Evolution of cooperation and network structure

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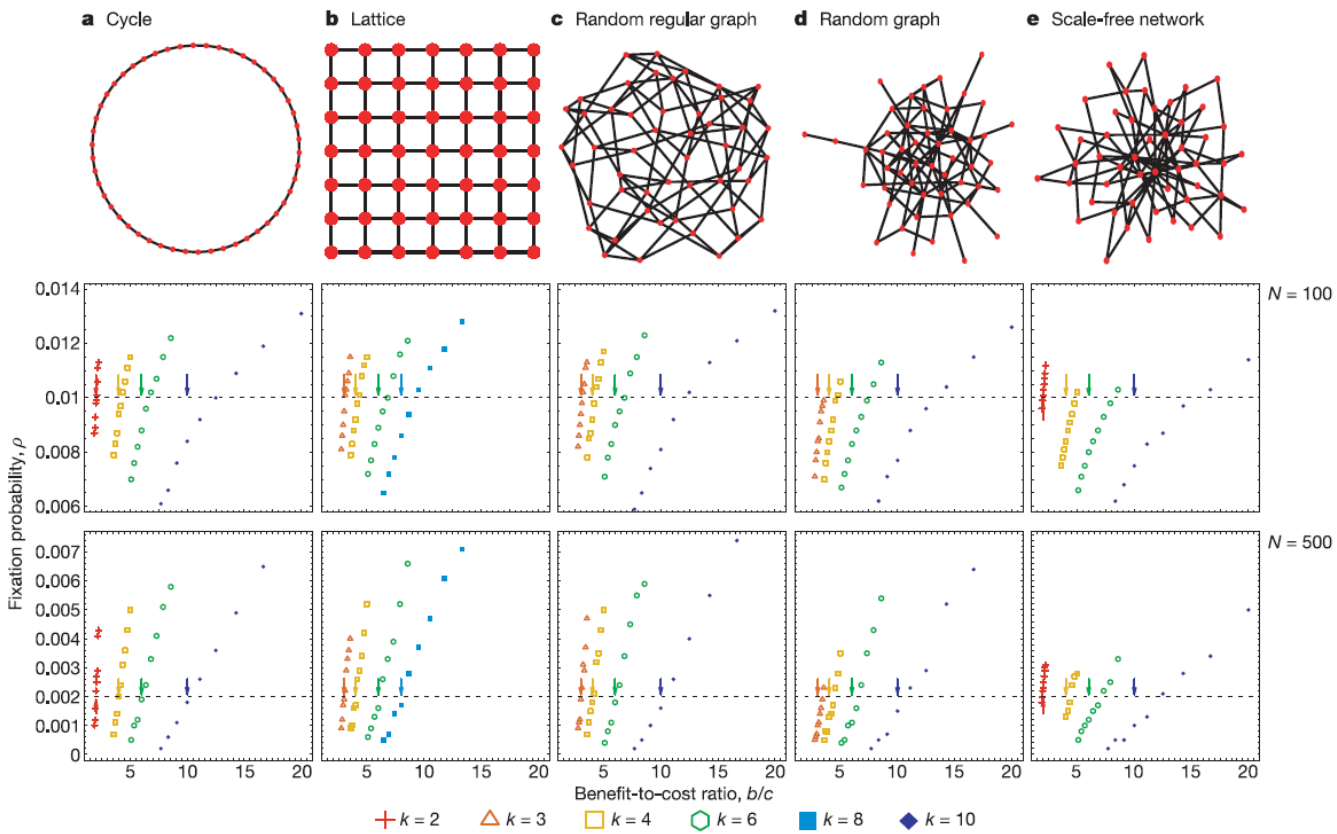
- 1. Introduction & Background
- 2. Games from regular to random networks
- 3. Effect of selection intensity
- 4. Effect of fixed contribution
- 5. Games in scale-free network

1. Introduction

- A simple rule for the evolution of cooperation on graphs and social networks (Ohtsuki et al. 2006)

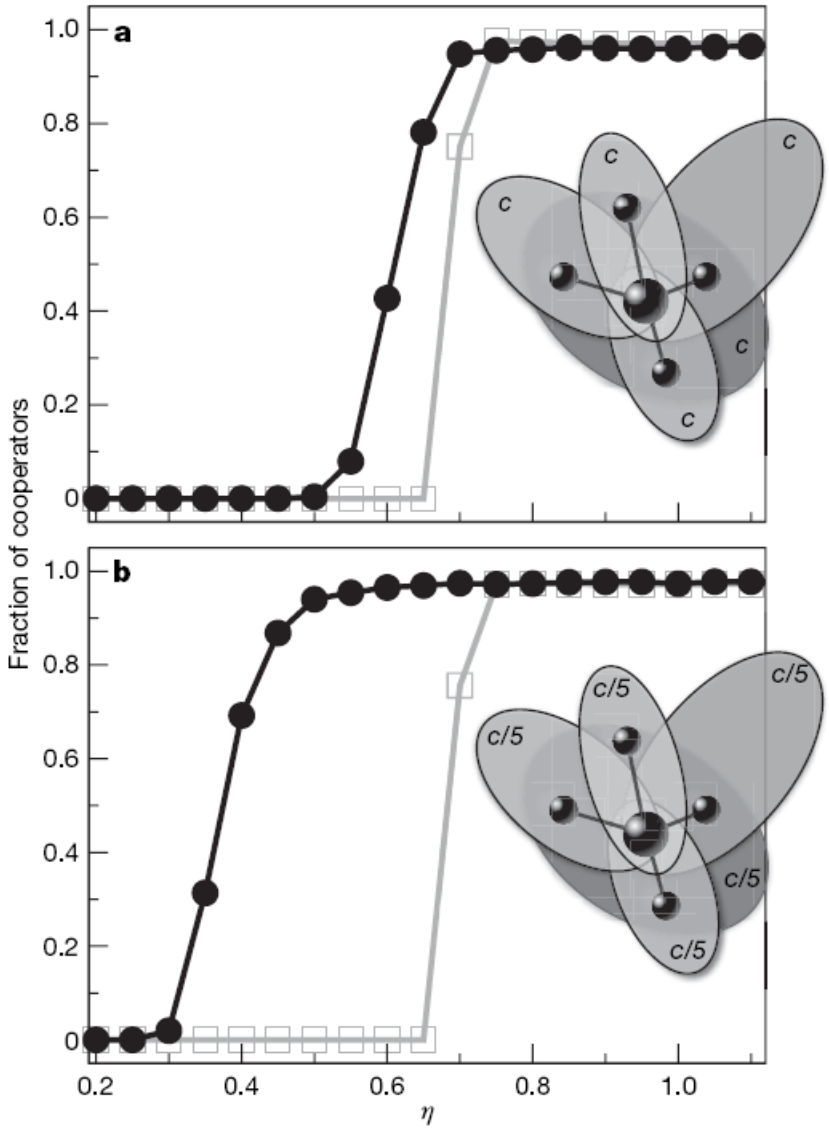
$$r > k$$



- Social diversity promotes the emergence of cooperation in public goods games

  (Santos et al. 2008)

  Fixed Contribution

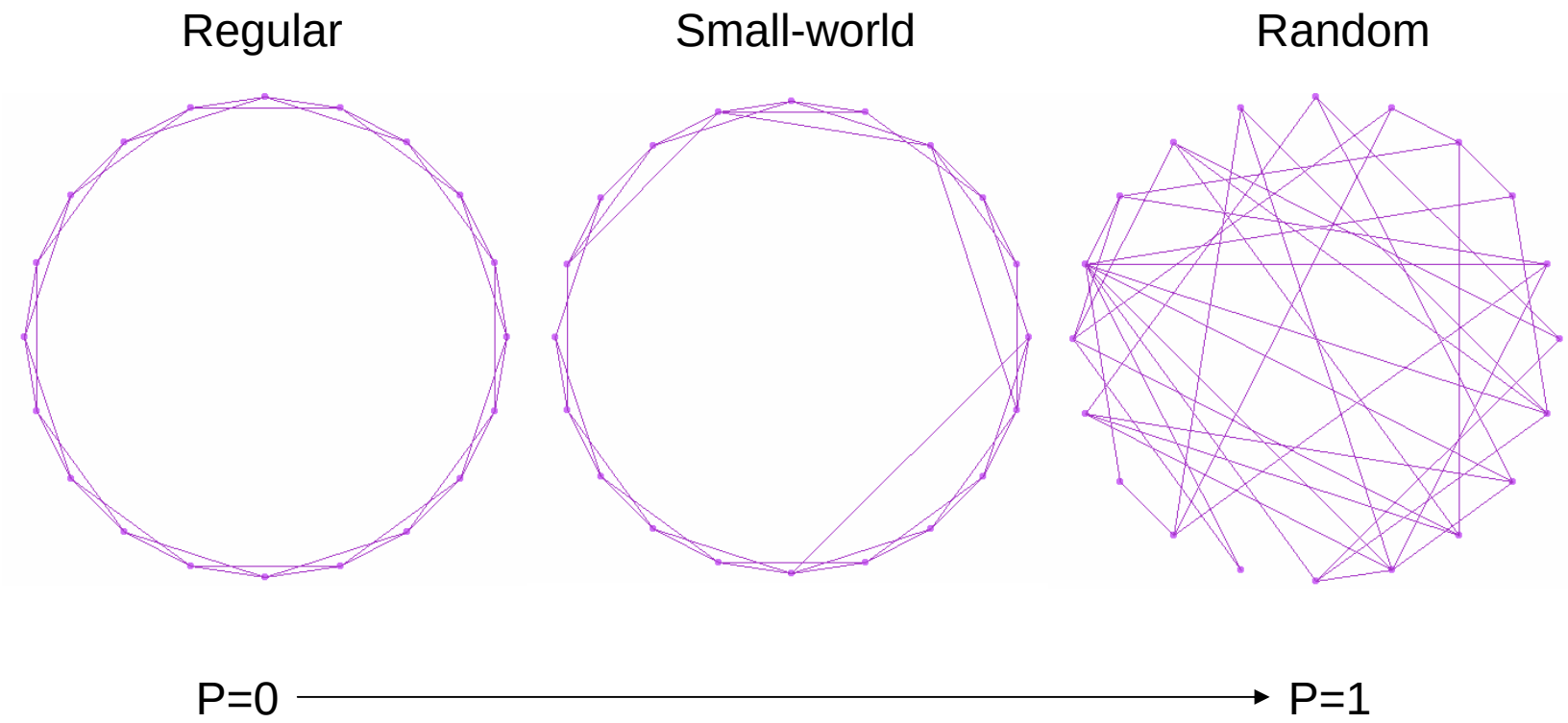


Questions:

