Linear Algebra Techniques in Combinatorics & Graph Theory (11w5033)

Jan. 30 - Feb, 4, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card—given to you at check-in—at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a **cash honor system**.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–9:45	Douglas Stinson
9:45–10:30	Charles J. Colbourn
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00-11:45	Felix Lazebnik
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00-14:15	Group Photo; meet on the front steps of Corbett Hall
14:15–15:00	Bojan Mohar
15:00 –15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30 –16:45	A. Mohammadian
16:45–17:00	Vladimir Nikiforov
17:00–17:45	Steve Kirkland
17:45–19:30	Dinner

Tuesday	
7:00–9:00	Breakfast
9:00–9:45	Bart De Bruyn
9:45–10:30	Aart Blokhuis
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45	Richard Wilson
11:45–13:30	Lunch
13:30-14:15	Christian Krattenthaler
14:15–15:00	Jack Koolen
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
1530–16:15	Shmuel Friedland
16:15–17:00	Ian Wanless
17:00–17:45	Mieko Yamada
17:45–19:30	Dinner
21:30–???	**Karaoke** at Wild Bill's Legendary Saloon
	Brave souls meet at 21:30 in 2nd floor lounge to walk to main drag of Banff
	Karaoke begins at 22:00
Wednesday	
7:00–9:00	Breakfast
9:00–9:45	Paul Terwilliger
9:45–10:30	Hajime Tanaka
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00-11:45	M. Mitjana
11:45–12:30	William J. Martin
12:30–13:30	Lunch
13:30–17:30	Free Afternoon to hike,wander, etc.
17:30–19:30	Dinner
Thursday	
7:00–9:00	Breakfast
9:00–9:45	Rob Craigen
9:45–10:30	Qiang (Steven) Wang
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45	Carlos Fonseca
11:45–13:30	Lunch
13:30-14:15	Yaokun Wu
14:15–15:00	Juan Rada
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
1530–16:15	Wei Wang
16:15–17:00	Behruz Tayfeh-Rezaie
17:00–17:30	Sejeong Bang
17:30–19:30	Dinner
Friday	
7:00–9:00	Breakfast
9:00-10:30	Problem Session and Informal Discussion
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon

** We are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although we are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Title: Geometric distance-regular graphs without 4-claws

Speaker: Sejeong Bang, Seoul National University

Abstract: A non-complete distance-regular graph Γ is called geometric if there exists a set $\mathcal{C}$ of Delsarte cliques such that each edge of Γ lies in a unique clique in $\mathcal{C}$. In this talk, we determine the non-complete distance-regular graphs satisfying $\max\{3, \frac{8}{3}(a_1+1)\} < k < 4a_1 + 10 - 6c_2$. To prove this result, we first show by considering non-existence of 4-claws that any non-complete distance-regular graph satisfying $\max\{3, \frac{8}{3}(a_1+1)\} < k < 4a_1 + 10 - 6c_2$ is a geometric distance-regular graph with smallest eigenvalue -3. Moreover, we classify the geometric distance-regular graphs with smallest eigenvalue -3. As an application, 7 feasible intersection arrays are ruled out.

Title: Maximal cocliques in Point-Hyperplane graphs

Speaker: **A. Blokhuis**, Eindhoven University of Technology

Abstract: We explain the proof of an Erdős-Ko-Rado type theorem for the Kneser graph on the point-hyperplane flags of a finite projective space. More precisely: If C is a set of point-hyperplane flags of $PG(n-1, q)$, such that for each pair (P_i, H_i), $i = 1, 2$ we have $P_1 \in H_2$ or $P_2 \in H_1$ (or both), then $|C| \leq 1 + 2q + 3q^2 + \cdots + (n-1)q^{n-2}$. Related results and problems will also be discussed.

