Adaptive FE discretization of the Navier-Stokes equations for turbulent flow

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Target: high Re turbulent flow

- Fully developed turbulent flow, shocks, boundary layers, complex geometry, fluid-structure interaction
- Full computational resolution in DNS impossible
- Existence of classical solution unknown
- Ad hoc mesh design: non-optimal and expensive
- State of the art: RANS, or LES for moderate Re

Our approach: Adaptive FEM DNS/LES

- Approximate turbulence as weak solutions by a finite element method (in the spirit of Leray)
  - No RANS/LES averaging/filtering
- Automatic mesh design: by (parallel) adaptive FEM based on a posteriori error control
- **Modeling of turbulent boundary layers by a skin friction boundary condition**
- Adaptive approximation of boundary with respect to exact geometry model

Adaptive FEM DNS/LES

- Ex: For (v,q) in W_h : find (U,P) in V_h = {p.w. linear in space-time}

$$(U_t + U\cdot\nabla U,v) + (\nu\nabla U,\nabla v) - (P,\nabla\cdot v) + (q,\nabla\cdot U)$$
$$+ (\delta(U\cdot\nabla U+\nabla P),U\cdot\nabla v+\nabla q) = (f,v)$$

- Slip velocity: $u\cdot n = 0$ (strong BC)
- Wall shear stress: $\tau = n^T\sigma t = \beta(u\cdot t)$ (weak BC: β friction coeff.)

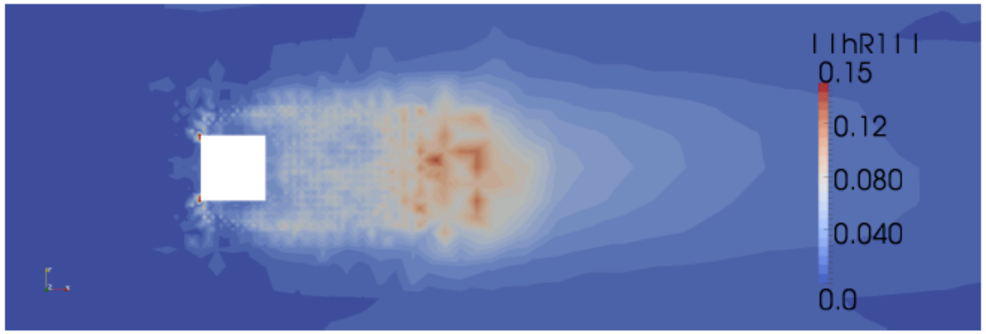
- Least squares stabilization of a residual: $U\cdot\nabla U+\nabla P$, with $\delta \sim h$
- No explicit (physics based) subgrid model of unresolved scales
- Dissipation: $- dK/dt = ||\beta^{1/2}u\cdot t||^2 + ||\nu^{1/2}\nabla U||^2 + ||\delta^{1/2}(U\cdot\nabla U+\nabla P)||^2$

[Hoffman SISC 05, JFM 06, CM 06, IJNMF 08, Hoffman/Johnson CMAME 06]

Adaptive FEM DNS/LES

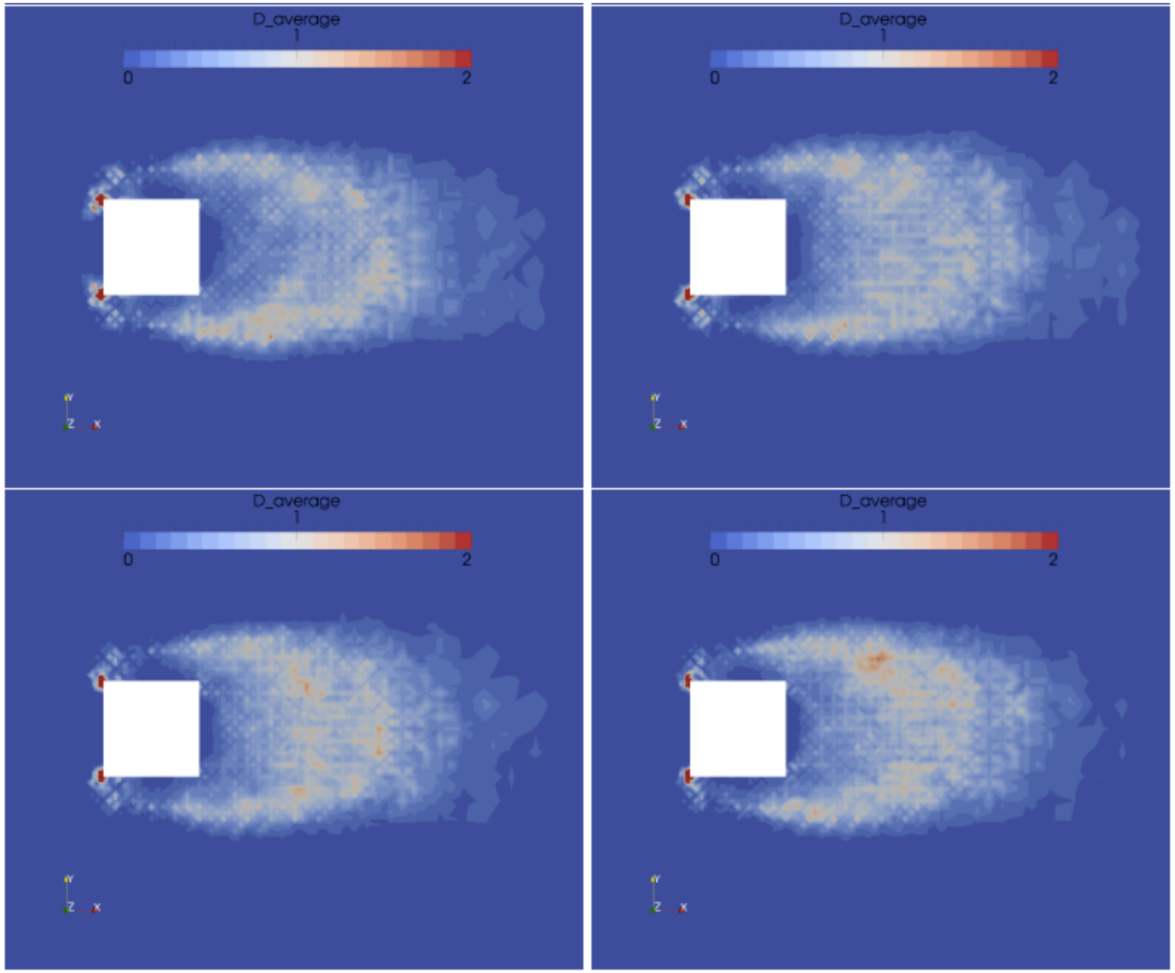
- A posteriori error estimate: $|M(u) - M(U)| \leq \sum_K E_K$ (cells K)
- Error indicator $E_K = S_K \times h_K R_K$ (S_K stability weight, R_K residual)
- Output sensitivity of $M(\cdot)$ by adjoint equation: stability weight S_K
- Adjoint equation: $-\partial\phi/\partial t - (u\cdot\nabla)\phi + \nabla U^T\phi + \nabla\theta = \psi, \quad \nabla\cdot\phi = 0$

Stability weight : S_K



Residual : $h_K R_K$

Law of finite dissipation: $D_h \rightarrow D_0 > 0$



Dissipation intensity $D_h = |\delta^{1/2}(U \cdot \nabla U + \nabla P)|^2$ under mesh refinement

Local energy estimate (ν = 0)

Theorem 2 *With $f = 0$, and noting that $\delta_i \leq Ch \leq Ch_{max}$ for $i = 1, 2$, we have the following local energy estimate for cG(1)cG(1), with $\phi_n(x, t)$ a smooth positive test function with local support, piecewise constant in time over I_n:*

$$| \sum_{n=1}^{N} [\int_\Omega (\frac{1}{2}(|U^n|^2 - |U^{n-1}|^2)k_n^{-1} + \nabla \cdot (\bar{U}^n(\frac{1}{2}|\bar{U}^n|^2 + P^n)))\phi_n \, dx] \, k_n$$
$$+ \sum_{n=1}^{N} [\int_\Omega (\delta_1 |\bar{R}_1(\bar{U}^n, P^n)|^2 + \delta_2 |\bar{R}_2(\bar{U}^n)|^2)\phi_n \, dx] \, k_n \, |$$
$$\leq Ch_{max,\phi,n}^{1/2}$$

with $h_{max,\phi,n} \equiv \max_{n: supp \ \phi_n \neq \emptyset} (\max_{x \in supp \ \phi_n} h(x))$

Turbulent boundary layer model

LES BL resolution >99% of mesh points -> need wall model!
[Piomelli/Balaras Annu. Rev. Fluid Mech. 02]

Typical LES wall modeling:

- Slip velocity: $u \cdot n = 0$
- Wall shear stress model: $\tau = n^T \sigma t = \beta (u \cdot t)$

How to implement this model?

- Complex geometry: What is the domain? What normal n?
- How to implement BC (weak or strong etc.)?
- How to choose the function/parameter β?

- How sensitive is the simulation to the above parameters?

Geometry model

First focus on implementation of slip BC : $u \cdot n = 0$

How to approximate the geometry?

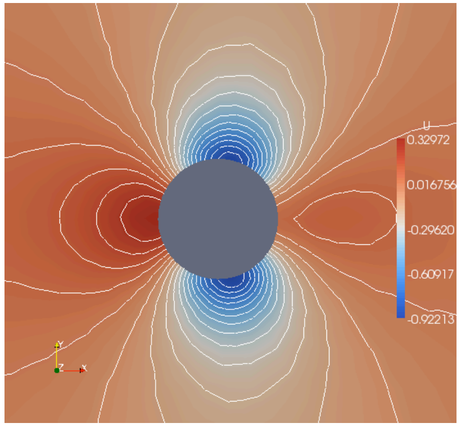
- Adaptive approximation: new nodes on exact geometry
- High order geometry: Isoparametric FEM, Isogeometric FEM [Hughes et.al. CMAME 05], NEFEM [Sevilla et.al. IJNMF 08]

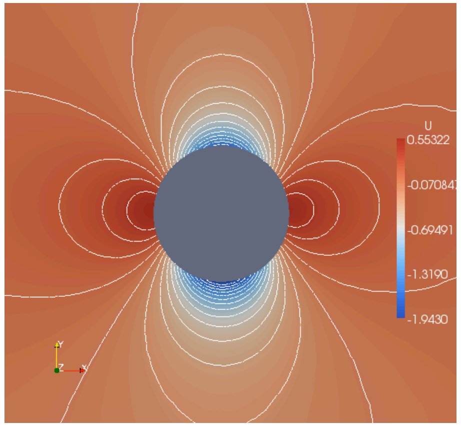
What is the normal n? Weak or strong implementation of BC?

