A space time model for the analysis of extremes from RCM output

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Some aspects on extreme values

- $X_1, X_2, X_3 \ldots$, is a sequence of measurements.
- Maxima over blocks of size n,

$$M_n = \max\{X_1, \ldots, X_n\}.$$

- The *Generalized Extreme Value (GEV)* distribution

$$H(z) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\}$$

- $\infty < \mu < \infty$ (loc.), $\sigma > 0$ (scale) y $-\infty < \xi < \infty$ (shape).

- *Domains of Atraction:*
  - $\xi > 0$ *Fréchet:*. 'Tail' of dist. follows power function.
  - $\xi < 0$ *Weibull:* Family with 'bounded' tail.
  - $\xi \to 0$ *Gumbel:* Family with 'tail' exponentially decreasing.
- *Fisher-Tippet-Gnedenko*: If a_n y b_n are such that $n \to \infty$

$$P\left[\frac{M_n - a_n}{b_n} \leq z\right] \to H(z)$$

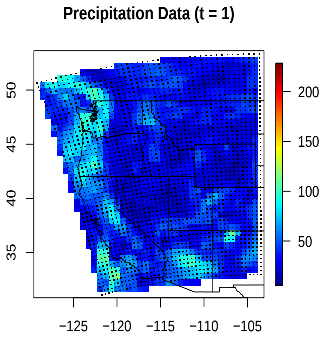
  then $H(z)$ is a GEV (Coles 2002).
- Alternative, *threshold methods* or *Poisson Processes*.

Spatial models for extremes

- Casson and Coles (1999) modelled wind speeds for hurracanes.
- Cooley, Nychka and Naveau (2007) risk map for an event of extreme precipitation.
- Huerta and Sansó (2007) space-time model for ozone maxima.
- Sang and Gelfand (2008) model for rainfall in South Africa.
- Cooley and Sain (2008) extreme precipitation generated by a RCM.
- Glenn Stark (current Ph.D grad student in Stats).

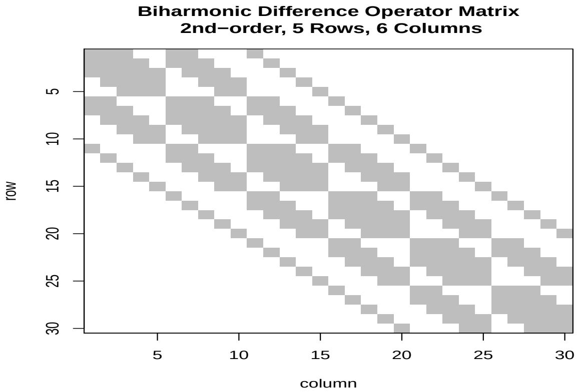
Numerical output from a regional climate model

- *Mesoscale* model developed by Penn State/NCAR (MM5).
- 20 years of extreme precipitation for a control run (“Winter”).
- The spatial domain is $56 \times 44 = 2464$.



Markov Random Fields

- Model with structure via neighbors (Markovian).
- *Precision* matrices are used not covariance.



Gaussian Markov Random Field

- Random vector X (dim. n) with parameters η and $\mathbf{Q}$.
- Graph $G = (V, E)$ with *nodes* and *vertices* such that

$$\mathbf{Q}_{ij} \neq 0 \iff i \sim j.$$

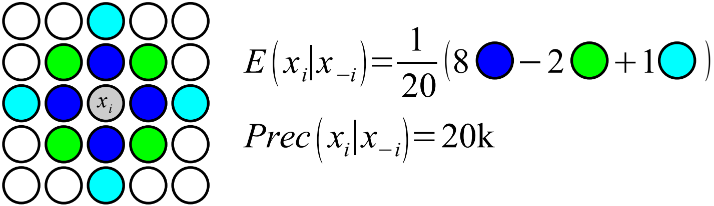
- The probability model of X is

$$f(x) = (2\pi)^{-\frac{n-k}{2}} (|\mathbf{Q}|^*)^{\frac{1}{2}} exp\left(-\frac{1}{2}(x-\eta)'\mathbf{Q}(x-\eta)\right)$$

- Improper case: $|\mathbf{Q}|^*$ is the product of the $n-k$ eigenvalues of $\mathbf{Q}$ different to zero.

Definition of **Q**

- The matrix defines structure in a r rows and c columns grid.
- Symmetric and semi-positive definite with range $r \times c - 3$.
- $X_i | X_{-i}$ is Normally distributed.