Title: A Handful of Sparse Testing Problems

Speaker: **Charles J. Colbourn**, Arizona State University

Abstract: Consider the following questions:

1. Given a population of n items, of which at most t are defective, what is the smallest number of ‘pools’ to determine the defective items?
2. Given a faulty system of n components, of which at most t interact to create the fault, what is the smallest number of ‘tests’ to determine the faulty interaction?
3. Given an implementation of a boolean function of n variables, of which at most t are relevant, what is the smallest number of ‘queries’ to determine the function?
4. In a set of n users, each is given a copy of a piece of software in which specified bits carry a unique fingerprint. Given that a coalition of at most t users can collude to locate (as far as possible) the bit locations of the fingerprint, and change them arbitrarily, what is the fewest bits so that no user can be ‘framed’ by the coalition?
5. Given an n-dimensional signal, of which at most t coordinates are significant, what is the smallest number of ‘measurements’ to determine the signal?

In each case, tests or samples are conducted nonadaptively, i.e., without reference to results of other tests or samples. Phrased in this way, these five problems sound very similar. All involve sparsity in an essential way. Yet they arise in quite disparate research fields: combinatorial group testing (union- and cover-free families; disjunct matrices; superimposed codes); interaction testing (covering arrays; surjective codes; qualitatively independent partitions); computational learning; combinatorial cryptography (frameproof codes); and compressive sensing.

In this presentation, underlying combinatorial similarities among these five problems are developed.

Title: A construction for Hadamard matrices using Unreal matrices.

Speaker: **R. Craigen**, University of Manitoba
Coauthors: B. Compton, W. de Launey

Abstract: In this remarkable construction Hadamard matrices are obtained from Butson-Hadamard matrices whose entries are complex sixth roots of unity, with an extra condition: that they are *unreal*, which means precisely what the name suggests: no entries are real. This was work done a couple of years ago with Warwick de Launey, one of the last things I did with him and his associate Bobby Compton. Although the method is quite new, it is directly tied to a classical way to obtain Hadamard matrices, namely the Williamson construction. Some elegant matrix algebra reveals the connection. I will speak more generally about the unusual condition these matrices must satisfy, and some potential ways to adapt the technique for more general use.

Title: Pseudo-embeddings and pseudo-hyperplanes of point-line geometries

Speaker: **Bart De Bruyn**, Ghent University

Abstract: Let $\mathcal{S} = (\mathcal{P}, \mathcal{L}, \mathrm{I})$ be a point-line geometry with point set $\mathcal{P}$, line set $\mathcal{L}$ and incidence relation $\mathrm{I} \subseteq \mathcal{P} \times \mathcal{L}$ having the property that the number of points on each line is finite and at least three.

Suppose V is a vector space over the field $\mathbb{F}_2$ of order 2. A *pseudo-embedding* of $\mathcal{S}$ into the projective space $\mathrm{PG}(V)$ is a mapping e from $\mathcal{P}$ to the point-set of $\mathrm{PG}(V)$ satisfying the following two properties: (1) $< e(\mathcal{P}) >= \mathrm{PG}(V)$; (2) if L is a line of $\mathcal{S}$ containing the points $x_1, x_2, \ldots, x_k$ and $\bar{v}_i$, $i \in \{1, 2, \ldots, k\}$, is the unique vector of V such that $e(p_i) =< \bar{v}_i >$, then $\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_{k-1}$ are linearly independent and $\bar{v}_1 + \bar{v}_2 + \cdots + \bar{v}_k = \bar{o}$.

A pseudo-embedding $e : \mathcal{S} \to \mathrm{PG}(V)$ is called *homogeneous* if for every automorphism θ of $\mathcal{S}$, there exists a (necessarily unique) projectivity ϕ_θ (arising from an element of $GL(V)$) such that $e(p^\theta) = e(p)^{\phi_\theta}$ for every point p of $\mathcal{S}$.

A *pseudo-hyperplane* of $\mathcal{S}$ is a set X of points of $\mathcal{S}$ such that $\mathcal{P} \setminus X$ has an even number of points in common with each line of $\mathcal{S}$.

In the talk, I will discuss several connections between the notions pseudo-embedding and pseudo-hyperplane. I will also discuss the pseudo-hyperplanes and homogeneous pseudo-embeddings of certain projective spaces, affine spaces and generalized quadrangles.