- Weak implementation: discontinuos face normals from mesh
  -> Artificial friction on curved boundaries! Or even no slip BC!
- Strong: nodal normals from weighted average of face normals
- Strong: exact geometry normals [Krivodonova/Berger JCP 06]

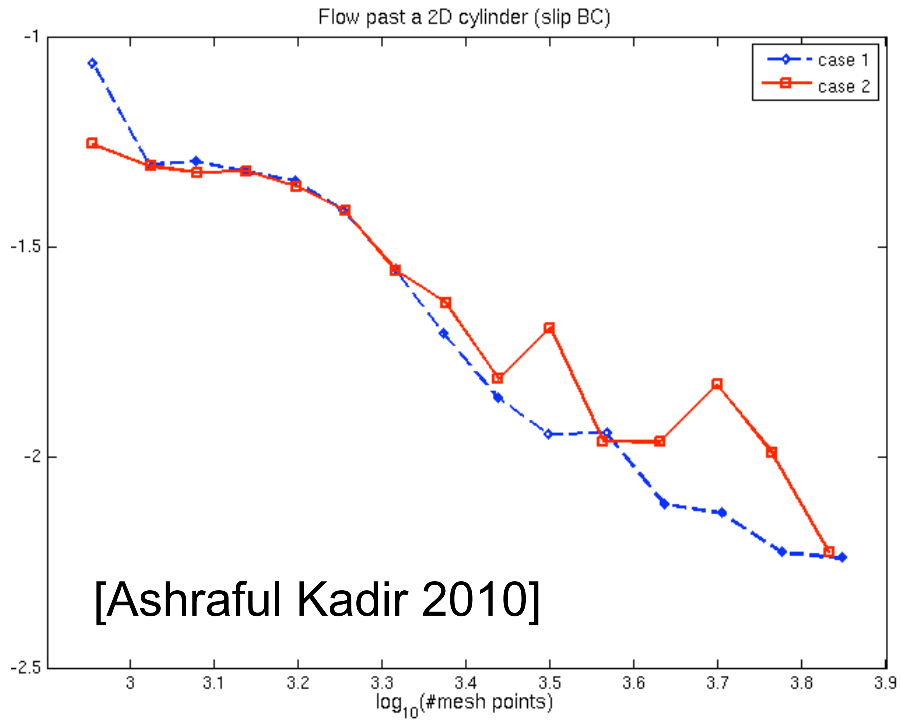
2D Euler flow: mesh vs. exact normals

Mesh locally refined with respect to error in drag force and adapted to geometry





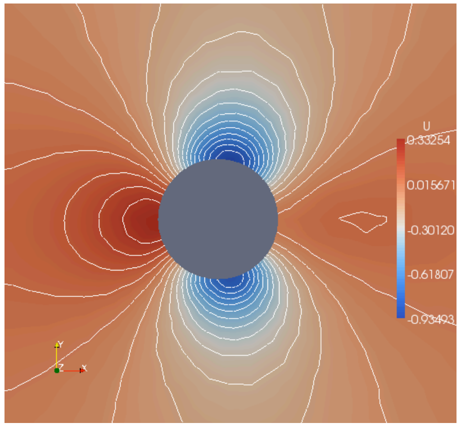
Lower: strong BC, mesh normals

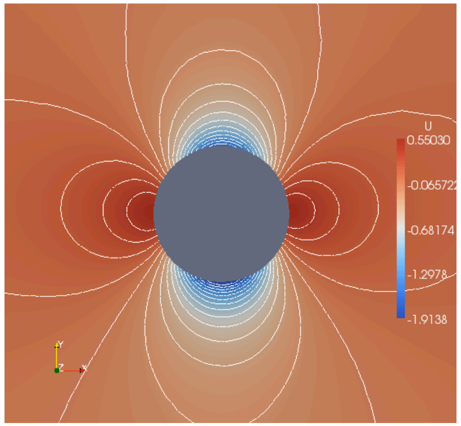


[Ashraful Kadir 2010]

~1500 vertices

~7000 vertices





Upper: strong BC, exact normals

Wall shear stress model $\tau = \beta(u \cdot t)$

How to choose the function/parameter β?

- Schumann [JCP 75]: “simple” parameter $\beta = \tau_{\text{skin friction}} / U_{\text{mean}}$
- Since then: towards increasingly complex β
- State of the art: hybrid methods LES-RANS (e.g. DES [Spalart et.al. 97])

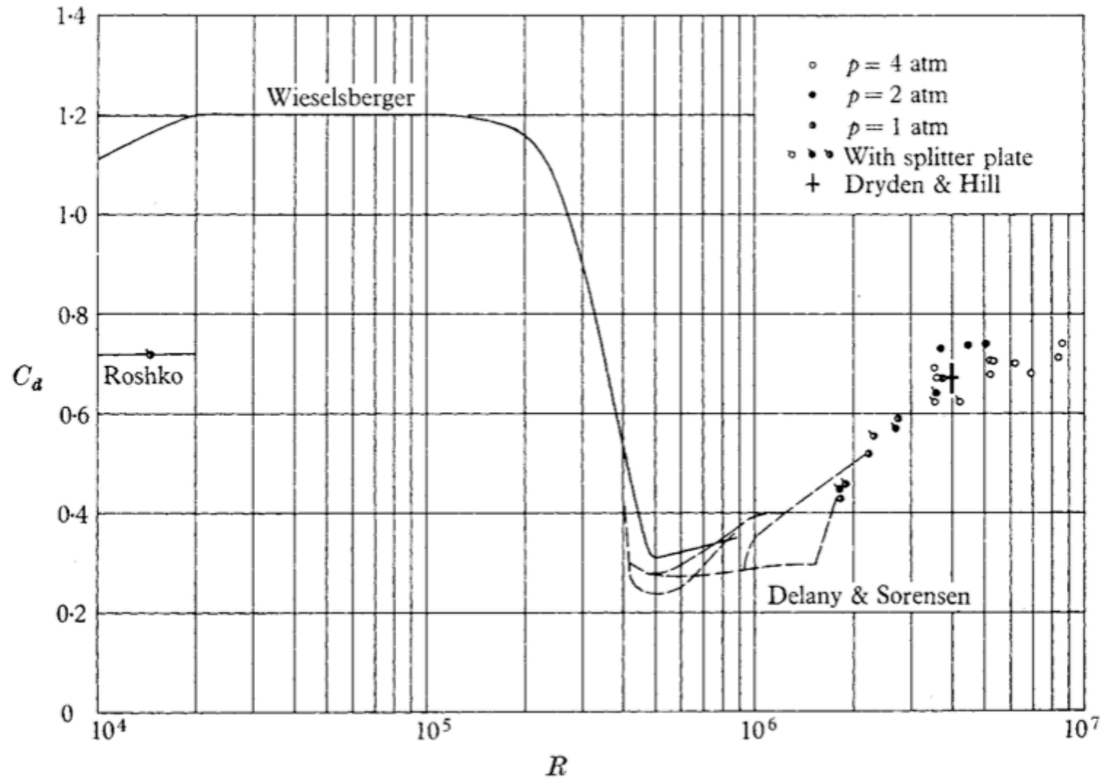
- BL thin ($\delta \sim \nu^{1/5}$), skin friction small ($c_f \sim Re^{-1/5}$)
- Wall shear stress τ decrease with increasing Re

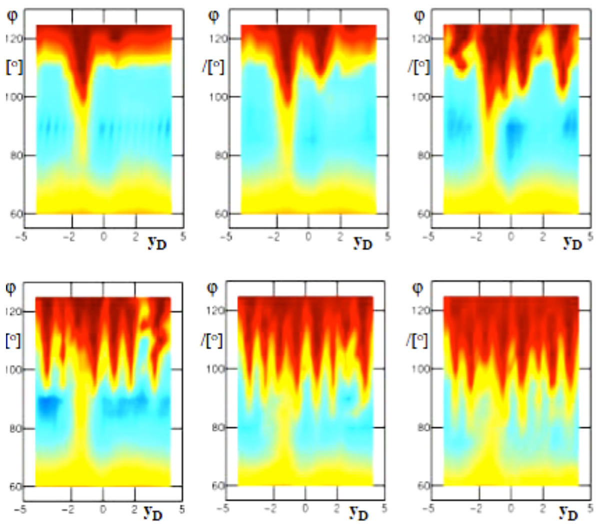
- High Re: is the result sensitive to τ?

- If not sensitive to τ: wall shear stress model not needed?

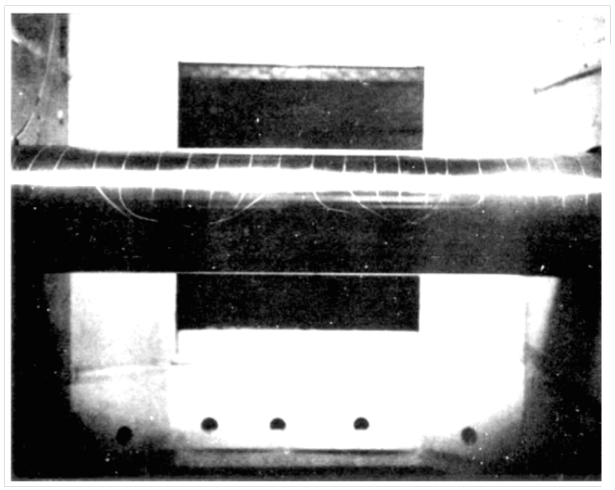
High Re cylinder

- Drag crisis: BL transition
- Drag coeff ~1.2 -> ~0.3-0.4
- Stable 3d cells at crit Re
- Cell diam ~ cylinder diam

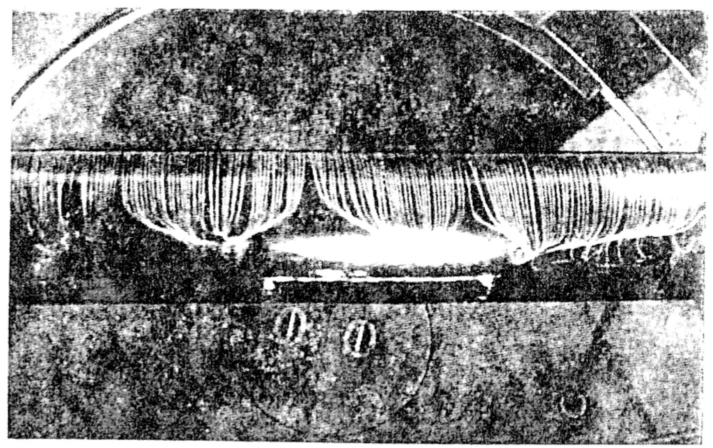




[Gölling Dissertation 01]



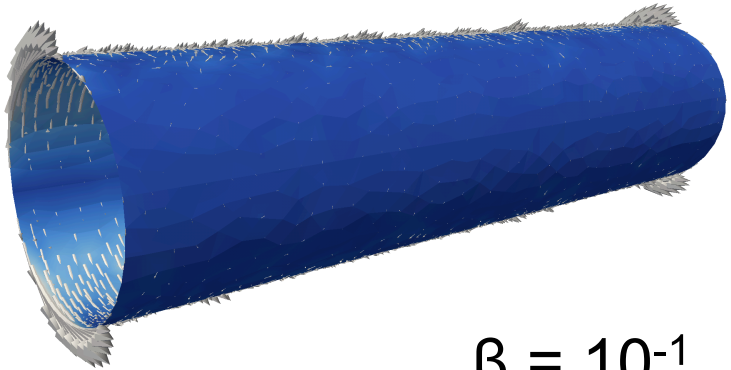
[Humphreys JFM 60]



