
PV

Outline

Definitions, unit

Climatology of PV and PV anomalies

Main characteristics: conservation, invertibility, partitioning

Cloud condensational PV production / destruction

Case study 1: North Atlantic cyclogenesis

Case study 2: extratropical transition of hurricane *Hanna* (2008)

Further reading

Note the gradation in definitions of potential vorticity

Full (Ertel) PV   PV α $\vec{\eta} \cdot \vec{\nabla}\theta$ with $\vec{\eta} = \vec{\nabla} \wedge \vec{u} + 2\vec{\Omega}$

Isentropic PV   IPV α $(\zeta + f) \,.\, \theta_z$

Quasi-geostrophic PV   q α $\zeta + \theta_z$

and for each definition there is a conservation principle, i.e.,

D{PV}/Dt = 0   for inviscid & adiabatic flow

where D/Dt denotes the appropriate material derivative

Definition of Ertel PV and unit

Definition

$$Q = \frac{1}{\rho}\,\vec{\eta}\cdot\vec{\nabla}\theta$$

good approximation
for synoptic scales

$$Q \simeq \frac{1}{\rho}\,(f+\zeta)\,\frac{\partial\theta}{\partial z}$$

Unit

$$\begin{aligned} Q &\simeq \frac{1}{\rho}\,f\,\frac{\partial\theta}{\partial z} \simeq 1\,\mathrm{kg}^{-1}\,\mathrm{m}^3\cdot 10^{-4}\,\mathrm{s}^{-1}\cdot 100\,\mathrm{K}/10^4\,\mathrm{m} \\ &= 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}\,\mathrm{K}\,\mathrm{kg}^{-1} \\ 1\,\mathrm{pvu} &= 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}\,\mathrm{K}\,\mathrm{kg}^{-1} \end{aligned}$$

PV characteristics

Conservation
for adiabatic, frictionless flows

$$\frac{D}{Dt} Q = 0,$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

is the total or material derivative

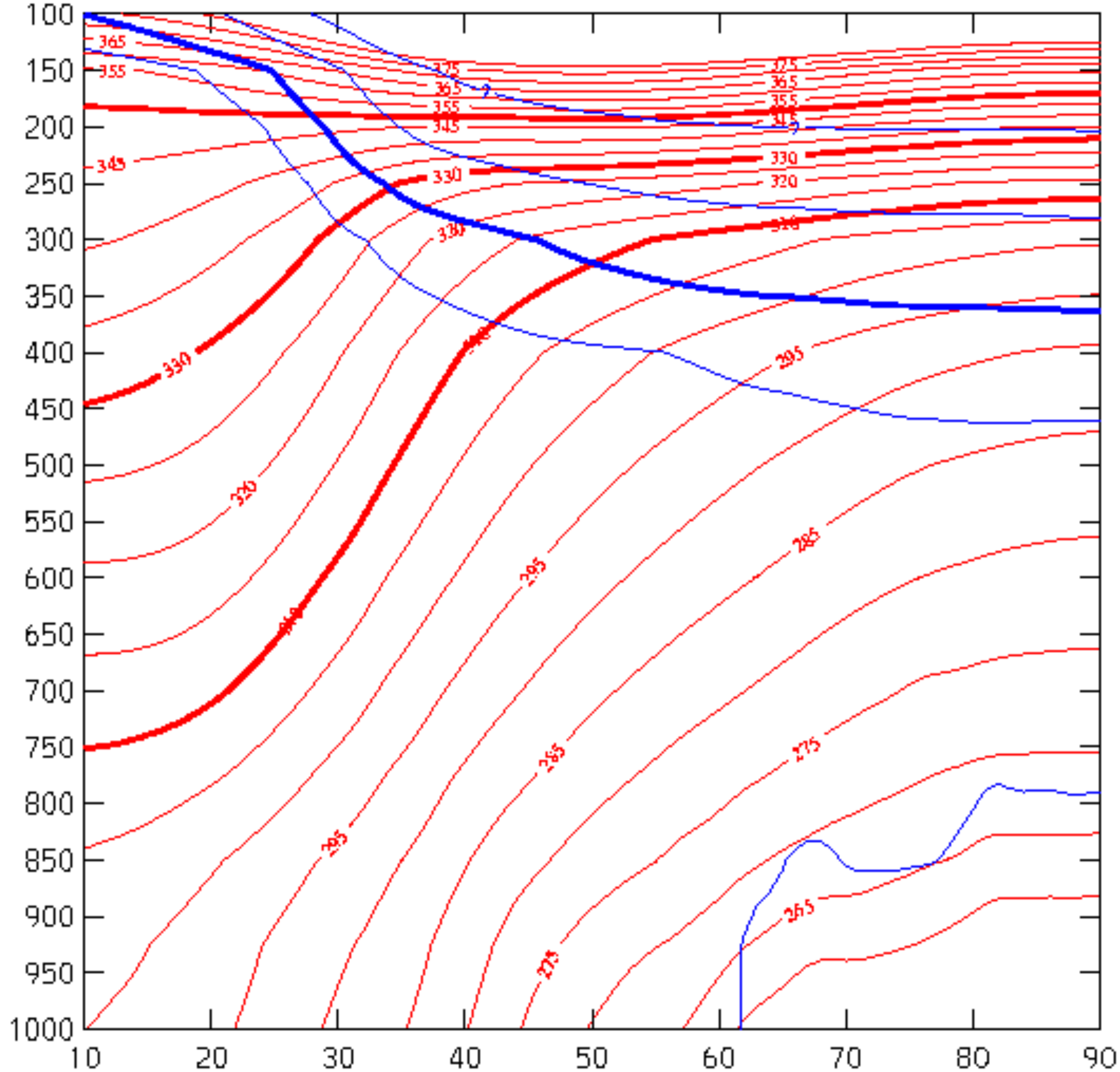
Invertibility

Q-distribution in atmosphere &
potential temperature at the surface
can be "inverted" to derive u,v,T,p
math: solve elliptic PDE

