Some definitorial suggestions for parameterized proof complexity

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joint with Jörg Flum,

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Parameterized complexity

(Q, κ) is a parameterized problem:

$Q \subseteq \{0,1\}^*$ is a classical problem, $\kappa : \{0,1\}^* \to \mathbb{N}$ is ptime.

(Q, κ) is fpt:

solvable in time $f(\kappa(x)) \cdot |x|^{O(1)}$.

R is a fpt reduction from (Q, κ) to (Q', κ'):

(1) R is a reduction from Q to Q',

(2) R is fpt computable (wrt κ),

(3) $\kappa'(R(x)) \leq g(\kappa(x))$.

Examples

p-VC
Input: graph G, $k \in \mathbb{N}$.
Parameter: k.
Question: does G contain a vertex cover of cardinality k?

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p-CLIQUE
Input: graph G, $k \in \mathbb{N}$.
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Examples

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p-CLIQUE

p-DS
Input: graph G, $k \in \mathbb{N}$.
Parameter: k.
Question: does G contain a dominating set of cardinality k?

$$p\text{-VC} \quad \leq_{\text{fpt}} \quad p\text{-CLIQUE} \quad \leq_{\text{fpt}} \quad p\text{-DS}$$

$$\not\equiv_{\text{fpt}} \qquad\qquad \not\equiv_{\text{fpt}}$$

if W[1] $\neq$ FPT   if W[2] $\neq$ W[1]

Parameterized proof systems

(P, κ) is a parameterized proof system:

P is a classical proof system, $\kappa : \{0, 1\}^* \to \mathbb{N}$ is ptime.

R is a fpt simulation of (P, κ) in (P', κ'):

(1) R is a simulation of P in P' (i.e. $P'(R(x)) = P(x)$),

(2) R is fpt computable (wrt κ),

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Examples

p-EF
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Parameter: number of extension axioms in π. $(e \leftrightarrow \sigma)$

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Proof: F-proof π.
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Proof: F-proof π.
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$$\mathsf{F} \quad \leq_{\mathsf{fpt}} \quad p\text{-EF} \quad \leq_{\mathsf{fpt}} \quad p\text{-SF}.$$

Examples

p-EF*
Proof: treelike EF-proof π.
Parameter: number of extension axioms in π.

p-SF*
Proof: treelike SF-proof π.
Parameter: number of substitution inferences in π.

(p-)F*
Proof: treelike F-proof π.
Parameter: 0.

$$\mathsf{F}^* \quad \leq_{\mathrm{fpt}} \quad p\text{-}\mathsf{EF}^* \quad \leq_{\mathrm{fpt}} \quad p\text{-}\mathsf{SF}^*.$$

Some results

Proposition p-$\mathsf{SF}^* \leq_{\mathsf{fpt}} p$-$\mathsf{EF}^*$.

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Some results

Proposition p-SF$^* \leq_{\mathsf{fpt}} p$-EF*.

Proposition p-EF $\leq_{\mathsf{fpt}} p$-EF*.

Proposition p-SF $\leq_{\mathsf{fpt}} p$-EF.

$$\mathsf{F} \quad \leq_{\mathsf{fpt}} \quad p\text{-}\mathsf{EF}^* \quad \equiv_{\mathsf{fpt}} \quad p\text{-}\mathsf{EF} \quad \equiv_{\mathsf{fpt}} \quad p\text{-}\mathsf{SF}^* \quad \equiv_{\mathsf{fpt}} \quad p\text{-}\mathsf{SF}.$$

Question p-EF $\leq_{\mathsf{fpt}}$ F ?

Finer reductions

R is a polynomial parameterized simulation of (P, κ) in (P', κ'):

(1) R is a simulation of P in P' (i.e. $P'(R(x)) = P(x)$),

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Questions

- p-EF $\leq_{\mathsf{fpt}}$ F ?

- p-SF $\leq_{\mathsf{p}}$ p-EF ?

In p-SF $\leq_{\mathsf{fpt}}$ p-EF we map k substitution inferences to $2^{O(k)}$ extension axioms.

- Can you do with $2^{o(k)}$ extension axioms?