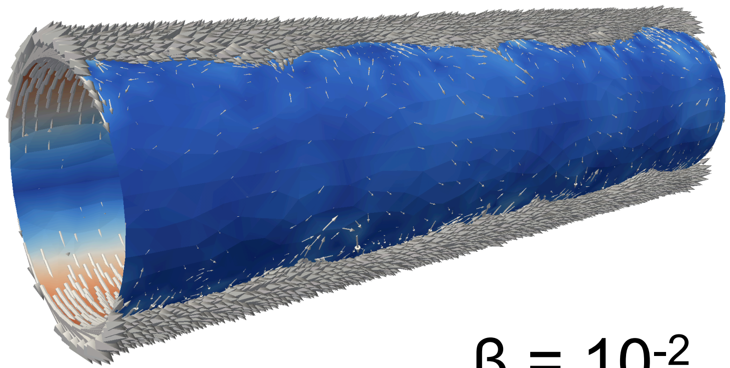
[Korotkin 76]

$\beta = 10^{-1}, 10^{-2}, 10^{-3}, 0$ (100k nodes, ν=0)

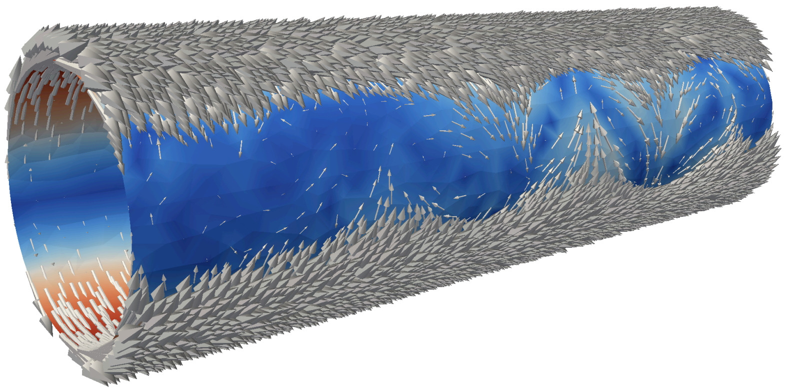
Mesh locally refined with respect to error in drag force and adapted to geometry



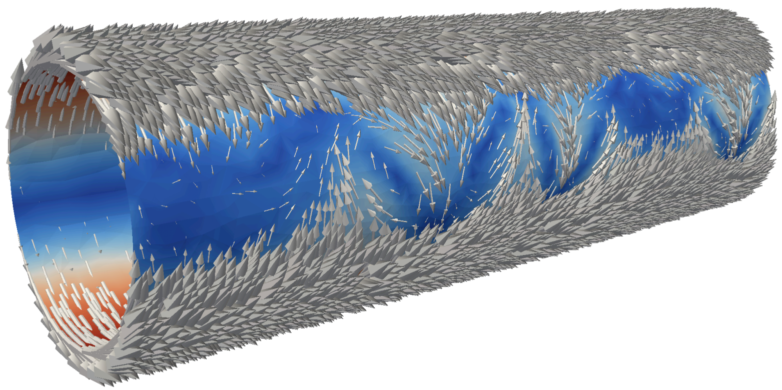
$\beta = 10^{-1}$



$\beta = 10^{-2}$



$\beta = 10^{-3}$

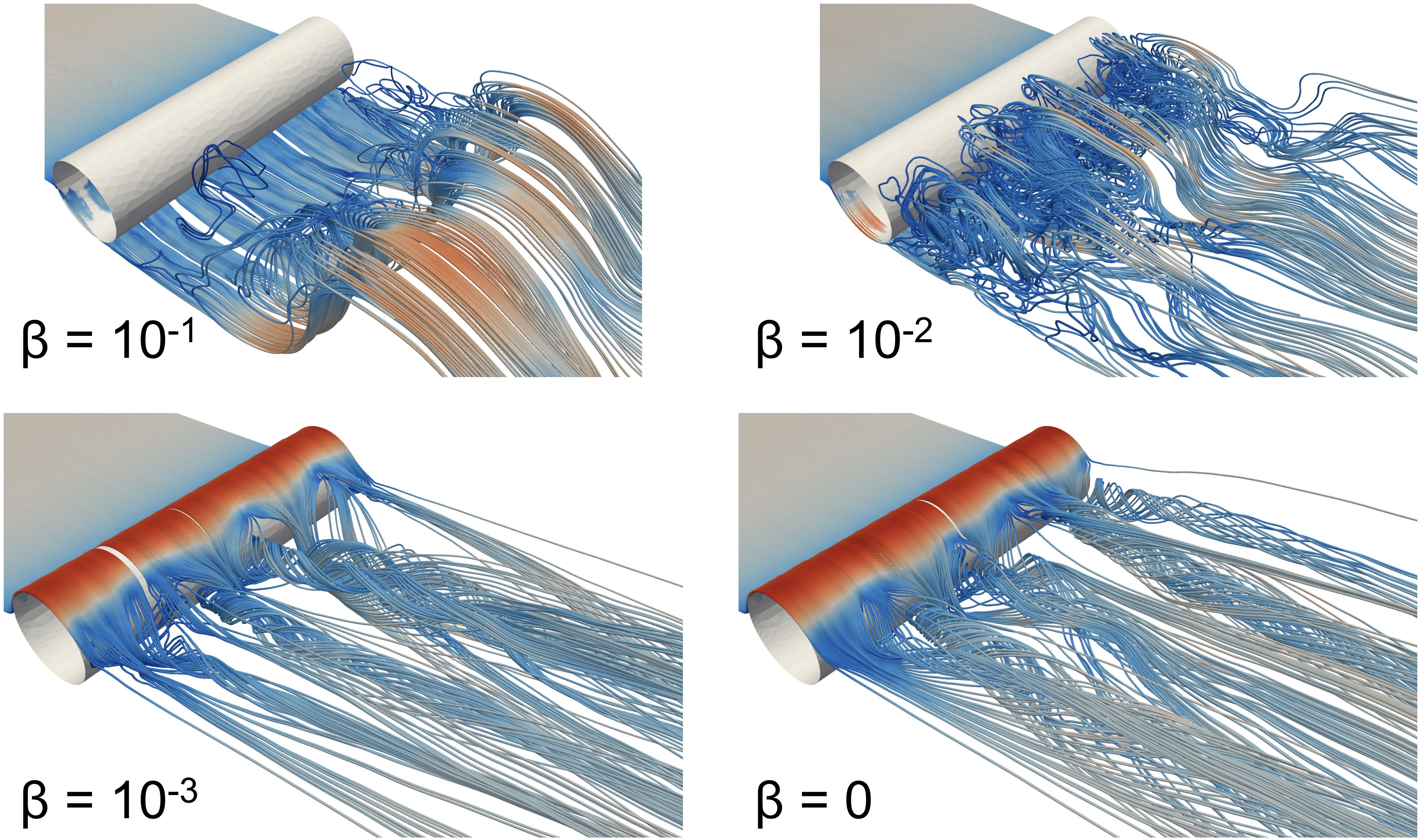


$\beta = 0$

[J.Hoffman/N.Jansson, proc. QLES'09]

$\beta = 10^{-1}, 10^{-2}, 10^{-3}, 0$ (100k nodes, $v=0$)

Mesh locally refined with respect to error in drag force and adapted to geometry



[J.Hoffman/N.Jansson, proc. QLES'09]

