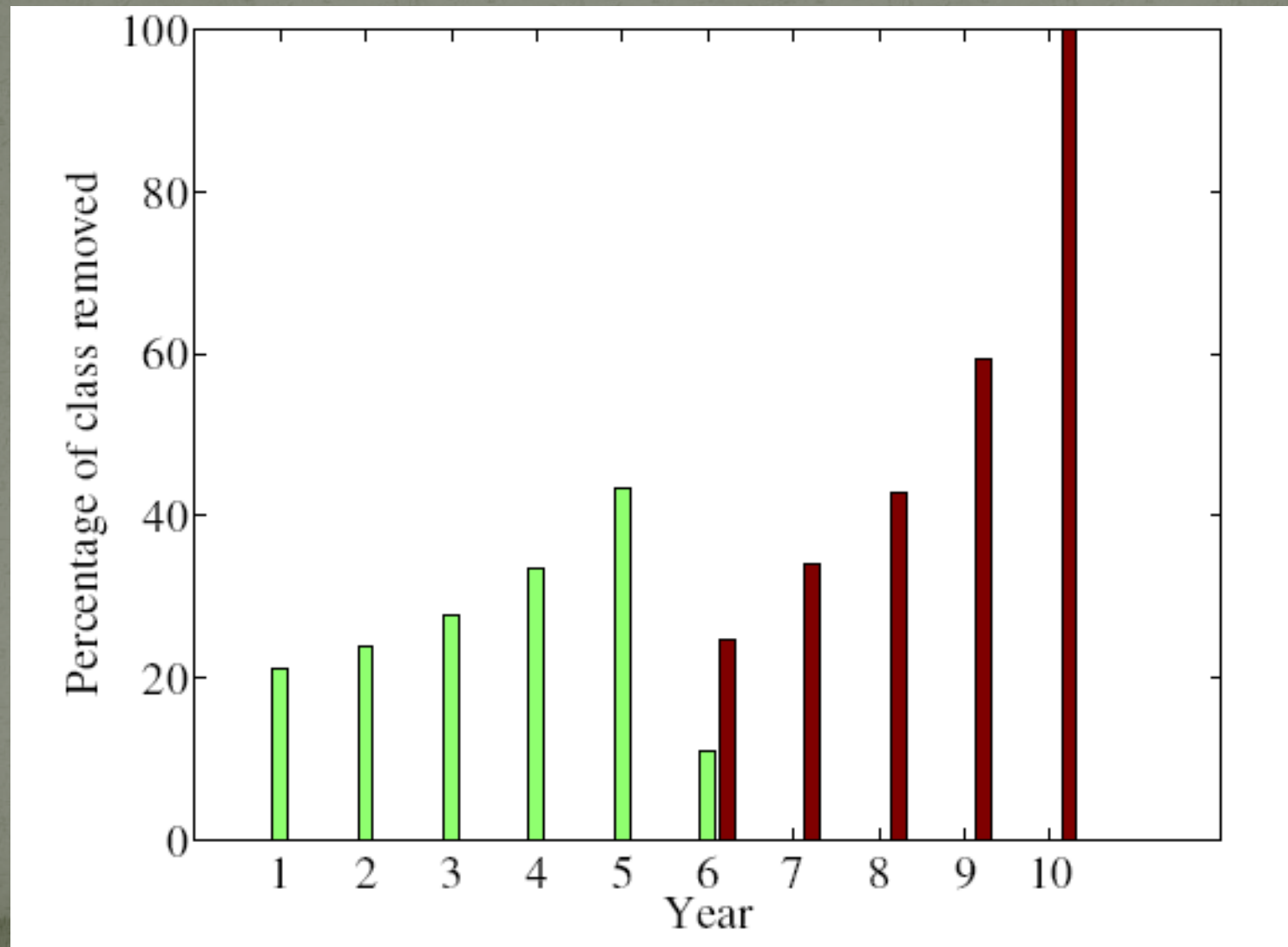
Population size as a function of time and the annual budget allocated to control, when the objective is to minimize the population within 10 years subject to budget constraint.

