Spatial modeling of annual minimum and maximum temperature

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Introduction

Understanding and quantifying the spatial and temporal nature of extreme in temperature is important for both design and safety issues and climate change research. How well isolated does a building have to be or how powerful does the air-conditioning in a house have to be to withstand possible extremes in temperature are questions regarding public health. The need for understanding the risk of humans being subject to extreme temperatures is apparent in light of the European heat wave 2003 which caused excessive death to thousands of people (Bosch, 2003) and the heat wave in Russia summer 2010. Quantifying the temporal change of extremes in temperature is important for climate change research and decisions in urban planning.

In Cooley et al. (2007) the intensity of daily precipitation above a high threshold at several weather stations is modeled with the generalized Pareto distribution, and the number of exceedances above the threshold at the stations are modeled as binomial random variables. In Cooley et al. (2006) lichen measurements are modeled with the generalized extreme value distribution. In both of these papers, a Bayesian hierarchical model is built to model the spatial structure of the parameters of the data distributions. Casson and Coles (1999) modeled the extremes of simulated hurricane wind speeds with the point-process extreme value distribution. They modeled the location parameter, the scale parameter and the shape parameter as spatial processes. In Sang and Gelfand (2009) a spatio-temporal model based on a multivariate Markov random field models are introduced and applied to precipitation data from South Africa. Schliep et al. (2010) modeled the three parameters of the generalized extreme value distribution spatially with a multivariate intrinsic autoregressive model.

Aim of study

The aim of this study is to develop models for data on annual minimum and maximum temperature which take into account both the spatial and temporal nature of the data as well as the fact that the observations are extremes. One of the desired outputs from these models are the 100-year maximum (and minimum) temperature at a given site which is defined as the number such that with probability 0.01 the temperature goes above (beneath) that number. The models are applied to data from Iceland observed from 1961 to 2009.

Data

The data consist of the observed maximum temperature in Iceland within periods defined from January 1st to December 30th and the observed minimum temperature over periods from July 1st to June 30th. The observations are from 72 sites over the period from January 1st 1961 to June 30th 2009. Most of the sites are close to the coast of Iceland. At some of the sites observations from all the 48 periods were collected while at the other sites observations are missing in one or more periods. The average number of observations per site is around 34.

Spatial predictions are made on a set of 280614 unobserved sites that are on a grid laid over Iceland. The grid is a latitude-longitude grid with resolution of half a minute. The average North-South distance is **0.93** km and the average East-West distance is **0.39** km. For each site on the grid and for the observed sites data on the longitude, latitude, altitude and distance to open sea are available. The distance to open sea is measured from a smooth reference line around Iceland which is at minimum 10 kilometers from the coast (Bjornsson et al., 2006). These variables will be used as explanatory variables except for longitude.

Model

The model for annual maximum is presented here. The model for minimum temperature is essentially the same. The cdf for the observed maximum temperature at site $\mathbf{i}$ and time $\mathbf{t}$, denoted by $\mathbf{y_{it}}$, is given by

$$F(y_{it}) = \exp\left[-\left\{1+\xi_i\left(\frac{y_{it}-\mu_{it}}{\sigma_i}\right)\right\}^{-1/\xi_i}\right],$$

if $1+\xi_i\left(\frac{y_{it}-\mu_{it}}{\sigma_i}\right) \geq 0$ and zero otherwise, where $\mu_{it} \in \Re$, $\sigma_i > 0$ and $\xi_i \in \Re$ are location, scale and shape parameters, respectively. The parameters σ_i and ξ_i modeled spatially with a Matérn correlation function. The model for μ_{it} is such that

$$\mu_{it} = \eta_i + \gamma_t$$
$$\eta_i = x_i^T\beta + \epsilon_i$$
$$\gamma_t = \Delta(t - t_0) + z_t$$

where

- x_i are explanatory variables at site $\mathbf{i}$
- β is a vector of unknown parameters
- ϵ_i is a mean zero site effect modeled spatially with a Matérn correlation function
- Δ is the annual average increase/decrease in maximum temperature
- t_0 is the central year in the data
- z_t is a mean zero year effect

The explanatory variables are

- $x_{1,i} = 1$: a constant
- $x_{2,i}$: latitude (not in the model for annual maximum temperature)
- $x_{3,i}$: altitude
- $x_{4,i}$: distance to open sea

The Bayesian approach and MCMC are applied for parametric inference.

Results - maximum temperature

Figure 1 shows the 100-year maximum temperature (the 99% percentile) in Iceland for the year 2010 based on spatial and temporal predictions. The lowlands in the Southern part, North-Eastern part and the Eastern part of the country yield the highest values in 100-year maximum temperature, ranging from **26**°C to **30**°C.

