Structure and representations of exceptional groups
July 4–July 9, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	**Buffet Dinner**, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	**Breakfast**
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–10:00	Bent Orsted, "Borel-de Siebenthal discrete series for exceptional groups"
10:00–10:30	**Coffee Break**, 2nd floor lounge, Corbett Hall
10:30–11:30	Jeff Adams, "Elliptic elements of the Weyl group of E_8"
11:30–13:00	**Lunch**
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:15–14:45	**Coffee Break**, 2nd floor lounge, Corbett Hall
14:45–15:45	Roberto Percacci, "Elements of a GraviGUT"
16:00–17:00	Gordan Savin, "Classifying discrete series representations of G_2 using minimal representations"
17:30–19:30	**Dinner**
19:30–21:00	Garrett Lisi,"Unification."

Tuesday

7:00–9:00	**Breakfast**
9:00–10:00	Hadi Salmasian, "Unitary representations of supergroups and the method of orbits"
10:00–10:30	**Coffee Break**, 2nd floor lounge, Corbett Hall
10:30–11:30	Karl-Hermann Neeb, "Semibounded representations of automorphism groups of Banach symmetric spaces"
11:30–13:30	**Lunch**
13:30–14:30	Bertram Kostant, "Experimental evidence for the occurrence of E_8 in nature and the radii of the Gossett circles"
14:30–15:00	**Coffee Break**, 2nd floor lounge, Corbett Hall
15:00–16:00	Birne Binegar, "W-graphs, nilpotent orbits, and primitive ideals"
16:15-17:15	Daniel Sternheimer, "Some instances of the reasonable effectiveness (and limitations) of symmetries and deformations in fundamental physics"
17:30–19:30	**Dinner**
19:30–21:00	Garrett Lisi, "A physicist's topology—a group effort"

Wednesday

7:00–9:00	**Breakfast**
9:00–10:00	Todor Milev, "Computing regular subalgebras of simple Lie algebras"
10:00-10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Skip Garibaldi, "There is no (interesting) 'Theory of Everything' inside E_8"
11:30–13:30	**Lunch**
13:31–17:29	Free Afternoon
17:30–19:30	**Dinner**

Thursday

7:00–9:00	**Breakfast**
9:00–10:00	Dan Barbasch, "The spherical unitary dual for the quasisplit group of type E_6"
10:00–10:30	**Coffee Break**, 2nd floor lounge, Corbett Hall
10:30–11:30	Dan Ciubotaru, "The Dirac operator for graded affine Hecke algebras"
11:30–13:30	**Lunch**
13:30–14:30	Michael Eastwood, "Representations from contact geometry"
14:30–15:00	**Coffee Break**, 2nd floor lounge, Corbett Hall
15:00–16:00	Alan Huckleberry, "The role of Kobayashi hyperbolicity in the study of flag domains"
16:15-17:15	Toshiyuki Kobayashi, title to be announced
17:30–19:30	**Dinner**
19:30–21:00	Garrett Lisi, "Massive speculation—trialities and tribulations"

Friday

7:00–9:00	**Breakfast**
9:00–10:00	Open discussion time
10:00–10:30	**Coffee Break**, 2nd floor lounge
10:30-11:30	Tearful farewells and Exchange of Facebook friend requests
11:30–13:30	**Lunch**
Checkout by 12 noon	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to check out by noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Jeff Adams** (University of Maryland)
Title: *Elliptic elements of the Weyl group of* E_8
Abstract: An element of a Weyl group W is elliptic if it has no fixed points in the reflection representation. An example is the Coxeter element, studied extensively by Kostant. Elliptic elements were classified by Carter in 1972, who discovered a relation with nilpotent conjugacy classes in the corresponding semisimple group G. Lusztig has recently studied this from a new point of view. Each elliptic conjugacy class in W is naturally a semisimple conjugacy class in G. I'll consider the elementary question: what is the map from elliptic conjugacy classes in G to W-orbits in T? I'll focus on the example of E_8.

Speaker: **Dan Barbasch** (Cornell University)
Title: *The spherical unitary dual for the quasisplit group of type* E_6
Abstract:

Speaker: **Birne Binegar** (Oklahoma State University)
Title: *W-graphs, nilpotent orbits, and primitive ideals*
Abstract:

Speaker: **Dan Ciubotaru** (University of Utah)
Title: *The Dirac operator for graded affine Hecke algebras*
Abstract: This is joint work with D. Barbasch and P. Trapa. We define an analogue of the Dirac operator for graded affine Hecke algebras of p-adic groups, and establish a version of Parthasarathy's Dirac operator inequality. We then prove a version of Vogan's conjecture for Dirac cohomology. The formulation of the conjecture depends on a uniform parametrization of spin representations of Weyl groups. We apply our results to prove new results about unitary representations of graded affine Hecke algebra.

Speaker: **Michael Eastwood** (Australian National University (Canberra))
Title: *Representations from contact geometry*
Abstract: Apart from $SL(2)$, each simple Lie group is the symmetry group of a contact manifold equipped with some extra geometric structure. This includes the exceptional groups. This fact can be used to give a geometric construction of the finite-dimensional representations of the simple groups, including the exceptional groups. I shall sketch how this works.