(a)





The fraction of each stage class (green for isolates, red for meadows) removed by control in each year under the optimal control strategy.



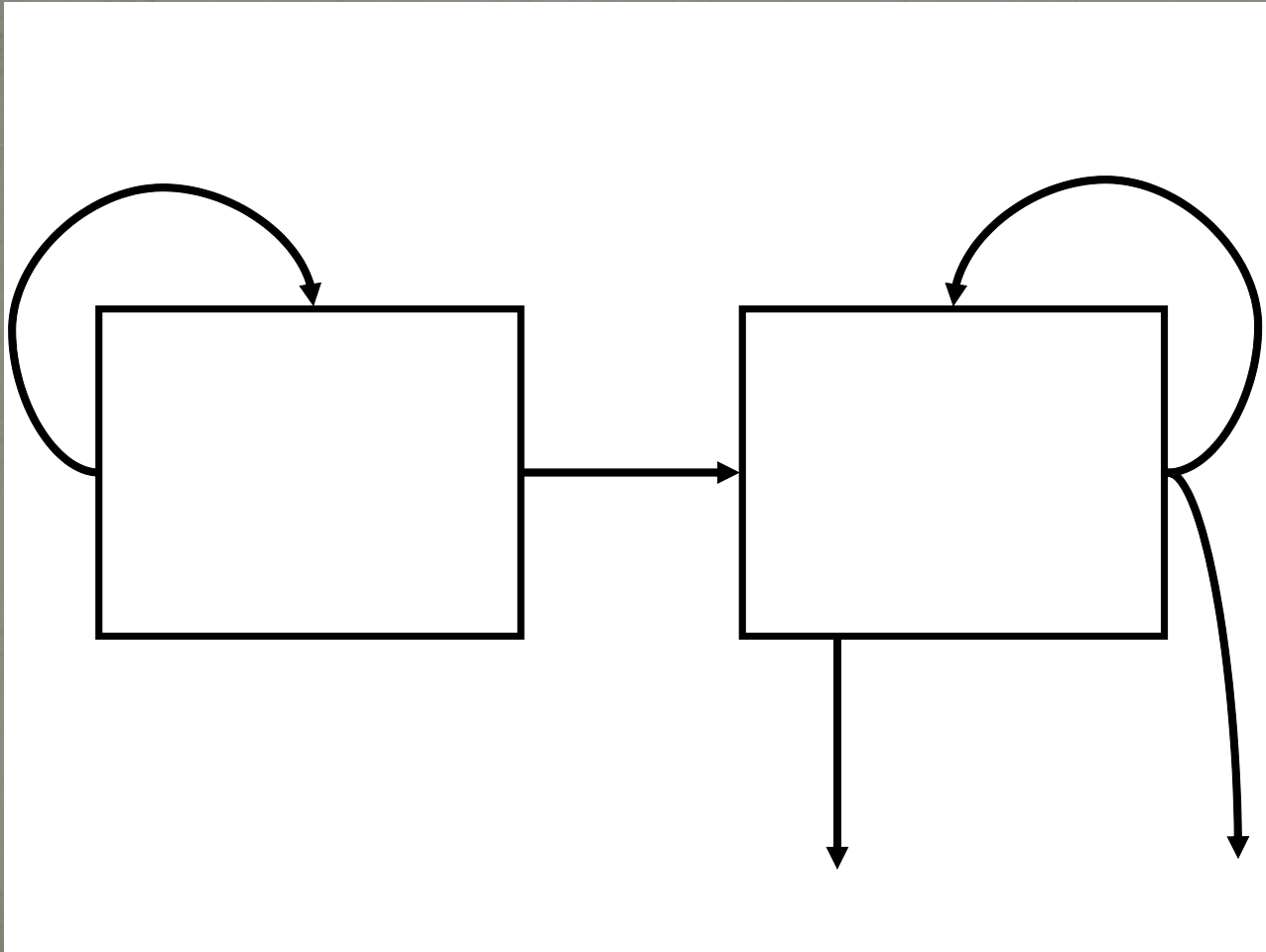
# Initial conclusions

- Optimal approach is time dependent
  - May be much more effective
- Cost of waiting
  - Overall cost of control can be much less when started earlier
- Since a LP problem solution is always at a vertex – focus on a single class unless budget large enough to remove an entire class, then add one more class

