Schedule for workshop on
"Analysis and Boundary Value Problems on
Real and Complex Domains"
BIRS, 26-20 July 2010

Locations

- All meals in the Dining Centre on the 4th floor of the Sally Borden Building
- All coffee breaks in room 5210, Corbett Hall
- All lectures in room 159, Max Bell Building

Monday, 26 July

7:30–9:30am: breakfast
10:00–10:30am: coffee
10:30–11:00am: announcements & introductions (Max Bell 159)
11:00–11:50am: lecture by Michael Christ
11:50–noon: questions & answers
noon–1:30pm: lunch
2:00–2:50pm: lecture by Evgeny Poletsky
2:50–3:00pm: questions & answers
3:00–3:30pm: group photo on steps of Corbett Hall / coffee
3:30–5:30pm: informal discussion
5:30–7:30pm: dinner

Tuesday, 27 July

7:30–9:00am: breakfast
9:00–10:30am: informal discussion
10:30–11:00am: coffee
11:00–11:50am: lecture by Michael Lacey
11:50am–noon: questions & answers
noon–1:30pm: lunch
2:00–2:50pm: lecture by Siqi Fu
2:50–3:00pm: questions & answers
3:00–3:30pm: coffee
3:30–5:30pm: informal discussion
5:30–7:30pm: dinner

Wednesday, 28 July

7:30–9:00am: breakfast
9:30–10:20am: lecture by Jurgen Leiterer
10:20–10:30am: questions & answers
10:30–11:00am: coffee
11:00–11:50am: lecture by Guy David
11:50am–noon: questions & answers
noon–1:30pm: lunch
**** Free afternoon ****
5:30–7:30pm: dinner

Thursday, 29 July

7:30–9:00am: breakfast
9:00–10:30am: informal discussion
10:30–11:00am: coffee
11:00–11:50am: lecture by Irina Mitrea
11:50am–noon: questions & answers
noon–1:30pm: lunch
2:00–2:50pm: lecture by Martin Dindos
2:50–3:00pm: questions & answers
3:00–3:30pm: coffee
3:30–5:30pm: informal discussion
5:30–7:30pm: dinner

Friday, 30 July

7:30–9:00am: breakfast
9:00–10:50am: lecture by Malabika Pramanik
10:50–10:00am: questions & answers
10:00–10:30am: coffee
10:30–11:20am: lecture by John Lewis
11:20–11:30am: questions & answers

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Titles and abstracts

Michael Christ: *On random tensors*

Abstract: A venerable theme is the smallness of the Fourier transform of a measure, when linear structure is absent. Prototypical examples include natural surface measures on curved submanifolds, and classes of random measures.

We investigate multilinear extensions of this theme, particularly in connection with two classes of matrices with random coefficients. Applications concerned with correlations, return times, and sparse subsequences in ergodic theory are given.

The techniques are elementary, relying principally on TT^*, Fourier transformation, independence, large deviations, and entropy considerations.

Guy David: *Some minimal cones of dimension 2 in R^4*

Abstract: A precise list of minimal cones of dimension 2 in R^n would be useful for improved regularity results on minimal or almost minimal sets in higher dimensions (think about soap films). We shall try to explain why, and present a new result from XiangYu Liang that says that the union of two almost orthogonal planes in R^4 is minimal.

Martin Dindos: *L^p and BMO solvability of the Dirichlet boundary value problem for general divergence-form elliptic equations*

Abstract: We will present a recent result (joint work with C. Kenig and J. Pipher) that establishes stability and equivalence of solvability of the Dirichlet problem with BMO boundary data.

It has been known before that the boundary value problem

$$Lu = div(A\, grad\, u) = 0, u|_{\partial\Omega} = f \in L^p$$

for p large is solvable if and only if the elliptic measure corresponding to the operator L belongs to the Muckenhaupt A_∞ class with respect to the surface measure on the boundary.

We have established that the correct end-point for this question is the BMO class, that is the elliptic measure corresponding to the operator L belongs to the Muckenhaupt A_∞ class with respect to the surface measure on the boundary if and only if the Dirichlet problem with the BMO data is solvable.

This result holds on a very large class of NTA (nontangetially accessible domains). This class in particular contains Lipschitz and polyhedral domains.

Siqi Fu: *Comparison of the Bergman and Szego kernels*

ABSTRACT: In this talk, we discuss boundary behavior of the quotient of the Szego and Bergman kernels for a smooth bounded pseudoconvex domain in C^n. Our analysis depends on the weighted L^2-estimates for the d-bar-operator by Hormander, Demailly, and Berndtsson. We also make essential use of Blocki's estimates for the pluricomplex Green function on hyperconvex domains. This talk is based on joint work with Boyong Chen.

