
Margaret Beck, Heriot Watt and Boston University
Title: "Rapid convergence to quasi-stationary states in the 2D Navier-Stokes equation."
Abstract: "For the 2D Navier-Stokes equation on the torus with small viscosity, certain quasi-stationary states consisting of localized vorticity play an important role in the long-time dynamics. Numerical studies indicate that the convergence to these states is rapid. A mechanism for this will be presented, motivated by work of Villani and of Gallagher, Gallay, and Nier: the linearization around certain explicit states leads to a non-self-adjoint operator that creates decay on a fast time-scale. The proof involves the construction of a new norm that is adapted to the so-called hypocoercivity of the operator."

Alan Champneys, University of Bristol
Title: Localised patterns via snaking, towards a multi-dimensional view
Abstract: This talk shall give an overview of the pheonomenon of so-called homoclinic snaking in PDE systems in 1D. Using a spatial dynamics approach, much progress has been made in recent years on understanding how localised patterns of arbitrary extent are organised in a "snakes and ladders" bifurcation diagram. Some recent extensions to these ideas will be discussed including; slanted snaking in the presence of a global mode, snaking in discrete systems and so-called defect mediated snaking. The latter part of the talk shall be a more open ended attempt to assess the extent to which these fundamentally 1D ideas extend to 2D systems.

Wan Chen, OCCAM, Oxford University
Title: Dynamics and Stability of Localized Spots in Gray-Scott Model
Abstract: The Gray-Scott model has been shown numerically and experimentally to exhibit a rich variety of localized spatio-temporal patterns including, standing spots, oscillating spots, self-replicating spots, etc. This talk concentrates on analyzing dynamics and stability of spot pattern in the semi-strong interaction regime where the diffusivity ratio of the two components is asymptotically small.
We use the matched asymptotic analysis to construct multi-spot solutions by summing an infinite-order logarithmic expansion in terms of a small parameter. An asymptotic differential algebraic system of ODE~@~Ys for the spot locations is derived to characterize the slow dynamics of a collection of spots. it is shown theoretically and from the numerical computation of certain eigenvalue problems that there are three main types of fast instabilities for a multi-spot solution including spot self-replication, spot annihilation due to overcrowding, and an oscillatory instability in the spot amplitudes. These instability mechanisms are studied in detail and phase diagrams in parameter space where they occur are computed and illustrated for various spatial configurations of spots in square and unit disk.

Jonathan Dawes, University of Bath
Title: Scaling laws for slanted snaking.
Abstract: Toy models such as the Swift-Hohenberg equation appear to describe aspects of the formation of localized states in continuum physical systems. However in cases such as pattern formation in vertically vibrated granular media, and thermal convection in the

presence of an imposed vertical magnetic field, an additional large-scale neutral mode exists due to a conservation law. In such cases the homoclinic snaking curves become stretched out into `slanted snaking' and localized states may exist far more subcritically than spatially extended patterns. The extent of this subcriticality is governed by scaling laws that can in some cases be deduced semi-analytically.

Irv Epstein, Brandeis Univ.
Title: Localized Patterns in Experimental Reaction-Diffusion Systems
I will review experiments on reaction-diffusion systems in which localized patterns occur. Phenomena of interest include stationary spots, oscillatory spots (oscillons), traveling spots, spot splitting, oscillatory clusters and wave segments. Localized patterns have been observed in aqueous solution, oil-water microemulsions, surface catalyzed reactions and gas discharges. Earlier studies are of 2D patterns. We recently developed a tomographic technique for studying 3D microemulsions, where model simulations suggest 3D localized phenomena should be observable. Noise may contribute to the occurrence and stabilization of localized behavior. Models have been proposed for many of the observed phenomena, but important theoretical questions remain.

