Some instances of the reasonable effectiveness (and limitations) of symmetries and deformations in fundamental physics

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[This talk summarizes many ideas and presents joint works (some, in progress) that would not have been possible without the deep insight, on the role of symmetries and of deformations in physics, of my friend and coworker for 35 years, Moshe Flato and, after his untimely death in Paris on 27 November 1998, the support of Noriko Sakurai, my companion for 9 years and wife for almost one, until her death on 16 October 2009 in Singapore.]

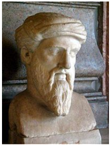
Moshe Flato (9/17/1937 – 11/27/1998) & Noriko Sakurai (2/20/1936 – 10/16/2009)



Abstract

A survey of some applications of group theory and deformation theory (including quantization) in mathematical physics. Rotation and discrete groups in molecular physics (“dynamical” symmetry breaking in crystals, Racah-Flato-Kibler); chains of groups and symmetry breaking. “Classification Lie groups” (“internal symmetries”) in particle physics. Space-time symmetries, relations with internal symmetries. Deformations of symmetries. Deformation quantization, quantum groups and quantized spaces. Field theories and evolution equations (from the point of view of nonlinear Lie group representations). Connections with some cosmology, including especially quantized anti-de Sitter groups and spaces. Prospects for future developments between mathematics and physics.

Epistemological comments

Pythagoras is the first to be recorded saying that *Mathematics is the way to understand the universe.*

Many developed similar ideas, including Sir James Hopwood Jeans:
“The Great Architect of the Universe now begins to appear as a pure mathematician.”
“We may as well cut out the group theory. That is a subject that will never be of any use in physics.” [Discussing a syllabus in 1910.]
Einstein in 1921: “As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.”
“Curse” of experimental sciences. Mathematical logic: if A and $A \to B$, then B. In real life, imagine model or theory A. If $A \to B$ and “B is nice” (e.g. verified & more), then A! [Inspired by Kolmogorov quote.] (It ain’t necessarily so.)

Three questions: Why, What, How?

Comments on the title

In 1960 Eugene P. Wigner wrote his famous provocative paper in Comm. Pure Applied Math. 13, 1-14, *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*, reproduced with many interesting essays in *Symmetries and Reflexions* (Indiana University Press 1967, MIT Press 1970).
Many elaborated on it, including the converse statement by Atiyah (*the unreasonable effectiveness of physics in mathematics*) on several occasions, lately with Dijkgraaf and Hitchin in Phil. Trans. R. Soc. A 2010 368, 913-926, *Geometry and physics*.

An aim of this talk is to indicate by examples dealing with symmetries and with deformation theory, important somewhat complementary aspects, that these effectivenesses are quite reasonable but have their limitations.

Symmetries in physics, a tachyonic outlook

“Dynamical” symmetries of (covariant) equations (e.g. Poincaré group and other space-time symmetries)
and spectroscopical symmetries.
In atomic and molecular physics know the forces and e.g. breaking of SU(2) symmetry by crystalline field is natural (Wigner, Racah methods, Flato’s master thesis continued by Kibler, and many more).
In nuclear physics one is lead to “classification symmetries.”
In particle physics more and more particles were discovered [Fermi botanist] exhibiting new (sometimes “strange”) properties, which lead again to “classification symmetries” (e.g. SU(3)) which (after some twists) made people invent new dynamics (QCD, on the basis of QED).
Connection? L.O’Raifeartaigh, Jost-Segal,
Coleman-Mandula (and super-generalizations),
our objections and counterexamples.

Rotation group. Bethe, Wigner, Racah




Crystal field theory: seminal work of Hans Bethe in 1929, who also introduced “double groups” (in SU(2), not SO(3)). The breaking SO(3) $>$ G is caused by an inhomogeneous electric field at the site of an ion [with N electrons on a layer d ($\ell = 2$) or f ($\ell = 3$). The breaking is created by ion environment (crystal, molecule, even biological). The interaction Hamiltonian has the symmetry of its causes (“Curie principle”).
Wigner’s book *Group theory and its application to the quantum mechanics of atomic spectra* (first German edition, 1931; expanded English version 1959). Together with Weyl (1928 book “Gruppentheorie und Quantenmechanik”) they spread the “GruppenPest” (Wigner was Hungarian then).

