On the local minima of the Mahler products

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Abstract

For a convex body K in $\mathbb{R}^n$, the volume product of K is given by

$$P(K) = \min_{z \in K} \mathrm{vol}(K)\mathrm{vol}(K^{*z})$$

where $K^{*z} = \{y \in \mathbb{R}^n; \langle y - z, x - z \leq 1 \text{ for all } x \in K\}$ is the polar body of K with respect to z. Similarly if $\phi : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ if a convex function such that $0 < \int e^{\phi} < +\infty$, the "Mahler" product of ϕ is defined by

$$P(\phi) = \min_{z \in \mathbb{R}^n} \int e^{-\phi} \int e^{-\mathcal{L}^z \phi}$$

where $\mathcal{L}^z \phi(y) = \sup_{x \in \mathbb{R}^n} \langle x - z, y - z \rangle - \phi(x)$ is the Legendre trnsform of ϕ with respect to z. We prove that on the set of all (centrally symmetric) convex bodies, the volume product has no local mimimum at any (centrally symmetric) body having positive Gauss curvature at some boundary point and that on the set of all convex functions (convex even functions) the Mahler product has no local minimum at any function (even function) having some regular point.