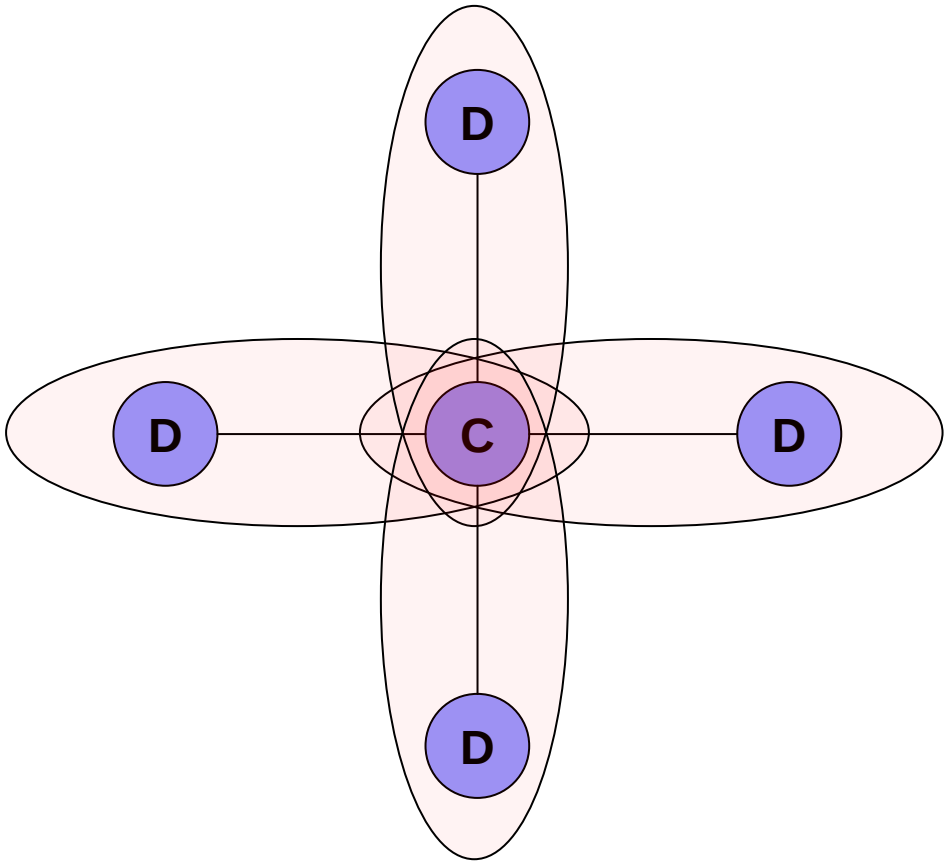
- How does the network structure work on the evolution of cooperation?
- Will different network structures work on the same way?
- Is there the best network structure for increasing cooperation?

2. Results from regular to random

- Small-World network structure (Watts, 1998)



- Games: Prisoner's Dilemma
- Payoff: Two strategy: C, D. All defectors (D) do nothing. C in PD, -1 → r, to each neighbor



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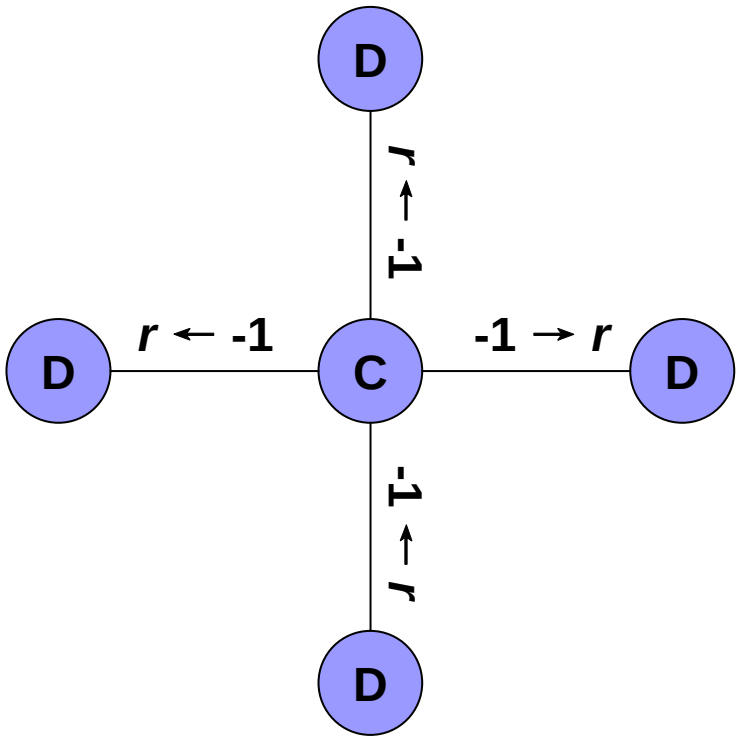

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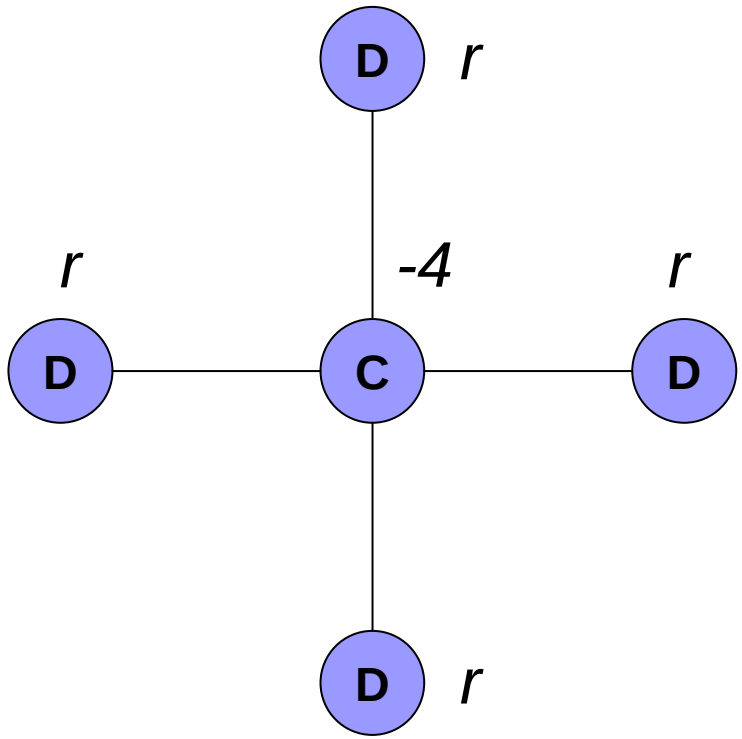
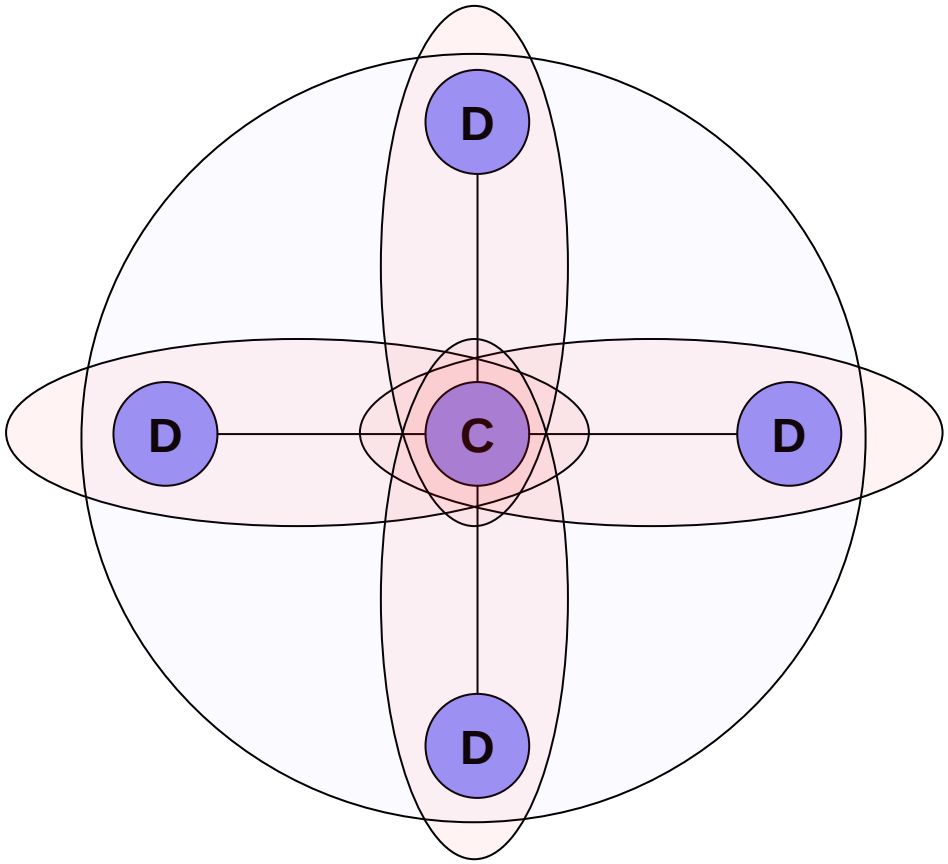


- Games: Public Goods
- Payoff: C in PG, -1 → r (r/(k+1) for each one), each neighborhood centred in vertex.



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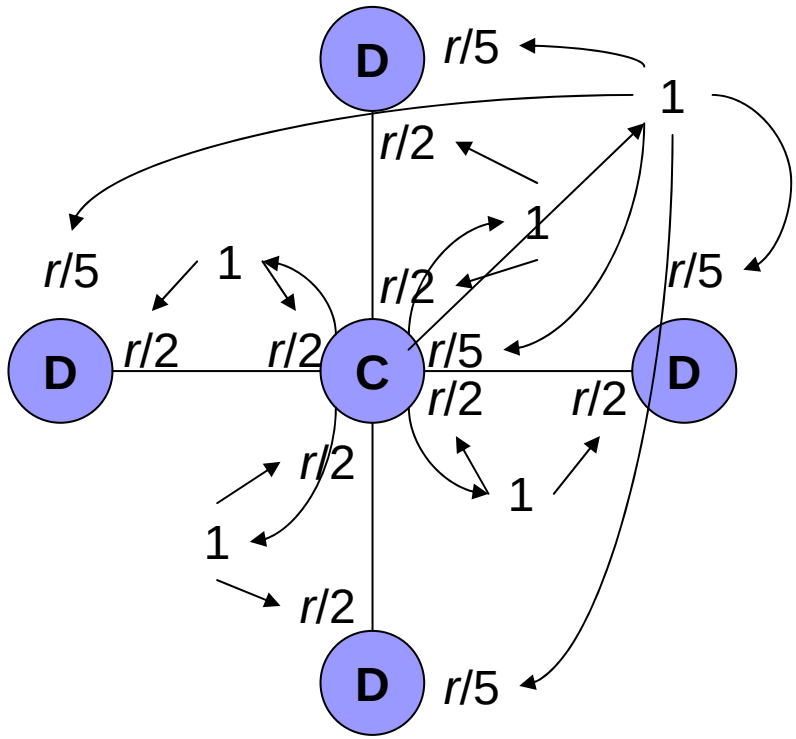