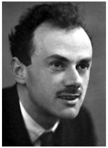
Ugo Fano, Giulio (Joel) Racah (book on “Irreducible tensor sets”) and Racah school in Jerusalem.

Crystal-Field theory. Racah, Flato, Kibler and many more

At the end of 50's (and early 60's) crystal-field theory and its applications to optical and magnetic properties of ions d^N or f^N in a finite symmetry G relied exclusively on the Wigner-Racah algebra of G or its double cover (in SU(2)): Y.Tanabe, S.Sugano, H.Kamimura in Japan and J.S.Griffith in UK.
The originality of the Racah-Flato-Kibler approach was to imbed the "physical" chain $SO(3) > G$ in a "classification" chain:
$SU(5) > SO(5) > SO(3) > G' > G$ for d^N ions $(2\ell + 1 = 5)$
$SU(7) > SO(7) > G_2 > SO(3) > G' > G$ for f^N ions $(2\ell + 1 = 7)$,
with an additional technical intermediate finite group $G' > G$ at the end [G' being the cubic group S_4, order 24, for trigonal S_3, order 6, or tetragonal crystals (order 12, stretch a cube along one axis to a rectangular prism).]
Chains include a physical part and on top of it a tower, useful to classify states and interactions, hence compute matrix elements (via Wigner-Eckhart type theorems expressing matrix elements of spherical tensor operators as product). Introducing groups before G simplifies computation of other interactions acting on electrons of ions (Coulomb, spin-orbit, etc.), a brilliant idea of Racah. The technique has then been widely used in molecular spectroscopy, for many crystal systems.

Such chains were later used in nuclear physics and are now found in Grand Unified Theories, e.g. (over $\mathbb{C}$) $E_7 > E_6 > SO(10) > SL(5) > SL(3)xGL(2)$.

Poincaré and anti de Sitter "external" symmetries



1930's: Dirac asks Wigner to study UIRs of Poincaré group. 1939: Wigner paper in Ann.Math. UIR: particle with positive and zero mass (and "tachyons"). Seminal for UIRs (Bargmann, Mackey, Harish Chandra etc.)

Deform Minkowski to AdS, and Poincaré to AdS group SO(2,3). UIRs of AdS studied incompletely around 1950's. 2 (most degenerate) missing found (1963) by Dirac, the singletons that we call Rac$= D(\frac{1}{2}, 0)$ and Di$= D(1, \frac{1}{2})$ (massless of Poincaré in 2+1 dimensions). In normal units a singleton with angular momentum j has energy $E = (j + \frac{1}{2})\rho$, where ρ is the curvature of the AdS_4 universe (they are naturally confined, fields are determined by their value on cone at infinity in AdS_4 space, AdS_4/CFT_3 correspondence). The massless representations of $\mathrm{SO}(2,3)$ are defined (for $s \geq \frac{1}{2}$) as $D(s+1, s)$ and (for helicity zero) $D(1,0) \oplus D(2,0)$. There are many justifications to this definition. They are kinematically composite:
$(\mathrm{Di} \oplus \mathrm{Rac}) \otimes (\mathrm{Di} \oplus \mathrm{Rac}) = (D(1,0) \oplus D(2,0)) \oplus 2 \bigoplus_{s=\frac{1}{2}}^{\infty} D(s+1, s)$.

Also dynamically (QED with photons composed of 2 Racs, FF88).

Generations, “internal” symmetries

At first, because of the isospin I, a quantum number separating proton and neutron introduced (in 1932, after the discovery of the neutron) by Heisenberg, $SU(2)$ was tried. Then in 1947 a second generation of “strange” particles started to appear and in 1952 Pais suggested a new quantum number, the strangeness S.
In 1975 a third generation (flavor) was discovered, associated e.g. with the τ lepton, and its neutrino ν_τ first observed in 2000.
In the context of what was known in the 1960's, a rank 2 group was the obvious thing to try and introduce in order to describe these “internal” properties. That is how in particle physics theory appeared $U(2)$ (or $SU(2) \times U(1)$, now associated with the electroweak interactions) and the simplest simple group of rank 2, $SU(3)$, which subsists until now but has taken different forms, mostly as “color” symmetry in QCD theory. We first classify empirically, and when a model is “nice” we invent the forces.

