Towards a Set of Visualization Tools for Convex Algebraic Geometry

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Introduction

with Ph. Rostalski.

- Today NOT: nice pictures.
- BUT: How to produce good visualizations by yourself!
- Tool: SURFEX (based on SURF, written by S. Endraß and others).
- …via the computer algebra software SINGULAR → library.
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Background on Visualization Strategies

- basic ray tracing:
  - fast, parallelizable (GPU, . . .),
  - does not show small connected components and lower-dimensional components,
  - cubic with $A^\bullet$, swallowtail (from our calendar).
- subdivision methods, e.g. using interval arithmetic
  - fast, parallelizable, yields triangulations,
  - too MANY points.
- critical points methods
  - very slow, yields triangulations,
  - may give guarenteed output.

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- ray tracer: several real algebraic surfaces
- curves on surfaces
- clip by a given set of real algebraic surfaces
- parameters

- Numerical issues!
- Examples: SURFEX: pillow ($2xyz - x^2 - y^2 - z^2 + 1 = 0$), cayley cubic (www)
- Simpler interface: SURFER, developed for our exhibition (with the MFO): www.imaginary-exhibition.com

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. . . in the form of a SINGULAR library.

- To visualize B. Sturmfels' example of a spectrahedron with 8 real nodes in its boundary:
  - show this in SURFEX from SINGULAR
  - $A(x) \geq 0 \iff$ all principal minors ≥ 0
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- Alternatively use the Renegar derivatives – what are they?

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Renegar Derivatives

$f \in k[x_0, \ldots, x_n]_d$ homogeneous of degree d, $p \in \mathbb{P}^n$ a point.

- $\text{polar}(f, p) = \sum_{i=0}^{n} p_i \cdot \frac{\partial f}{\partial x_i}$
  - $\deg \leq \deg(f) - 1$
  - passes through sing. pts. with multiplicity one less
  - cuts the curve exactly in the points where the tangents through p touch f
- iteratively: n-th polar.
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- allow computations before the visualization
  - dual curve (anim from calendar) / surface,
  - Hessian (anim from calendar)
  - whole surface,
  - ...
- produce many such pictures and produce a anim
  - example: sections of a 6-dim. spectrahedron: whole surface anim

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- basic idea:
  - optimize in enough directions
  - this yields points on the boundary of the convex set
  - compute a triangulation with these points as vertices
  - visualize this
- examples:
  - screenshot
  - the pillow
  - Bernd's ex. with 8 nodes
  - a projection of a spectrahedron
  - a family of projections of spectrahedra
  - a random cut through conv$G(3,6)$

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  - this yields points on the boundary of the convex set
  - compute a triangulation with these points as vertices
  - visualize this
- examples:
  - screenshot
  - the pillow
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  - a projection of a spectrahedron
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  - a random cut through conv$G(3, 6)$

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To Do List

- show whole variety, but highlight the convex set we're interested in: curves (PIC 1, PIC 2), surfaces (PIC 1)
- improve SDP-visualizations (both quality and speed)
- MatLab $\rightarrow$ surfex interface
- improve/finish MATLAB / SINGULAR ToolKits
- more clipping surfaces in SURFEX
- show curves on surfaces only on those parts of the surface satisfying some inequalities (e.g. 6 lines on the pillow)
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Anything you'd like to add to this list? Contact us!

Thank You

Thank you for your attention.

Oliver Labs

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www.surfex.AlgebraicSurface.net
www.Calendar.AlgebraicSurface.net