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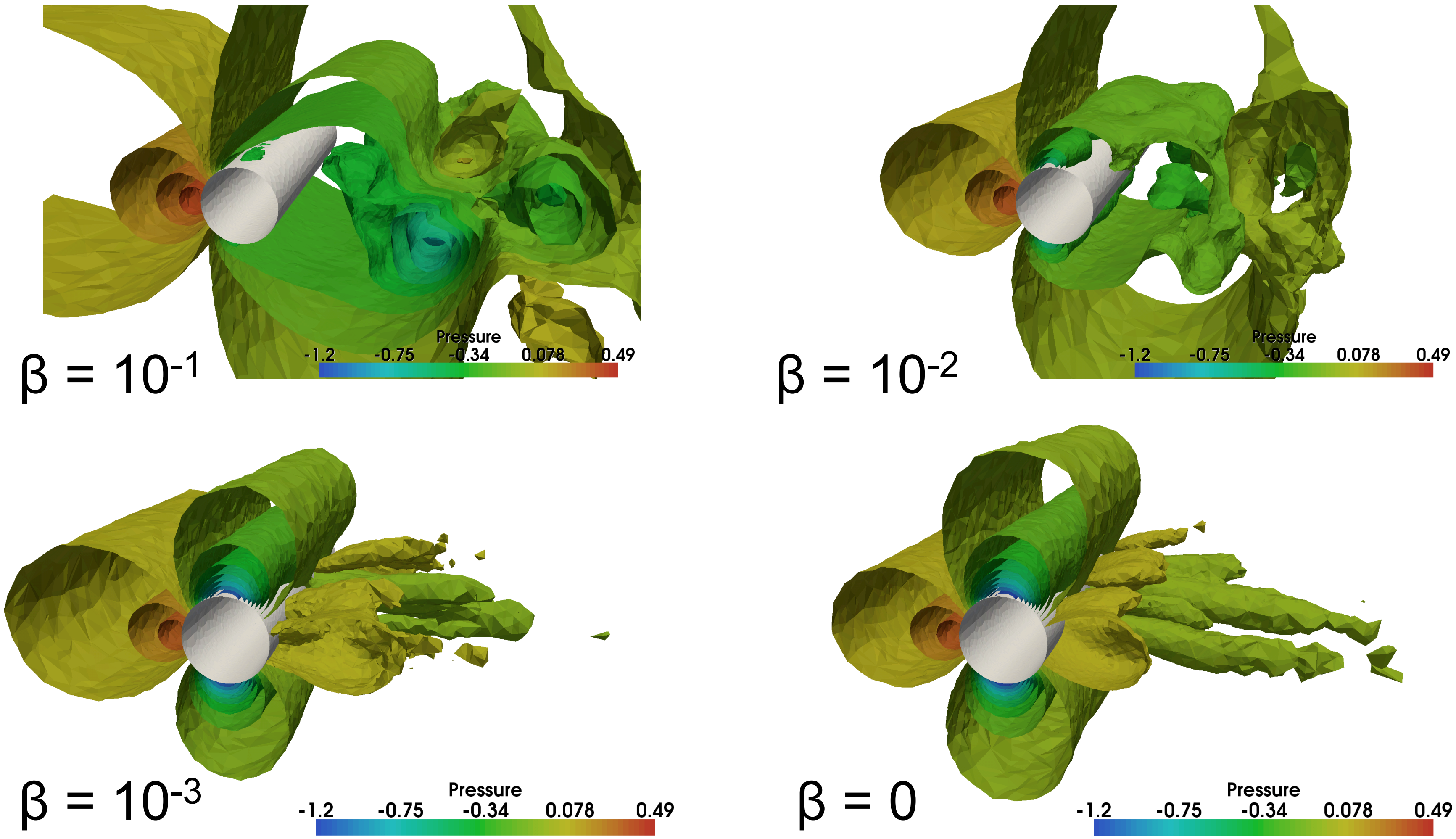
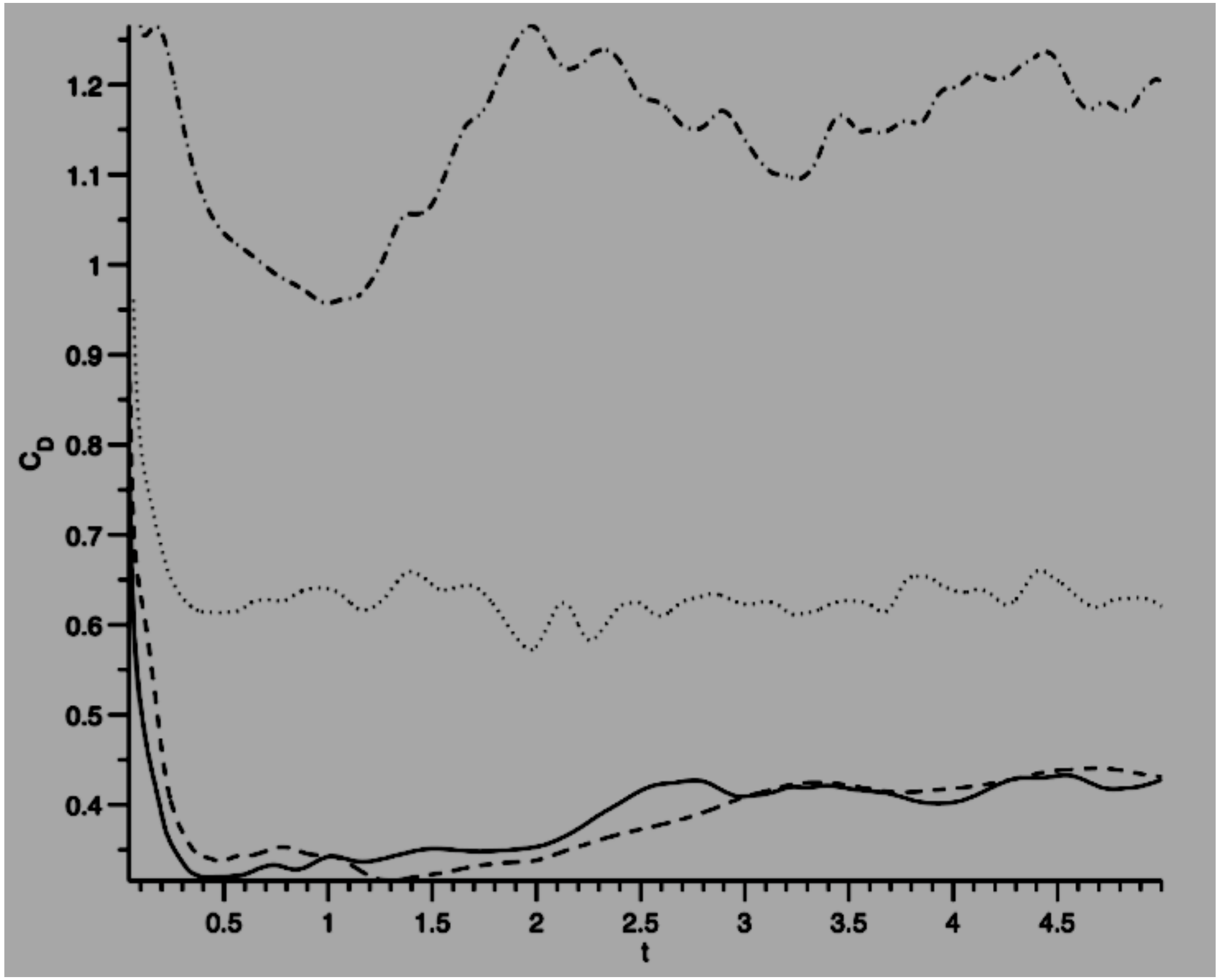
Mesh locally refined with respect to error in drag force and adapted to geometry



[J.Hoffman/N.Jansson, proc. QLES'09]

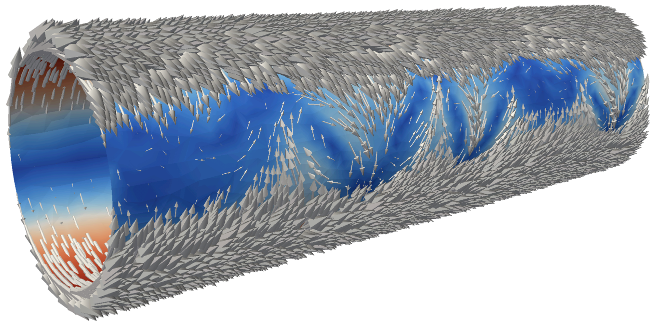
$\beta = 10^{-1}, 10^{-2}, 10^{-3}, 0$ (100k nodes)

Drag c_D



[J.Hoffman/N.Jansson, proc. QLES'09]

Linear stability analysis

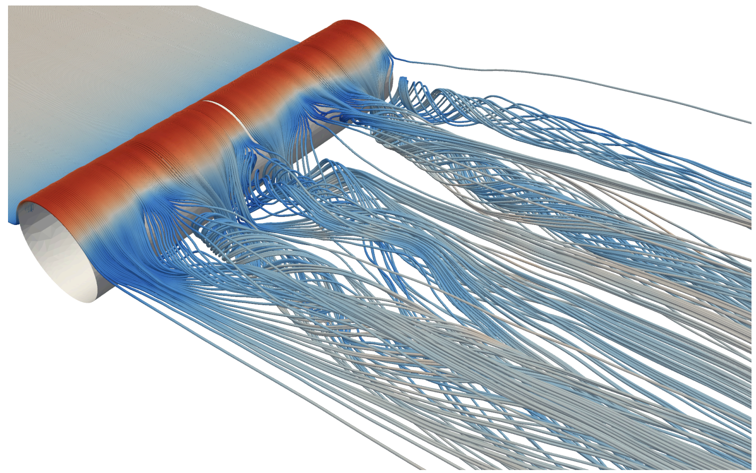


- Linearized equations at potential flow:

$\partial \phi/\partial t + (u\cdot\nabla)\phi + (\phi\cdot\nabla)u + \nabla\theta = 0, \quad \nabla\cdot\phi=0$

- Vorticity equations:

$\partial \omega/\partial t + (u\cdot\nabla)\omega - (\omega\cdot\nabla)u = 0, \quad \omega=\nabla\times u$



- Key for stability: solution gradient ∇u
- At separation: ∇u = [2 0 0; 0 -2 0; 0 0 0]

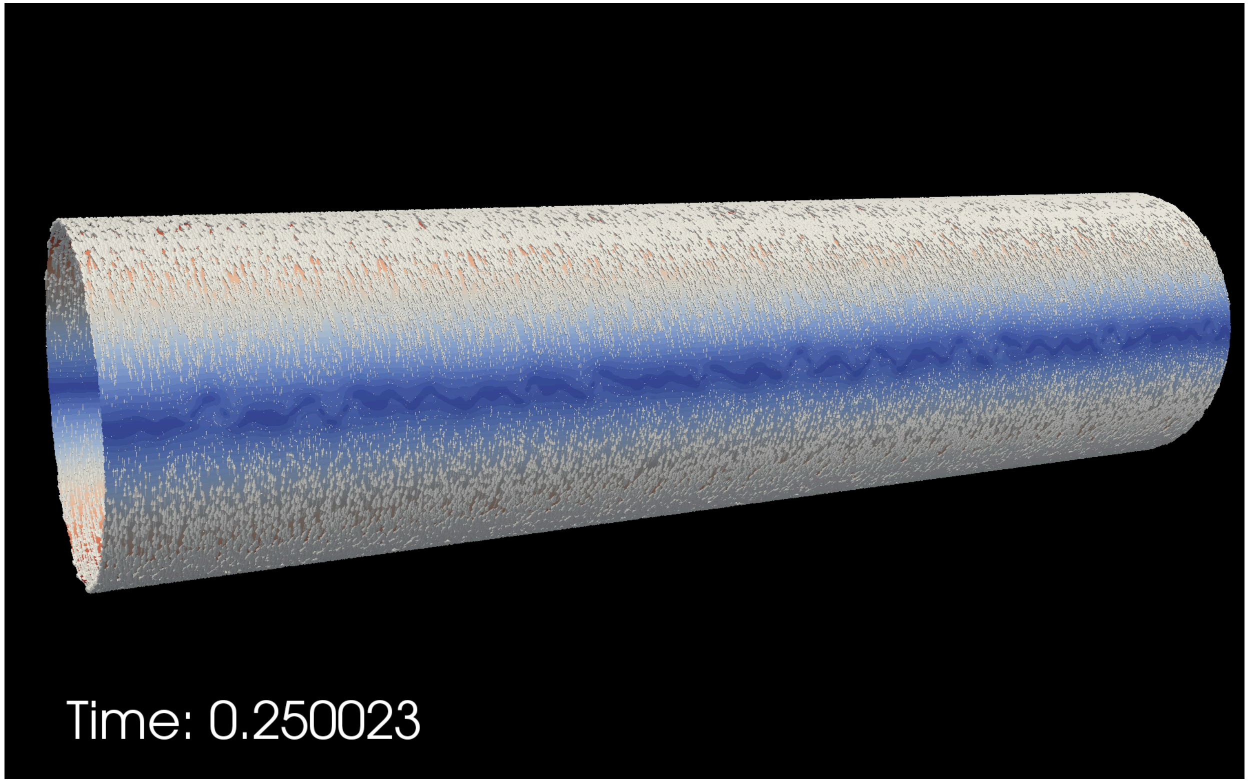
Potential solution is exponentially unstable at separation:

1. $\partial \phi_2/\partial t + (u\cdot\nabla)\phi_2 + \partial \iota/\partial_2 = 2\phi_2$ (exponential growth of ϕ_2)
2. $\partial \omega_1/\partial t + (u\cdot\nabla)\omega_1 = 2\omega_1$ (exponential growth of ω_1)

[J.Hoffman/C.Johnson, Springer 07, BIT 08, JMFM 10]