Connection with space-time symmetries? (O'Raifeartaigh no-go “theorem” and FS counterexamples.) Reality is (much) more complex.

careful with no-go theorems

Natural question: study the relation (if any) of internal world with space-time (relativity). That was, and still is a hard question. (E.g., combining the present Standard Model of elementary particles with gravitation is until now some quest for a Holy Grail.) Negating any connection, at least at the symmetry level, was a comfortable way out. For many, the proof of a trivial relation was achieved by what is often called the O'Raifeartaigh Theorem, a "no go theorem" stating that any finite-dimensional Lie algebra containing the Poincaré Lie algebra and an "internal" Lie algebra must contain these two as a direct product. Proof was based on nilpotency of Poincaré energy-momentum generators but implicitly assumed the existence of a common invariant domain of differentiable vectors, which Wigner was careful to state as an assumption in his seminal 1939 paper and was proved later for Banach Lie group representations by "a Swedish gentleman". We showed in a provocative PR Letter that the result was not proved in the generality stated, then exhibited a number of counterexamples. The sophisticated Coleman-Mandula attempt to prove a direct product relation contained an implicit hypothesis, hidden in the notation, that presupposed the result claimed to be proved. One should be careful with no-go theorems. Also when negating a model.

We now know that the situation is much more complex, especially when dynamics has to be introduced in the theory. So a fortiori one cannot and should not rule out a priori any relation between space-time and internal symmetries.

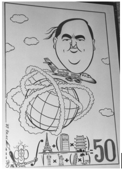
Dirac quote

"... One should examine closely even the elementary and the satisfactory features of our Quantum Mechanics and criticize them and try to modify them, because there may still be faults in them. The only way in which one can hope to proceed on those lines is by looking at the basic features of our present Quantum Theory from all possible points of view. Two points of view may be mathematically equivalent and you may think for that reason if you understand one of them you need not bother about the other and can neglect it.
But it may be that one point of view may suggest a future development which another point does not suggest, and although in their present state the two points of view are equivalent they may lead to different possibilities for the future. Therefore, I think that we cannot afford to neglect any possible point of view for looking at Quantum Mechanics and in particular its relation to Classical Mechanics. Any point of view which gives us any interesting feature and any novel idea should be closely examined to see whether they suggest any modification or any way of developing the theory along new lines.

A point of view which naturally suggests itself is to examine just how close we can make the connection between Classical and Quantum Mechanics. That is essentially a purely mathematical problem – how close can we make the connection between an algebra of non-commutative variables and the ordinary algebra of commutative variables? In both cases we can do addition, multiplication, division..." **Dirac**, *The relation of Classical to Quantum Mechanics* (2^{nd} Can. Math. Congress, Vancouver 1949). U.Toronto Press (1951) pp 10-31.

Flato's deformation philosophy





Physical theories have domain of applicability defined by the relevant distances, velocities, energies, etc. involved. The passage from one domain (of distances, etc.) to another doesn't happen in an uncontrolled way: experimental phenomena appear that cause a paradox and contradict [Fermi quote] accepted theories. Eventually a new fundamental constant enters, the formalism is modified: the attached structures (symmetries, observables, states, etc.) *deform* the initial structure to a new structure which in the limit, when the new parameter goes to zero, "contracts" to the previous formalism. The question is, in which category to seek for deformations? Physics is conservative: if start with e.g. category of associative or Lie algebras, tend to deform in same category. But there are important generalizations:

The framework of deformation quantization

Poisson manifold (M, π), deformations of product of functions.