# 'Easy' extensions

- Dependence on habitat
- Spatial extent
- Dependence on tidal height

# Damage (Hall and Hastings, JTB)



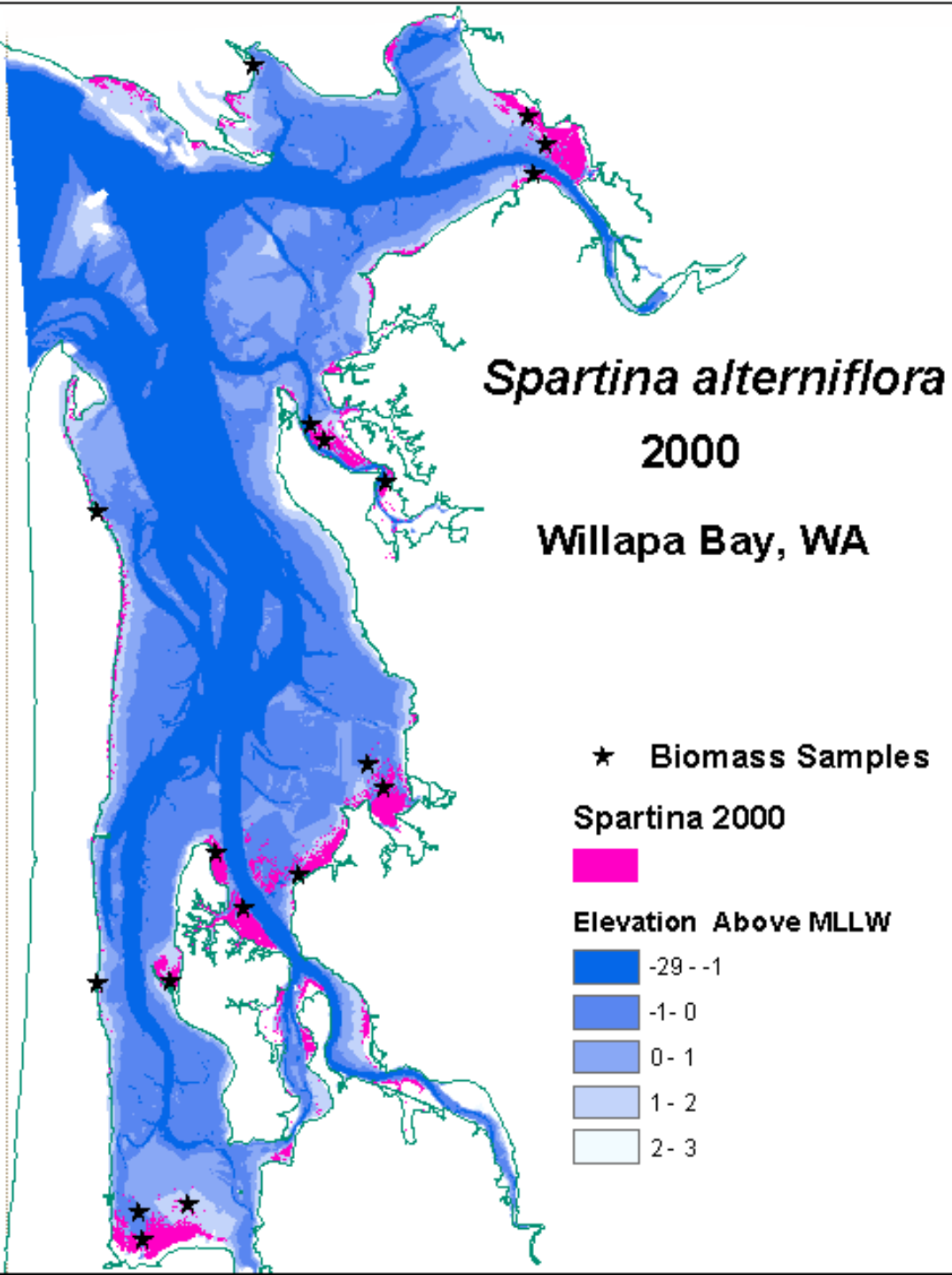
# Conclusions

- Allee effect slows down invasion considerably
- Best control strategy is to remove clones first if budget is low or if minimizing for cost only
- If minimizing for risk and budget is high, removing meadows first is best strategy
- Meadow first strategy is risky especially if budgets for future years are unpredictable.

# Where's the data?

- Willapa Bay
  - Analysis of aerial photographs
- SF Bay
  - Remote sensing data