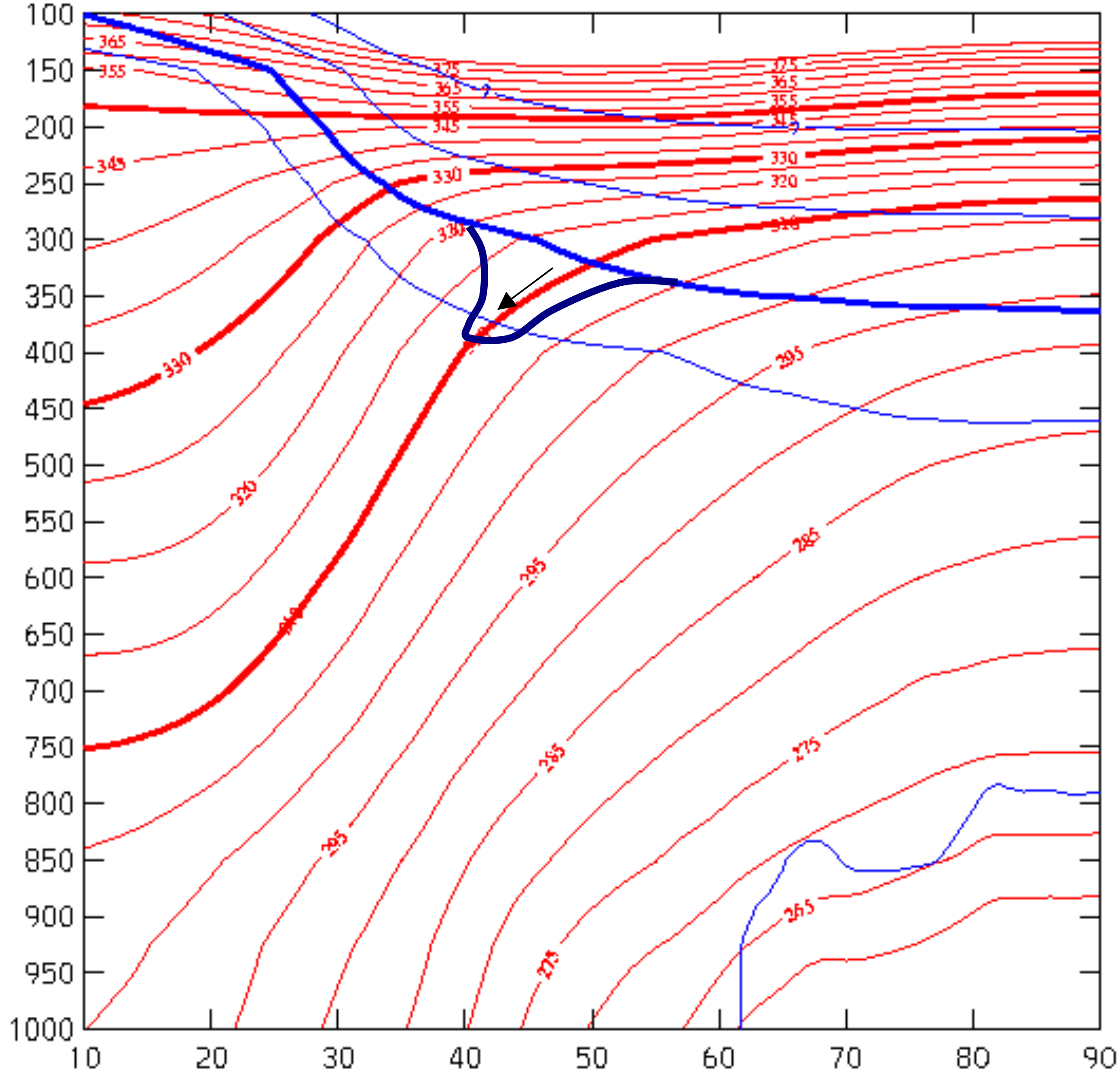
Partitioning

Q-distribution can (often) be partitioned
in distinct "anomalies", which interact
with each other