Inspired by deformation philosophy, based on Gerstenhaber's deformation theory [Flato, Lichnerowicz, Sternheimer; and Vey; mid 70's] [Bayen, Flato, Fronsdal, Lichnerowicz, Sternheimer, LMP '77 & Ann. Phys. '78]

• $\mathcal{A}_t = C^\infty(M)[[t]]$, formal series in t with coefficients in $C^\infty(M) = A$. Elements: $f_0 + tf_1 + t^2 f_2 + \cdots$ (t formal parameter, not fixed scalar.)
• Star product $\star_t : \mathcal{A}_t \times \mathcal{A}_t \to \mathcal{A}_t$; $f \star_t g = fg + \sum_{r \geq 1} t^r C_r(f, g)$
- C_r are bidifferential operators null on constants: $(1 \star_t f = f \star_t 1 = f)$.
- $\star_t$ is associative and $C_1(f, g) - C_1(g, f) = 2\{f, g\}$, so that $[f, g]_t \equiv \frac{1}{2t}(f \star_t g - g \star_t f) = \{f, g\} + O(t)$ is Lie algebra deformation.

Basic paradigm. Moyal product on $\mathbb{R}^{2n}$ with the canonical Poisson bracket P: $F \star_M G = \exp\left(\frac{i\hbar}{2} P\right)(F, G) \equiv FG + \sum_{k \geq 1} \frac{1}{k!}\left(\frac{i\hbar}{2}\right)^k P^k(F, G)$.

Applications and Equivalence

Equation of motion (time τ): $\frac{dF}{d\tau} = [H, F]_M \equiv \frac{1}{i\hbar}(H \star_M F - F \star_M H)$
Link with Weyl's rule of quantization: $\Omega_1(F \star_M G) = \Omega_1(F)\Omega_1(G)$

Equivalence of two star-products $\star_1$ and $\star_2$.

- Formal series of differential operators $T(f) = f + \sum_{r\geq 1} t^r T_r(f)$.
- $T(f \star_1 g) = T(f) \star_2 T(g)$.

For symplectic manifolds, equivalence classes of star-products are parametrized by the 2nd de Rham cohomology space $H^2_{dR}(M)$: $\{\star_t\}/\sim\, = H^2_{dR}(M)[[t]]$ (Nest-Tsygan [1995] and others). In particular, $H^2_{dR}(\mathbb{R}^{2n})$ is trivial, all deformations are equivalent.
Kontsevich: $\{$Equivalence classes of star-products$\} \equiv \{$equivalence classes of formal Poisson tensors $\pi_t = \pi + t\pi_1 + \cdots\}$.

Remarks: - The choice of a star-product fixes a quantization rule.
- Operator orderings can be implemented by good choices of T (or ϖ).
- On $\mathbb{R}^{2n}$, all star-products are equivalent to Moyal product (cf. von Neumann uniqueness theorem on projective UIR of CCR).

Existence and Classification

Let (M, π) be a Poisson manifold. $f \tilde{\star} g = fg + t\{f, g\}$ does not define an associative product. But $(f \tilde{\star} g) \tilde{\star} h - f \tilde{\star} (g \tilde{\star} h) = O(t^2)$.

Is it always possible to modify $\tilde{\star}$ in order to get an associative product?

Existence, symplectic case:
– DeWilde-Lecomte [1982]: Glue local Moyal products.
– Omori-Maeda-Yoshioka [1991]: Weyl bundle and glueing.
– Fedosov [1985,1994]: Construct a flat abelian connection on the Weyl bundle over the symplectic manifold.

General Poisson manifold M with Poisson bracket P:
Solved by Kontsevich [1997, LMP 2003]. "Explicit" local formula: $(f, g) \mapsto f \star g = \sum_{n \geq 0} t^n \sum_{\Gamma \in G_{n,2}} w(\Gamma) B_\Gamma(f, g)$, defines a differential star-product on $(\mathbb{R}^d, P)$; globalizable to M. Here $G_{n,2}$ is a set of graphs Γ, $w(\Gamma)$ some weight defined by Γ and $B_\Gamma(f, g)$ some bidifferential operators.