Michael Lacey: *A2 Linear Bound for Calderon-Zygmund Operators*

ABSTRACT: We show that in all dimensions, weight w in Muckenhoupt class $A2$, and L^2 bounded, normalized, smooth Calderon-Zygmund operators T, T maps $L^2(w)$ to $L^2(w)$ with norm bounded by the $A2$ characteristic of w to the first power. This is the sharp power on the $A2$ characteristic, and represents a culmination of a line of investigation started by Hunt-Muckenhoupt-Wheeden in 1973. The proof depends upon a profound extension of the David-Journe $T1$ theorem to the $A2$ setting, due to Perez-Treil-Volberg. Our contribution is to verify the testing conditions in this $T1$ Theorem. Central to this is appropriate decomposition of T, placing a variant of the John-Nirenberg inequality at ones disposal. Joint with Tuomas Hytonen, Maria Carmen Reguera, Eric Sawyer, Ignacio Uriate-Tuero, and Armen Vagharshakyan.

Jurgen Leiterer: *Estimates for the splitting of holomorphic cocycles*

ABSTRACT: It is well-known that any holomorphic vector bundle over a domain in the complex plane or over a convex domain in $\mathbb{C}^n$ is trivial. In other words, if $\{U_\mu\}$ is an open covering of such a domain, and $\{f_{\mu\nu}\}$ is a cocycle of holomorphic matrix functions $f_{\mu\nu} : U_\mu \cap U_\nu \to GL(r, \mathbb{C})$, then there exists a family $\{f_\mu\}$ of holomorphic functions $f_\mu : U_\mu \to GL(r, \mathbb{C})$ with $f_{\mu\nu} = f_\mu f_\nu^{-1}$ on $U_\mu \cap U_\nu$. The topic of the talk is to find a solution $\{f_\mu\}$ with estimates. In general, this seems to be very difficult. But for cocycles which are sufficiently close to the unit cocycle, such estimates can be proved. We will discuss possible applications.

John Lewis: *Where we are at with p harmonic measure*

ABSTRACT: In this talk I will discuss recent work with coauthors concerning p harmonic measure in simply connected domains $\subset \mathbf{R}^2$ and p harmonic measure in Reifenberg flat domains - Wolff snowflakes $\subset \mathbf{R}^n, n \geq 3$. Our goal is to eventually obtain endpoint analogues of work on harmonic measure (the $p = 2$ case) due to P. Jones, N. Makarov, and T. Wolff. I hope to convince the audience that we are not too far from our goal.

Irina Mitrea: *Harmonic analytic and geometric measure theoretic methods in several complex variables*

ABSTRACT: Practice has shown that the combination of Harmonic Analysis, Geometric Measure Theory and Complex Analysis is an extremely fertile and potent mix in the complex plane, with many notable achievements whose degree of technical sophistication is breathtaking. In sharp contrast with these successes the case of Several Complex Variables has been very little explored from the perspective of the latest advances of Harmonic Analysis and Geometric Measure Theory. The aim of this talk is to discuss some recent progress in a program whose goal is to study the extent to which tools and methods from Harmonic Analysis and Geometric Measure Theory may yield a qualitative upgrade of some of the most fundamental results in Several Complex Variables.

Malabika Pramanik: *Diagonal estimates for the Bergman kernel on certain domains in* $\mathbb{C}^n$

ABSTRACT: We study three classes of complex domains with the common feature that their defining functions are of monomial type. Not surprisingly, the geometry of monomial polyhedra governs the sharp diagonal estimates for the Bergman kernel on such domains. We describe how the blowup of the kernel near the boundary depends on the structure of these polyhedra. As an application, we are able to estimate the Bergman kernel on the diagonal for certain general classes of Reinhardt and weakly pseudoconvex domains, in particular the "cross of iron".

This is joint work with Alexander Nagel.

Evgeny Poletsky: *Weak and strong limit values*

ABSTRACT: The classical results about the boundary values of holomorphic or harmonic functions on a domain state that under additional integrability assumptions these functions have limits along specific sets

approaching boundary. The proofs of these results are based on properties of smooth boundaries which are used to define the approach regions and on estimates of representing kernels along these regions.

We will discuss the situation when there are representing kernels but no assumptions about the boundary smoothness are made and, consequently, no natural definitions of approach regions could be given.