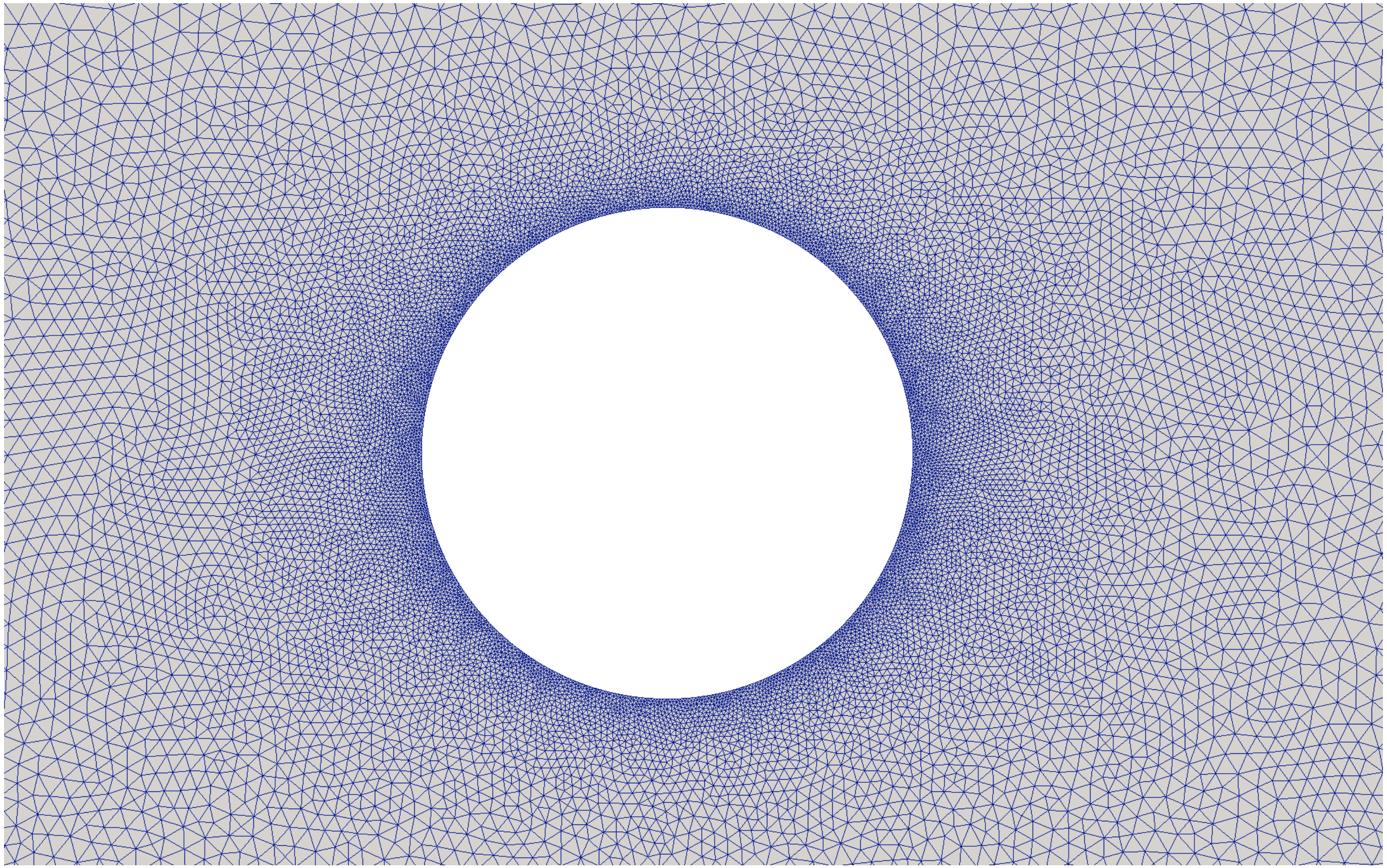
Large computation (β = 0), long time [264 cores/Cray XT6m]

- Mesh: 1 503 094 nodes, 7 348 169 elements

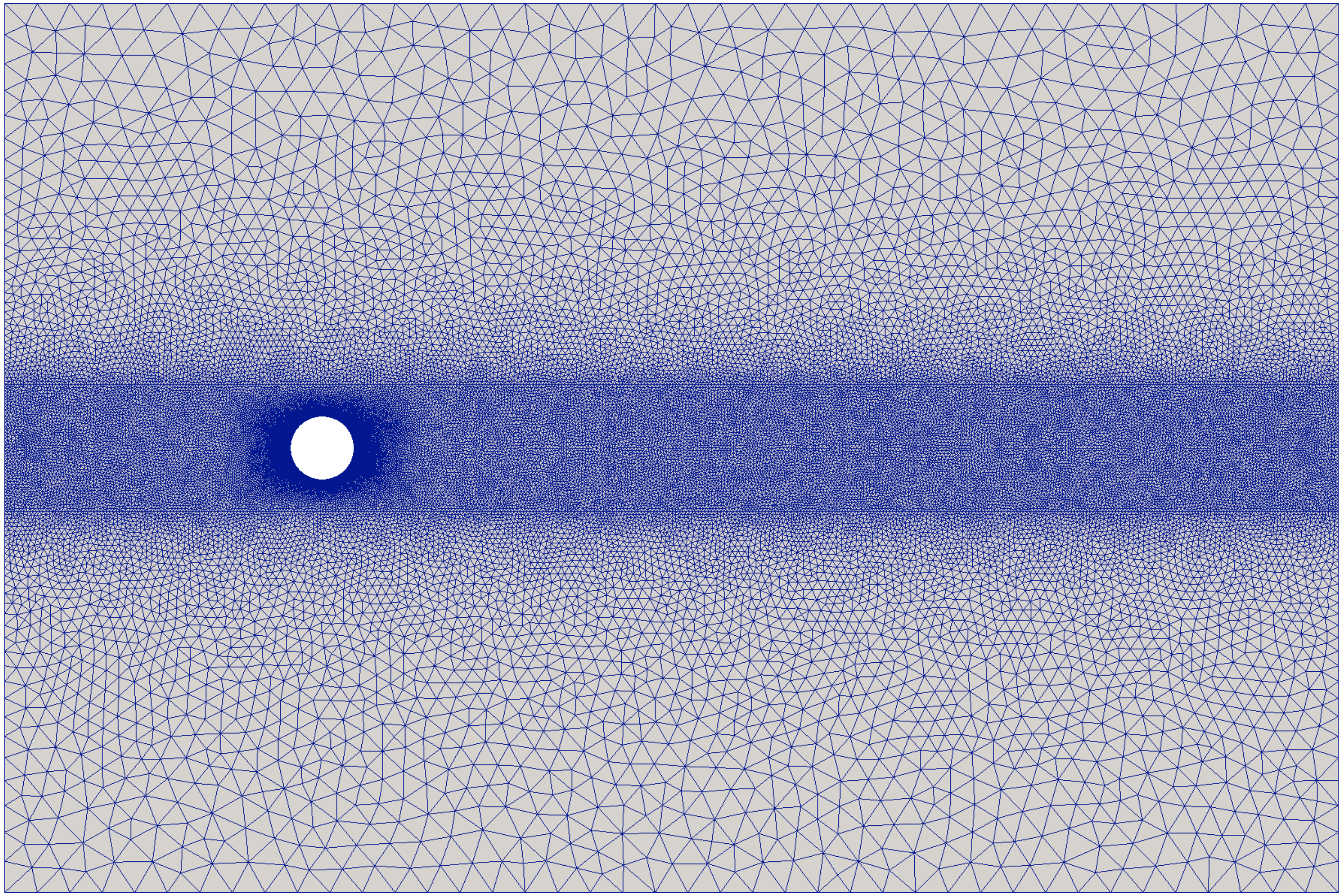


[N.Jansson/J.Hoffman, 2010]