Anna Ghazaryan, Miami University
Title: Unstable fronts in high Lewis number combustion model
Abstract: I will discuss high Lewis number combustion model. Posed on one-dimensional space, it has been studied in detail by different researchers. Unique combustion fronts are shown to exist. For all regimes, fronts are unstable: some absolutely unstable, while others convectively unstable. On the nonlinear level, the convective instability has been captured analytically using semigroup theory and exponential weights. The same model posed on two-dimensional space has also been studied. There exist unstable planar fronts. The question I want to discuss is whether the techniques developed for the nonlinear stability analysis of the one-dimensional case will work for the planar case.

Mariana Haragus, Universite de Franche-Comte, France
Title: Existence of defects in the Swift-Hohenberg equation
Abstract: We show the existence of grain boundaries and dislocations in the classical Swift-Hohenberg equation and in an anisotropic Swift-Hohenberg equation, respectively. While grain boundaries are found as steady waves connecting roll patterns with different orientations, dislocations are constructed as traveling waves connecting roll patterns with different wavenumbers. The analysis relies upon a spatial dynamics formulation of the bifurcation problem, a local center-manifold reduction, and normal form theory. We also discuss possible extensions and limitations of our approach.
joint work with Arnd Scheel (University of Minnesota)

Makoto Iima, Hokkaido University
Title: Strong interaction of localized convection cells and pattern formation in binary fluid mixture
Abstract: We study strong interaction among spatially localized convection patterns in binary fluid convection. We discuss bifurcation diagram and its system-size dependency for the spatially localized

convection cells which is traveling (traveling pulse). The solution is time-periodic in the frame moving with the traveling pulse. Using the solution, the strong interaction (collision) between the traveling pulses was analyzed; Most of the output were single traveling pulses. Based on these results, we discuss pattern formation process in a huge system size. This work is a collaboration with Dr. Takeshi Watanabe and Prof. Yasumasa Nishiura. -- Makoto Iima (makoto.iima@gmail.com)

Mat Johnson, Indiana University

Title: Nonlinear Modulational Stability of Periodic Traveling Waves of a Generalized Kuramoto-Sivashinsky Equation

Abstract: In this talk, we consider the stability of periodic traveling wave solutions of a generalized Kuramoto-Sivashinsky equation. In special cases it has been known since 1976 that, when subject to small localized perturbations, spectrally stable solutions of this form exist. Although numerical time-evolution experiments indicate that these waves should also be nonlinearly stable to such perturbations, an analytical verification of this result has only recently been provided. Here, I will discuss this result and, if time allows, I will also discuss the stability of such waves to small nonlocalized perturbations asymptotic to constant shifts in phase.

Theodore Kolokolnikov, Dalhousie University

Title: Particle interaction models of biological aggregation

Abstract: Animal groups often form striking aggregation patterns. Examples include from schools of fish to locust swarms, to patterns in bacterial cultures. In this talk, we discuss a very simple model of swarming based on pairwise particle interactions with short-range attraction and long-range repulsion, which can lead to very complex and intriguing patterns in two or three dimensions. Depending on the relative strengths of attraction and repulsion, a multitude of various patterns are observed, from nearly-constant density swarms to annular solutions, to complex N-fold symmetry patterns.

We show that annular-type patterns may form if short-range repulsion is sufficiently weak. A Turing-type analysis of such annular states further reveals a wealth of possible instabilities which often lead to complicated and beautiful patterns. We also classify two-ring, annular and triangular patterns which arise when the ring becomes unstable. In three dimensions, we classify the stability of a sphere using spherical harmonics.

Finally, we consider the inverse problem: given a target pattern, how to custom-design the force interaction to obtain said pattern as a steady state.

