Combinatorial Games Workshop 11w5073

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1 Overview of the Field

Combinatorial game theory deals principally with two person games of perfect information and without chance elements. It provides techniques for determining who can win from a game position and how. While ad hoc techniques for such analysis are as old as humanity, the modern theory employs more powerful and general tools such as the notion of a game-theoretic value. From a mathematical standpoint, these techniques can be brought to bear to completely solve a wide variety of games. From the algorithmic perspective (that is, when the efficiency of the analysis is considered), there are efficient methods to analyze certain GO endgames and capture races, and various classes of impartial games. In addition, complexity results abound and when a game is provably NP-hard, there are heuristic methods that can be employed. In a hot game, the first player to move can gain a big advantage. Such games, like GO and AMAZONS, frequently decompose into independent sub-games where playing the best local move in a sub-game might be sub-optimal in the global context. The goal is to combine the local considerations into an optimal, or near optimal, global strategy. Indeed, the ancient Asian game of GO is one the last of the classical games of skill in which computer performance has not caught up with humans.

2 Recent Developments and Open Problems

Three important topics are: Temperature Theory; misere-play games; and complexity of impartial games. There have been recent strides forward in all three areas of the subject, many directly attributable to collaborations started in previous 'Games of No Chance' workshops at MSRI and BIRS.

1. Temperature theory: how to play when large gains can be made, is a direction of research that has played a central role in combinatorial game theory for many decades. Over the past fifteen years there have been several major advances in temperature theory, and many promising directions for further research have emerged see [2, 6, 8–11, 13–15] for a small selection. Mean and temperature are fundamental invariants of partizan games that quantify the value and urgency of a position. They were studied in the 1950s by Milnor and Hanner, but in the context of modern combinatorial game theory the first rigorous construction was given by Berlekamp, Conway and Guy in the late 1970s [3]. The Berlekamp–Conway–Guy construction applied only to short games, in which repetition is prohibited

and play must end after a finite number of moves. In the ensuing years there has been increasing interest in extending this theory to a wide class of loopy games, those in which repetition is permitted. One such extension was introduced by Elwyn Berlekamp [2] at the first 'Games of No Chance' conference in 1994, and several more recent advances have dramatically increased the scope and sophistication of the theory. These include Fraser's [6] extension of temperature to games with long cycles, and the dogmatic theory of hyperactive positions due to Berlekamp and Spight [15]. These advances suggest numerous possibilities for further research, including a better understanding of cold loopy games; generalizations of heating and cooling to the loopy context; and ultimately, a unified temperature theory that applies to all loopy games. One of the most exciting elements of this theory is its application to the Asian board game GO, which has been a motivating influence in the development of combinatorial game theory. This connection has attracted attention from numerous mathematicians and GO players from Japan and Korea, several of whom have attended previous 'Games of No Chance' conferences (see [12] for example). A further interplay comes from a new heuristic that has been so effective in GO, that of Monte Carlo Tree Search (MCTS) methods (see [5,7] for example). These are sample-based search approaches using Monte-Carlo simulations and selective tree search. They have recently lead to a breakthrough, and considerably closed the gap to the best human players.

2. Misere-play Impartial Games: this was a major topic of the 2008 BIRS Games workshop. Recent work (July 2009) by Guo and Miller with Plambeck, Siegel and Weiemerskirsh looking on, suggest a radical change in the mathematical setting and tools. Instead of regarding the games as quotient monoids with a bi-partition they are now encoded as lattice points in rational convex polyhedra. Encodings provided by these lattice games can be made particularly efficient for octal games. The setting of lattice games naturally allows for misere play, where 0 is declared a losing position. Lattice games also allow situations where larger finite sets of positions are declared losing. Generating functions for sets of winning positions provide data structures for strategies of lattice games. The main conjecture is that every lattice game has a rational strategy: a rational generating function for its winning positions. Another conjecture is that every lattice game has an affine stratification: a partition of its set of winning positions into a finite disjoint union of finitely generated modules for affine semigroups. If true, this would give an effective algorithm for solving the games and would represent a great step forward In 2009, Meghan Allen [1] extended to misere partizan games the monoid approach of Plambeck and Seigel that has worked well in impartial games. This involves a tetra-partition of the misere quotient as opposed to a bipartition in the impartial case. Her techniques worked well in characterizing all misere quotients that are isomorphic to that generated by $* = \{0|0\}$. Two important questions that are central to furthering the techniques are: if the monoids for S and T are isomorphic when are they isomorphic to that of $S \cup T$? For which class of games is $* + * = 0 (\mathrm{mod}\ g)$ for all g in the class?