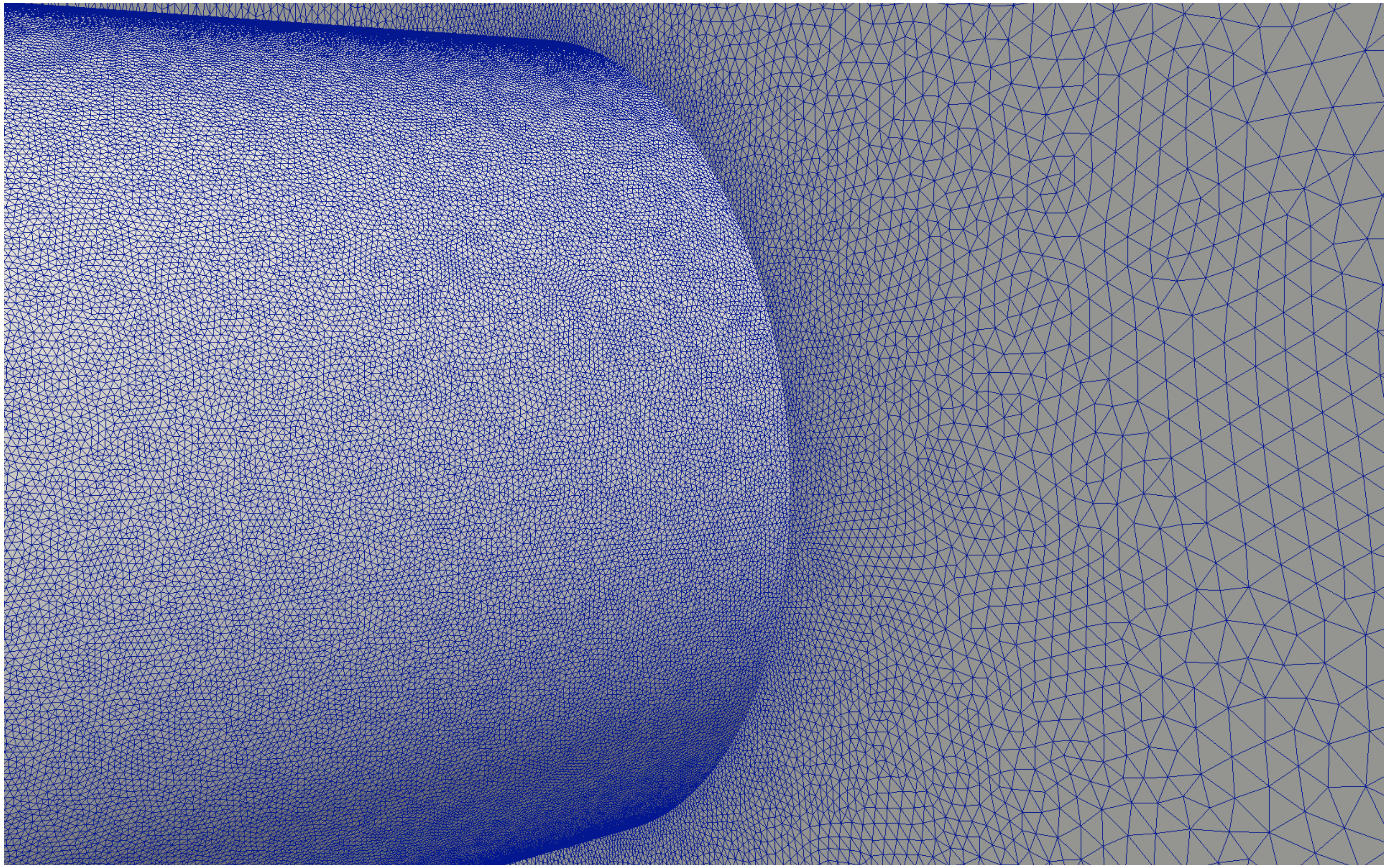
Ad hoc refined mesh



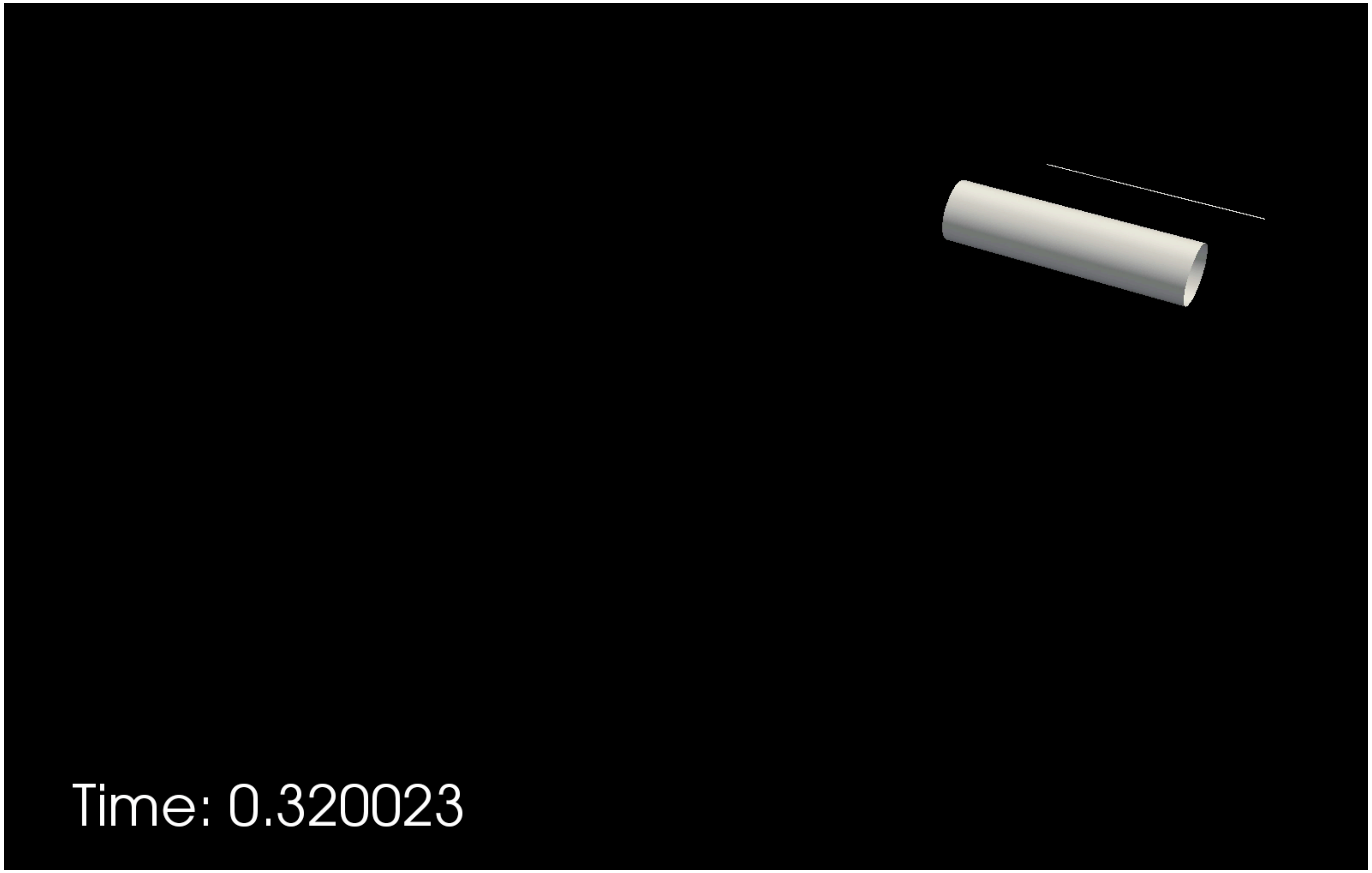
Ad hoc refined mesh



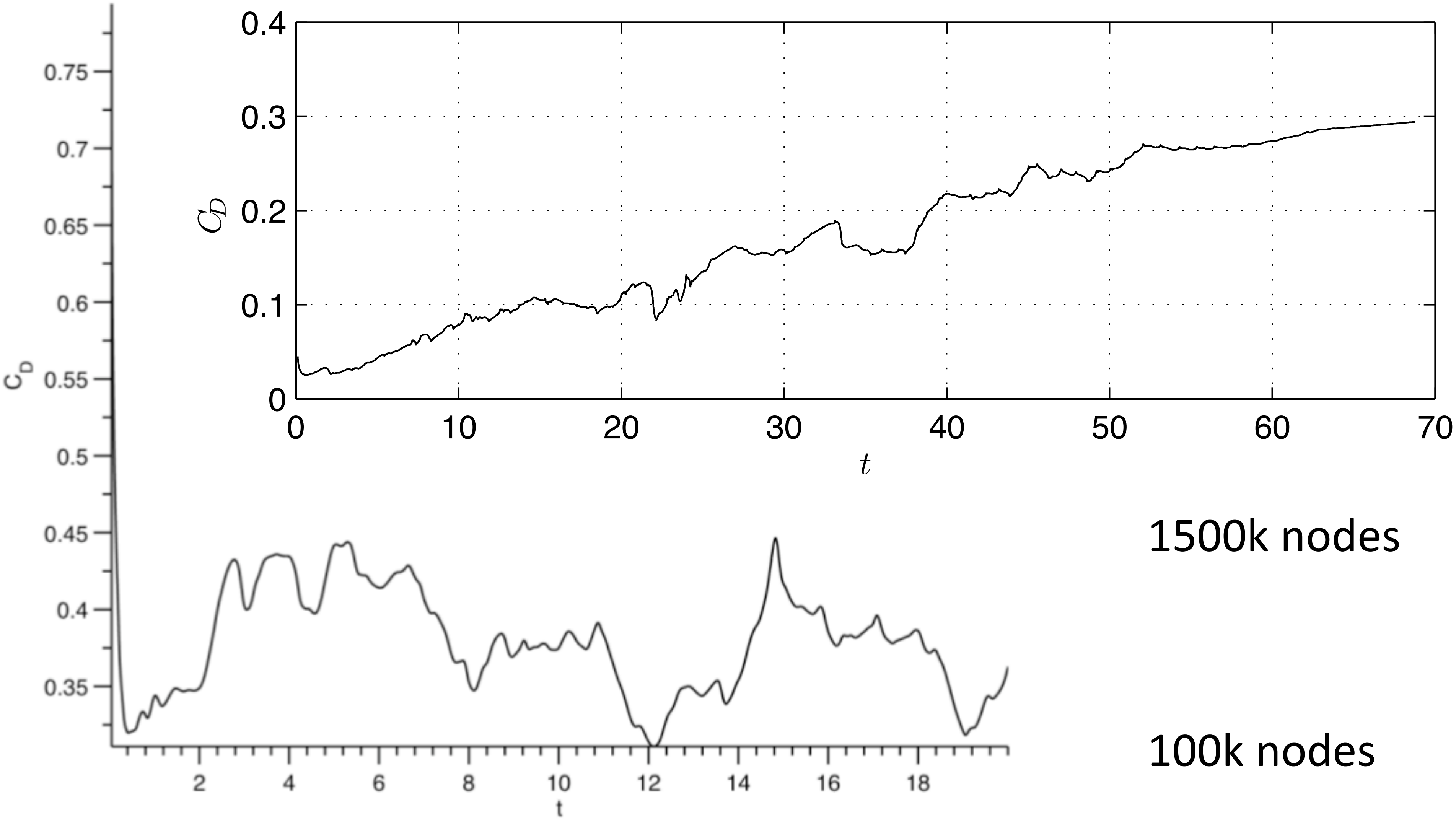
Ad hoc refined mesh



Particle paths



Finer mesh: long start-up, same limit

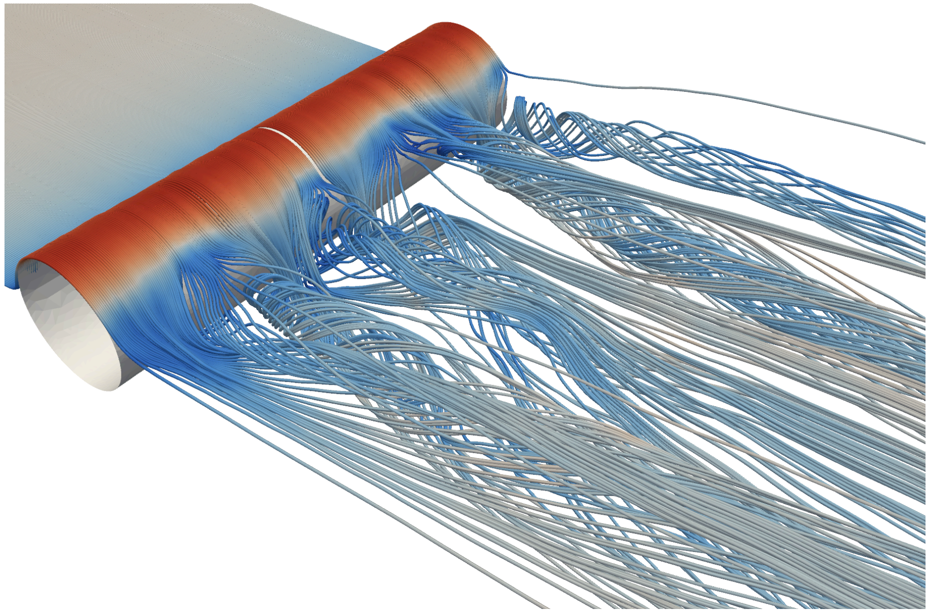


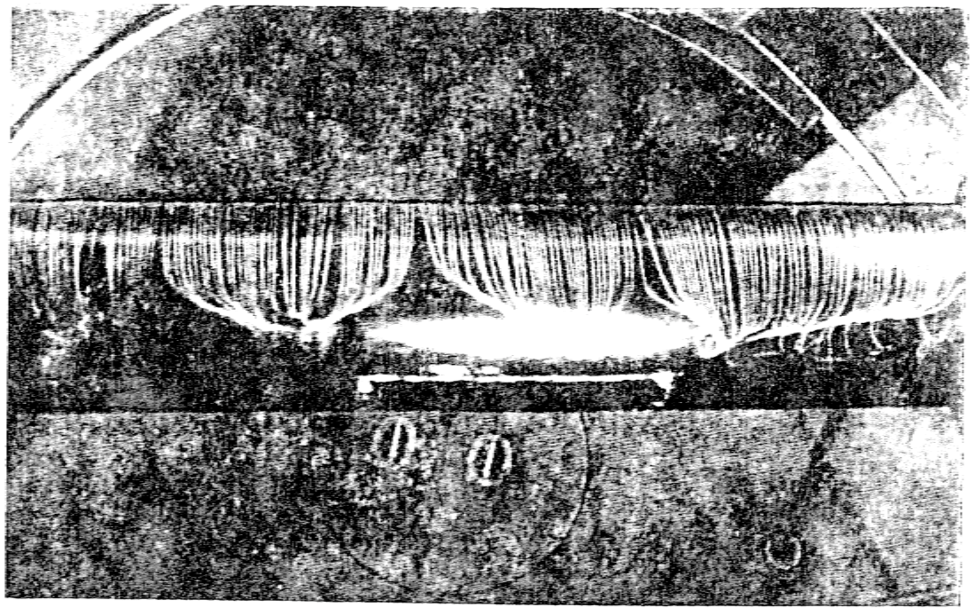
1500k nodes

100k nodes

High Re cylinder, slip bc ($\beta = 0$)

- Drag coefficient: 1.2 -> 0.3-0.4 (drag crisis)
- Streamwise vorticity forms stable cells
- Vorticity cell diameter ~ cylinder diameter
- Independent of skin friction $\beta < 10^{-3}$
- Inviscid separation mechanism – no boundary layer!