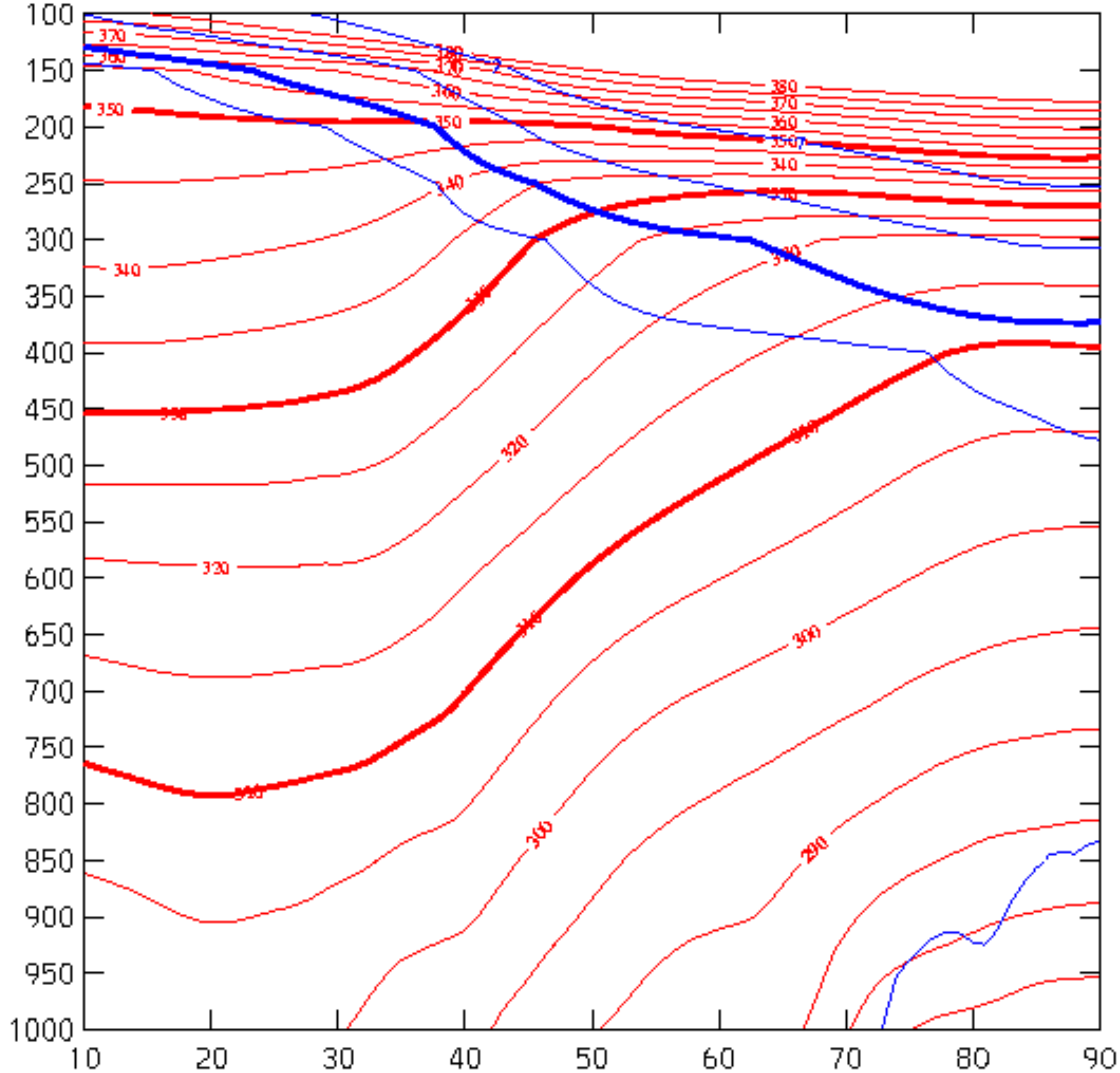
winter climatology of PV and θ



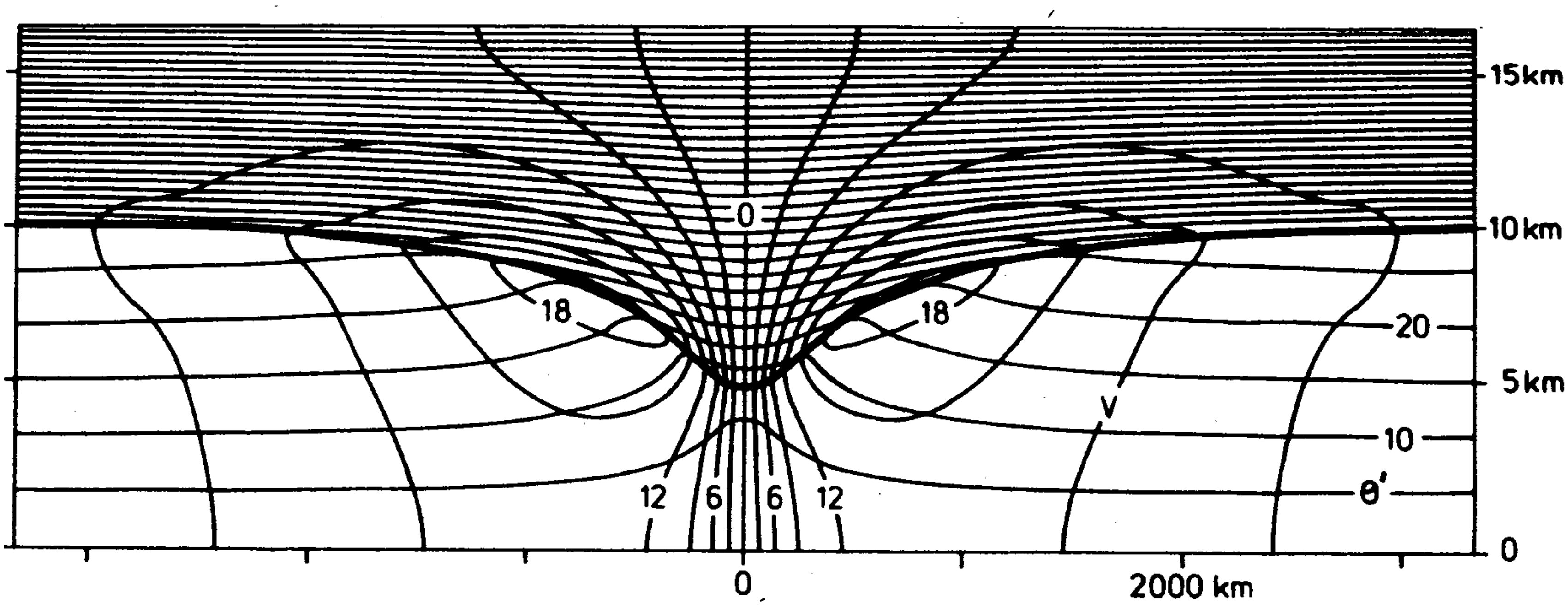
winter climatology of PV and θ



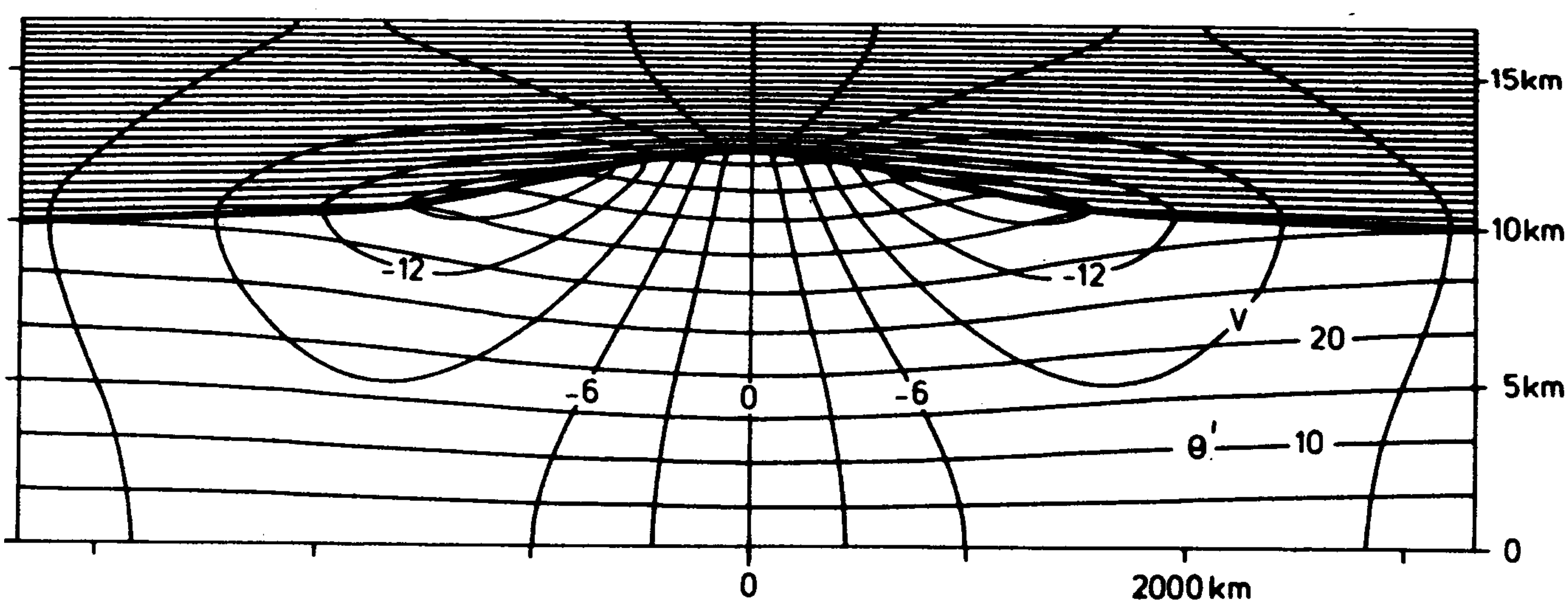
summer
climatology
of PV and θ



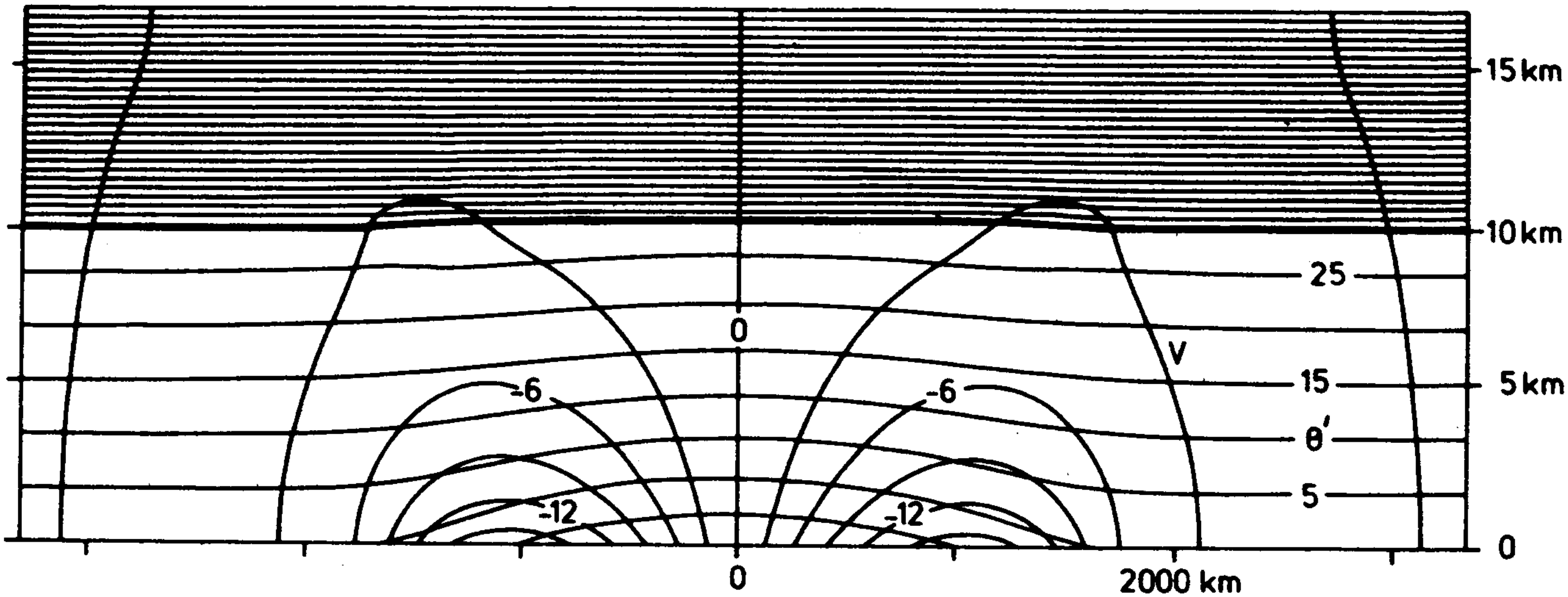
PV inversion of
positive upper-level PV anomaly