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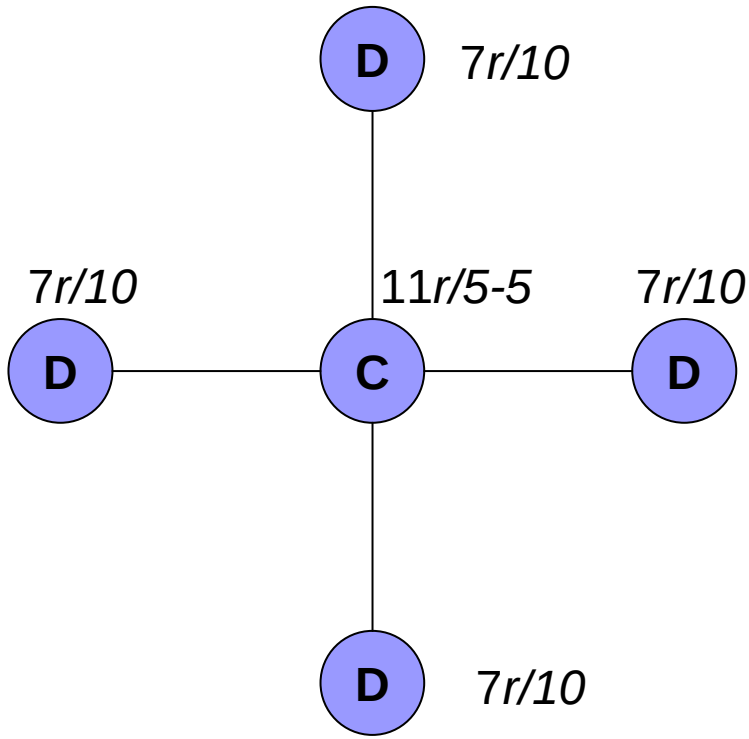
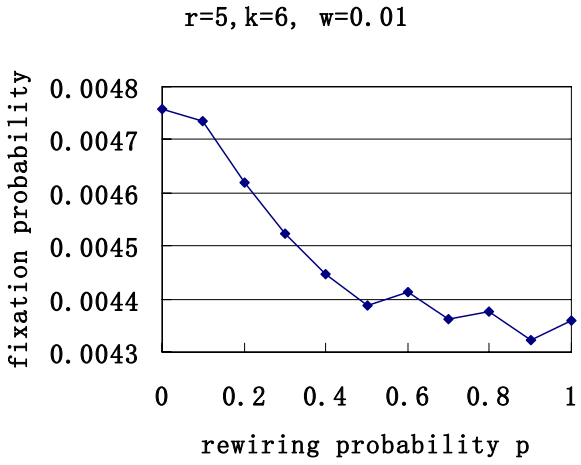


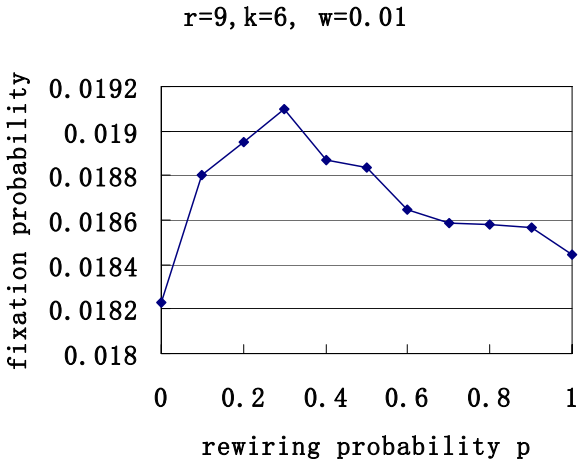
- Fitness=(1-w)+w×Payoff
- Evolution model: Death-Birth process
- Fixation Probability of a C invade an all D's network.
- N=100, times=10^6, 10 different graphs for each parameter (*p*, *k*, *r*)

Results under the weak selection

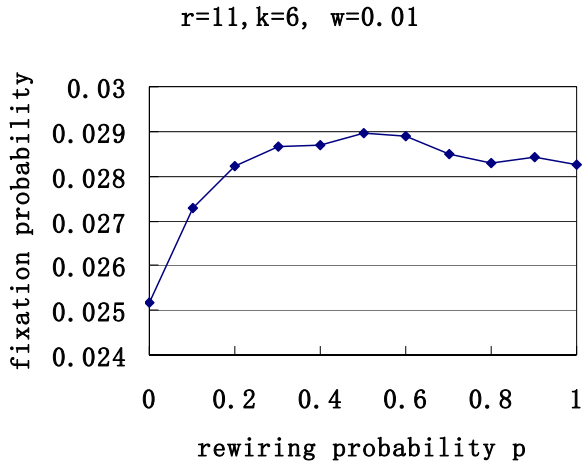
PD game *r*<*k*



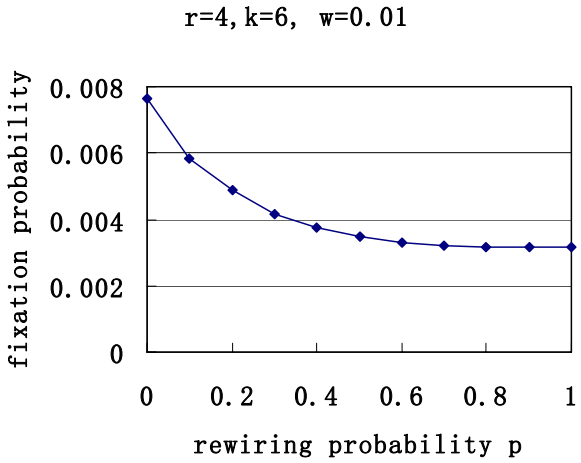
PD game *r*>*k*



PD game *r*>>*k*



PG game

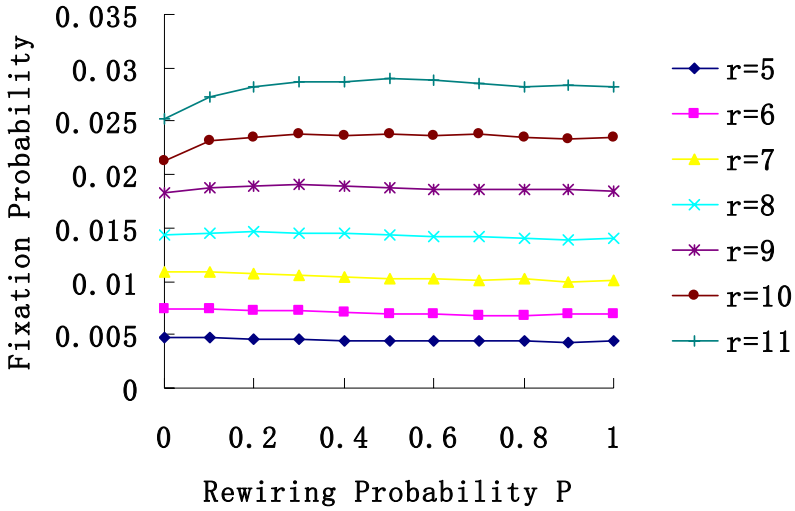


PD game : 1. decrease by *p*
2. increase first, then decrease
3. increase to a level, then stable

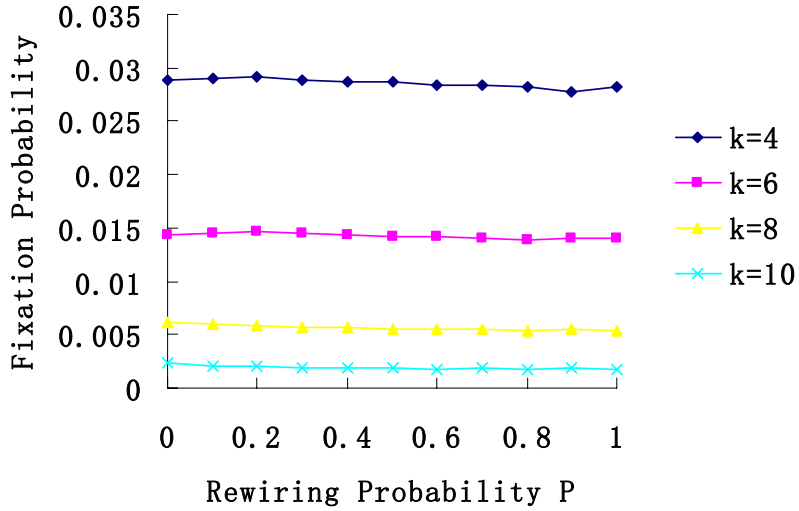
PG game: decrease by *p*

PD games for different *r* and *k*

k=6, w=0.01



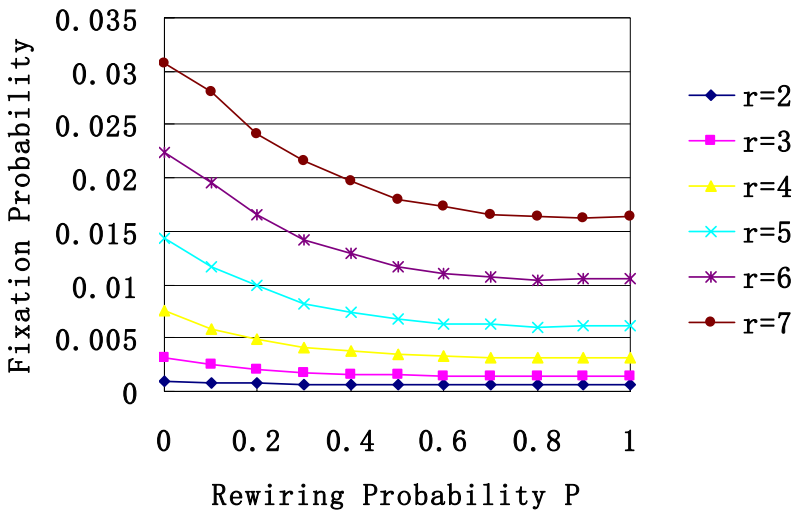
r=8, w=0.01



1. The change of fixation probability with the increase of *p* is small for given *r* and *k*.
2. This implies that for PD game, the effect of network structure on cooperation should be small under the weak selection.