Joint work with Hui Sun, James von Brecht, David Uminsky and Andrea Bertozzi

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Shigeru Kondo , Osaka University

Title : Turing pattern in the real biological system

Abstract: Pigmentation patterns on fish skin keep changing during their growth in the way the simulation of Turing model (reaction-diffusion system) predicts. We have been investigating the underlying mechanism that generates the beautiful patterns. Recently we observed the in vitro behavior of pigment cells, which clarified the cellular interactions that generates the pigment pattern. Surprisingly, there is no diffusible reagent that acts as the activator or inhibitor of traditional RD model, but a specific structure and the behavior of pigment cells replace the rolls of the

putative diffusible molecules in the Turing model.

Gregory Kozyreff, Free University of Brussels
Title: Exponential asymptotic for fronts connecting an homogenous state and an hexagonal pattern.
Abstract In the first part of this talk we discuss the standard multiples-scale analysis of a slow front connecting a homogenous solution and a 1D small-amplitude periodic pattern. Beyond all orders of the formal calculation, the exponentially small terms responsible for the snaking bifurcation diagram can be found. These terms are intimately related to the complex singularities of the front. In the second part, we explain how to apply the same procedure in two dimensions. For certain orientations, analytic formulas for the fronts can be obtained and complex singularities can be located. This opens the way to analytically determine the pinning range of localized hexagonal patterns.

David Lloyd, University of Surrey, U.K.
Title: Some Problems with Multi-dimensional Localised Patterns
Abstract: In my talk I will present three problems involving mutli-dimensional localised patterns. The first problem will look at how one might ascertain the preferred large-scale structure of hexagon/roll patches in the Swift-Hohenberg equation by the calculation of Wulff structures (a technique used in crystallography to find the preferred crystalline shape of fixed volume inside a separate phase). The second problem will look at homoclinic snaking near the singular limit in a crime-hotspot model where we move away from the shadowing limit. The third problem looks at localised hexagon structures on the surface of a ferrofluid. We present numerical results of the full ferrofluid equations with a linear magnetisation law that suggest homoclinic snaking can explain the hexagon patches observed in experiments.

Scott McCalla, Brown U.
Title: Patterns in the Swift-Hohenberg equation
The existence of three different families of localized radially symmetric patterns in the Swift-Hohenberg equation will be discussed; the dimension will be treated as a continuous parameter in the equation. Snaking occurs for the one dimensional solutions. However, in higher dimensions the bifurcation curves break up in interesting ways. An analytic approach to proving the existence of these solutions at onset will also be outlined.

Yana Nec, Univ. British Columbia
Title: Existence and stability of spike-type solutions to one dimensional Gierer-Meinhardt model with sub-diffusion
Abstract: Spike-type solutions are shown to exist for a reaction -- sub-diffusion system with Gierer-Meinhardt kinetics. Sub-diffusion induces an asymmetry in the algebraic-differential system governing the evolution of an n-spike pattern, and causes a loss of inflexion invariance in the spike motion. The system depends on the anomaly index and is stable, conforming to $o(1)$ eigenvalues of the original system.
Full stability analysis, conforming to ${\mathcal O}(1)$ eigenvalues, yields a fractional non-local eigenvalue problem. For a pattern of n identical spikes the system is decoupled, and the stability onset

threshold and eigenvalue locations are shown to change in a non-intuitive way.

Keith Promislow, Michigan State University
Title: Network formation in Charged Polymer/solvent mixtures
Abstract: We present a non-intuitive, highly nonlinear reformulation of the Cahn-Hilliard energy for binary mixtures which allows the incorporation of the fundamental effects of electrostatics in charge-polymer solvent interaction. These effects are manifested as solvation energies, entropy, and self- energies which are not accessible from the classical Cahn-Hilliard formulations. The associated gradient flows shows a stunning diversity of network structures that have strong parallels to a wide host of organic structures (the Golgi apparatus, Streptococcus) as well as to membrane casting processes. We present sharp-interface reductions that lead to higher order Ricci curvature flows that couple the dynamical systems structure of the underlying network structures to the geometric flow. We discuss extensions to multi-phase mixtures, which is actually much more intuitive than for the Cahn-Hilliard energy.