Workshop on Benchmark problems for Airframe Noise Computations (BANC-I)

Landing gear test case
[Boeing/Nasa Test and Evaluation]

Re = 10^6
Boundary layers tripped to
assure turbulent separation

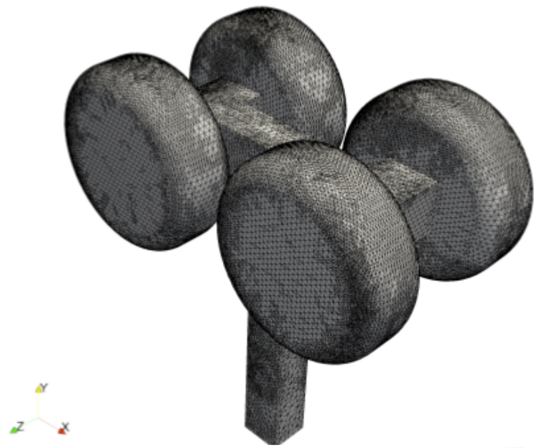
In conjunction with AIAA meeting:
June 2010, Stockholm

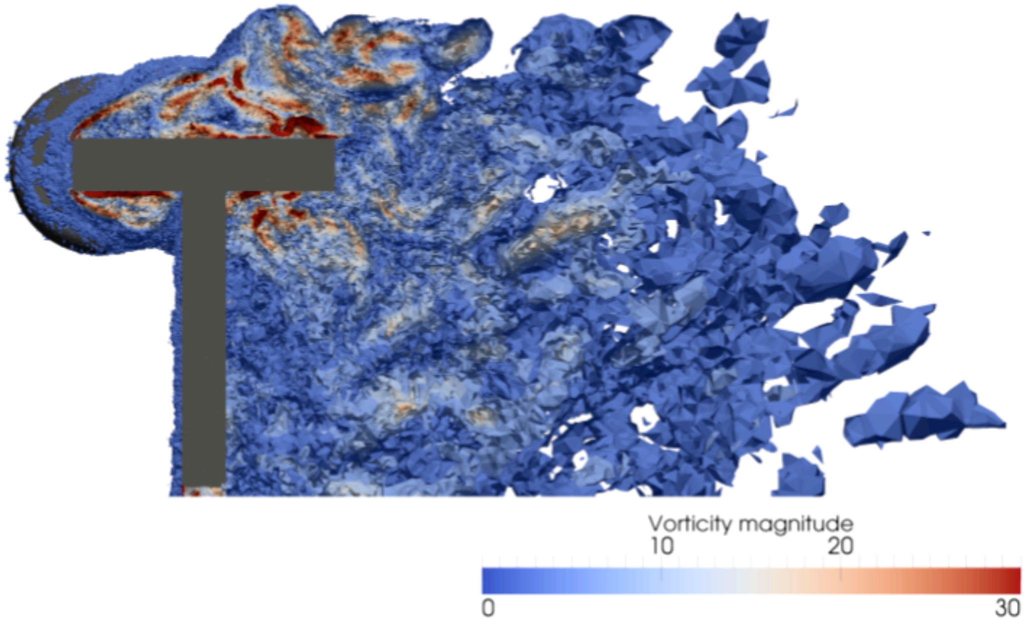
Planned follow up in BANC-II

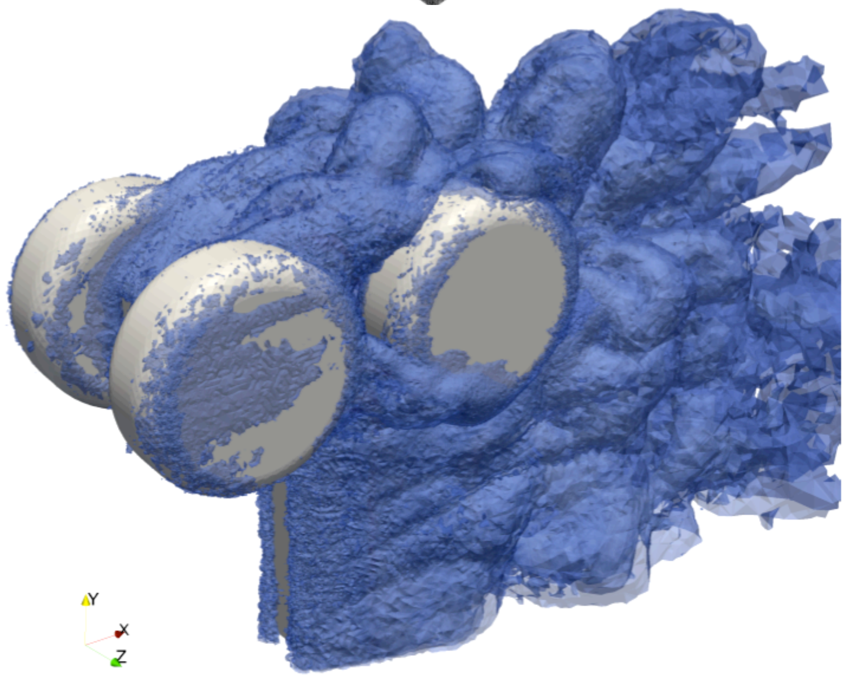
[Vilela De Abreu/Jansson/Hoffman 2010]

Adaptive FEM DNS/LES, slip bc

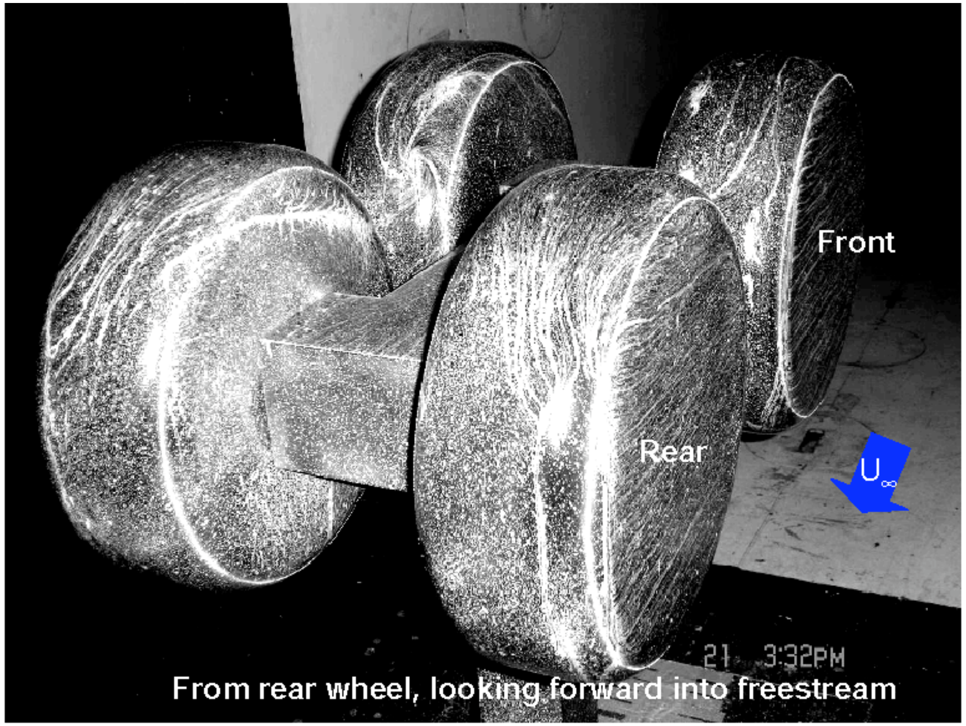
Mesh locally refined 7 times with
respect to error in drag force:
final mesh ca. 1 000 000 nodes