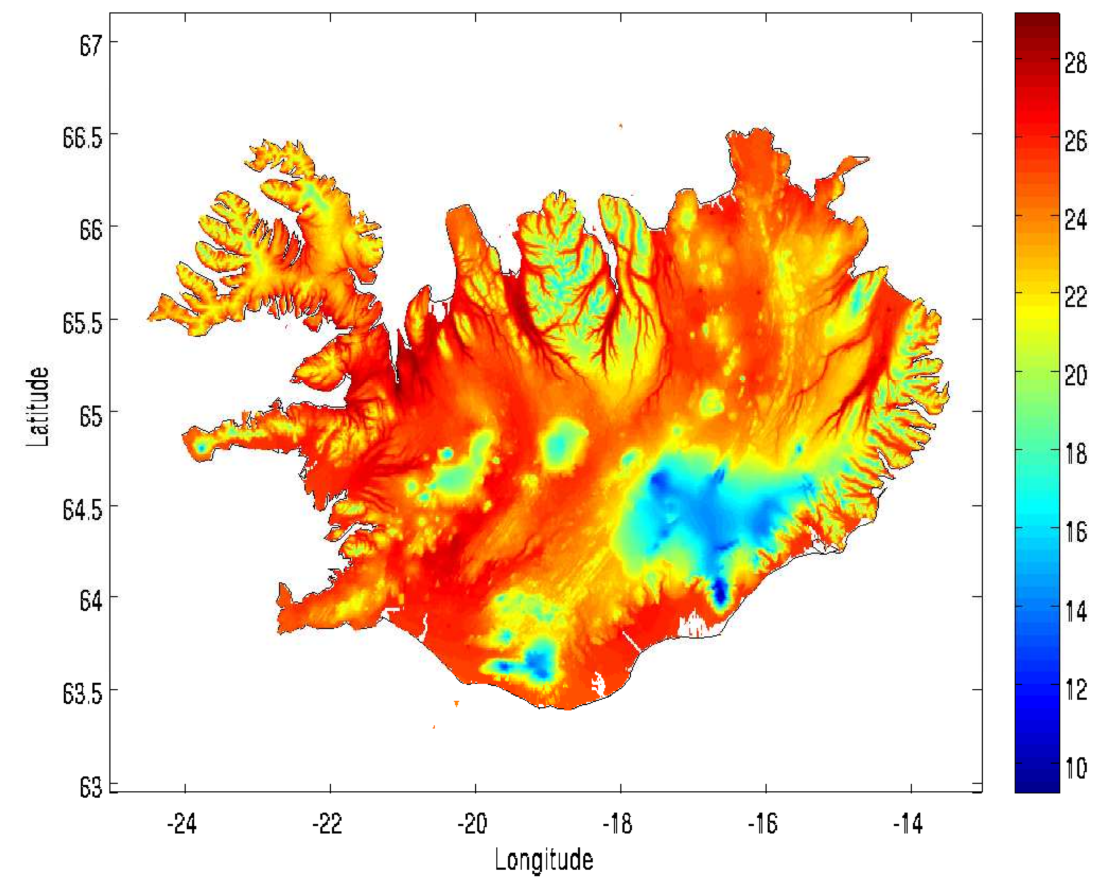

Figure: The the 100-year maximum temperature in Iceland for the year 2010.

Results - minimum temperature

Figure 2 shows the 100-year minimum temperature (the 1% percentile) in Iceland for the year starting on July 1st 2010 and ending on June 30th 2011. The spatial and temporal predictions reveal that the 100-year minimum temperature exceeds $-\mathbf{36}$°C in the central part of Iceland and ranges from $-\mathbf{14}$°C to $-\mathbf{24}$°C around the coast of Iceland with the lowest coastal temperatures in the Northern part and the Northeastern part.