Title: Some results on P-vertices of trees

Speaker: **Carlos Fonseca**, University of Coimbra, Portugal

Abstract: We consider some open questions presented recently by Kim and Shader on a continuity-like property of the P-vertices of nonsingular matrices whose graph is a path. The case of the stars is also considered. Some general results are provided for trees.

Title: Some open problems in matchings in graphs

Speaker: **Shmuel Friedland**, University of Illinois at Chicago.

Abstract: In the first part of this lecture we discuss the conjecture for the maximal number of k-matching in r-regular bipartite graph on $2n = 2mr$ vertices. Namely, the maximum number of k-matchings is achieved for a disjoint union of m $K_{r,r}$, the complete regular bipartite graphs on $2r$-vertices. (This is known for $r = 3$.) The perfect matching case, i.e. $k = n$ follows from Bregman's inequality.

In the second part we study the analog of the van der Waerden conjecture for a weighted complete graph on $2n$ vertices, K_{2n}. That is, let $\Psi_{2n} \subset R_+^{n\times n}$ be a convex hall of all adjacency matrices representing the subgraphs of perfect matchings in K_{2n}. So Ψ_{2n} is a subset of symmetric doubly stochastic matrices, with zero diagonal. which satisfyies Edmond's condition. Namely, it is necessary and sufficient that the sum of the weighted edges going out of any odd number set of vertices is at least 1. Let $haf(A)$ be the haffnian of $A \in \Psi_{2n}$, i.e. the sum of all the weights of all perfect matchings in K_{2n}. Let $\mu_n = \min_{A\in\Psi_{2n}} haf(A) = haf(L_{2n})$. Is L_{2n} has all off-diagonal entries equal to $\frac{1}{2n-1}$? (Probably not.) Is $\liminf(\mu_n)^{\frac{1}{n}} > \frac{1}{3}$.

Title: Bipartite subgraphs and the signless Laplacian matrix

Speaker: **Steve Kirkland**, National University of Ireland Maynooth

Abstract: For a connected graph G, we consider the corresponding signless Laplacian matrix $Q = D + A$, where A is the adjacency matrix of G, and D is the diagonal matrix of vertex degrees. Motivated by the observation that G is bipartite if and only if the smallest eigenvalue of Q is zero, we investigate how, for a nonbipartite connected graph G, a vector x yielding a small Rayleigh quotient for Q can be used to identify a bipartite subgraph that is only weakly connected to the rest of G. Specifically, for such a vector x, we let H be the subgraph induced by the nonzero entries of x, and we give a sufficient condition in order for H to be bipartite. We also provide an upper bound on the number of vertices of H that are adjacent to at least one vertex in $G \setminus H$. Our results are applied to some graphs with degree sequences approximately following a power law distribution, and to a graph arising from protein-protein interaction.

Title: Hoffman graphs and graphs with smallest eigenvalue -3

Speaker: **Jacobus H. Koolen**, Postech, S. Korea Abstract: Hoffman give a structured way to construct

graphs with smallest at least some fixed number. In this talk I will present some recent work on Hoffmann graphs and will discuss the class of graphs with smallest eigenvalue -3 in more details. (This talk is based on joint work with A. Munemasa, T. Taniguchi, H. Jang and H. Yu)

Title: Determinants and Pfaffians in Enumerative Combinatorics

Speaker: **C. Krattenthaler**, Universität Wien

Abstract: In this talk I shall explain why many enumerative combinatorialists are fascinated by determinants — obviously from a strongly biased personal perspective. The particular sources where determinants arise in combinatorial contexts are non-intersecting lattice paths, orthogonal polynomials, and (combinatorial) representation theory (among others). My personal interest comes especially from the fact that this connection can be used with great advantage in the enumeration of rhombus tilings. In the talk, I shall explain this connection, the relevant enumeration theorems, giving rise to determinants and Pfaffians, and present some attractive sample applications in the enumeration of rhombus tilings, among which some old ones and some more recent ones.

Title: Alon-Tarsi conjecture on a nowhere-zero point in linear mappings: a survey.