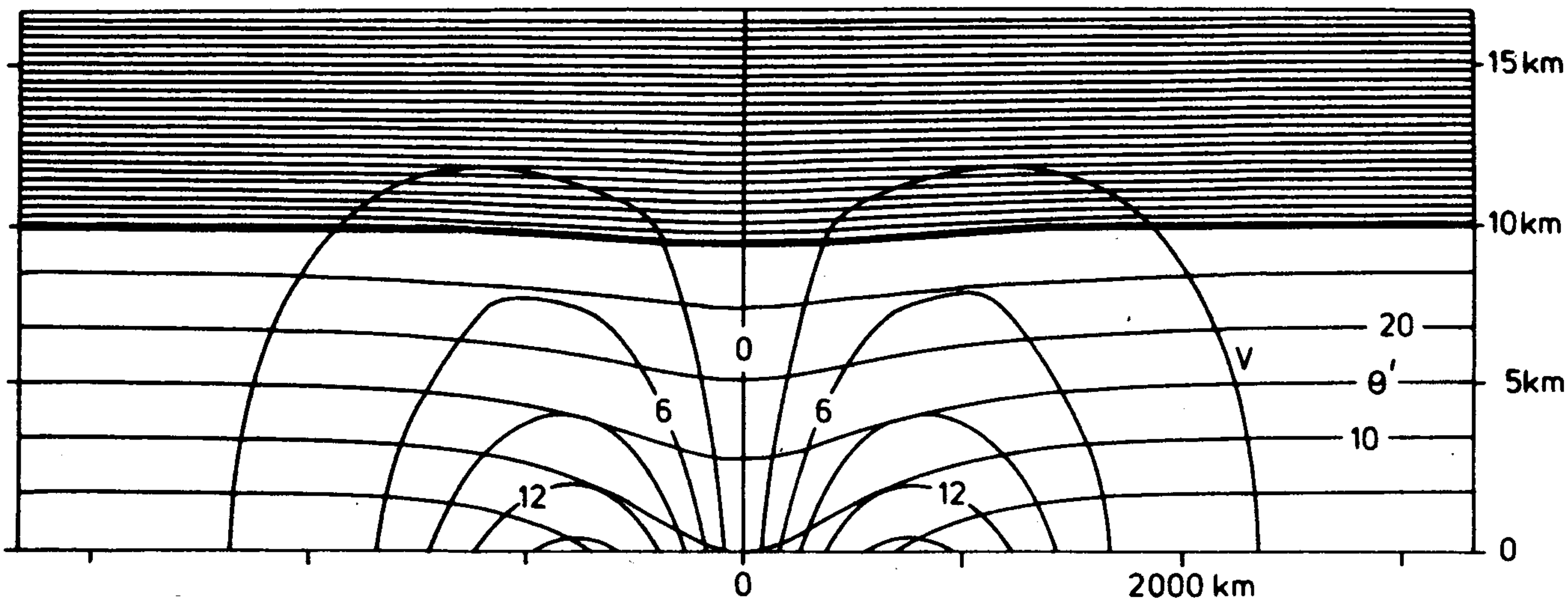
negative upper-level PV anomaly



negative surface θ-anomaly



positive surface θ-anomaly



PV non-conservation

PV non-conservation in the presence of frictional and diabatic processes:

$$\frac{D}{Dt}\,Q = -g\,\vec{\eta}_p \cdot \vec{\nabla}_p\dot{\theta} - g\,\vec{\nabla}_p\theta \cdot (\vec{\nabla}_p \wedge \vec{F}).$$

gradient of diabatic heating rate

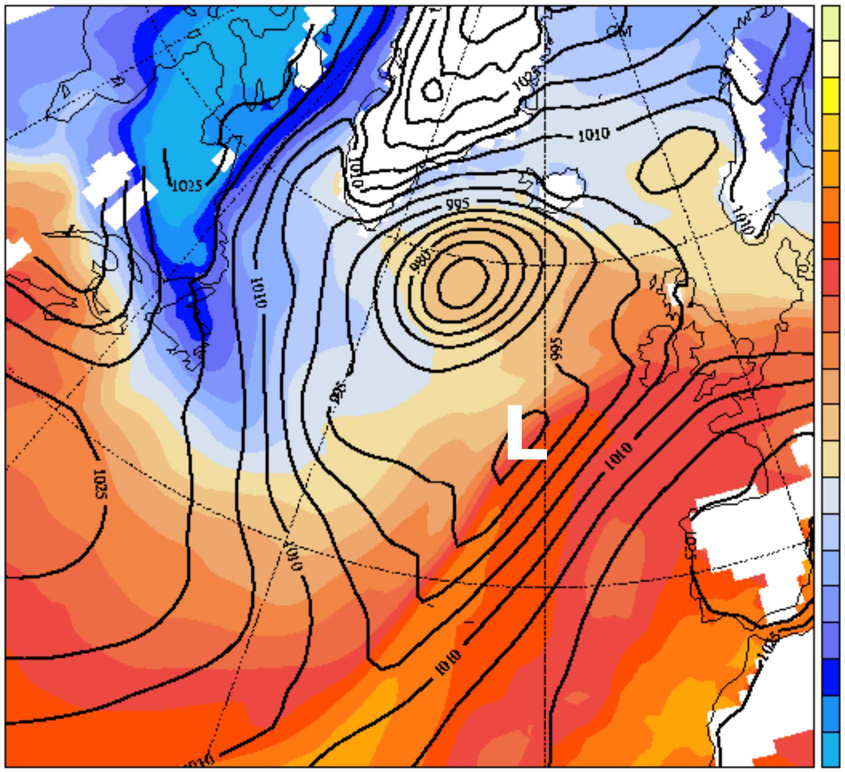
non conservative forces (e.g., friction)

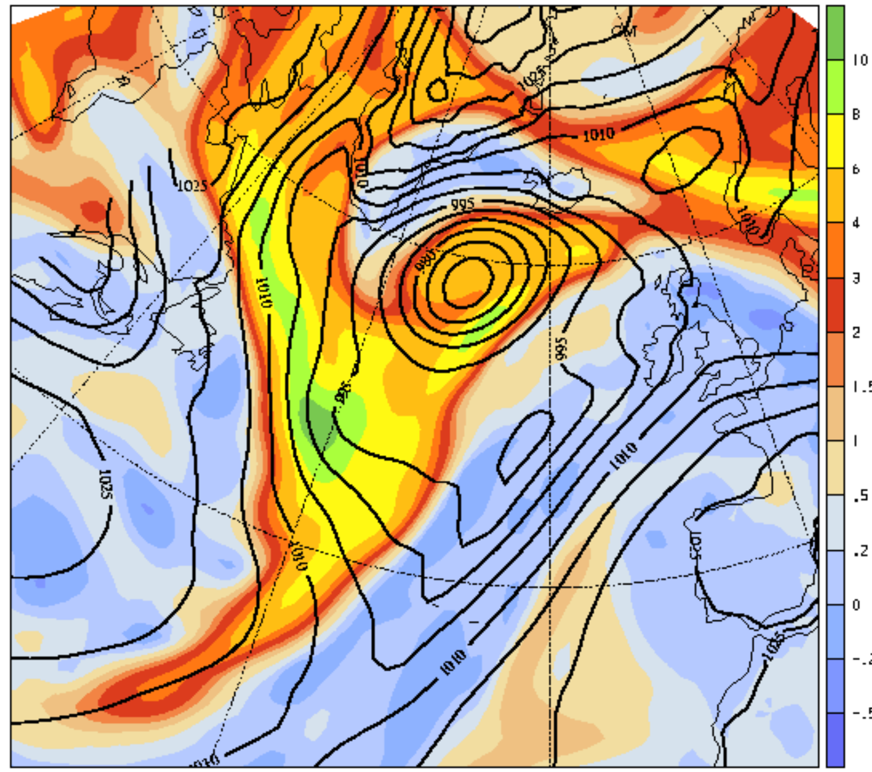
good approximation on synoptic scales

$$\frac{D}{Dt}\,Q \simeq -g\,(f+\zeta)\,\frac{\partial\dot{\theta}}{\partial p}.$$