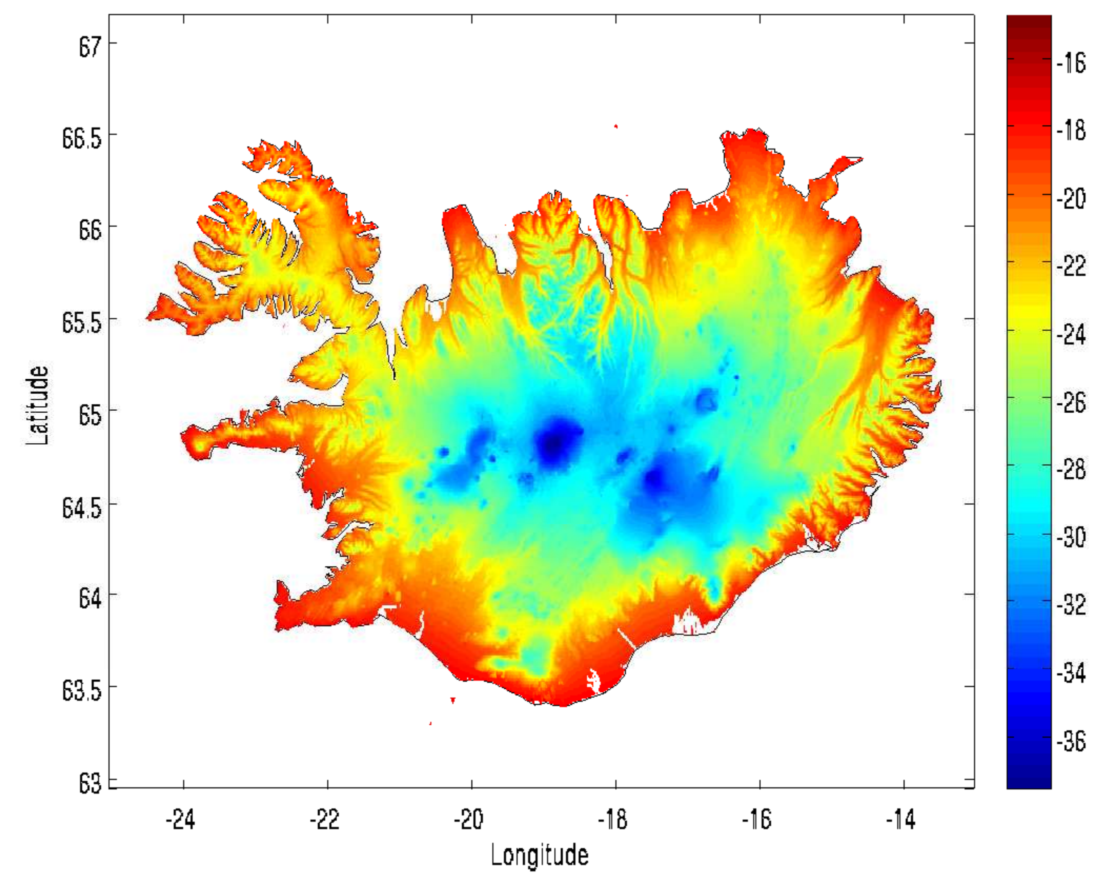

Figure: The the 100-year minimum temperature in Iceland for the year long period from July 1st 2010 to June 30th 2011.

Results - trend

The overall temporal effect for maximum temperature, γ_t, is shown in Figure 3 (left panel). The average increase in maximum temperature is $\mathbf{10\Delta = 0.48}$°C per decade. The overall temporal effect for minimum temperature, γ_t, is shown in Figure 3 (right panel). The average increase in minimum temperature is $\mathbf{10\Delta = 0.72}$°C per decade.

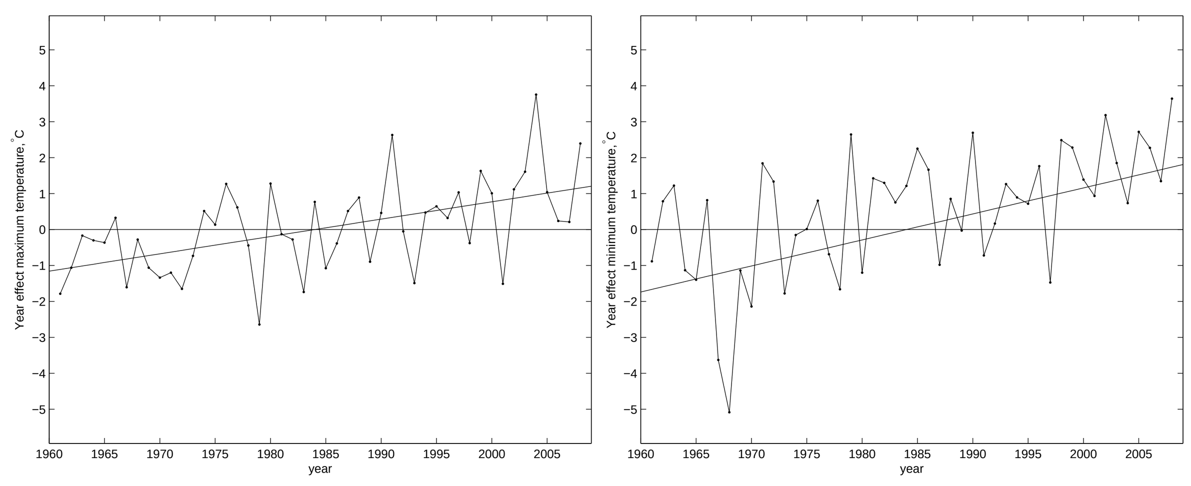

Figure: The overall trend in maximum and minimum temperature in Iceland over the years 1961 to 2009.

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