Speaker: **Felix Lazebnik**, University of Delaware

Abstract: In this talk I will survey results related to the following conjecture by N. Alon and M. Tarsi (1989): *Let A be a nonsingular n by n matrix over the finite field* $\mathbf{F}$, *$q \geq 4$, then there exists a vector x in* $\mathbf{F}^n$ *such that both x and Ax have no zero component.*

Title: Combinatorial objects as finite varieties

Speaker: **W.J. Martin**, Worcester Polytechnic Institute

Abstract: We propose to introduce tools from algebraic geometry in the study of combinatorial configurations. Let k be a field and let X be a finite subset of k^m. The *ideal* of X is the set of all polynomials in $k[Z_1, \ldots, Z_m]$ which vanish on all points of X. We consider two parameters $g(X)$ and $h(X)$: the first is defined as the smallest total degree of any nonzero polynomial in I; the second is defined by taking the minimum, over all possible generating sets S for ideal I, of the maximum degree of a polynomial in S. For example, any spherical configuration X in Euclidean space has $g(X) = 2$ and many of these, such as the unit hypercubes, have $h(X) = 2$ as well.

Title: The M–property for distance–regular graphs

Speaker: **M. Mitjana**, Universitat Politècnica de Catalunya (Spain)

Abstract: We analyze when the Moore–Penrose inverse of the combinatorial Laplacian of a graph is a M–matrix; that is, it has non–positive off–diagonal elements or, equivalently when the Moore–Penrose inverse of the combinatorial Laplacian of a graph is also the combinatorial Laplacian of another network. When this occurs we say that the graph has the M–property. We prove that only distance–regular graphs with diameter up to three can have the M–property and we give a characterization of the graphs that satisfy the M–property in terms of their intersection array. Moreover, we exhaustively analyze strongly regular graphs having the M–property and we give some families of distance regular graphs with diameter three that satisfy the M–property. Roughly speaking, we prove that all distance–regular graphs with diameter one; about half of the strongly regular graphs; only some imprimitive distance–regular graphs with diameter three, and no distance–regular graphs with diameter greater than three, have the M–property. In addition, we conjecture that no primitive distance–regular graph with diameter three has the M–property.

Joint work with: E. Bendito, A. Carmona and A.M. Encinas

Title: Non-regular Graphs with Four Distinct Laplacian Eigenvalues

Speaker: **A. Mohammadian**, Institute for Research in Fundamental Sciences (IPM)

Abstract: In this talk, we investigate connected non-regular graphs with four distinct Laplacian eigenvalues. We characterize all such graphs which are bipartite or have exactly one multiple Laplacian eigenvalue. Other examples of interest are also presented.

Title: Spectrally degenerate graphs

Speaker: **Bojan Mohar**, Simon Fraser University

Abstract: It is well known that the spectral radius of a tree whose maximum degree is D cannot exceed $2\sqrt{D-1}$. Similar upper bound holds for arbitrary planar graphs, whose spectral radius cannot exceed $\sqrt{8D}+10$, and more generally, for all d-degenerate graphs, where the corresponding upper bound is $\sqrt{4dD}$. Following this, we say that a graph G is *spectrally d-degenerate* if every subgraph H of G has spectral radius at most $\sqrt{d\Delta(H)}$, where $\Delta(H)$ denotes the maximum degree of the graph. A rough converse of the above-mentioned results will be presented. It will be shown that each spectrally d-degenerate graph G contains a vertex whose degree is at most $4d\log_2(\Delta(G)/d)$ (if $\Delta(G) \geq 2d$). It will be shown that the dependence on Δ in this upper bound cannot be eliminated. The proofs involve probabilistic techniques. It may also be proved that the problem of deciding if a graph is spectrally d-degenerate is co-NP-complete. This is joint work with Zdeněk Dvořák.

Title: Extremal graph theory and the signless Laplacian

Speaker: **V. Nikiforov**, University of Memphis

Abstract: In recent years, most of the classical extremal graph results related to the Turán theorem have been stated in a stronger form in terms of the spectral radius of the adjacency matrix. This talk will present similar results about $q(G)$, the spectral radius of the signless Laplacian of a graph G.