3. Normal-play Impartial Games: The lattice game approach of Guo and Miller for misere-play games also provides a framework for normal-play impartial games. It provides an algorithm to compute strategies for many heap games in a natural setting. How effective this is for all impartial games is yet to be seen. At the same time Fraenkel & Peled [5] found another computationally effective approach for many impartial games. A pair of integer sequences that split the positive integers I is often—especially in the context of impartial games—defined recursively by $A_n = \mathrm{mex}\{A_i, B_i : 0 \leq i < n\}$, $B_n = A_n + C_n, n \geq 0$, where $\mathrm{mex}(S)$ is the smallest nonnegative integer not in S, and $C : I \to I$. Given $x, y \in I$, a typical problem is to decide whether $x = A_n, y = B_n$. For general functions C_n, the best algorithm for this decision problem was until now exponential in the input size $\omega(logx + logy)$. Very recently it was proved constructively that the problem is actually polynomial for the wide class of approximately linear functions C_n. This solves constructively and efficiently the complexity question of a number of previously analyzed take-away games of various authors. It is of interest to extend this result in new directions, such as: (i) relax the requirment of C_n being "approximately linear"; (ii) consider the case where A_n and C_n are not necessarily additively related. These extensions may lead to classes of new games not yet imagined.

3 Presentation Highlights and Scientific Progress Made

Monte Carlo methods in combinatorial games (see [4, 7] for example) was a major topic at the workshop. Olivier Teytaud's keynote address, *Monte Carlo Tree Search Method*, was an excellent introduction to the subject by one of the world's experts. He gave a wonderful overview. The easy format afforded by BIRS allowed time for the audience, a mix of computer scientists, mathematicians and assorted graduate and undergraduate students, to ask pertinent questions and to have discussions afterwards. The talk and later discussions gave sufficient details so that several computer scientists where able to produce a Monte Carlo based program for the computer version of the games tournament. Even more impressive was the fact that Jiang Zhujiu, a professional GO player attending the conference, later was able to comment on games played by a GO-playing, Monte Carlo based, program. He also played against the computer and gave comments on the evaluation techniques of the program, providing some very valuable insights.

Formally, Olivier's talk was backed up by presentations by Cazenave and by Mueller both of whom attempting to use Monte Carlo methods to quickly approximate the temperature of a game. Temperature approximation is something that the professional players do very quickly (a top-down approach) but the professional 'scientists' only have a cumbersome, and time consuming, bottom up approach. Again, Jiang Zhujiu's comments were very insightful. There were informal meetings every afternoon to discuss all the Monte-Carlo approaches.

Games form a partial order so it is of no surprise that lattices arise. A major difficulty is that there are few classes of games known which give rise to lattices and in these classes the order grows so large so quickly that little intuitive insight can be gained. (In general, the classes are stratified by 'games born by day n'.) Angela Siegel's talk, *Distributive and other lattices in CGT*, presented two very new and exciting results. A result of Albert & Nowakowski (2011) shows that any closed set of short games generates a distributive lattice. She extended this result to include a subclass of loopy games. She then presented a class of games whose 'born by day n' subclass gives a planar but not distributive lattice, the first non-distributive lattice found in the area. Immediately, a special afternoon session was set up, the outcome of which will be a paper with eight authors. In addition, enough excitement was generated that one of other the graduate students has changed her PhD topic from algebra to lattices in combinatorial game theory.