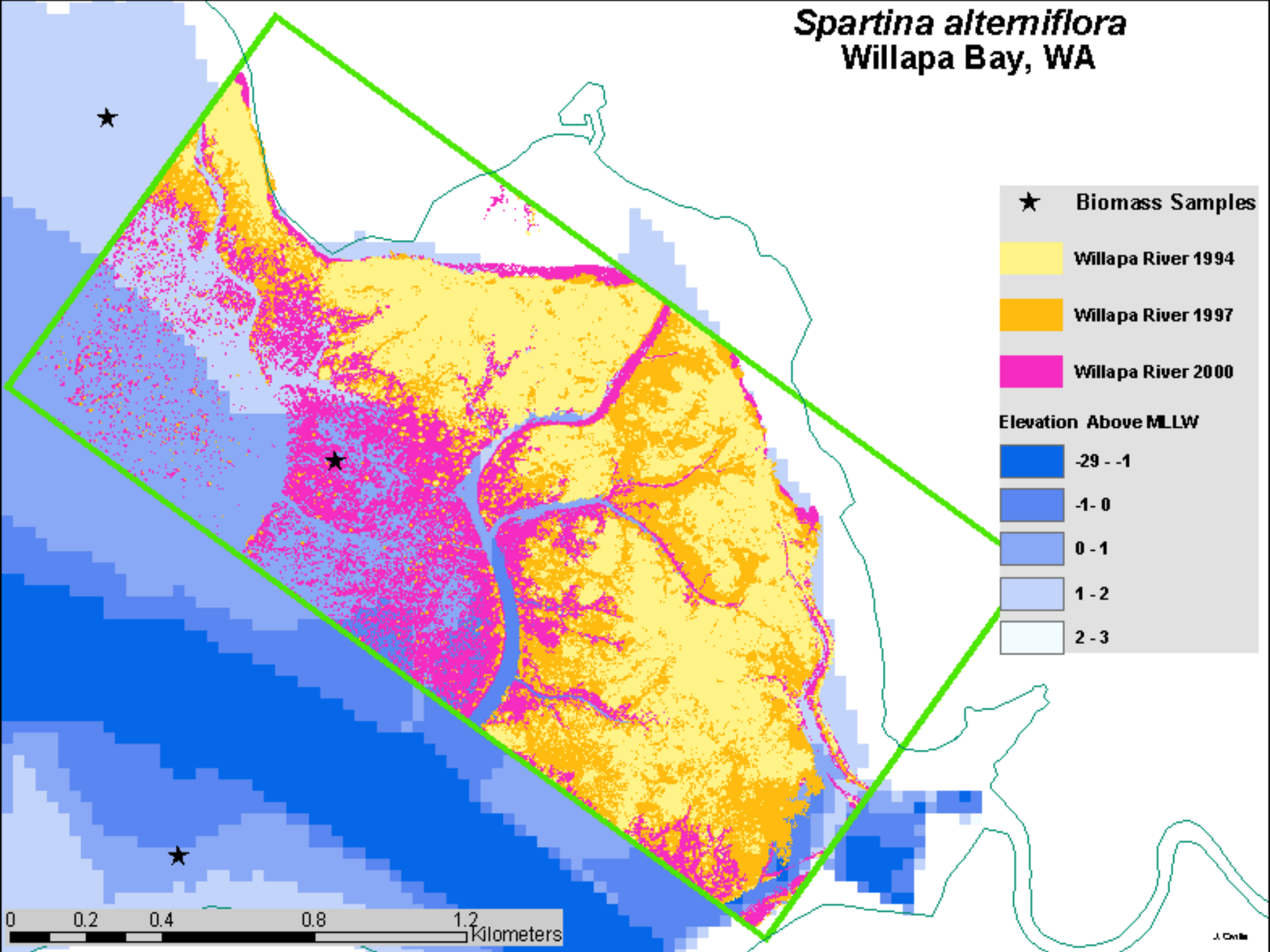


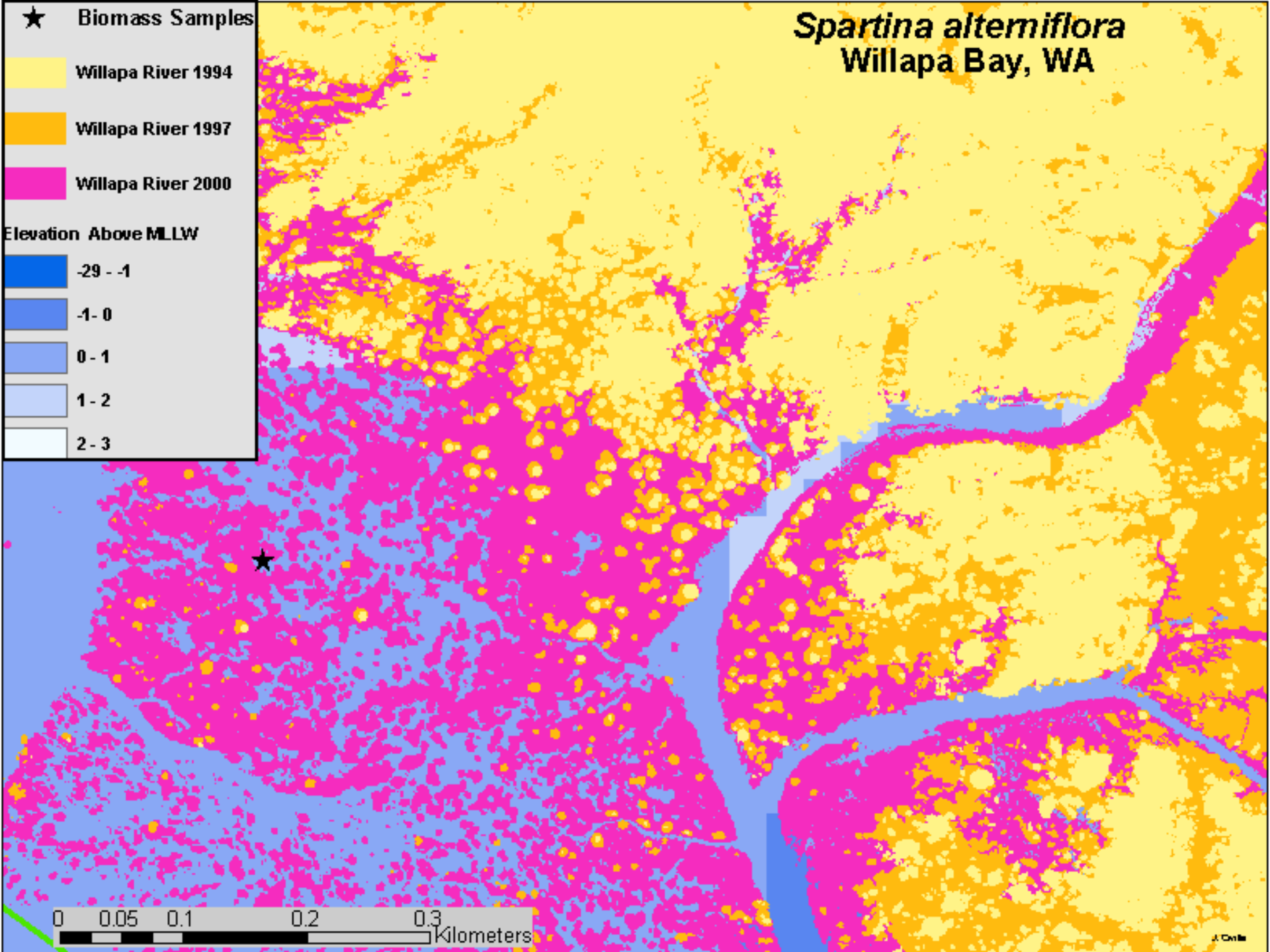


# Approach in Willapa Bay

- Initial steps
  - (orthorectify, etc.)
- With aid of GIS software, identify clones
  - By hand
- Match up successive years

# *Spartina alterniflora* Willapa Bay, WA



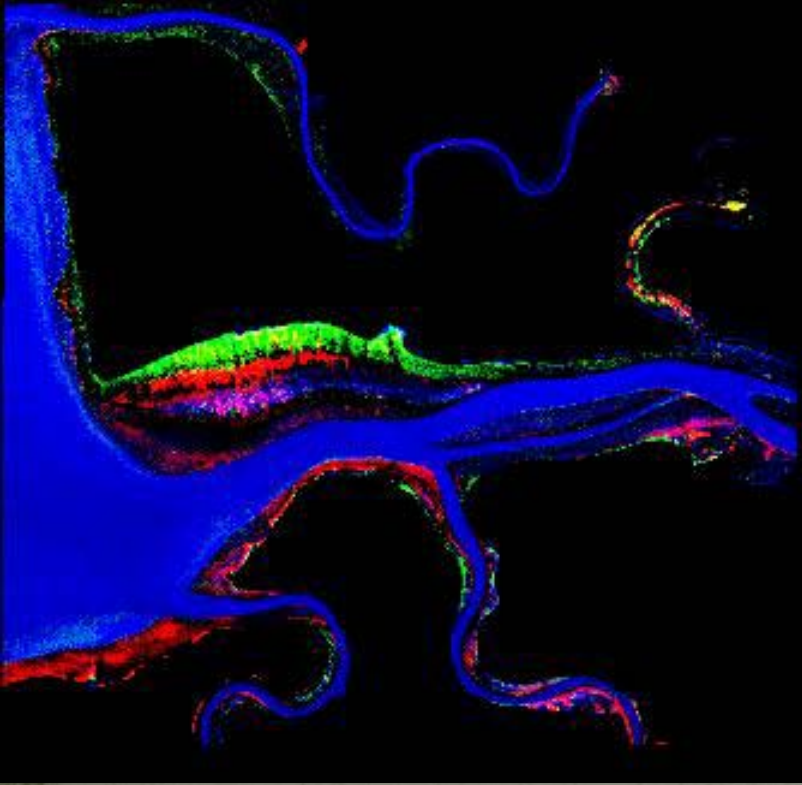




# Approach in SF Bay

- Low resolution, high bandwidth data
- Identify components by 'spectral signature'
  - Ground truthing
  - Choose number of components to identify
    - Mud
    - Water
    - Spartina
    - Other vegetation

(Rosso, P. H., Ustin, S. L. & (2005)  
International Journal of Remote Sensing 26: 5169 –  
5191)



Picture is an Aviris image (pixel size, 17x17 m approx.) of Coyote Creek area marsh, in the southern tip of San Francisco Bay. Image is from August 1999.

Colors indicate the percentage of each component, Spartina (red), Salicornia (green) and water (blue), present at each pixel as determined by a spectral unmixing approach. The unmixing was done on the basis of eight endmembers (reference spectra). Five plant species, water and open mud.

# Comparison of approaches

- Willapa

- High resolution, low bandwidth
- High accuracy
- Labor intensive
- Data expensive
- Works well with invasion into bare mud

- SF Bay

- Low resolution, high bandwidth
- Lower accuracy
- After difficult initial steps, easier to implement
- Can handle multiple types