The general extremal problem that will be discussed is the following one: How large can $q(G)$ be, if G is a graph of order n, with no subgraph isomorphic to a given graph F? Somewhat surprisingly, this problem has a tight asymptotic solution for all graphs F.

The first step of the solution is a fairly general theorem giving asymptotics and effective bounds of $q(G)$, when G belongs to some infinite abstract class. This result, coupled with other techniques, gives a tight upper bound on $q(G)$ in terms of the order and the clique number of G. In particular, this bound implies the Turán theorem and solves a conjecture of Hansen and Lucas.

Finally, it turns out that if G of order n and $q(G)$ is slightly above the value implying that G contains a complete graph of order r, then G contains a complete r-partite graph in which every part has size $\Omega(\log n)$. This fact, in particular, implies the asymptotic solution of the general extremal problem outlined above.

Part of this research is joint with Nair de Abreu.

Title: Bounds for the low energy of a digraph

Speaker: **Juan Rada**, Universidad Simon Bolivar

Abstract: The energy of a graph G was defined by I. Gutman in 1978 as

$$E(G) = |\lambda_1| + \cdots + |\lambda_n|$$

where n is the number of vertices of G and $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of G, which are real numbers. Recently, this concept was extended to digraphs as

$$e(D) = |Re(z_1)| + \cdots + |Re(z_n)|$$

where $z_1, \ldots, z_n$ are the eigenvalues of D. This definition was motivated by Coulson's integral formula

$$e(D) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \left[n - \frac{ix\Phi_D'(ix)}{\Phi_D(ix)} \right] dx$$

where Φ_D denotes the characteristic polynomial of D. $e(D)$ was called the low energy of D.

The aim of this talk is to give some bounds for the low energy of a digraph. More specifically, we will give sharp bounds for any digraph D:

$$\sqrt{2c_2} \le e(D) \le \sqrt{\frac{1}{2} n (a + c_2)}$$

where c_2 is the number of walks of length 2 and a is the number of arcs.

When D is a normal digraph (i.e. its adjacency matrix is a normal matrix), the bounds mentioned above can be improved as follows:

$$\sqrt{a + c_2} \le e(D) \le \frac{a}{n} + \sqrt{(n-1)\left[\frac{a + c_2}{2} - \left(\frac{a}{n}\right)^2\right]}$$

We also discuss when equalities occur.

Finally, we will study the problem of which digraphs have at most three eigenvalues. This is related to the problem of digraphs with minimal energy.

Title: Two spectral characterizations of regular, bipartite graphs with five eigenvalues

Speaker: **Dragan Stevanović**, University of Niš, Serbia

Abstract: Graphs with a few distinct eigenvalues usually possess an interesting combinatorial structure. We show that regular, bipartite graphs with at most six distinct eigenvalues have the property that each vertex belongs to the constant number of quadrangles. This enables to determine, from the spectrum

alone, the feasible families of numbers of common neighbors for each vertex with other vertices in its part. For particular spectra, such as $[6, 2^9, 0^6, -2^9, -6]$ (where exponents denote eigenvalue multiplicities), there is a unique such family, which makes it possible to characterize all graphs with this spectrum.

Using this lemma we also to show that, for $r \geq 2$, a graph has spectrum $[r, \sqrt{r}^{r(r-1)}, 0^{2(r-1)}, -\sqrt{r}^{r(r-1)}, -r]$ if and only if it is a graph of a 1-resolvable transversal design $TD(r, r)$, i.e., if it corresponds to the complete set of mutually orthogonal Latin squares of size r in a well-defined manner.

Title: Retransmission Permutation Arrays

Speaker: **Douglas R. Stinson**, University of Waterloo

Abstract: Li, Liu, Tan, Viswanathan, and Yang introduced a technique for resolving overlapping channel transmissions that used an interesting new type of combinatorial structure. In connection with this problem, they provided an example of a 4×4 array having certain desirable properties. We define a class of combinatorial structures, which we term *retransmission permutation arrays*, that generalise the example that Li *et al.* provided. We show that these arrays exist for all possible orders. We also define some extensions having additional properties, for which we provide some partial results.
This talk is based on joint work with Jeff Dinitz, Maura Paterson and Ruizhong Wei.