PG games for different *r* and *k*

k=6, w=0.01



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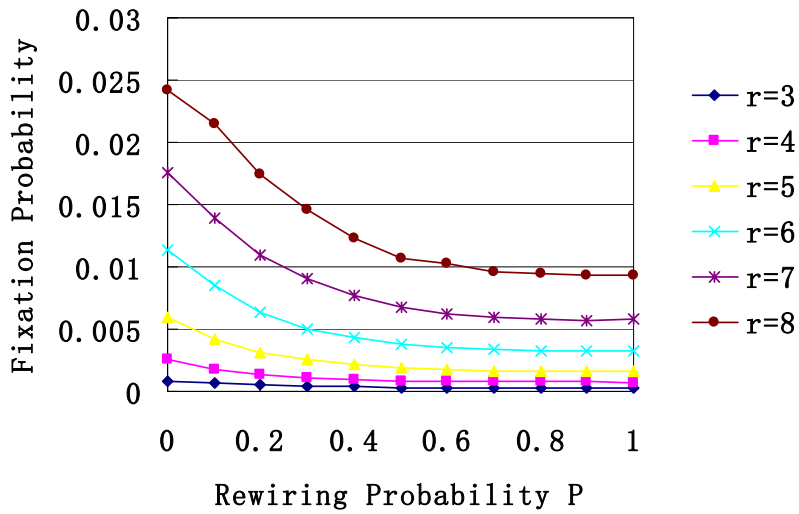
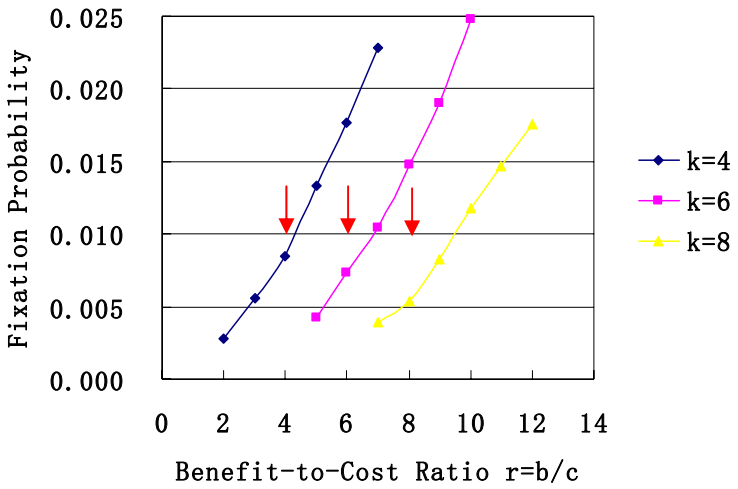


1. The fixation probability decreases with the increase of *p*.
2. Why?

3. Effect of selection intensity

- Fixation probability under the weak selection:

w=0.01, regular, PD game



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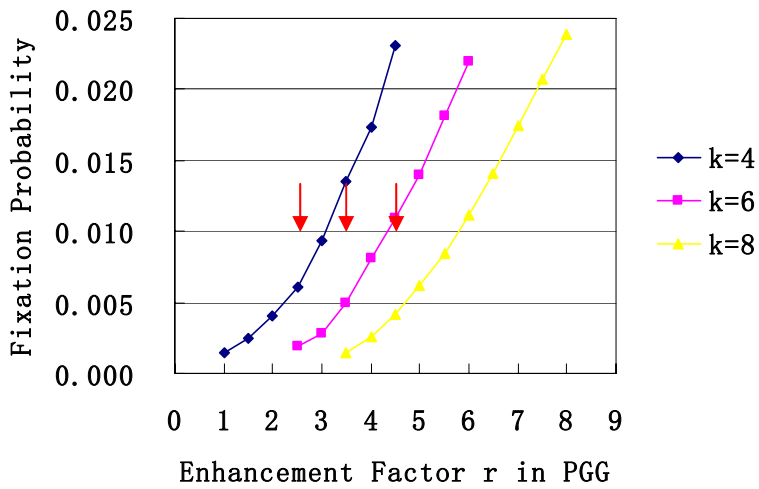
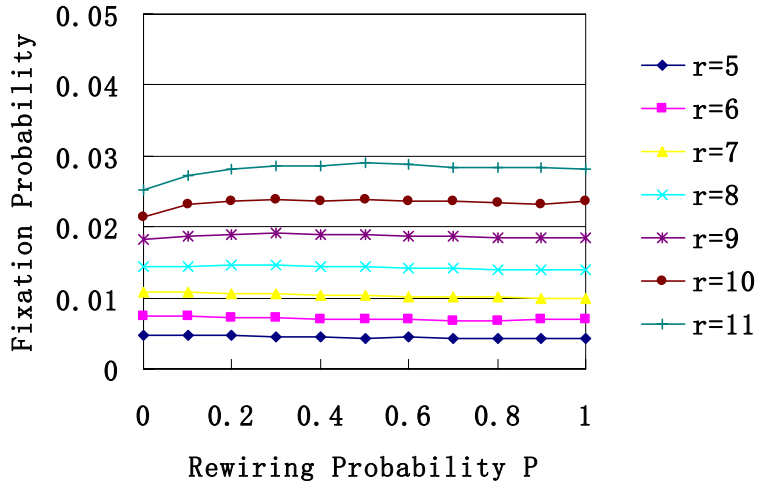


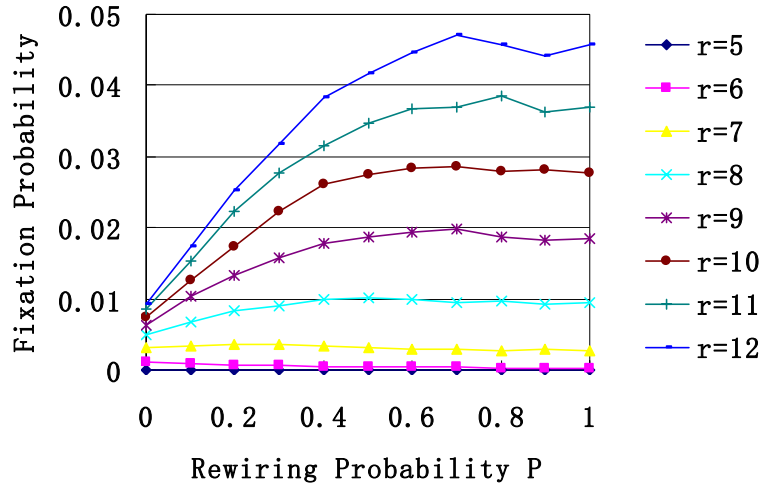
*The Benefit-to-Cost Ratio $r>k$ rule in PGG means $r>(k+1)/2$

PD games for different *w* values

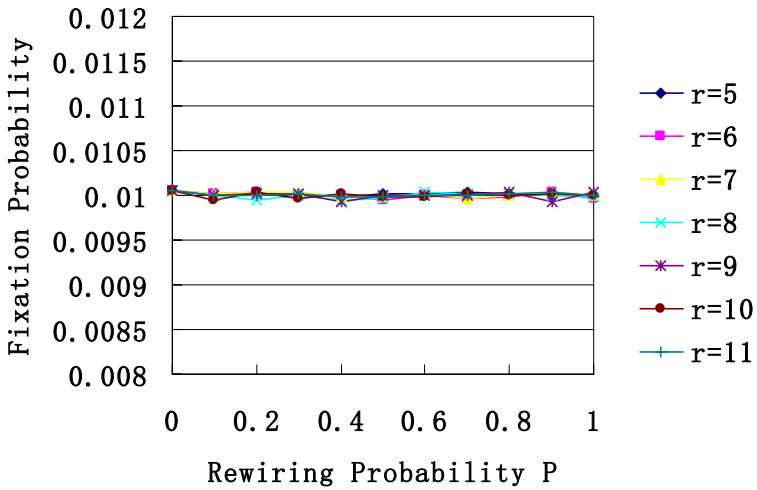
k=6, *w*=0.01



k=6, *w*=0.1



k=6, *w*=0

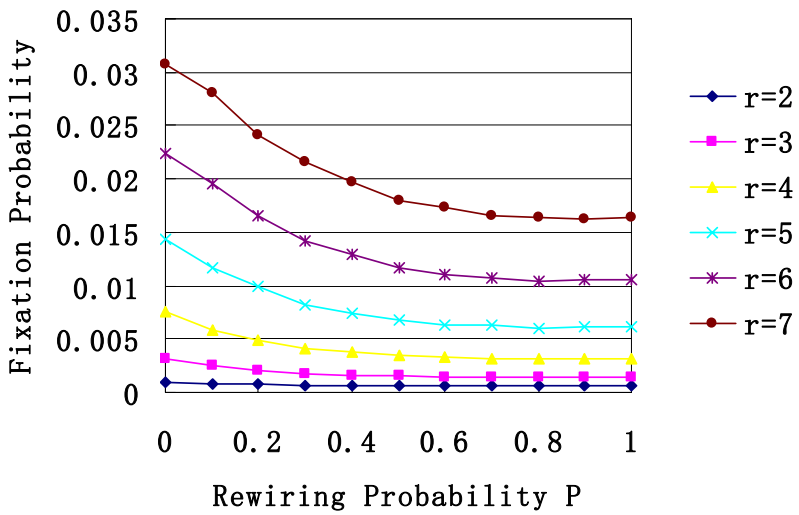


Small-World and random network promote cooperation more efficiently than regular under the strong selection