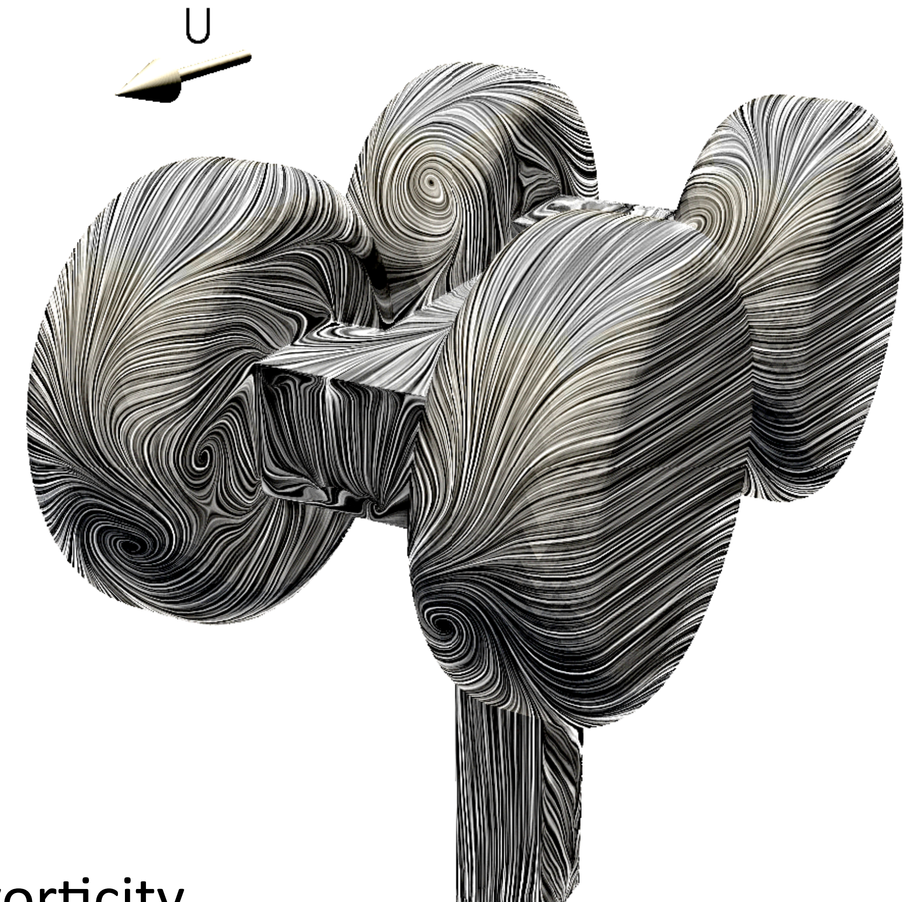






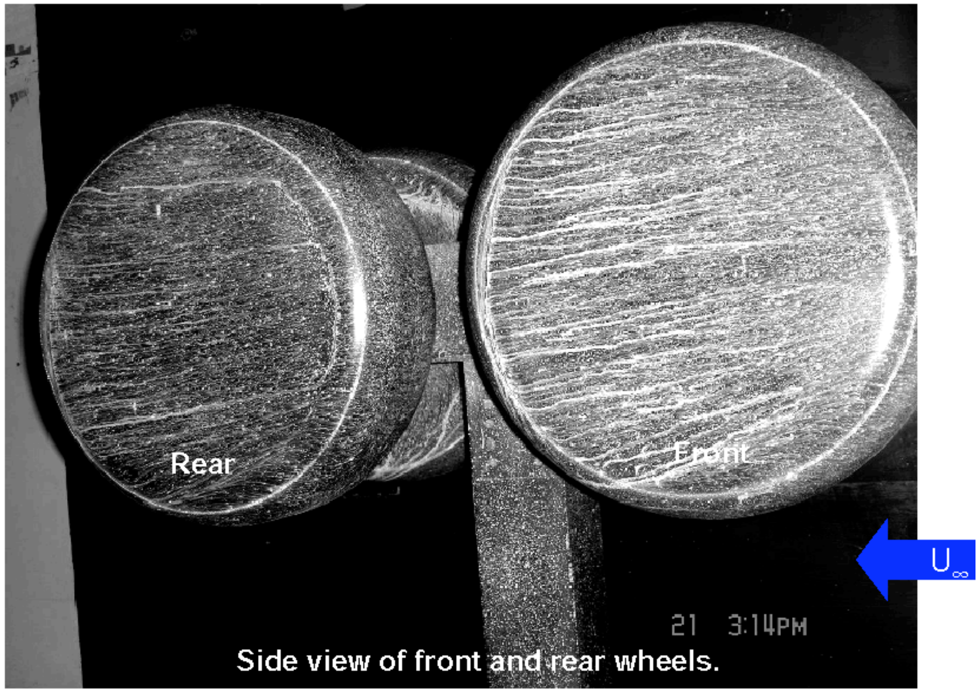
Oil film vs. mean field streamlines

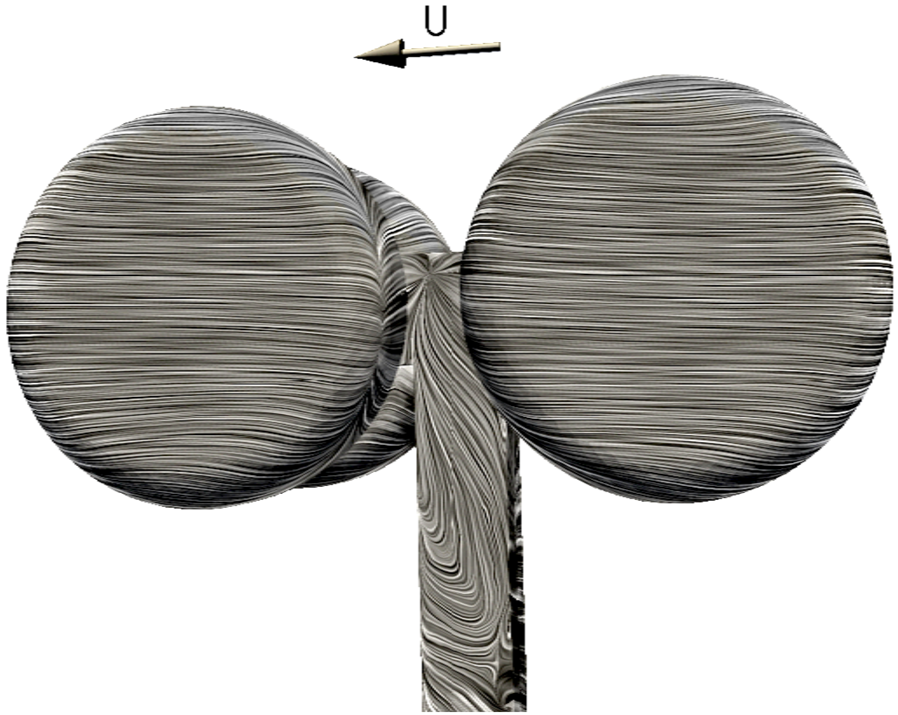




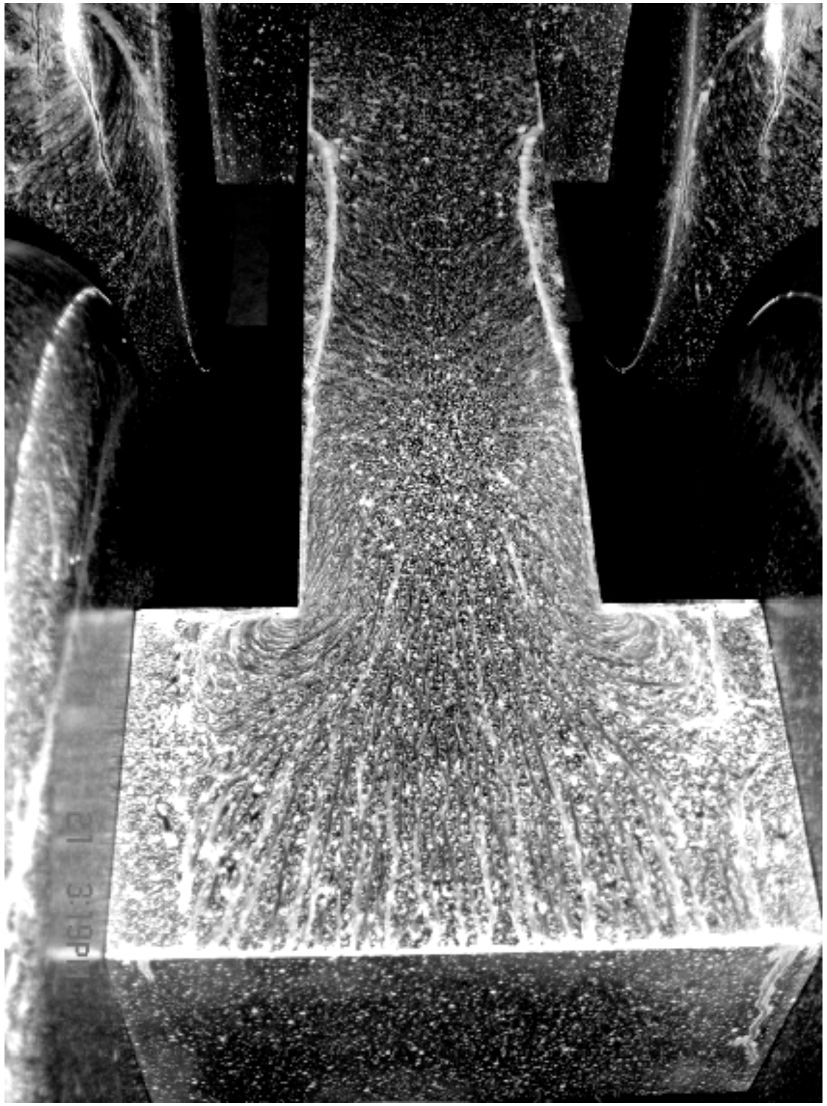
Note vortex separation patterns:
Inviscid separation in streamwise vorticity

Oil film vs. mean field streamlines



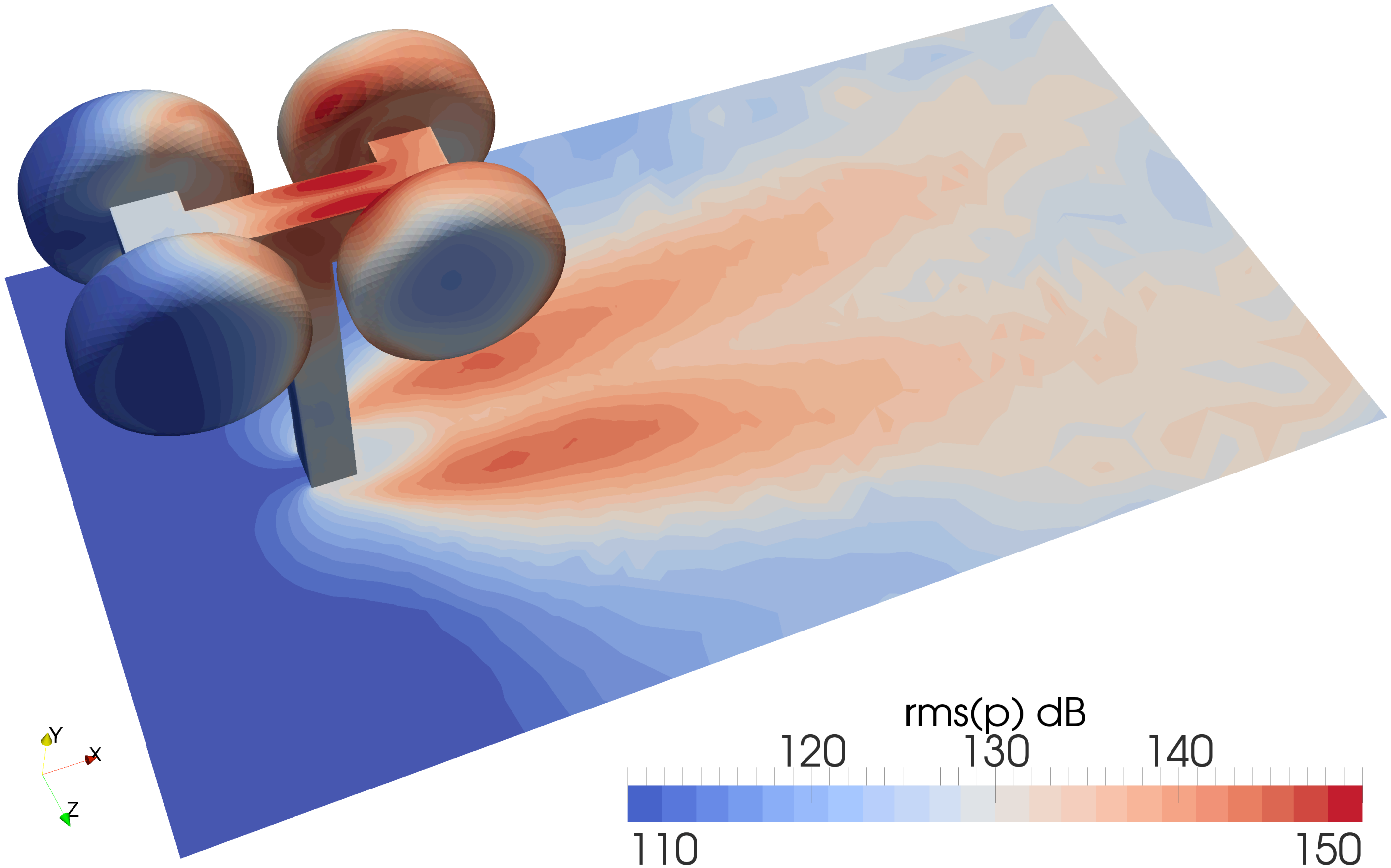


Oil film vs. mean field streamlines



Acoustic sources: pressure rms [dB]

Compares well with DES simulations
[Spalar/Shur/Strelet/Travin 2010]



Summary

Adaptive FEM DNS/LES for high Re flow:

- Parallel adaptive FEM with a posteriori error control
- Strong implementation of slip BC, with skin friction
- High Re turbulent boundary layers: inviscid separation, can be modeled by zero skin friction (slip bc)
- No subgrid model, no wall model: no empirical parameters
- Great opportunities: ongoing quantitative validations
- Implemented in Unicorn at www.fenicsproject.org

Computational Technology Laboratory: www.csc.kth.se/ctl
Unicorn open source FEM solver: www.fenicsproject.org

Vetenskapsrådet
Energimyndigheten