Particular case of Formality Theorem. Operadic approach

This is Quantization

A star-product provides an *autonomous* quantization of a manifold M. BFFLS '78: Quantization is a deformation of the composition law of observables of a classical system: $(A, \cdot) \to (A[[t]], \star_t)$, $A = C^\infty(M)$.

Star-product $\star$ ($t = \frac{i}{2}\hbar$) on Poisson manifold M and Hamiltonian H; introduce the star-exponential: $\mathrm{Exp}_\star(\frac{\tau H}{i\hbar}) = \sum_{r \geq 0} \frac{1}{r!}(\frac{\tau}{i\hbar})^r H^{\star r}$.

Corresponds to the unitary evolution operator, is a singular object i.e. belongs not to the quantized algebra $(A[[t]], \star)$ but to $(A[[t, t^{-1}]], \star)$. Singularity at origin of its trace, Harish Chandra character for UIR of semi-simple Lie groups.

Spectrum and states are given by a spectral (Fourier-Stieltjes in the time τ) decomposition of the star-exponential.

Paradigm: Harmonic oscillator $H = \frac{1}{2}(p^2 + q^2)$, Moyal product on $\mathbb{R}^{2\ell}$.

$\mathrm{Exp}_\star(\frac{\tau H}{i\hbar}) = \left(\cos(\frac{\tau}{2})\right)^{-1} \exp\left(\frac{2H}{i\hbar}\tan(\frac{\tau}{2})\right) = \sum_{n=0}^{\infty} \exp\left(-i(n + \frac{\ell}{2})\tau\right)\pi_n^\ell$.

Here ($\ell = 1$ but similar formulas for $\ell \geq 1$, L_n is Laguerre polynomial of degree n)

$\pi_n^1(q,p) = 2\exp\left(\frac{-2}{\hbar}H(q,p)\right)(-1)^n L_n\left(\frac{4}{\hbar}H(q,p)\right)$.

Conventional vs. deformation quantization

- It is a matter of practical feasibility of calculations, when there are Weyl and Wigner maps to intertwine between both formalisms, to choose to work with operators in Hilbert spaces or with functional analysis methods (distributions etc.) Dealing e.g. with spectroscopy (where it all started; cf. also Connes) and finite dimensional Hilbert spaces where operators are matrices, the operatorial formulation is easier.
- When there are no precise Weyl and Wigner maps (e.g. very general phase spaces, maybe infinite dimensional) one does not have much choice but to work (maybe "at the physical level of rigor") with functional analysis.

Contrarily to what some (excellent physicists) assert, deformation quantization is quantization and not a mere reformulation: it permits (in concrete cases) to take for $\hbar$ its value, when there are Weyl and Wigner maps one can translate its results in Hilbert space, and e.g. for the 2-sphere there is a special behavior when the radius of the sphere has quantized values related to the Casimir values of $SO(3)$.

Cohomological renormalization in deformation quantization

Starting with some star-product $\star$ (e.g. similar to the normal star-product on a manifold of initial data), we would like to interpret various divergences appearing in the theory in terms of coboundaries (or cocycles) for the relevant Hochschild cohomology. If we suspect that a term in a cochain of the product $\star$ is responsible for the appearance of divergences, applying an iterative procedure of equivalence, we can try to eliminate it, or at least get a lesser divergence, by subtracting at the relevant order a divergent coboundary; we would then get a better theory with a new star-product, "equivalent" to the original one. Furthermore, since in this case we expect to have at each order an infinity of non equivalent star-products, we can try to subtract a cocycle and then pass to a nonequivalent star-product whose lower order cochains are identical to those of the original one. We would then make an analysis of the divergences up to order $\hbar^r$, identify a divergent cocycle, remove it, and continue the procedure (at the same or hopefully a higher order). Along the way one should preserve the usual properties of a quantum field theory (Poincaré covariance, locality, etc.) and the construction of adapted star-products should be done accordingly. The complete implementation of this program should lead to a cohomological approach to renormalization theory. For $\lambda\phi^4_2$-theory a λ-dependent star-product formally equivalent to normal permits to gain one order in perturbation theory.

