Q: Why don't I care about conditions on the poset P?

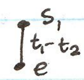
Because I start with varieties. In fact, almost everything I do comes from familiar posets:

Bruhat graphs, their subgraphs, their quotient graphs

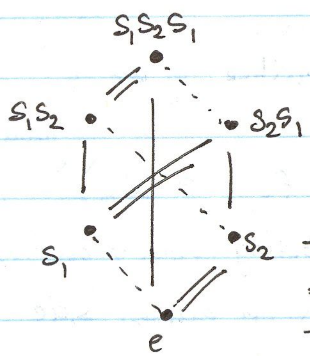
flag varieties → Bruhat graphs

Grassmannians → quotient graphs

EG: GL_2/B



Better eg: X = flag variety GL_n/B $\quad G/B$



vertices	permutations w	$w \in W$
edges	$(ij)w \leftrightarrow w$	$s_\alpha w \leftrightarrow w$
direction	from longer to shorter in Bruhat ~~graph~~ order	
label on edge	$t_i - t_j$ for $i<j$	α for $\alpha > 0$

— $= t_1-t_3$ yellow

═ $= t_2-t_3$ pink

--- $= t_1-t_2$ green

Remark: In this context, the basis elements we discovered for $n=2$ are better known as Schubert classes

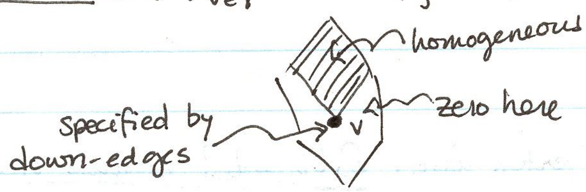
Intrinsic characterisation of basis elements from graph:

Flow-up ~~basis~~ elements $\{f_v\}_{v\in V}$ ~~is~~ satisfy the following

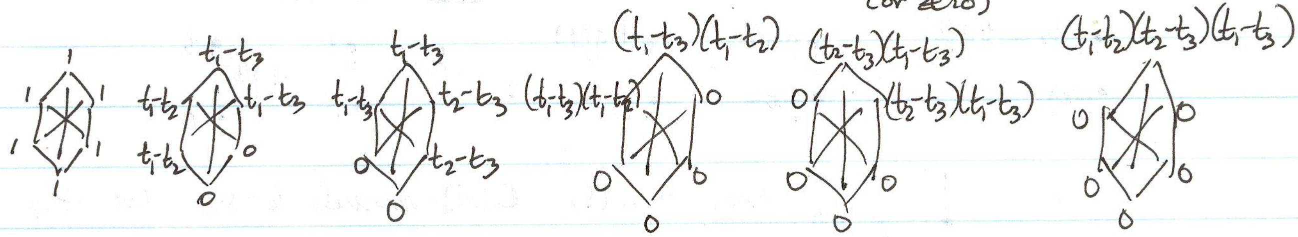
① $f_v(v) = \prod_{\substack{\text{edges down} \\ \text{from } v}} (\text{label on edge})$

② $f_v(w) = 0$ if $w \not\geq v$

③ else $f_v(w)$ is homogeneous of same degree as $f_v(v)$ (or zero)



EG:



3. Open questions (if time, we'll do more detail on 1-2)

1. (Billey) Characterize graphs for which there is a flow-up basis, or unique flow-up basis
2. Find explicit formulas for basis elements for specific families of graphs (eg: Billey for G/P)
3. Find basis for subvarieties $Y \subseteq X$, when X is known to have good basis (Harada-Tymoczko)
4. Equivariant Pieri rules (or LR rules): $\sigma_{\square\square\square} \cdot \sigma_\lambda = \sum c^{\nu}_{\square\square\square\,\lambda} \sigma_\nu$ — want explicit, positive formula (eg counting paths in graph)
   Knutson-Tao, Robinson
5. Affine Grassmannians: The graphs P_X look like crystal graphs. Realize this geometrically / construct representations
   Kamnitzer →