A case study of North Atlantic cyclogenesis

06 UTC 22 Nov

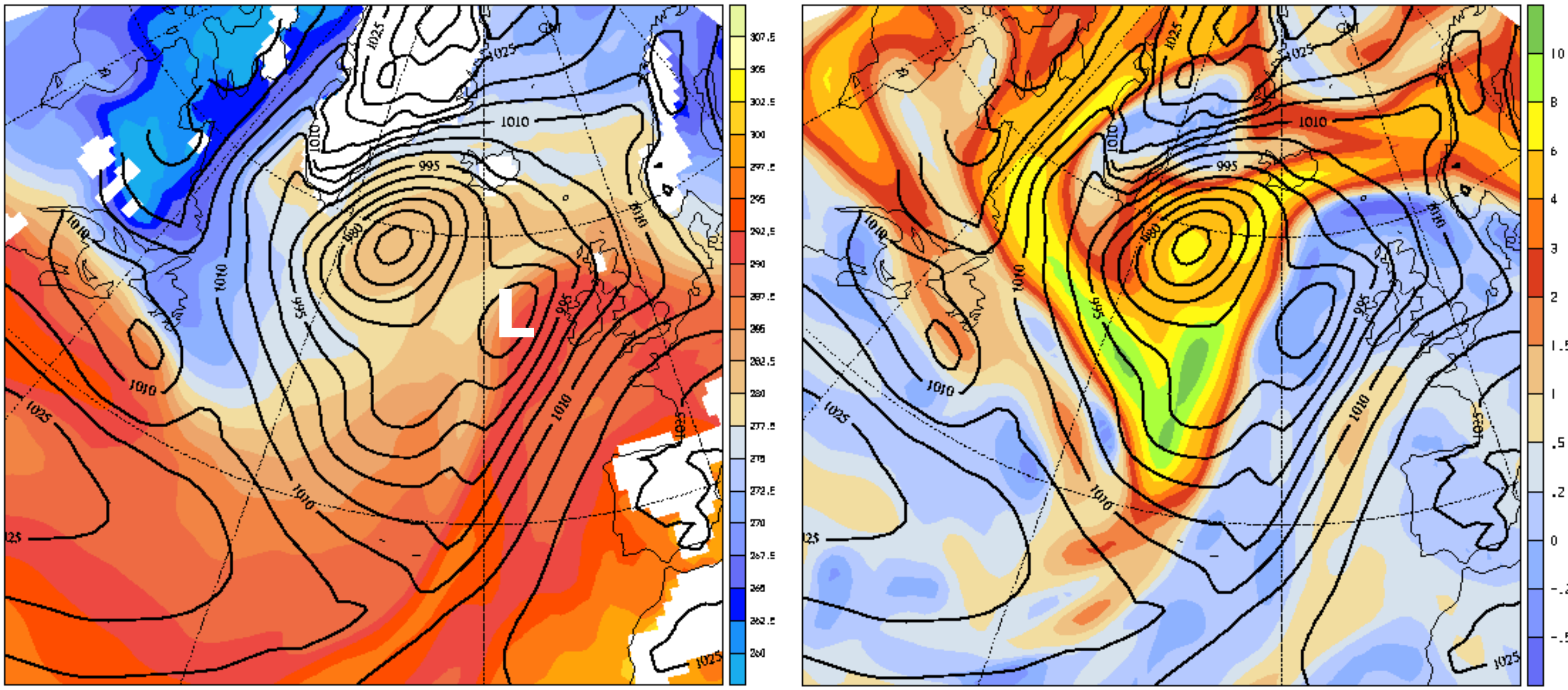




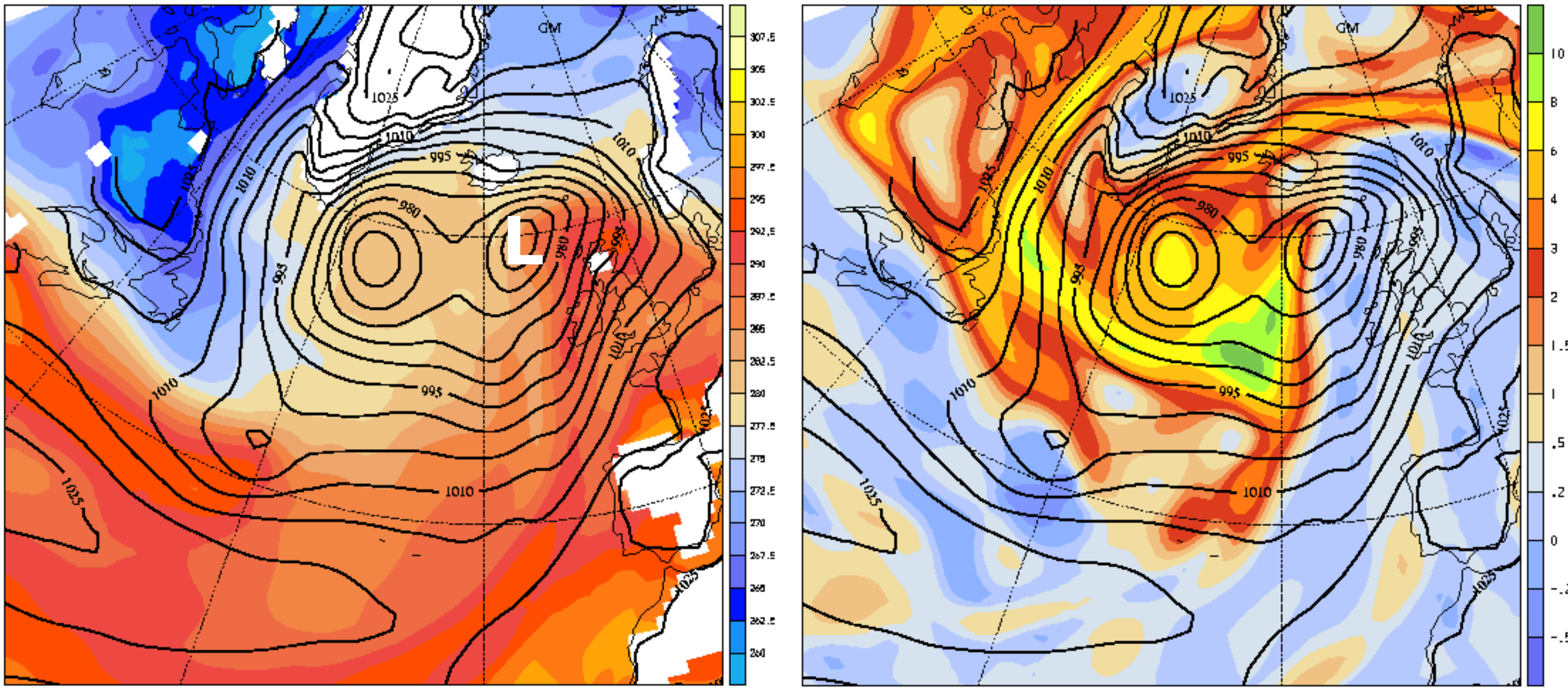
potential temperature on 850 hPa
sea level pressure (SLP)

potential vorticity on 315 K
sea level pressure (SLP)