The Connes–Kreimer rigorous renormalization procedure might fit in this pattern.

Nonlinear group representations and evolution equations

A cohomological (formal), then analytical, study of nonlinear Lie group representations was started by us about 33 years ago (FPS77). Nonlinear representations can be viewed as successive extensions of their linear part S^1 by its (symmetric) tensorial powers $\otimes^n S^1$, $n \geq 2$: first S^1 by $S^1 \otimes S^1$, then the result by $\otimes^3 S^1$ and so on. Cohomology plays a natural role. E.g. it is sufficient to have at least one invertible operator in the representation of the center of the enveloping algebra for the corresponding 1-cohomology to vanish, rendering trivial an associated extension. Spectacular applications to covariant nonlinear partial differential equations, e.g. nonlinear Klein-Gordon and especially the coupled Maxwell-Dirac equations (first-quantized electrodynamics, cf. e.g. FST AMS Memoir 606, 1997). In such equations the nonlinearity appears as coupled to the linear (free) equations, with a coupling constant that plays the role of deformation parameter.

Once the classical covariant field equations are studied enough in details one can think of studying their quantization along the lines of deformation quantization, e.g. by considering the quantized fields as functionals over the initial data of the classical equations.

Composite electrodynamics

Photon (composite QED) and new infinite dimensional algebras.

Flato, M.; Fronsdal, C. *Composite electrodynamics*. J. Geom. Phys. 5 (1988), no. 1, 37–61.

Singleton theory of light, based on a pure gauge coupling of scalar singleton field to electromagnetic current. Like quarks, singletons are essentially unobservable. The field operators are not local observables and therefore need not commute for spacelike separation, hence (like for quarks) generalized statistics. Then a pure gauge coupling generates real interactions – ordinary electrodynamics in AdS space. Singleton field operator $\phi(x) = \sum_j \phi^j(x) a_j$ +h.c. Concept of normal ordering in theory with unconventional statistics is worked out; there is a natural way of including both photon helicities.

Quantization is a study in representation theory of certain infinite-dimensional, nilpotent Lie algebras (generated by the a_j), of which the Heisenberg algebra is the prototype (and included in it for the photon). Compatible with QED.

Singleton-based field theory in AdS

Dis and Racs and around (mostly M. Flato & C. Frønsdal)
Interacting singletons. Lett. Math. Phys. 44 (1998), no. 3, 249–259. (MF, CF)
Singleton fields, in the context of strings and membranes, have been regarded as topological gauge fields that can interact only at the boundary of anti-de Sitter space. At spatial infinity they may have a more physical manifestation as constituents of massless fields in spacetime. The composite character of massless fields is expressed by field-current identities that relate ordinary massless field operators to singleton currents and stress-energy tensors. Naive versions of such identities do not make sense, but when the singletons are described in terms of dipole structures, such constructions are at least formally possible. The new proposal includes and generalizes an early composite version of QED, and includes quantum gravity, super gravity and models of QCD. Unitarity of such theories is conjectural.

Singleton field theory and neutrino oscillations in AdS

Singletons, Physics in AdS Universe and Oscillations of Composite Neutrinos,
Lett. Math. Phys. 48 (1999), no. 1, 109–119. (MF, CF, DS)

The study starts with the kinematical aspects of singletons and massless particles. It extends to the beginning of a field theory of composite elementary particles and its relations with conformal field theory, including very recent developments and speculations about a possible interpretation of neutrino oscillations and CP violation in this context. This framework was developed since the 70's. Based on our deformation philosophy of physical theories, it deals with elementary particles composed of singletons in anti-de Sitter spacetime.