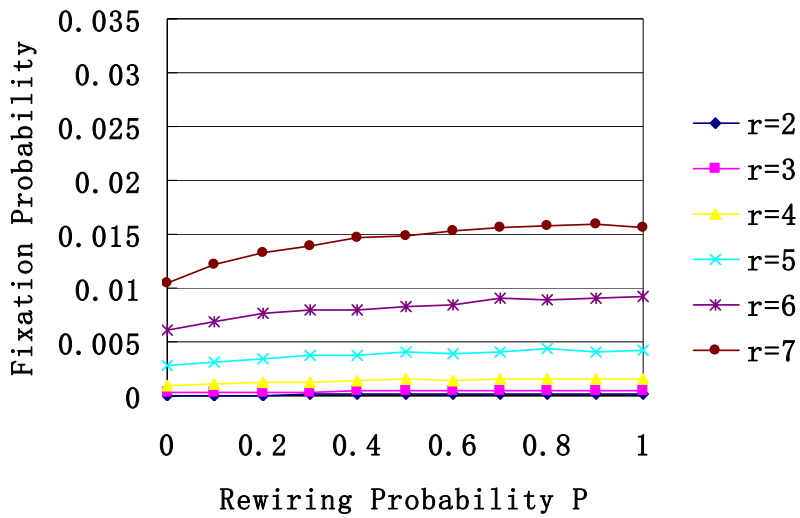
No selection

PG games for different *w* values

k=6, *w*=0.01



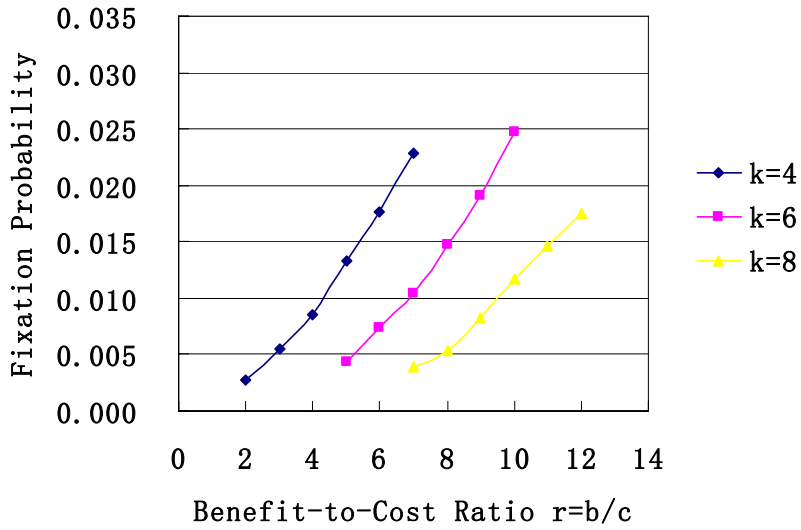
k=6, *w*=0.1



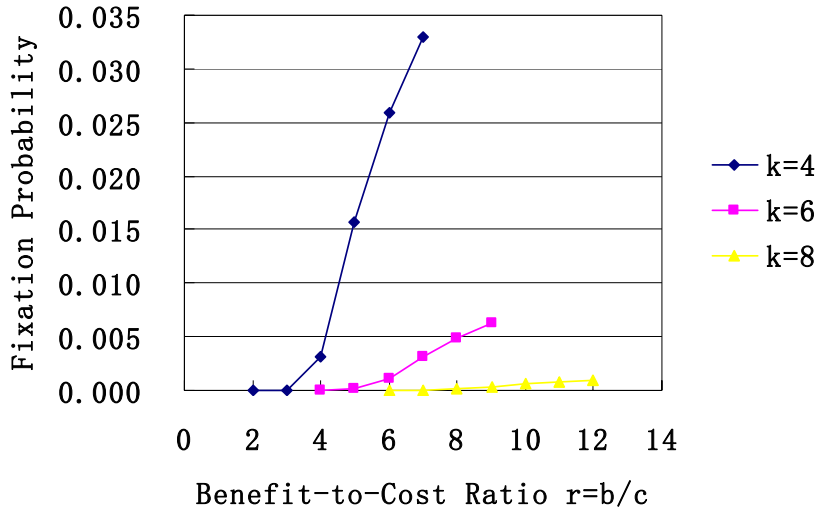
Small-World and random networks promote cooperation more efficiently than regular under the strong selection.

Effect of *w* and *r*>*k* rule

w=0.01, regular, PD game



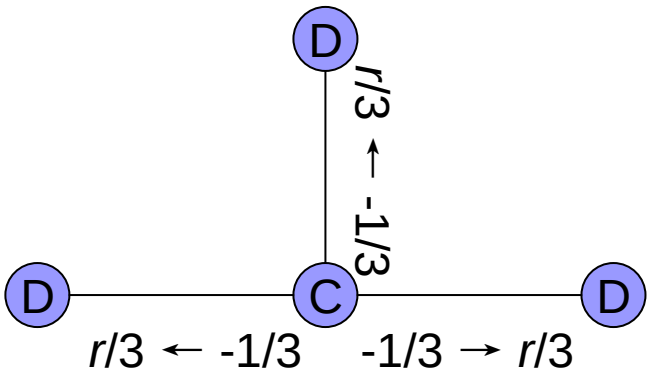
w=0.1, regular, PD game



Strong selection will result in the rapid increase of the fixation probability with the increase of r if *r*>>*k*, and the increase of the fixation probability with the increase of *r* will be slow if *r* is only little larger than *k*.

4. Fixed Contribution

In each game, the cost of C is 1/*k* in PD, 1/(*k*+1) in PG, and the benefit to his neighbor is *r*/*k* in PD, *r*/(*k*+1) to each neighbourhood in PG.



PD game

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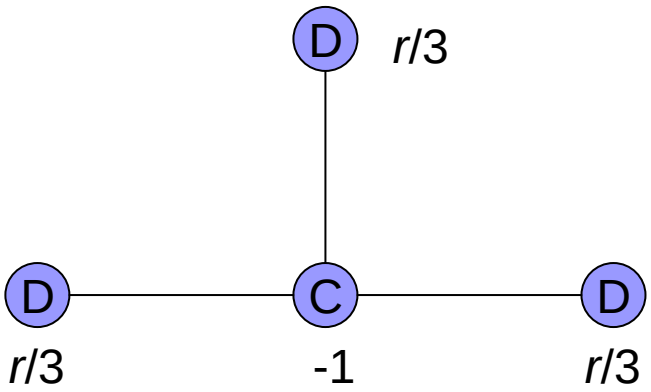
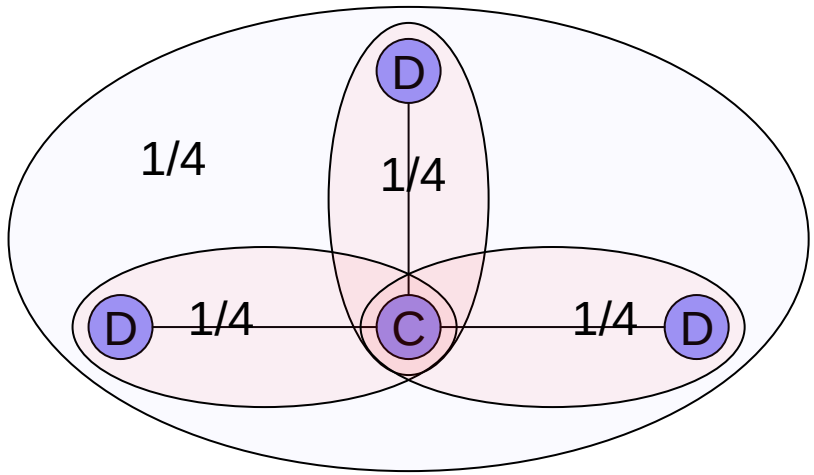


PD game



PG game

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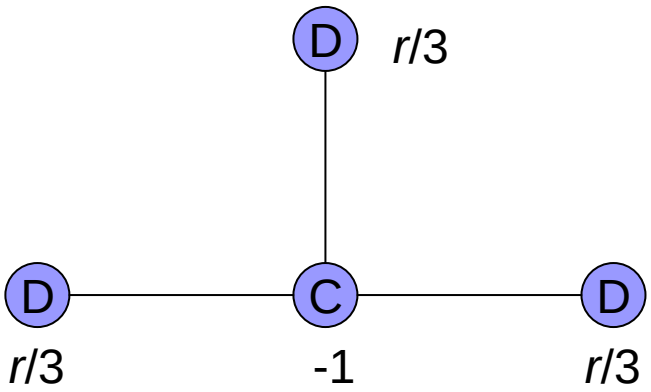
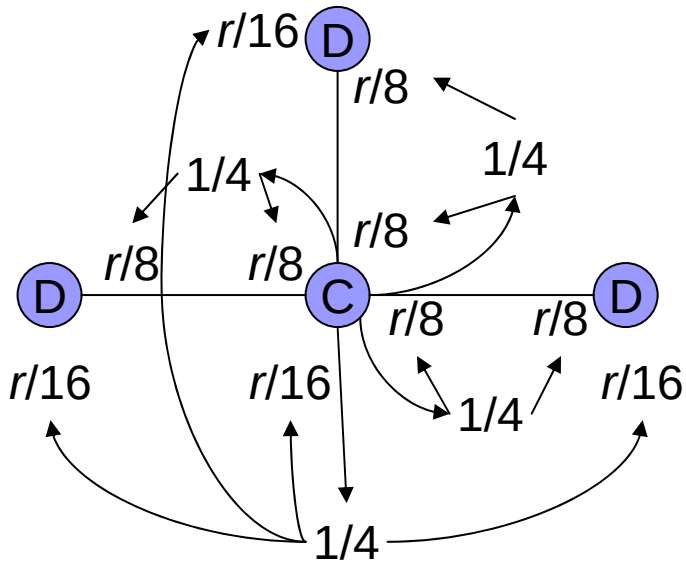


PD game



PG game

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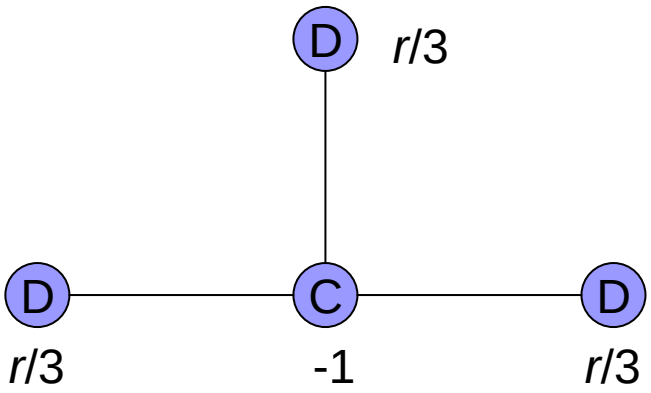
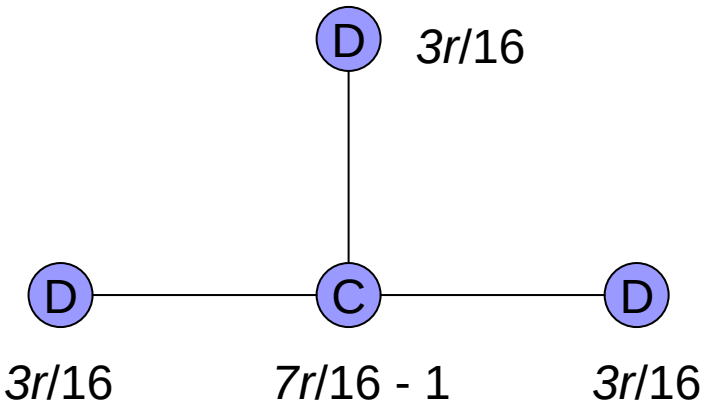


PD game

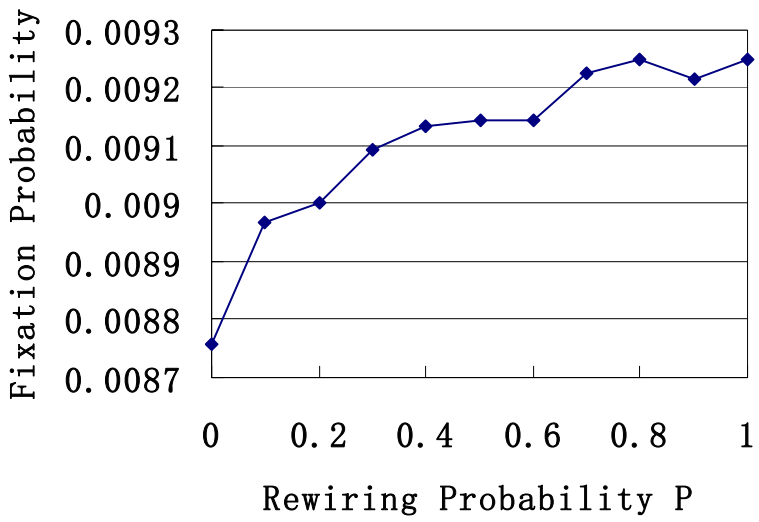


PG game

result

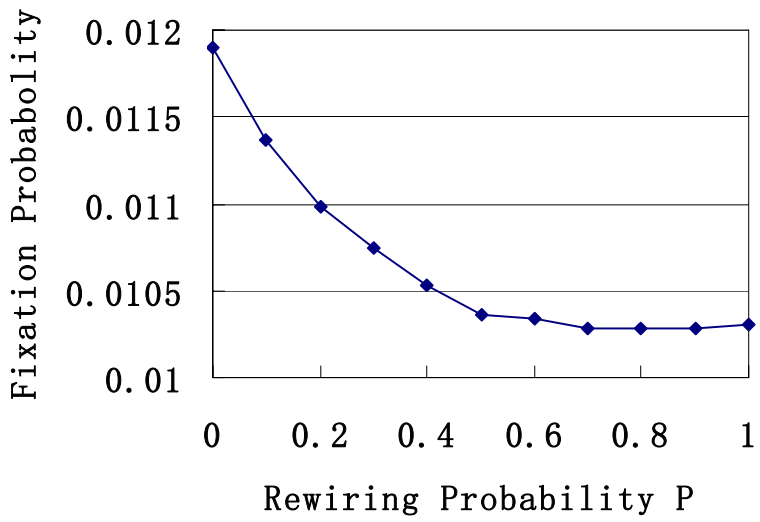
In each game, the cost of C is 1/*k* in PD, 1/(*k*+1) in PG, and the benefit to his neighbor is *r*/*k* in PD, *r*/(*k*+1) to each neighbourhood in PG.

