There is no Theory of Everything inside E_8

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I’m an algebraist.

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There is No "Theory of Everything" Inside E_8

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Abstract: We analyze certain subgroups of real and complex forms of the Lie group E_8, and deduce that any "Theory of Everything" obtained by embedding the gauge groups of gravity and the Standard Model into a real or complex form of E_8 lacks certain representation-theoretic properties required by physical reality. The arguments themselves amount to representation theory of Lie algebras in the spirit of Dynkin's classic papers and are written for mathematicians.

Background

Establish notation

GraviGUT outline

(from Percacci's talk)

1. Identify GraviGUT group E
2. Fit particles into a representation of E
3. Write $\mathcal{G}$-invariant action
4. Explain symmetry breaking
5. Check that new particles not seen at low energies have high mass

Pause

Groups & representations

Example: Nesti-Percacci

- G = Spin(10)
- E = Spin(11,3)
- V = 64 + 64 + 64

	m=1	m=2	m=3
n=1	0	16+16 +16	0
n=2	$\underline{16}$+$\underline{16}$+ $\underline{16}$	0	0
n=3	0	0	0

Example: Lisi (June 2010)

- G = Spin(10)
- $E = E_{8(-24)}$
- V = Lie(E)

	m=1	m=2	m=3
n=1	45+10+ 10+1	16+$\underline{16}$	1
n=2	$\underline{16}$+16	1+10 +1	0
n=3	1	0	0

Our paper

No Theory of Everything inside E_8

ToE inside E_8

Our task: fit all fields of the Standard Model and gravity tightly in E_8, with only a handful of new particles

ToE inside E_8

- G = your favorite compact connected real group
- E = real form of E_8
- V = Lie(E)
- Concoct a map G x Spin(3,1) into E with finite kernel, "so that V is a good representation of G"

Easy observation

You <u>can't</u> get 3 generations of fermions.

- 3 generations of fermions implies $\dim V_{1,2}$ is $\geq 3 \cdot 16 = 48$
- $\dim(2 \otimes 1 \otimes V_{2,1} + 1 \otimes 2 \otimes V_{1,2}) \geq 192$
- But: $Spin(3,1) = SL(2,\mathbf{C})$ has center ± 1, and -1 acts on this subspace as -1. By E. Cartan (or Serre), the -1-eigenspace has $\dim \leq 128$.

ToE inside E_8

new, easier

some

Our task: fit ~~all~~ fields of the Standard Model and gravity tightly in E_8, with only a handful of new particles

Theorem (Distler-G)

- Take E = $E_{8(-24)}$, $E_{8(8)}$, or $R_{\mathbf{C}/\mathbf{R}}(E_{8,\mathbf{C}})$
- If $V_{m,n} = 0$ for all (m,n) with $m \geq 4$ or $n \geq 4$, then $V_{1,2}$ is *not* a complex representation of G.

Definition of "complex"

- Let *G* be a real group, and fix a representation of *G* x **C** on some complex vector space *A*. Three possibilities:
- *A* is defined over **R**: *A* is <u>real</u>
- *A*+*A* is defined over **R** but *A* is not: *A* is <u>pseudoreal</u> ("quaternionic")
- *A*+*A* is not defined over **R**: *A* is <u>complex</u>

If $V_{m,n} = 0$ for all (m,n) with $m \geq 4$ or $n \geq 4$, then $V_{1,2}$ is not a complex representation of G.

Why is that bad?

- You want G_{SM} to embed in G.
- Standard Model requires $V_{1,2}$ to be a complex representation of G_{SM}.
- If $V_{1,2}$ is not a complex representation, then you get a profusion of extra particles and new theoretical challenges.

Theorem (Distler-G)

- Take $E = E_{8(-24)}$, $E_{8(8)}$, or $R_{\mathbf{C}/\mathbf{R}}(E_{8,\mathbf{C}})$
- If $V_{m,n} = 0$ for all (m,n) with $m \geq 4$ or $n \geq 4$, then $V_{1,2}$ is *not* a complex representation of *G*.
- Note: does not depend on choice of compact group *G*

How to prove it?

- Complexify to get $SL_{2,\mathbf{C}} \times SL_{2,\mathbf{C}}$ embedded in $E \times \mathbf{C}$ = complex E_8
- $V_{m,n} = 0$ for $m \geq 4$ or $n \geq 4$ implies both copies of $SL_{2,\mathbf{C}}$ have Dynkin index 1 or 2
- Both copies have the same Dynkin index

Dynkin index 2 case

- centralizer of one $SL_{2,\mathbf{C}}$ is $Spin_{13,\mathbf{C}}$
- $Spin_{13,\mathbf{C}}$ has two index 2 $SL_{2,\mathbf{C}}$'s
- One gives $(SL_{2,\mathbf{C}} \times SL_{2,\mathbf{C}})/(-1,-1)$ in $E_{8,\mathbf{C}}$ (ignore it); other is $SL_{2,\mathbf{C}} \times SL_{2,\mathbf{C}}$
- centralizer of full $SL_{2,\mathbf{C}} \times SL_{2,\mathbf{C}}$ is $Sp_{4,\mathbf{C}} \times Sp_{4,\mathbf{C}}$

How to determine the real forms?

- G is contained in G_{max}, the maximal compact subgroup of $Z_E(\mathrm{Spin}(3,1))$
- If $V_{1,2}$ is not complex for G_{max}, then it is not complex for G
- We know $Z_E(\mathrm{Spin}(3,1)) \times \mathbf{C}$; need to determine the real form (hence G_{max}) and restrict $V_{1,2}$ to G_{max}

How to determine the real forms?

- Two tools: (a) we know how the Galois action permutes the summands of *V* as a representation of Spin(3,1) x Z_E(Spin(3,1))
- (b) use the Killing form on *E* to control the real form of Z_E(Spin(3,1))

Case: Dynkin index 1

- $Z_E(\mathrm{Spin}(3,1))$ is $\mathrm{Spin}(12\text{-}a,a)$ for some $0 \leq a \leq 6$
- $V_{1,2}=S_+$, $V_{2,1}=S_-$ interchanged, so $a=1,3,5$
- If $a=5$, by rank $E=E_{8(8)}$
- If a=1,3, -1 in Spin(3,1) centralizes so(12,4) in Lie(E), so $E=E_{8(-24)}$

Table of possibilities

E	G_{max} (contains G)	$V_{2,3}$	$V_{1,2}$
$E_{8(8)}$	Spin(5) x Spin(7)	0	$4 \otimes 8$
	Spin(5)	4	4+16
$E_{8(-24)}$	Spin(11)	0	32
	Spin(9) x SU(2)	0	$16 \otimes 2$
$R_{\mathbf{C}/\mathbf{R}}(E_{8,\mathbf{C}})$	E_7 (simply conn.)	0	56
	Spin(12)	0	32+32'
	Spin(13)	0	64
	Spin(5) x Spin(5)	$(4 \otimes 1) + (1 \otimes 4)$	$(4 \otimes 5) + (5 \otimes 4)$
	SU(2) x Spin(9)	$2 \otimes 1$	$(2 \otimes 9) + (2 \otimes 16)$

These representations are all non-complex

QED

Elevator summary

If you try to fit gravity and the Standard Model -- even just some of the fermions -- into E_8,

- you cannot get the known 3 generations of fermions, and
- you will find a profusion of new particles.