Composite neutrinos' oscillations

Developing a field theory of composite neutrinos (neutrinos composed of singleton pairs with, e.g., three flavors of singletons) it might be possible to correlate oscillations between the three kinds of neutrinos with the AdS_4 description of these 'massless' particles. Of course any reasonable estimate of the value of the cosmological constant rules out a direct connection to the value of experimental parameters like PC violation coupling constants or neutrino masses. PC violation is a feature of composite electrodynamics and any direct observation of singletons, even at infinity, will imply PC violation. If more than one singleton flavor is used, as is appropriate in the context of neutrinos, then PC invariance can be restored in the electromagnetic sector, but in that case, neutrino oscillations will imply PC violation. The structure of Anti de Sitter field theory, especially that of singleton field theory, may provide a natural framework for a description of neutrino oscillations.

Composite leptons and flavor symmetry

The electroweak model is based on “the weak group”, $S_W = SU(2) \times U(1)$, on the Glashow representation of this group, carried by the triplet $(\nu_e, e_L; e_R)$ and by each of the other generations of leptons.

Suppose that:
(a) There are three bosonic singletons $(R^N R^L; R^R) = (R^A)_{A=N,L,R}$ (three “Rac”s) that carry the Glashow representation of S_W;

(b) There are three spinorial singletons $(D_\varepsilon, D_\mu; D_\tau) = (D_\alpha)_{\alpha=\varepsilon,\mu,\tau}$ (three “Di”s). They are insensitive to S_W but transform as a Glashow triplet with respect to another group S_F (the “flavor group”), isomorphic to S_W;

(c) The vector mesons of the standard model are Rac-Rac composites, the leptons are Di-Rac composites, and there is a set of vector mesons that are Di-Di composites and that play exactly the same role for S_F as the weak vector bosons do for S_W: $W^B_A = \bar{R}^B R_A$, $L^A_\beta = R^A D_\beta$, $F^\alpha_\beta = \bar{D}_\beta D^\alpha$.

These are initially massless, massified by interaction with Higgs.

Composite leptons massified

Let us concentrate on the leptons ($A = N, L, R$; $\beta = \varepsilon, \mu, \tau$)

$$(L^A_\beta) = \begin{pmatrix} \nu_e & e_L & e_R \\ \nu_\mu & \mu_L & \mu_R \\ \nu_\tau & \tau_L & \tau_R \end{pmatrix} . \qquad (1)$$

A factorization $L^A_\beta = R^A D_\beta$ is strongly urged upon us by the nature of the phenomenological summary in (1). Fields in the first two columns couple horizontally to make the standard electroweak current, those in the last two pair off to make Dirac mass-terms. Particles in the first two rows combine to make the (neutral) flavor current and couple to the flavor vector mesons. The Higgs fields have a Yukawa coupling to lepton currents, $\mathcal{L}_{\mathrm{Yu}} = -g_{\mathrm{Yu}} \bar{L}^\beta_A L^B_\alpha H^{\alpha A}_{\beta B}$. The electroweak model was constructed with a single generation in mind, hence it assumes a single Higgs doublet. We postulate additional Higgs fields, coupled to leptons in the following way, $\mathcal{L}'_{\mathrm{Yu}} = h_{\mathrm{Yu}} L^A_\alpha L^B_\beta K^{\alpha\beta}_{AB} + \mathrm{h.c.}$. This model predicts 2 new mesons, parallel to the W and Z of the electroweak model (Frønsdal, LMP 2000). But too many free parameters.

Do the same for quarks (and gluons), adding color?

Questions and facts

Even if know "intimate structure" of particles (as composites of quarks etc. or singletons): How, when and where happened "baryogenesis"? [Creation of 'our matter', now 4% of universe mass, vs. 74% 'dark energy' and 22 % 'dark matter'; and matter–antimatter asymetry, Sakharov 1967.] Everything at "big bang"?! [Shrapnel of 'stem cells' of initial singularity?]

Facts: $SO_q(3,2)$ at even root of unity has finite-dimensional UIRs ("compact"?).

Black holes à la 't Hooft: can communicate with them, by interaction at surface.

Noncommutative (quantized) manifolds. E.g. quantum 3- and 4-spheres (Connes with Landi and Dubois-Violette); spectral triples $(\mathcal{A}, \mathcal{H}, D)$).