Speaker: **Skip Garibaldi** (Emory University)
Title: *There is no (interesting) 'Theory of Everything' inside* E_8*"*
Abstract: In joint work with Jacques Distler, we prove that the real forms of E8 and the realification of the complex form do not have subgroups with certain properties. The physical interpretation of this result is that the "Exceptionally Simple Theory of Everything" conflicts with well-accepted representation-theoretic properties of the Standard Model. This interpretation is robust, in that our result also shows that a whole family of related Theories of Everything also conflict with these same properties of the Standard Model.

Speaker: **Alan Huckleberry** (Ruhr-Universität Bochum)
Title: *The role of Kobayashi hyperbolicity in the study of flag manifolds*
Abstract: Open orbits D of simple real forms G_0 in flag manifolds $Z = G/Q$ of their complexifications G

are considered. For any choice K_0 of a maximal compact subgroup of G_0, the minimal K_0-orbit in the *flag domain* D is a compact complex manfold referred to as the base cycle C_0 with respect to the choice of K_0. It can be regarded as a point in the Chow (or Barlet space) $\mathcal{C}_q(Z)$ of all cycles of the same dimension q. It is known that $\mathcal{C}_q(Z)$ is smooth at C_0 and therefore it makes sense to consider the irreducible compoment of $\mathcal{C}_q(Z)$ which contains C_0 and the open subset $\mathcal{C}_q(D)$ of those cycles which are contained in D. The complex geometry of $\mathcal{C}_q(D)$ is the theme of the talk. For example, using analytic properties of the intersection of the cycles with certain special Schubert varieties, the Kobayashi hyperbolicity of $\mathcal{C}_q(D)$ is proved. This sheds light on the complex geometry of D, e.g., leading to a precise desciption of its group of holomorphic automorphisms. It should be emphasized that for fixed G_0 the cycle space $\mathcal{C}_q(D)$ varies tremendously as D and Z vary, making it a rich source of interesting complex varieties with the potential of realilzing nontrivial G_0-representations in a holomorphic context. A preprint (arXiv:1003:5974) is available.

Speaker: **Toshiyuki Kobayashi** (Kyoto University)
Title: *TBA*
Abstract:

Speaker: **Bertram Kostant** (MIT)
Title: *Experimental evidence for the occurrence of E_8 in nature and the radii of the Gossett circles*
Abstract: A recent experimental discovery involving spin structure of electrons in a cold one-dimensional magnet points to a validation of a (1989) Zamolodchikov model involving the exceptional Lie group E_8. The model predicts 8 particles and predicts the ratio of their masses. The conjectures have now been validated experimentally, at least for the first five masses. The Zamolodchikov model was extended in 1990 to a Kateev-Zamolodchikov model involving E_6 and E_7 as well.

In a seemingly unrelated matter, the vertices of the 8-dimensional Gosset polytope identify with the 240 roots of E_8. Under the famous two-dimensional (Peter McMullen) projection of the polytope, the image of the vertices are arranged in 8 concentric circles, hereafter referred to as the Gosset circles. The McMullen projection generalizes to any complex simple Lie algebra (in particular not restricted to *A-D-E* types) whose rank is greater than 1. The Gosset circles generalize as well. Applying results in my AJM 1959 paper, I found some time ago a very easily defined operator A on a Cartan subalgebra, the ratios of whose eigenvalues are exactly the the ratios of squares of the radii r_i of the generalized Gosset circles.

The two matters considered above relate to one another in that the ratio of the masses in the E_6, E_7, and E_8 Kateev-Zamolodchikov models are exactly equal to the ratios of the radii of the corresponding generalized Gosset circles.

Speaker: **Garrett Lisi**
Title: *Group-theoretic models in gravity, the standard model, and old-and-new ideas about unification*
Abstract: Titles of the individual talks are "Unification;" "A physicist's topology—a group effort;" and "Massive speculation—trialities and tribulations."

Speaker: **Todor Milev** (Jacobs Universität Bremen)
Title: *Computing regular subalgebras of simple Lie algebras*
Abstract: Let $\mathfrak{g}$ be a finite dimensional simple Lie algebra and $\mathfrak{h}$ be a fixed Cartan subalgebra. Let $\mathfrak{l}$ be a subalgebra containing $\mathfrak{h}$ (non-zero nilradicals allowed) and let $\mathfrak{k} \supset \mathfrak{h}$ be the reductive part of $\mathfrak{l}$.

A Fernando-Kac subalgebra of $\mathfrak{g}$, associated to an infinite dimensional $\mathfrak{g}$-module M, is defined as the set $\mathfrak{g}[M]$ of locally finitely acting elements of $\mathfrak{g}$. A subalgebra $\mathfrak{l}$ for which there exists an irreducible module M with $\mathfrak{g}[M] = \mathfrak{l}$ is called a Fernando-Kac subalgebra of $\mathfrak{g}$. A Fernando-Kac subalgebra is of finite type if there exists a module as above for which the Jordan-Hölder $\mathfrak{k}$-multiplicities of all simple $\mathfrak{k}$-modules are finite. A root-system criterion describing all $\mathfrak{l} \supset \mathfrak{h}$ that are Fernando-Kac of finite type was conjectured by my advisor I. Penkov based on his joint work with V. Serganova and G. Zuckerman and a paper of S. Fernando. My Ph. D. thesis proves this criterion for all finite dimensional simple Lie algebras except E_8

(the case $sl(n)$ was already proved in [PSZ]). The proofs for exceptional Lie algebras F_4, E_6, E_7 involved a computer computation.