18 UTC 22 Nov



06 UTC 23 Nov



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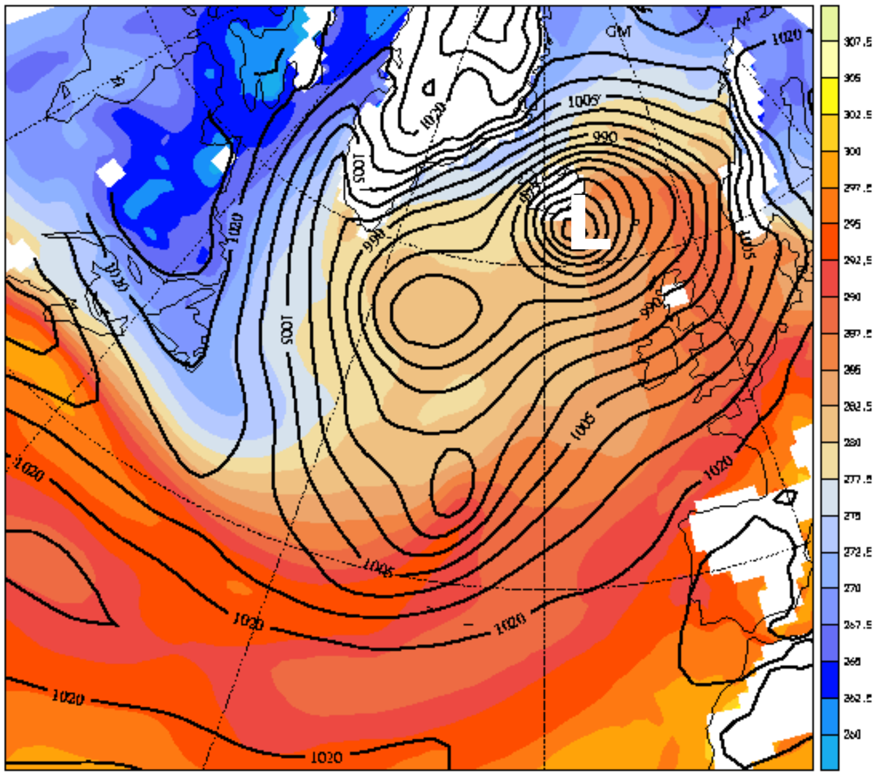
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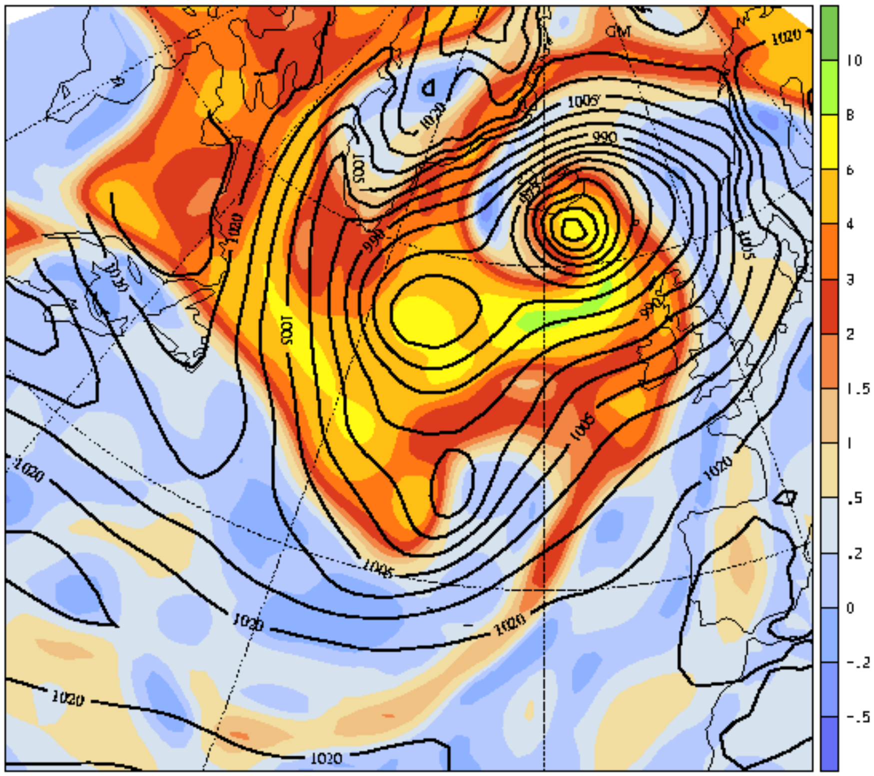
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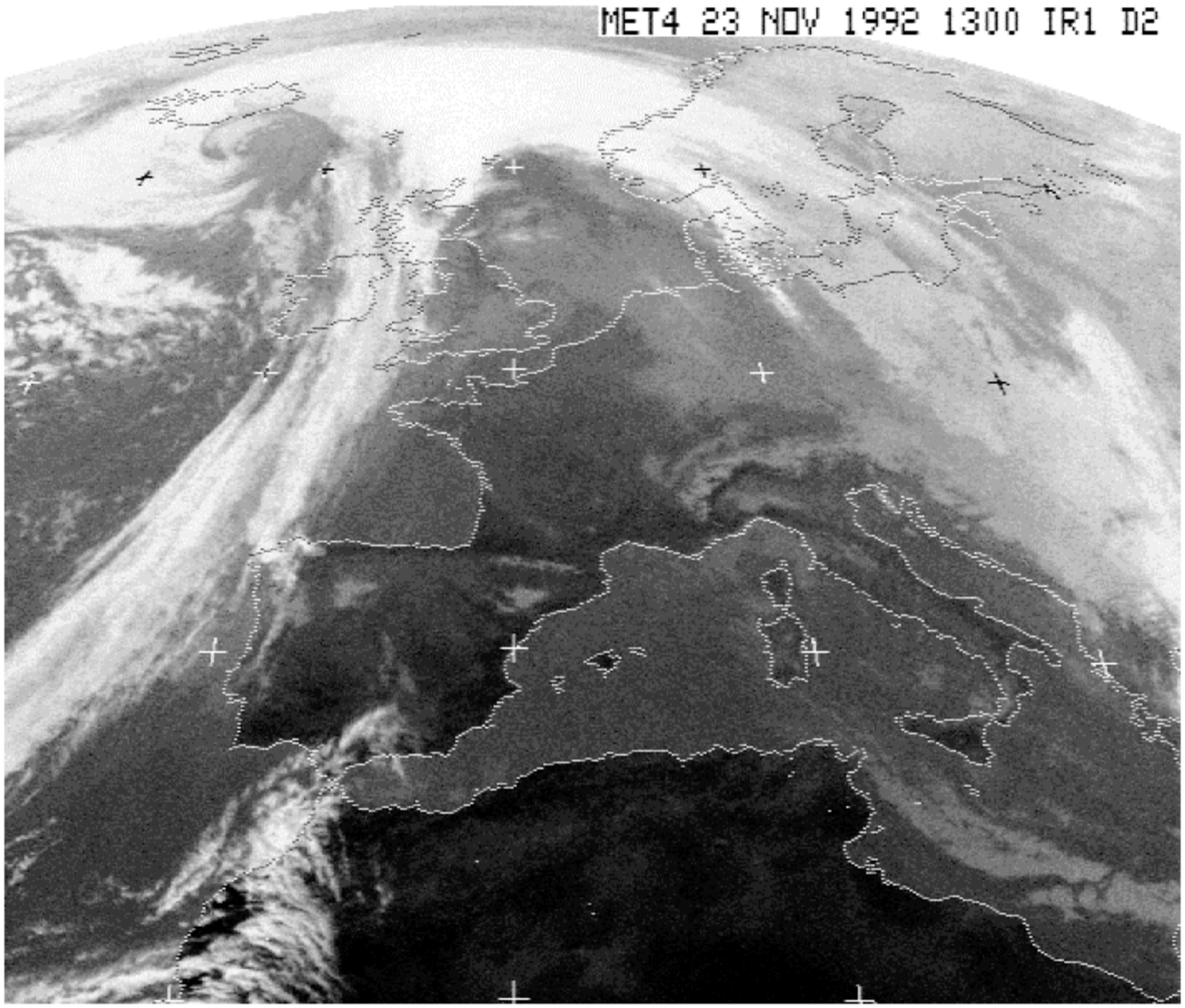
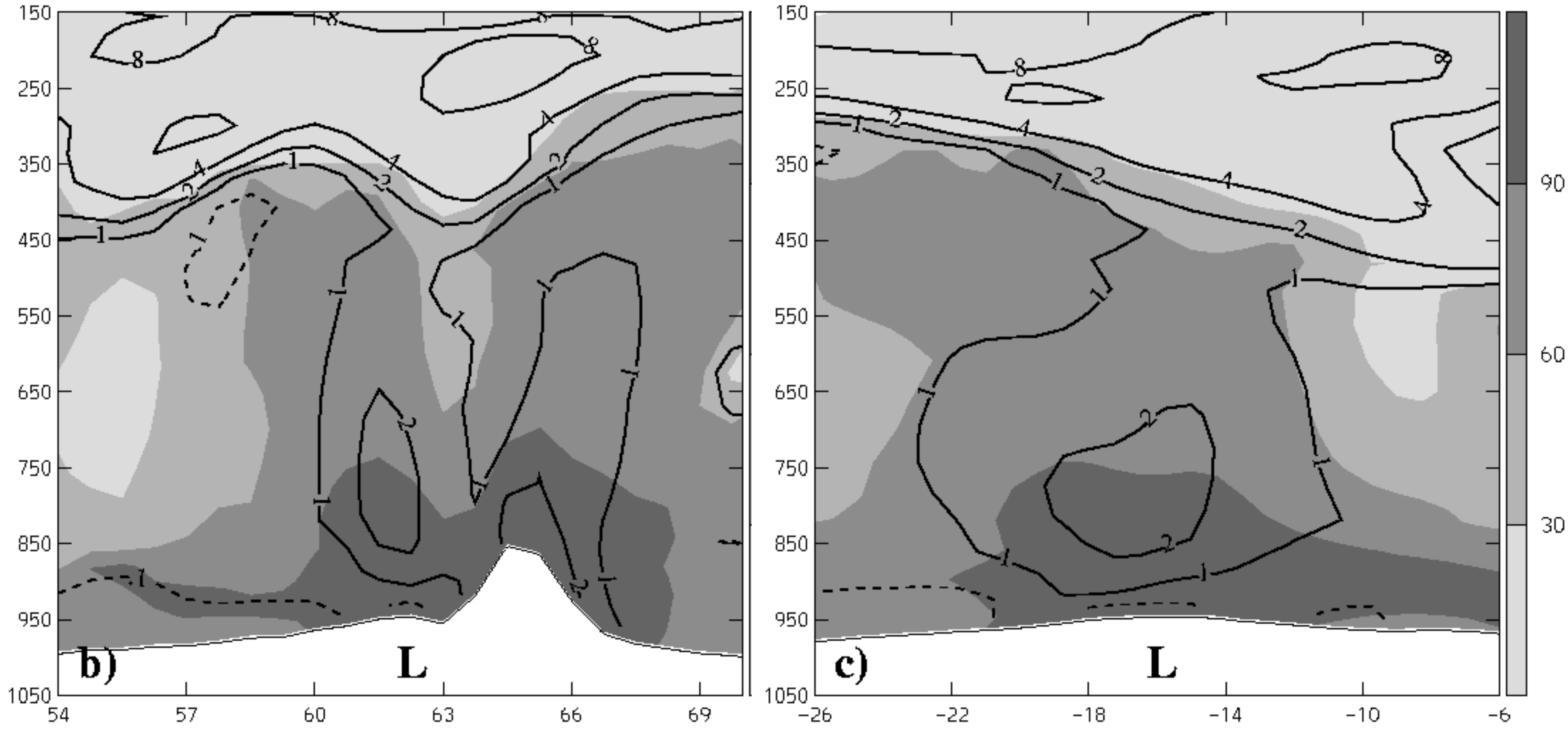
IR satellite image at 13 UTC 23 Nov