Title: Vertex subsets with minimal width and dual width in Q-polynomial distance-regular graphs

Speaker: **Hajime Tanaka**, University of Wisconsin & Tohoku University

Abstract: Every face (or facet) of a hypercube is again a hypercube. We generalize this situation and introduce the concept of a *descendent* of a Q-polynomial distance-regular graph. Many examples of descendents arise within the intrinsic geometric structure of the graph. Special attention will be paid to the convexity of descendents. We also discuss an application to the Erdős–Ko–Rado theorem in extremal set theory.

Title: Integral trees of odd diameters

Speaker: **B. Tayfeh-Rezaie**, Institute for Research in Fundamental Sciences (IPM), Iran

Abstract: A graph is called integral if all eigenvalues of its adjacency matrix consist entirely of integers. Recently, Csikvári proved the existence of integral trees of any even diameter. In the odd case, integral trees have been constructed with diameter at most 7. In this paper, we show that for every odd integer $n > 1$, there are infinitely many integral trees of diameter n. This is a joint work with E. Ghorbani and A. Mohammadian.

Title: Tridiagonal pairs and distance-regular graphs

Speaker: **Paul Terwilliger**, U. Wisconsin-Madison

Abstract: Let $\mathbb{F}$ denote a field and let V denote a vector space over $\mathbb{F}$ with finite positive dimension. We consider a pair of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy the following conditions:

(i) Each of A, A^* is diagonalizable.

(ii) There exists an ordering $\{V_i\}_{i=0}^d$ of the eigenspaces of A such that

$$A^* V_i \subseteq V_{i-1} + V_i + V_{i+1} \qquad (0 \leq i \leq d),$$

where $V_{-1} = 0$ and $V_{d+1} = 0$.

(iii) There exists an ordering $\{V_i^*\}_{i=0}^{\delta}$ of the eigenspaces of A^* such that

$$A V_i^* \subseteq V_{i-1}^* + V_i^* + V_{i+1}^* \qquad (0 \leq i \leq \delta),$$

where $V_{-1}^* = 0$ and $V_{\delta+1}^* = 0$.

(iii) There is no subspace W of V such that $AW \subseteq W$, $A^*W \subseteq W$, $W \neq 0$, $W \neq V$.

We call such a pair a *tridiagonal pair* on V. In the first part of the talk we classify up to isomorphism the tridiagonal pairs over an algebraically closed field. In the second part of the talk we discuss how tridiagonal pairs arise in algebraic graph theory. The connection is summarized as follows. For each tridiagonal pair the members of the pair satisfy two cubic polynomial relations called the *tridiagonal relations*. The corresponding *tridiagonal algebra* T is defined by two generators subject to those relations. The algebra T is noncommutative and infinite-dimensional. Let Γ denote a Q-polynomial distance-regular graph with vertex set X. Fix $x \in X$. Then there exists a tridiagonal algebra T over $\mathbb{C}$ and a representation $\rho : T \to \mathrm{Mat}_X(\mathbb{C})$ such that both (i) $\rho(A)$ is the $(0,1)$-adjacency matrix of Γ; (ii) $\rho(A^*)$ is diagonal with (y,y)-entry θ_i^*, where i denotes the path-length distance between x, y and θ_i^* is the ith dual eigenvalue of the Q-polynomial structure. The image $\rho(T)$ coincides with the subconstituent algebra of Γ with respect to x. This is joint work with Tatsuro Ito and Kazumasa Nomura.

References

[1] P. Terwilliger. The subconstituent algebra of an association scheme I. *J. Algebraic Combin.* **1** (1992) 363–388.

[2] T. Ito, K. Tanabe, and P. Terwilliger. Some algebra related to P- and Q-polynomial association schemes, in: *Codes and Association Schemes (Piscataway NJ, 1999)*, Amer. Math. Soc., Providence RI, 2001, pp. 167–192; `arXiv:math.CO/0406556`.