A regular subalgebra of a simple Lie algebra can be defined as a subalgebra that can be written in the form $[\mathfrak{k}, \mathfrak{k}]$. Regular subalgebras were classified in Dynkin's fundamental paper "Semisimple Lie algebras of semisimple Lie algebras" (there are 75 proper isomorphism classes in E_8). Dynkin's classification automatically applies to root reductive subalgebras. In order to enumerate all possible nilradicals up to isomorphism one should first compute the $\mathfrak{k}$-module decomposition of $\mathfrak{g}$.

All algorithms described in the talk (and more) have been implemented in my C++ "vector partition" program. Tables with all possibilities (up to isomorphism) for $\mathfrak{k}$ and the $\mathfrak{k}$-decomposition of E_8 (and any other simple Lie algebra of rank ≤ 8) are generated by the web server of my program `http://vector-partition.jacobs-university.de/cgi-bin/vector_partition_linux_cgi?rootSAs`. The table for E_8 is precomputed at `http://vector-partition.jacobs-university.de/tmp/E8/rootHtml.html` .

The C++ project is relatively large (at the time of writing approximately 30 000 lines of code without the web server and the user interface). The program is open source and you can download (and modify, copy, etc.) the source from the public source code repository at `http://vectorpartition.sourceforge.net`; however, due to the under-staffing of my (one-man) team its setup is not user-friendly yet.

Speaker: **Karl-Hermann Neeb** (Universität Erlangen)
Title: *Semibounded representations of automorphism groups of Banach symmetric spaces*
Abstract: In this talk we discuss separable unitary representations of the automorphism group of a Hilbert hermitian symmetric space and its central extensions. We assume that the representations are semibounded in the sense that, some element of the Lie algebra has a neighborhood on which the operators of the derived representation are uniformly bounded above. The methods to analyze such representations come from three sources: (1) Pickrell's regularity results on separable representations of orthogonal and unitary groups, (2) some recent insights in the structure of invariant open convex cones in orthogonal and unitary Lie algebras, and (3) procedures to realize representations in Hilbert spaces of holomorphic sections of complex Hilbert bundles over the symmetric space.

Speaker: **Bent Ørsted** (Aarhus University)
Title: *Borel-de Siebenthal discrete series for exceptional groups*
Abstract: For a semisimple Lie group admitting discrete series representations, it remains an interesting problem to find explicit realizations. In this lecture, based on joint work with Joe Wolf, we consider the Borel-de Siebenthal discrete series, giving details about the geometry of the corresponding coadjoint orbits. In particular for some exceptional groups we find models allowing continuation in the discrete series parameter.

Speaker: **Alessandra Pantano** (University of California Riverside)
Title: *Unitarizable principal series of double covers of split exceptional groups*
Abstract:

Speaker: **Roberto Percacci** (International School for Advanced Studies, Trieste)
Title: *Elements of a GraviGUT*
Abstract: A GraviGUT would be a theory where gravity is unified with the other interactions in a way that directly generalizes what is done in the grand unified theories of particle physics. I will describe what one would need to do to construct such a theory, and the steps that have been successfully carried through so far. I will concentrate on the case when the unifying group is $SO(3, 11)$.

Speaker: **Hadi Salmasian** (University of Ottawa)
Title: *Unitary representations of supergroups and the method of orbits*
Abstract: The main goal of this talk is to show that ideas of the orbit method can be applied to describe

unitary representations of Lie supergroups. We define Lie supergroups and their unitary representations (in a global sense) and prove that for nilpotent Lie supergroups there exists a bijective correspondence between irreducible unitary representations and nonnegative coadjoint orbits. A simple branching rule for irreducible unitary representations to the even part follows.

Speaker: **Gordan Savin** (University of Utah)
Title: *Classifying discrete series representations of G_2 using minimal representations*
Abstract:

Speaker: **Daniel Sternheimer** (Keio University and Université de Bourgogne)
Title: *Some instances of the unreasonable effectiveness (and limitations) of symmetries and deformations in fundamental physics*
Abstract: A survey of some applications of group theory and deformation theory (including quantization) in mathematical physics. Rotation and discrete groups in molecular physics ("dynamical" symmetry breaking in crystals, Racah-Flato-Kibler); chains of groups and symmetry breaking. "Classification Lie groups" ("internal symmetries") in particle physics. Space-time symmetries, relations with internal symmetries. Deformations of symmetries. Deformation quantization, quantum groups and quantized spaces. Field theories and evolution equations (from the point of view of nonlinear Lie group representations). Connections with some cosmology, including especially quantized anti-de Sitter groups and spaces. Prospects for future developments between mathematics and physics.