Connes' Standard Model with neutrino mixing, minimally coupled to gravity. Space-time is Riemannian compact spin 4-manifold (Barrett has Lorentzian version) $\times$ finite (32) NCG. More economical than SUSYSM and predicts Higgs mass at upper limit (SUSYSM gives lower). [Recent with Marcolli and Chamseddine. (Aug. 2009) Marcolli's early universe "Linde" models from NCG, with negative gravity & dark matter models with sterile neutrinos.]

Conjectures and a speculative answer

[Odessa Rabbi anecdote] Space-time could be, at very small distances, not only deformed (to AdS_4 with tiny negative curvature ρ, which does not exclude at cosmological distances to have a positive curvature or cosmological constant, e.g. due to matter) but also "quantized" to some $qAdS_4$. Such $qAdS_4$ could be considered, in a sense to make more precise (e.g. with some measure or trace) as having "finite" (possibly "small") volume (for q even root of unity). At the "border" of these one would have, for most practical purposes at "our" scale, the Minkowski space-time, obtained by $q\rho \to 0$. They could be considered as some "black holes" from which "q-singletons" would emerge, create massless particles that would be massified by interaction with dark matter or dark energy. That could (and should, otherwise there would be manifestations closer to us, that were not observed) occur mostly at or near the "edge" of our universe in accelerated expansion. These "qAdS black holes" ("inside" which one might find compactified extra dimensions) could be a kind of "shrapnel" resulting from the Big Bang (in addition to background radiation) and provide a clue to baryogenesis.

A NCG model for qAdS$_4$

To AdS$_n$, $n \geq 3$, we associate *naturally* a symplectic symmetric space (M, ω, s). The data of any invariant (formal or not) deformation quantization on (M, ω, s) yields canonically universal deformation formulae (procedures associating to a topological algebra $\mathbb{A}$ having a symmetry $\mathcal{G}$ a deformation $\mathbb{A}_\theta$ in same category) for the actions of a non-Abelian solvable Lie group $\mathcal{R}_0$ (one-dimensional extension of the Heisenberg group $\mathcal{H}_n$), given by an oscillatory integral kernel.

Using it we (P.Bieliavsky, LC, DS & YV) define a noncommutative Lorentzian spectral triple $(\mathcal{A}^\infty, \mathcal{H}, D)$ where $\mathcal{A}^\infty := (L^2_{\text{right}}(\mathcal{R}_0))^\infty$ is a NC Fréchet algebra modelled on the space $\mathcal{H}^\infty$ of smooth vectors of the regular representation on the space $\mathcal{H}$ of square integrable functions on $\mathcal{R}_0$, and D a Dirac operator acting as a derivation of the noncommutative bi-module structure, and for all $a \in \mathcal{A}^\infty$, the commutator $[D, a]$ extends to $\mathcal{H}$ as a bounded operator. The underlying commutative limit is endowed with a causal black hole structure (for $n \geq 3$) encoded in the $\mathcal{R}_0$-group action.

Perspectives and cosmological speculations

1. Define within the present Lorentzian context the notion of causality at the operator algebraic level.
2. Representation theory for $SO_q(2, n)$ (e.g. new reps. at root of unity, analogs of singletons, 'square root' of massless reps. of AdS or Poincaré, etc.) Also maybe quantized exceptional groups.
3. Define a kind of trace giving finite "q-volume" for qAdS at even root of unity (possibly in TVS context).
4. Find analogs of all the 'good' properties (e.g. compactness of the resolvent of D) of Connes' spectral triples in compact Riemannian case, possibly with quadruples $(\mathcal{A}, \mathcal{E}, D, \mathcal{G})$ where $\mathcal{A}$ is some topological algebra, $\mathcal{E}$ an appropriate TVS, D some (bounded on $\mathcal{E}$) "Dirac" operator and $\mathcal{G}$ some symmetry.
5. Limit $\rho q \to 0$ ($\rho < 0$ being AdS curvature)?
6. Unify (groupoid?) Poincaré in Minkowski space (possibly modified locally by the presence of matter) with these $SO_q(2, n)$ in the qAdS "black holes".
7. Field theory on such q-deformed spaces, etc.