Hans-Georg Purwins, Institut fur Angewandte Physik, Universitatt Munster, Germany (http://physik.uni-muenster.de/AP/Purwins/)
Title: Dissipative Solitons and the FitzHugh-Nagumo Equation
Abstract After defining the term ``Dissipative Soliton (DS)'', the emergence of the FitzHugh-Nagumo (FHN) equation and its generalization are discussed. This is followed by an intuitive discussion of the formation and stabilization of DSs in 1-dimensional space. Next the equation is used to describe qualitatively DS related pattern in a certain class of electrical transport systems. This leads to generalization of the FHN equation. The resulting 3-component equation qualitatively takes account of all important experimental observations. The talk finishes with general remarks on the state of the art of the detection of DSs in experimental systems and their modeling by evolution equations.
Literature:
``Dissipative Solitons'' by H.-G. Purwins, H. U. Bdeker and Sh. Amiranashvili, Advances in Physics 59 485 - 701 (2010)

Jens Rademacher, CWI Amsterdam
Title: Coherent structures in nanomagnets with aligned field
Abstract: We study Landau-Lifschitz-Gilbert equations with a gauge symmetry induced by alignment of external field and material anisotropy. We prove existence of a large class of coherent structures, which are built up from wavetrains or constant states in analogy to those studied in Ginzburg-Landau equations. Moreover, we discuss the stability of wavetrains.
joint work with Christof Melcher (RWTH Aachen).

Xiaofeng Ren, George Washington University
Title: Ansatz solutions to a problem of mean curvature and Newtonian potential
Abstract: Pattern formation problems arise in many physical and biological systems as orderly outcomes of self-organization principles. Examples include animal coats, skin pigmentation, and morphological phases in block copolymers. Recent advances in singular perturbation theory and asymptotic analysis have made it possible to study these problems rigorously. In this

talk I will discuss a central theme in the construction of various patterns as solutions to some well known PDE and geometric problems: how a single piece of structure built on the entire space can be used as an ansatz to produce a near periodic pattern on a bounded domain. We start with the simple disc and show how the spot pattern in morphogenesis and the cylindrical phase in diblock copolymers can be mathematically explained. More complex are the ring structure and the oval structure which can also be used to construct solutions on bounded domains. Finally we discuss the newly discovered smoke-ring structure and the toroidal tube structure in space. The results presented in this lecture come from joint works with Kang, Kolokolnikov, and Wei.

Arnd Scheel, U. Minnesota

Title: Pattern Selection Through Invasion in Cahn-Hilliard and Phase-Field Models

Abstract: I will discuss spinodal decomposition initiated by localized disturbances from a uniform state in Cahn-Hilliard and phase-field models. Typically, disturbances grow in the form of invasion fronts that create a pattern. We analyze the existence of such invasion fronts, in particular in regard to the selected wavenumber in the wake of the front. It turns out that the patterns selected in this fashion are very different from patterns selected from random initial conditions via the fastest-linear-mode principle. In particular, phase-field fronts select finite wavenumbers even in the limit of vanishing interfacial energy, while temporal selection predicts infinitely small scales. We also comment on coarsening processes, spatial resonances, and period-doubling sequences.

Tobias Schneider, Harvard University

Title: From Swift-Hohenberg to Navier-Stokes: Homoclinic Snaking in Plane Couette Flow

Abstract: For shear flows exact coherent solutions of the Navier-Stokes equations play key roles in the transition to turbulence and the turbulent dynamics itself. Here we examine a new class of spatially localized solutions to plane Couette flow. These solutions exhibit a sequence of saddle-node bifurcations similar to the ↑homoclinic snaking' phenomenon observed in the Swift-Hohenberg equation. The localized solutions exist over a wide range of Reynolds numbers and bifurcate off the known spatially periodic states. This is joint work with John F. Gibson, University of New Hampshire and John Burke, Boston University

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Kenneth Showalter, University of West Virginia

Title: Collective Behavior in Excitable Media: Interacting Particle-Like Waves

Abstract: We describe studies of interacting particle-like waves in the photosensitive Belousov-Zhabotinsky reaction. Unstable waves are stabilized by global feedback that affects the overall excitability of the medium, and the motion of these waves is controlled by imposing excitability gradients that are regulated by a secondary feedback loop. Waves interact via a Lennard-Jones type potential in which there are attractive forces at long distances and repulsive forces at short distances.