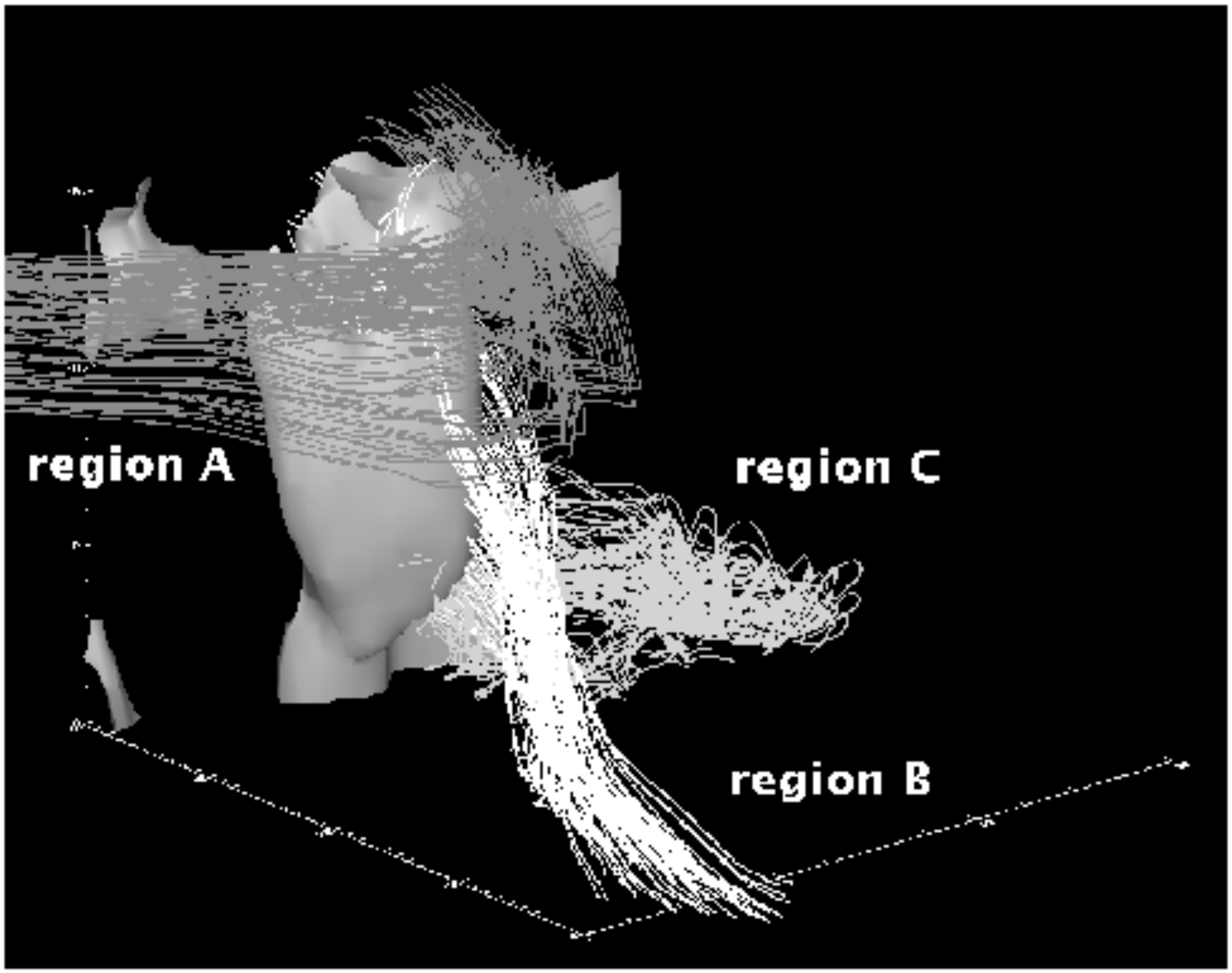
The PV tower
at 18 UTC 23 Nov (PV and RH)

S-N cross section

W-E cross section



The PV tower
at 18 UTC 23 Nov (PV and backward trajectories)



Recommended further reading

Kleinschmidt (1950, Meteorol. Zeitschrift): A historical application of PV concepts

Hoskins et al. (1985, QJRMS): The key paper that launched „PV thinking"

Bishop and Thorpe (1994, QJRMS): analogy between PV and electrostatics

Haynes and McIntyre (1987, 1990, JAS): concept of „PV substance"