PD game k=6, r=5, w=0.01



FP increases in PD

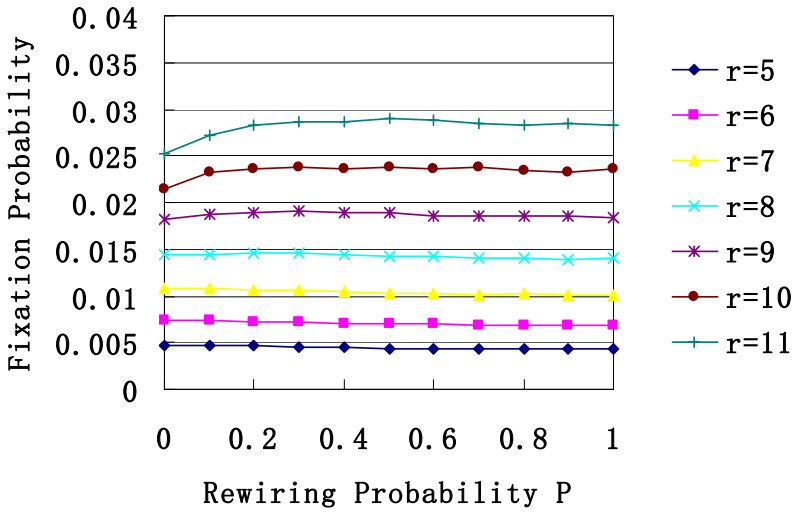
PG game k=6, r=6, w=0.01



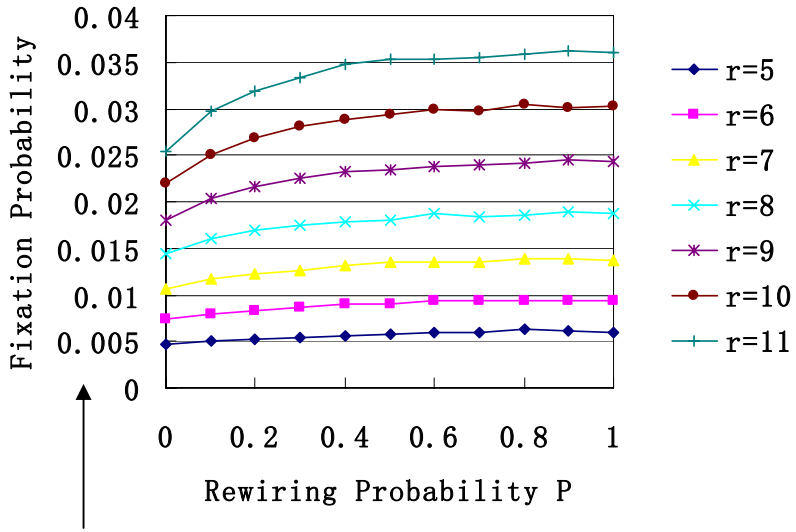
FP decreases in PG

PD Game

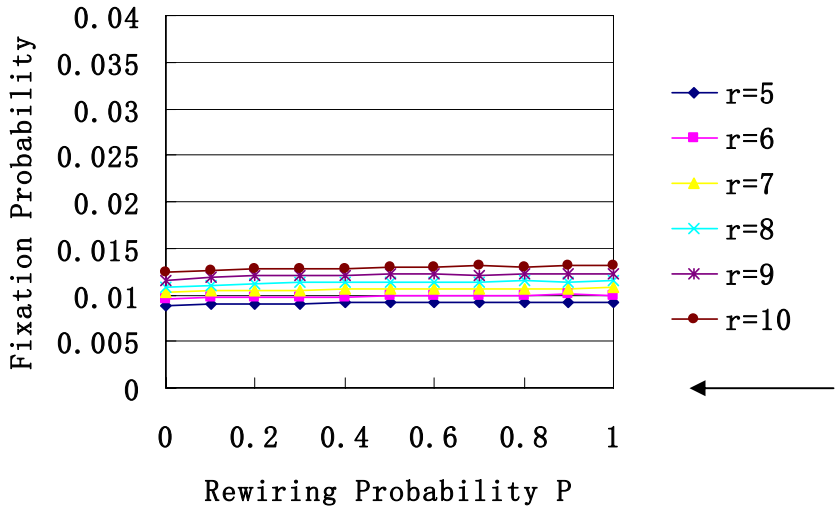
k=6, w=0.01, Control



k=6, w=0.06, Fixed Contribution



k=6, w=0.01, Fixed Contribution

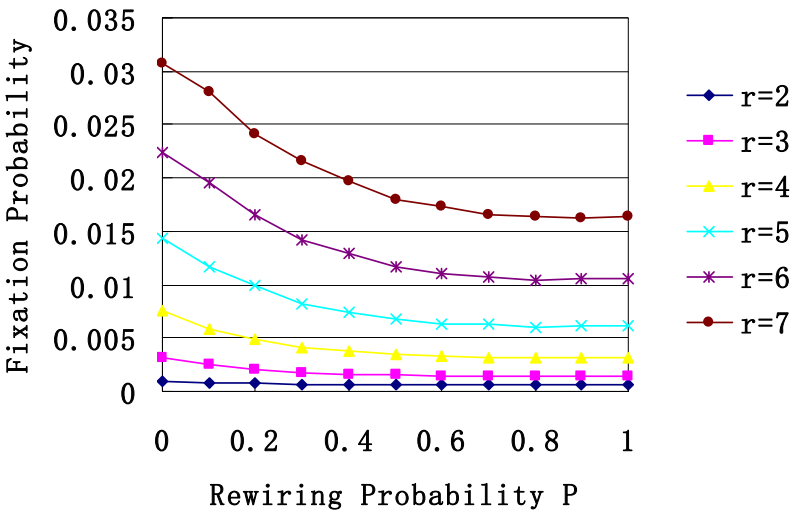


The fixed contribution promotes the cooperation in Small-World and random network for PD game.

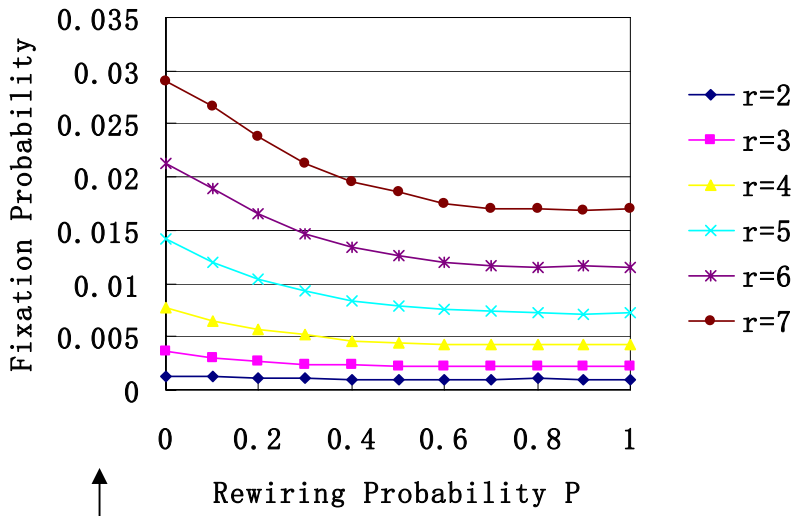
The effect of fixed contribution is similar to that of *w*, i.e., the *k* times of *w* is equivalent to the *k* times of the fixed contribution (originally, *FC*=1).

PG game

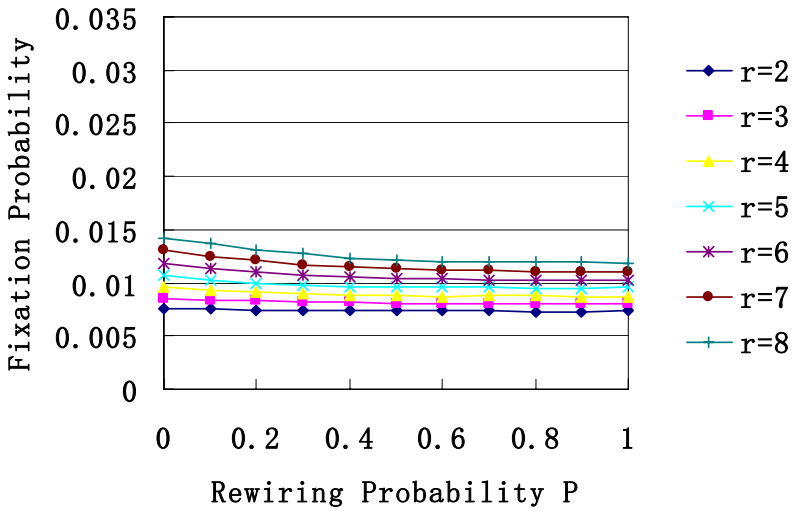
k=6, *w*=0.01, Control



k=6, *w*=0.06, Fixed Contribution



k=6, *w*=0.01, Fixed Contribution



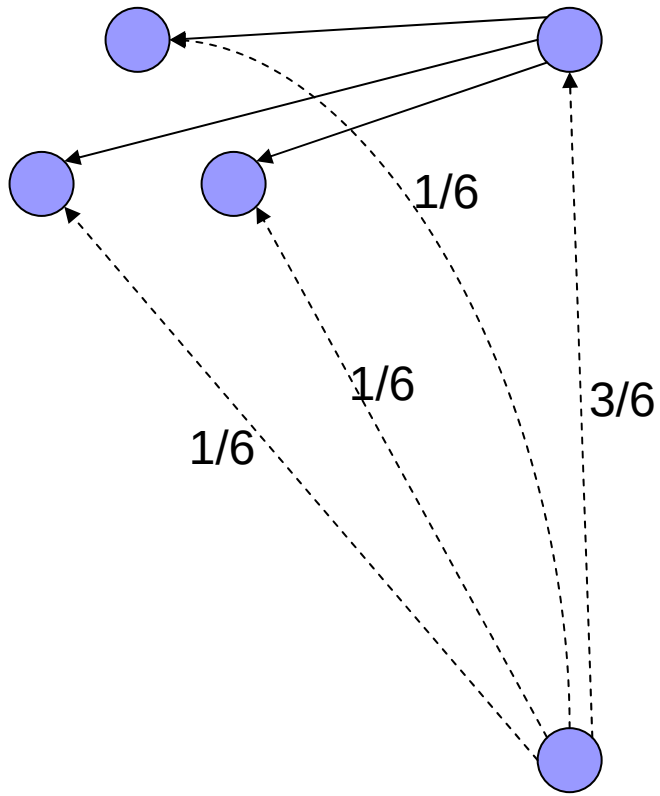
Under the weak selection, fixed contribution can't promote cooperation in PG game. This is different from the result of strong selection (Santos, 2008)

Conclusion of Small-World

- 1.Under weak selection network structure will not make any difference to promote cooperation in PD game, but regular does best in PG game.
- 2.Under strong selection the promotion of cooperation is sensitive to network structure.
- 3.Fixed contribution works in Small-world and random in PD, but doesn’t work in PG under weak selection.