A. J. Steele, M. Tinsley, and K. Showalter, Chaos 18, 026108 (2008).

M. R. Tinsley, A. J. Steele, and K. Showalter, Eur. Phys. J. Special Topics 165, 161 (2008).

Takashi Teramoto, Chitose Institute of Science and Technology

Title: Deformation-induced spot dynamics in reaction-diffusion systems

What is the origin of rotational motion? An answer is presented through the study of the dynamics for spatially localized spots near codimension 2 singularity consisting of drift and peanut instabilities. The drift instability causes a head-tail asymmetry in spot shape, and the peanut one implies a deformation from circular to peanut shape. Rotational spot motion can be produced by combining these instabilities in a class of three-component reaction-diffusion systems. Partial differential equations dynamics can be reduced to a finite dimensional one by projecting it to slow modes. Such a reduction clarifies the bifurcational origin of variety of spot motion in two dimensions in close analogy to the standard dynamics of 1:2 mode interactions.

Peter Van Heijster, Brown University

Title: Stable traveling spots in a planar three-component FitzHugh-Nagumo system

Abstract: In this presentation, we analyze the bifurcation of a stationary planar spot to a traveling planar spot in a three-component reaction- diffusion system. Formally, we are able to determine a drift bifurcation line for an asymptotically small parameter epsilon using singular perturbation theory. We will check numerically, using AUTO, that this line is indeed approached for decreasing epsilon. Moreover, AUTO shows that the drift bifurcation is supercritical, which indicates that the traveling spot is stable. Finally, we compare the by AUTO constructed stable traveling spot with a stable traveling spot obtained from a direct solver.

This is joint work with B. Sandstede

Juncheng Wei, Chinese U. of Hong Kong

Title: Localized Solutions in Cross-Diffusion and Chemotaxis

Abstract: I will discuss our recent work on localized solutions in the cross-diffusion system and chemotaxis system. The latter system models crime hotspots. In both systems, we find stable localized solutions but the stability analysis is quite different. (Joint work with T. Kolokolnikov and M. Ward)

Michael I. Weinstein, Columbia University

Title: Emergence of Periodic Patterns in the Design of Metastable States

Abstract: In many optical, mechanical and quantum systems, it is = desirable to spatially confine energy in a particular mode for a long time interval. We first discuss the mathematics of energy confinement in = energy-conserving extended systems, in particular, the notions of scattering loss and radiation damping. We then present analytical and computational results on optimal = energy-confining=20 structures, e.g. potentials in the Schroedinger equation=20 or dielectric coefficients in the Helmholtz equation. A feature of the optimization is the emergence of periodic patterns.

Tomohiko Yamaguchi, Hitoshi Mahara and Kosuke Suzuki, Nanosystem Research Institute, AIST

Title: Thermodynamic Viewpoint of Emergence in Chemical Systems

Abstract: We deal with an entropy issue in two non-equilibrium chemical systems. One is the reversible Gray-Scott system. Self-duplication and the succeeding time-development of spots are interpreted in terms of spatial inhomogeneity of entropy production which is induced by diffusion. The other is a dewetting system in a simple laboratory experiment. An evaporating thin film of fullerene (C60) solution leaves a periodical band pattern of C60 microcrystals on a flat plate due to stick-slip motion. The wavelength of this pattern is proportional to the entropy production. Introducing these as examples, we would like to share some thermodynamic viewpoints in the process of localized pattern formation.