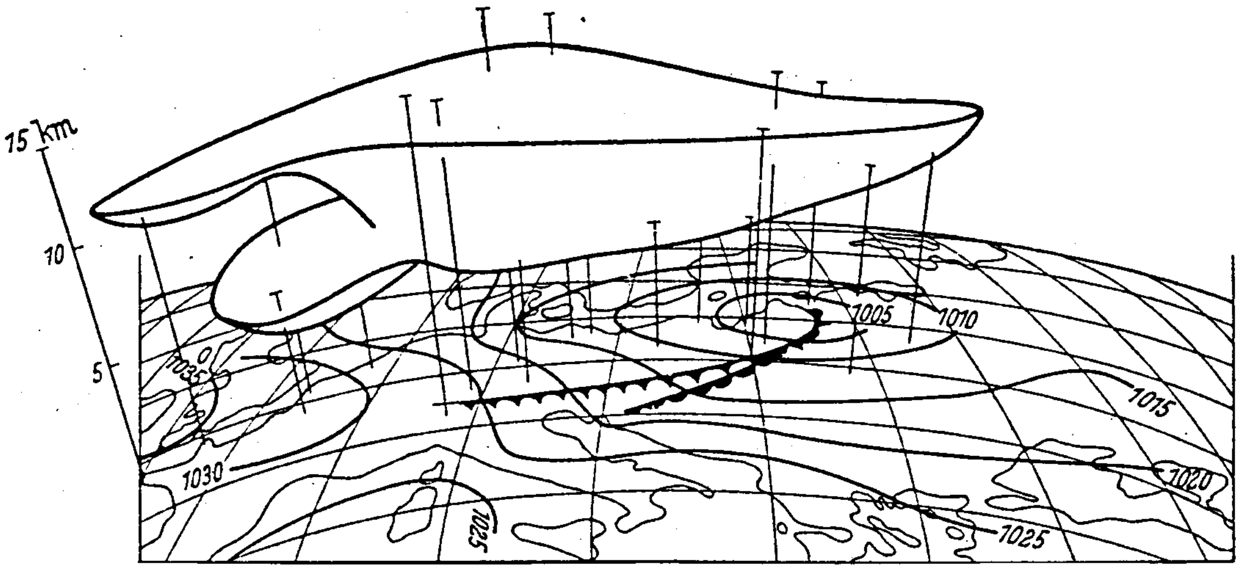
Schär (1990, JAS): PV flux and the Bernoulli function

Davis and Emanuel (1991, MWR): PV analysis of cyclogenesis

Rossa et al. (2000, MAP): cyclones and PV towers

... and many others!!

Kleinschmidt (1950)



Upper-level „Höhenkörper“ (= positive PV anomaly) associated with surface cyclogenesis

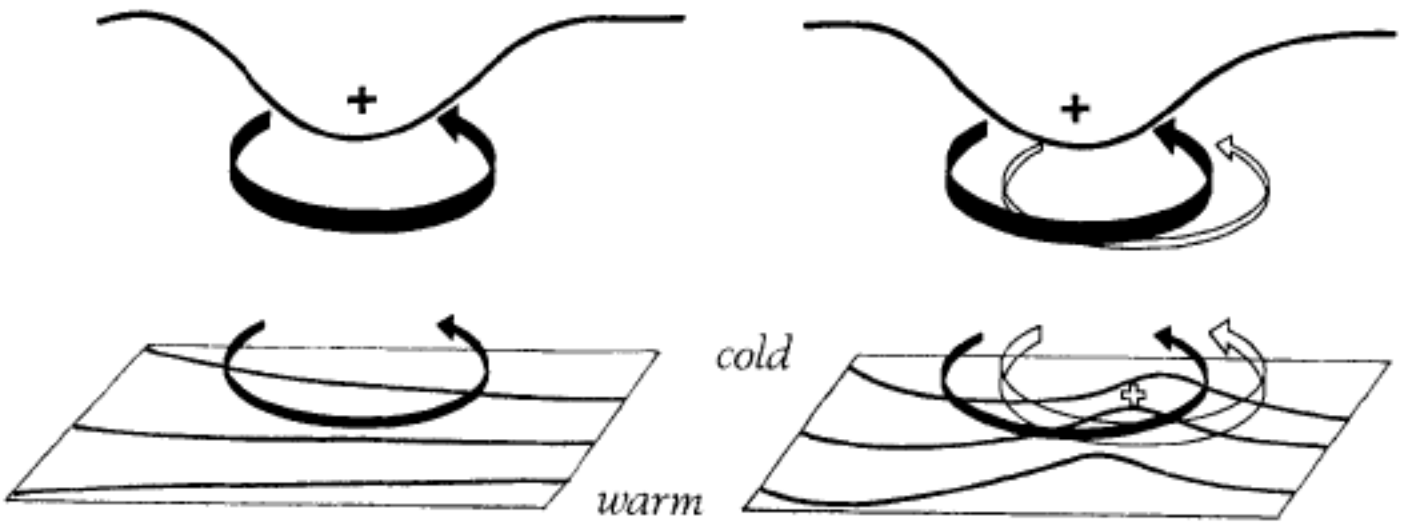
QUARTERLY JOURNAL
OF THE
ROYAL METEOROLOGICAL SOCIETY

Vol. 111 OCTOBER 1985 No. 470

Quart. J. R. Met. Soc. (1985), **111**, pp. 877–946 551.509.3:551.511.2:551.511.32

On the use and significance of isentropic potential vorticity maps

By B. J. HOSKINS[1], M. E. McINTYRE[2] and A. W. ROBERTSON[3]



Basic schematic of upper-level induced extratropical cyclogenesis

Q. J. R. Meteorol. Soc. (1994), **120**, pp. 713–731 551.511.32:537.2

Potential vorticity and the electrostatics analogy: Quasi-geostrophic theory

By CRAIG H. BISHOP and ALAN J. THORPE*
University of Reading, UK

$$q' = \overline{\nabla} \cdot \mathbf{D}'_q$$

$$\mathbf{D}'_q = \left(\frac{\partial \psi'}{\partial x}, \frac{\partial \psi'}{\partial y}, \frac{\sigma_0^2}{\sigma^2} \frac{\partial \psi'}{\partial \bar{z}} \right)$$

PV can be expressed as divergence of a vector field – exactly as the relationship in electrostatics between charge and the electric displacement field.

Atmosphere is analogous to anisotropic dielectric material, which implies the existence of „bound PV charges" at the boundaries.

On the Evolution of Vorticity and Potential Vorticity in the Presence of Diabatic Heating and Frictional or Other Forces

P. H. HAYNES AND M. E. MCINTYRE

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England

$$\frac{\partial(\sigma Q)}{\partial t} + \nabla \cdot \mathbf{J} = 0,$$

where (in isentropic coordinates)

$$\mathbf{J} = (u, v, 0)\sigma Q + \mathbf{J}_{\dot\theta} + \mathbf{J}_{\mathrm{F}},$$

$$\mathbf{J}_{\dot\theta} = \{\dot\theta \partial v/\partial\theta, -\dot\theta \partial u/\partial\theta, 0\},$$

$$\mathbf{J}_{\mathrm{F}} = \{-G, F, 0\}$$

There is no cross-isentropic flux of „PV substance“ (= σQ) also in the presence of diabatic effects and/or frictional forces.

For instance, diabatically produced PV anomalies emerge through dilution and concentration of „PV substance“.

NOTES AND CORRESPONDENCE

A Generalization of Bernoulli's Theorem

CHRISTOPH SCHÄR*

Department of Atmospheric Sciences, University of Washington, Seattle, Washington

$$\frac{\partial}{\partial t}(\rho Q) + \nabla \cdot \mathbf{J} = 0,$$

For statistical steady state conditions (but in the presence of diabatic heating and/or frictional forces), the flux is given by

$$\mathbf{J} = \nabla\theta \times \nabla B.$$

where *B* denotes the Bernoulli function

$$B = c_p T + \frac{1}{2}\mathbf{u}^2 + gz.$$

For non-steady conditions, the flux is given by

$$\mathbf{J} = \nabla\theta \times \left(\nabla B + \frac{\partial \mathbf{u}}{\partial t}\right) - \boldsymbol{\omega}\frac{\partial \theta}{\partial t}$$

Summary

PV is key variable of (large & synoptic-scale) atmospheric dynamics

Positive (negative) PV anomalies induce cyclonic (anticyclonic) wind field

PV is materially conserved in frictionless flows outside of clouds

PV can be produced / destroyed near regions of cloud condensational heating

Cyclogenesis can be regarded as formation of vertically coherent columns of positive PV anomalies