[3] T. Ito, K. Nomura, P. Terwilliger. A classification of the sharp tridiagonal pairs. *Linear Algebra Appl.* Submitted 2010; `arXiv:1001.1812`.

Title: Orthogonal arrays, dual codes, and divisibilities of polynomials over finite fields

Speaker: **Qiang (Steven) Wang**, Carleton University

Abstract: Consider a maximum-length shift-register sequence generated by a primitive polynomial f over a finite field. The set of its subintervals is a linear code whose dual code is formed by all polynomials divisible by f. Since the minimum weight of dual codes is directly related to the strength of the corresponding orthogonal arrays, we can produce orthogonal arrays by studying divisibility of polynomials. Munemasa (Finite Fields Appl., 4(3):252-260, 1998) uses trinomials over $\mathbb{F}_2$ to construct orthogonal arrays of guaranteed strength 2 (and almost strength 3). That result was extended by Dewar, Moura, Panario, Stevens and Wang (Des. Codes Cryptogr., 45:1-17, 2007) to construct orthogonal arrays of guaranteed strength 3 by considering divisibility of trinomials by pentanomials over $\mathbb{F}_2$.

In this talk we review the above results and then extend some of the above results by dropping either the primitivity restriction or the binary field restriction. We also briefly comment on the combinatorial applications. In particular, we show how we can use linear algebraic techniques (i.e., system of linear equations) to obtain some of our results. This is a joint work with Daniel Panario, Olga Sosnovski, and Brett Stevens.

Title: On some families of graphs determined by their generalized characteristic polynomials

Speaker: **Wei Wang**, Department of Mathematics, Xi'an Jiaotong University

Abstract: For a given graph $G = (V, E)$ with adjacency matrix A_G, the generalized characteristic polynomial of G is defined to be $\phi_G = \phi(\lambda, t) = det(\lambda I - A_G + tD_G)$, where D_G is the degree diagonal matrix of graph G. The bivariate polynomial ϕ_G generalizes some well known characteristic polynomials of graph G, and has also an elegant combinatorial interpretation as being equivalent to the Bartholdi zeta function- a generalization of the Ihara-Selberg zeta function, of graph G.

In this talk, we are mainly concerned with the problem of characterizing a given graph G by its generalized characteristic polynomial (or equivalently, by its Bartholdi zeta function). Some invariants of graphs with the same generalized characteristic polynomial are derived, and in particular, we show that the degree sequence of a graph G is determined by ϕ_G. Based on these, a unified approach is proposed to show that some families of graphs are characterized by ϕ_G. We also give a method for constructing graphs with the same generalized characteristic polynomial.

Title: Results towards the Dittert Conjecture

Speaker: **Ian Wanless**, Monash University

Abstract: The Dittert conjecture is a generalisation of van der Waerden's famous conjecture (now a theorem) on the minimimum permanent of doubly stochastic matrices.

Let K_n denote the convex set consisting of all real nonnegative $n \times n$ matrices whose entries have sum n. For $A \in K_n$ with row sums $r_1, \ldots, r_n$ and column sums $c_1, \ldots, c_n$, define $\phi(A) = \prod_{i=1}^n r_i + \prod_{j=1}^n c_j - \text{per}(A)$. Dittert's conjecture asserts that the maximum of ϕ on K_n occurs uniquely at $J_n = [1/n]_{n \times n}$.

In this talk I'll report that a counterexample to Dittert's conjecture must be fully indecomposable, and that its zeroes cannot form a single block. Also, in a sense that I'll make precise, if the conjecture fails then it doesn't fail by much!

Title: Smith form of incidence matrices related to t-uniform hypergraphs

Speaker: **Richard Wilson**, Caltech

Abstract: Given a t-uniform hypergraph H with vertex set X, we may consider the incidence matrix N whose rows are indexed by the t-subsets of X and whose columns are indexed by all isomorphic images of H with vertex set X . The invariant factors of N are understood when H has at least t isolated vertices.