5. Games in Scale-Free Network

- 1. Scale-invariant distribution
- 2. Preferential attachment



Scale-Free k=6

5. Games in Scale-Free Network

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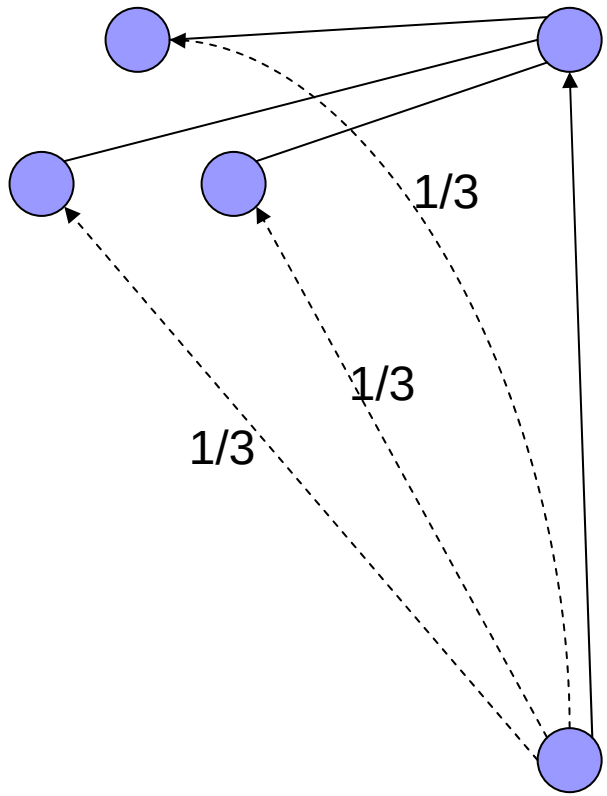

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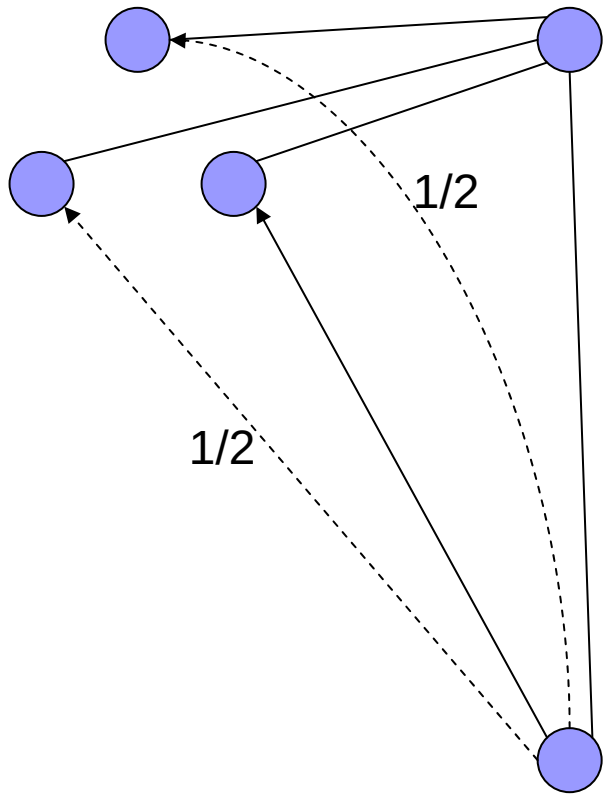

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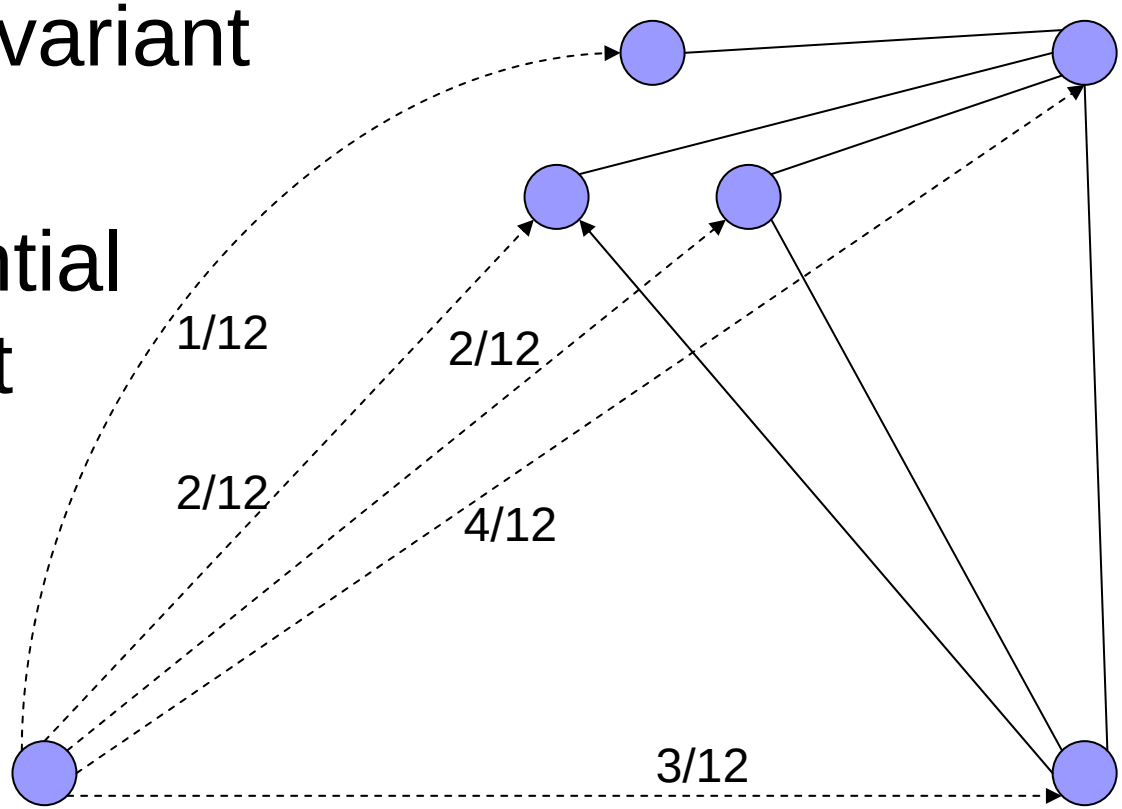

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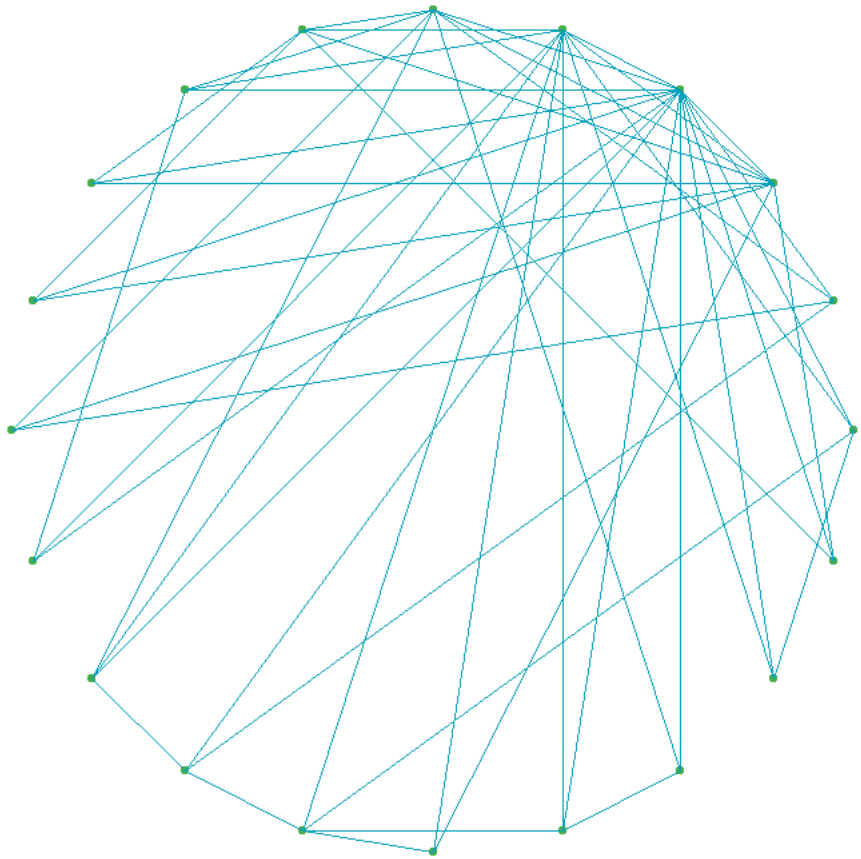
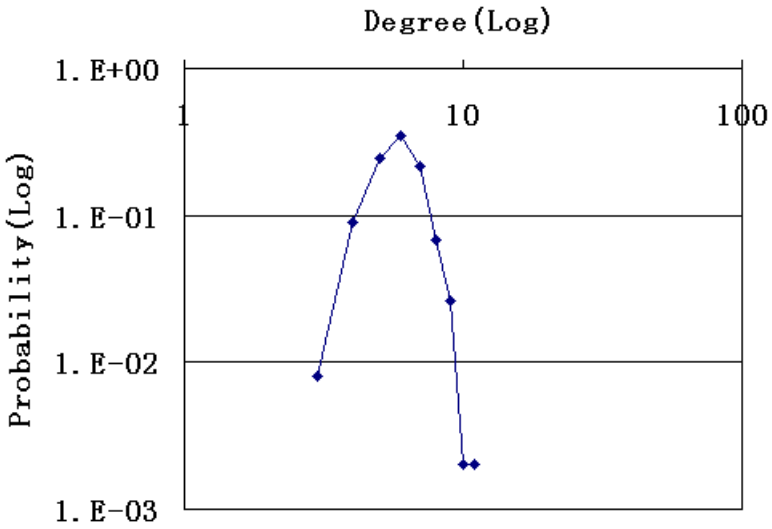
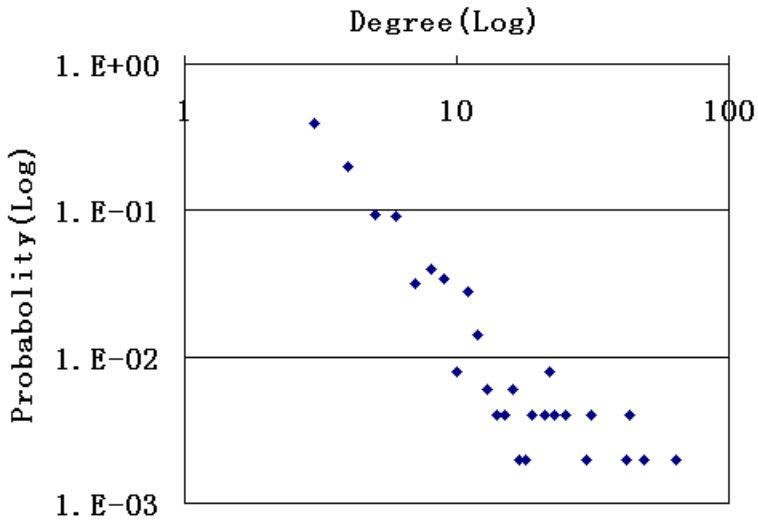