The researchers favouring impartial games held very active and long informal sessions every afternoon and also during the evenings. A special mention must go to Alan Guo & Mike Weimerskirch's talk, advertised as *Lattice point methods in misere games*. The lattice point method, developed by Ezra Miller& Alan Guo, has more general applications than just misere games. Indeed, during the week it was realized that a problem of Aviezri Fraenkel could be approached with this method. Fraenkel and Weimerskirch led several informal and successful, sessions and a paper is in the works. During the workshop, a class of games was discovered that are, in some sense, minimal in not being covered by Miller and Guo's methods. There is now a PhD student whose topic is to generalize the lattice point method to larger classes of games.

The game of GO is always an important topic. Two linked main themes have always been the implementation of strategies via computers and evaluation of temperature. There was much overlap with the Monte-Carlo methods talks and discussions but there were two non-computer related talks of great interest. Elwyn Berlekamp gave a report of the *coupon GO* tournament held two months earlier in Korea. 'Coupons' are a way of providing a rich environment, an environment that allows a smoother way of evaluating the temperature of a game. Starting from a position in a famous GO game, 6 professional GO players played a round robin tournament playing each other as black and as white. The results were a surprise to the professionals 'players' as well as to the 'scientists'. Each day, the results were also reported in the daily newspapers! The analysis of the games is still on-going. Teigo Nakamura talked about the extensions of his remarkable results first reported in the 2008 BIRS Workshop. His approach to complicated capturing races was to 'cool by 2'. He reported on several (general) situations that have defeated professional GO players even outside of tournament play. Another interesting sidelight was the exposition of the SEKI game by Gurevich but reported by Vladimir Oudalov. This was to take situations that were explicitly not covered by Nakamura's approach and to, essentially, turn the problem into one with linear algebra aspects.

All of the talks resulted in some discussions. We point out just three other highlights.

The first general result in combinatorial game theory is that every impartial game is equivalent to a game of of NIM with exactly one heap and there is a well-defined and algorithmic way of finding the requisite NIM heap. No such game was known for the general partizan theory. That changed with Carlos Santos's talk, *A Non-trivial Surjective Map onto the Short Conways Group* showed that every short game is equivalent to position in GENERALIZED KONANE and gave a technique for creating the position.

Neil McKay's talk, *Ordinal Sums with base* , re-started the discussion of how to play HACKENBUSH flowers. This is a long standing question (40 years) of Berlekamp's. This discussion occupied many evenings.

Kyle Burke, a recent graduate and new to academia, had the very pleasant experience of mentioning, in his talk, a complexity problem that interested Erik Demaine. The next day, Erik was able to announce that he and Kyle had solved the problem.

4 Outcome of the Meeting

One of the main goals of the Workshop was to create and strength ties between researchers and between fields. In this, the meeting was a huge success. New collaborations happened during the Workshop which will result in several papers. Since the Workshop, this collaboration continued and others have started and now there is active collaboration between groups in North America and Europe where before there was none.

In addition, three graduate students found topics for their PhD theses with the world's experts giving their 'blessing' and advice. The students have already become part of the community and have an international audience for their results. This has given the students a greater sense of pride in and resolve for their work.

Much of the success is due to the BIRS staff who made it easy for us to conduct the business of the Workshop without having to worry about other minor or major details. Given the diverse nature of the researchers in the area of Combinatorial Game Theory, the format, talks in the morning, informal sessions in the afternoon, is ideal. The formal talks allowed the topics for discussion to be laid out then the various well appointed rooms, allowed formation of the informal groups to discuss any of the topics. People could wander between groups and groups could merge when appropriate. The Lounge, itself, deserves to be a co-author on all of the papers that will result from the Workshop.

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