We consider cases when H has fewer than t isolated vertices. It is not only the invariant factors that interest us, but also the construction of explicit unimodular matrices E and F so that ENF is diagonal. Applications are given to a certain zero-sum Ramsey-type problem. This is joint work with Tony Wong.

Title: A possible extension of Sutner's All-Ones Theorem

Speaker: **Yaokun Wu**, Shanghai Jiao Tong University

Abstract: Take a positive integer n and let $[n]$ denote the set $\{1, 2, \ldots, n\}$. Let A be an $n \times n$ symmetric matrix over the binary field $\mathbb{F}_2$, namely a map from $[n] \times [n]$ to $\mathbb{F}_2$ such that $A(i,j) = A(j,i)$. The linear space of binary column vectors of length n is $\mathbb{F}_2^n = \{\chi_S : S \subseteq [n]\}$, where χ_S is the binary column vector of length n whose support is S. Write $Im(A)$ for $\{Ax : x \in \mathbb{F}_2^n\}$, which is a subspace of $\mathbb{F}_2^n$.

For any $i \in [n]$, let $f_i = I + A\chi_i\chi_i^\top$, where I is the identity matrix. Note that $Im(A)$ is an invariant subspace for each f_i. For any $x, y \in \mathbb{F}_2^V$, we write $x \to^* y$ provided there is a word $w_1 w_2 \cdots w_m$ over $[n]$ such that $f_{w_m} f_{w_{m-1}} \cdots f_{w_1} x = y$.

Let $L = \{i \in [n] : A(i,i) = 1\} \neq \emptyset$. Sutner's All-Ones Theorem, proved earlier by Bagchi and Sastry, says that χ_L is not a Garden-of-Eden for the σ-automata, namely $\chi_L \in Im(A) \setminus \{0\}$. Motivated by this result, we want to ask whether or not $\chi_L \to^* x$ and $x \to^* 0$ always hold for any $x \in Im(A)$. Some related results and problems will be discussed in this talk.

This is a joint work with John Goldwasser and Xinmao Wang.

Title : Difference sets and Reed-Muller codes over Galois rings of characteristic 2^n

Speaker: **Mieko Yamada**, Faculty of Mathematics and Physics, Institute of Science and Engineering, Kanazawa University

Abstract: An extension ring of $\boldsymbol{Z}/2^n\boldsymbol{Z}$ with the extension degree s is called a Galois ring and denoted by $GR(2^n, s)$. $R_n = GR(2^n, s)$ is a local ring and has a unique maximal ideal $p_n = 2R_n$. Every ideal of R_n is $p_n^l = 2^l R_n, 0 < l < n$.

In this talk, we treat two topics, difference sets and Reed-Muller codes, over Galois rings $GR(2^n, s)$.

First, we give a family of $(2^{ns}, 2^{ns/2-1}(2^{ns/2} - 1), 2^{ns/2-1}(2^{ns/2-1} - 1)$ difference sets over Galois rings $GR(2^n, s)$ for even power of 2 and of any extenstion degrees.

It is well known that if the subset is a (v, k, λ) difference set over an abelian group of order v a power of 2, the parameters have to be $v = 2^{2t}, k = 2^{t-1}(2^t - 1), ?\lambda = 2^{t-1}(2^{t-1} - 1)$. Such difference sets are known to exist and many examples were given over various kind of algebraic structures.

Though we obtain no difference sets with new parameters, we notice that the difference set over $GR(2^n, s)$ is embedded in the ideal part of the difference set over $GR(2^{n+2}, s)$ when fixed an extension degree. It means that there exists an infinite family of difference sets with the embedding system over Galois rings $GR(2^n, s)$.

To prove this theorem, we introduce a new operation into Galois rings. Then $GR(2^n, s)$ is an abelian group with respect to the new operation. The Gauss sums associated with the multiplicative characters defined by the subgroups with respect to the new operation, play an important role.

Next, we define Reed-Muller codes $RM(r, s)$ of order r over $GR(2^n, s)$ similarly to the definition of Reed-Muller codes over finite fields and show the properties of them. Furthermore we construct symmetric association schemes from $RM(1, s)$. Codes and symmetric association schemes also hold an embedding systems.