Scale-Free *N*=20 *k*=6

- SF is different from SW in the probability distribution of degree



Small-World, k=6, N=500, p=0.3

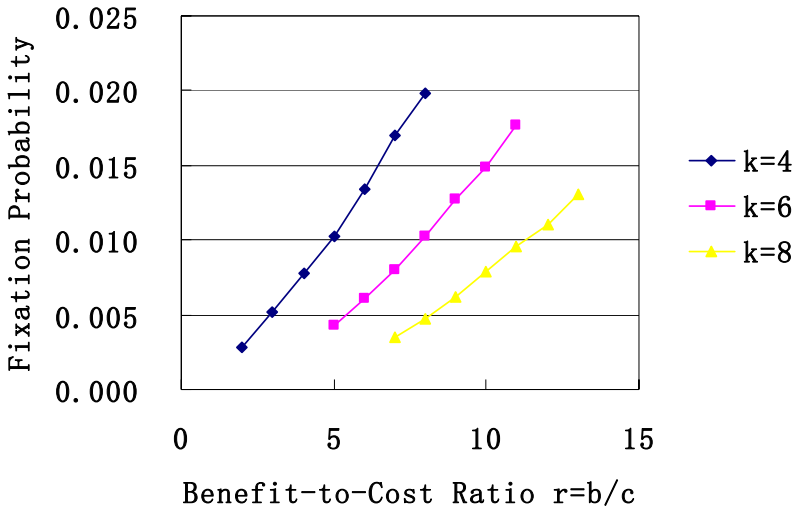


Scale-Free, k=6, N=500

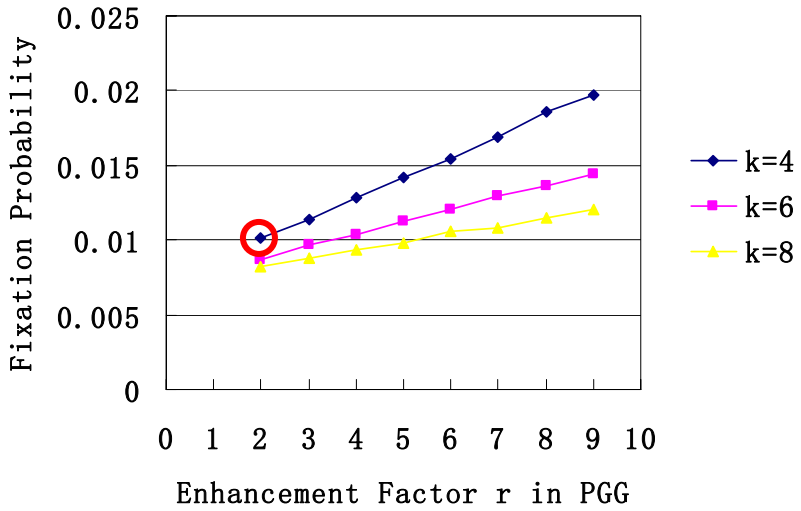
1. More vertices with degree=k/2 in Scale-Free not k in Small-World
2. Vertices with great numbers of degree (VIP)

Classical model (Ohtsuki et al., 2006)

PD game, Scale-Free, *w*=0.01



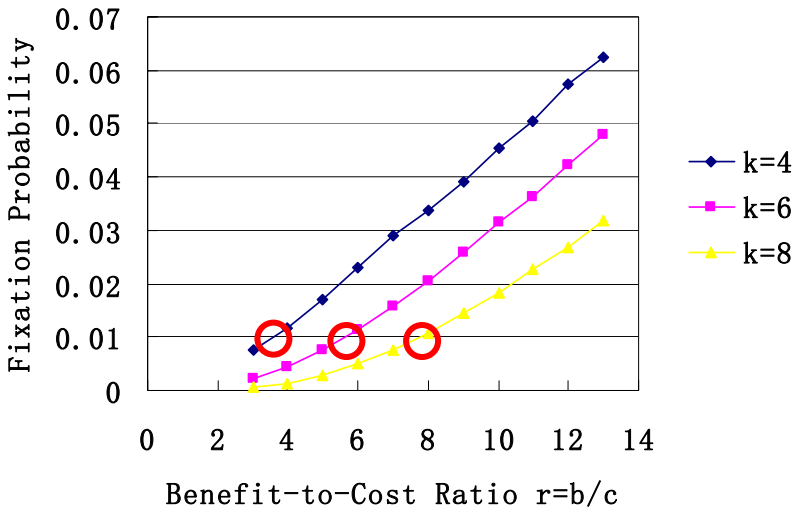
PG game, Scale-Free, *w*=0.01



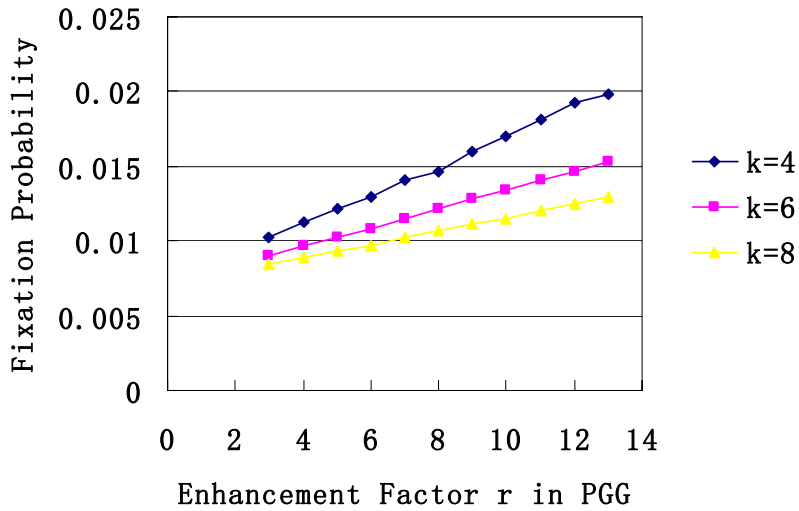
In public goods game, when *k*=4, the fixation probability does not obey the *r*>*k* rule, and the scale-free network promote cooperation more efficiently than regular, small-world and random networks.

Fixed contribution

PD game, Scale-Free, *w*=0.01



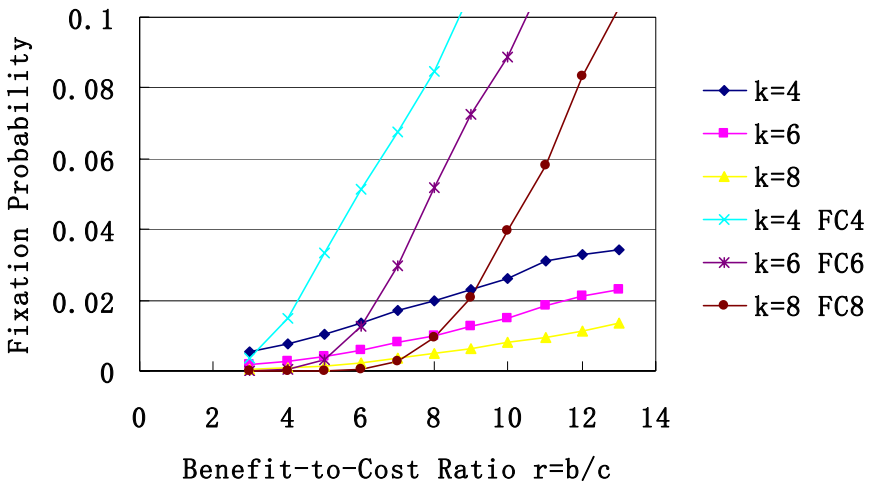
PG game, Scale-Free, *w*=0.01



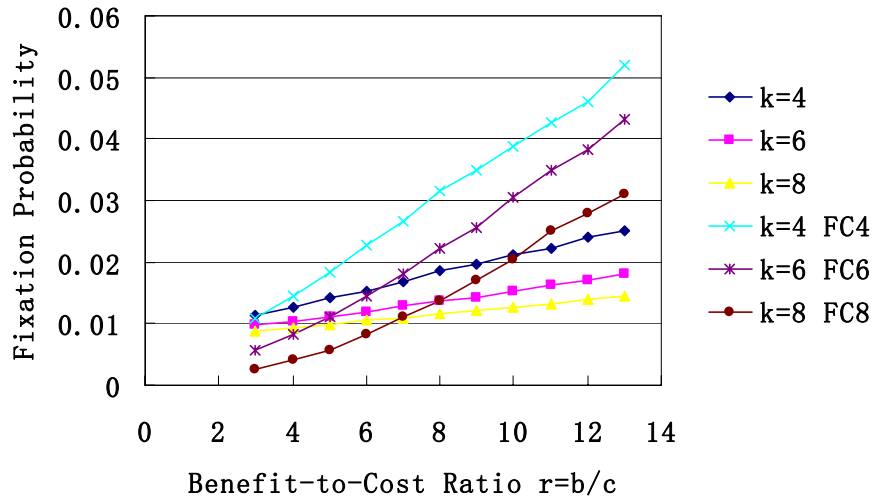
In PD game, for *k*=4 and *k*=6, the *r*>*k* rule also does not work.

Classical model vs. Fixed Contribution

PD game, Scale-Free



PG game, Scale-Free



I. Classical model with w=0.01, FC with w=0.01$\times k$.

II. For both PD and PG games, fixed contribution can promote cooperation efficiently.

Works in the future

- 1. Some theoretical result in detail.
- 2. More realistic network.
- 3. The effect of group size N, an anti-intuitional result on fixation contribution.

Thank you!