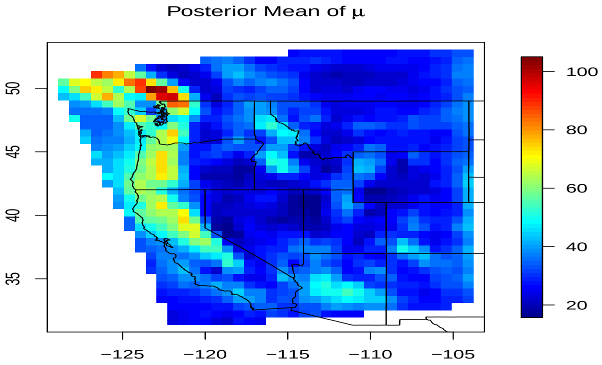


A Hierarchical Model

- First level:
  - $Y_{s,t} \sim GEV(\mu^*_{s,t}, \sigma, \xi)$; $s = 1, \ldots, 2464$; $t = 1, \ldots, 20$.
  - $\mu^*_{s,t} = \mu_s + \phi_t$ has a spatial and a temporal term. $\boldsymbol{\mu}$=$(\mu_1, \ldots, \mu_{2464})$
- Second level:
  - $\boldsymbol{\mu} \sim GMRF(\mathbf{0}, \theta\mathbf{Q})$ where $\mathbf{Q}$ is a *second order* precision matrix.
  - $\phi_1 = 0$, $\phi_t \sim N(\mu_\phi, \tau_\phi)$; $t = 2, \ldots, 20$.
- Prior distributions:
  - $\sigma \sim LN(m_\sigma, s_\sigma)$,
  - $\xi \sim N(m_\xi, s_\xi)$,
  - $\mu_\phi \sim N(0, 10^{-6})$; $\tau_\phi \sim Gamma(1, 1)$.
  - $\theta \sim LN(m_\theta, s_\theta)$.

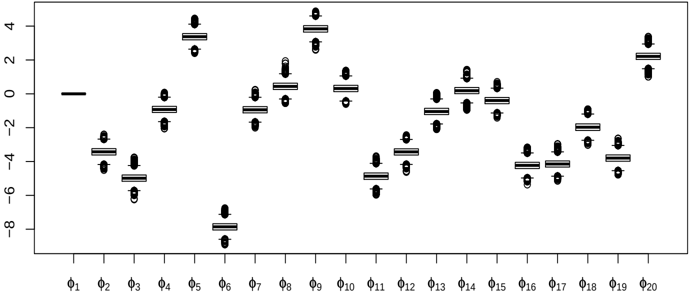
Results of 'Space-Time' model

- Monte Carlo (MCMC) with 20000 iterations, 10000 burn-in.
- Mean of posterior distribution of μ.



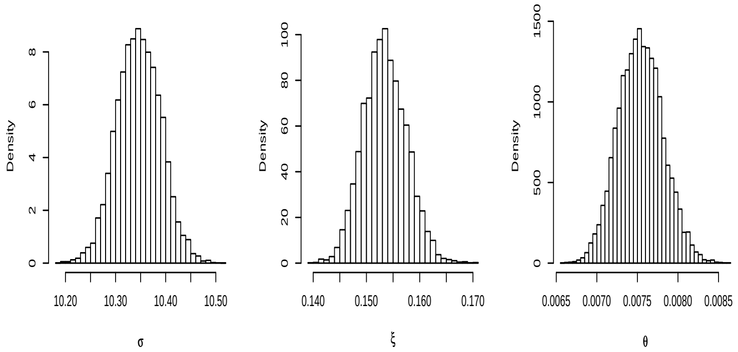
Box-plots for ϕ_i

- Based Monte Carlo samples.
- Measures temporal variability.



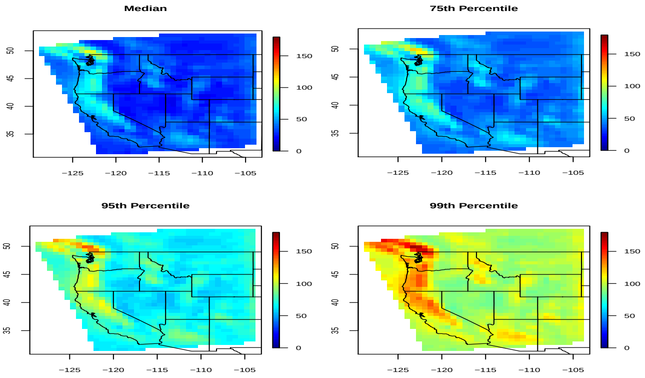
Posterior distributions for σ, ξ and θ

- Distributions are *unimodal*.
- We have a *Fréchet* case.

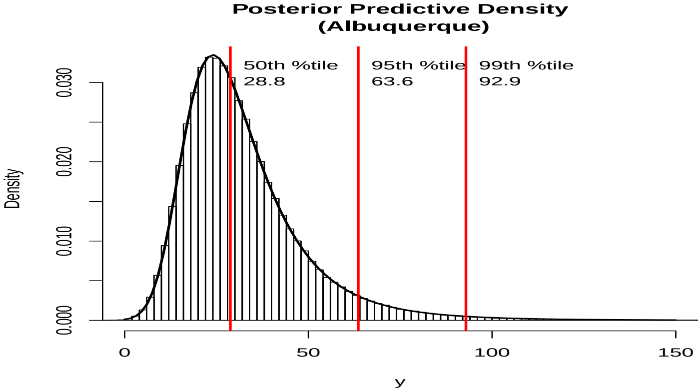


Quantile Estimation

- Based on *predictive distribution*.
- Conceptually clear but intensive.



Predictive distribution for Albuquerque



Spatial variability in σ and ξ

- Pointwise estimates of GEV distribution.
- Maps of parameters estimate for scale and shape.