Analysis of nonlinear wave equations and applications in engineering
August 9-14, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

When your schedule is finalized, please e-mail it to the BIRS Station Manager birsmgr@birs.ca by 12 noon on the Thursday before your arrival.

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:45	Gadi Fibich, *Singular standing-ring solutions of nonlinear partial differential equations*
9:50–10:35	Jared Bronski, *An index theorem for the stability of periodic waves of Korteweg-de Vries type*
10:40–11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45	Benjamin Schlein, *Tutorial 1: Derivation of effective evolution equations from first principle quantum dynamics*
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:10–14:55	Benjamin Schlein, *Tutorial 2: Derivation of effective evolution equations from first principle quantum dynamics*
14:55–15:15	Coffee Break, 2nd floor lounge, Corbett Hall
15:15–16:00	Milena Stanislavova, *Conditional stability theorems for Klein-Gordon type equations*
16:05–16:50	Yoshio Tsutsumi, *Stability of cavity soliton for the Lugiato-Lefever equation with additive noise*
16:55–17:40	Keith Promislow, *Tutorial: Pore Formation in Polymer Electrolytes*
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	Mathieu Lewin, *Variational models for infinite quantum systems: example of the crystal with defects*
9:50-10:35	Walid K. Abou Salem, *Microscopic derivation of the magnetic Hartree equation*
10:35-10:55	Coffee Break, 2nd floor lounge, Corbett Hall
10:55-11:40	Bjorn Sandstede, *Pointwise estimates and nonlinear stability of waves*
11:40–13:30	Lunch
13:30–14:15	Justin Holmer, *Dynamics of KdV solitons in the presence of a slowly varying potential*
14:15–14:35	Coffee Break, 2nd floor lounge, Corbett Hall
14:35–15:20	Natasa Pavlovic, *The quintic NLS as the mean field limit of a Boson gas with three-body interactions*
15:25–16:10	Eduard Kirr, *Asymptotic stability and resonances in subcritical NLS*
16:15–17:00	Stephen Gustafson, *Schroedinger and Landau-Lifshitz maps of low degree*
17:30–19:30	Dinner
19:30–21:00	Poster session
	Gideon Simpson, *Numerical simulations of the energy-supercritical Nonlinear Schrodinger equation*

Wednesday

7:00–9:00	Breakfast
11:30–13:30	Lunch
17:30–19:30	Dinner
19:30–20:15	Roy Goodman, *Bifurcations of Nonlinear Defect Modes*
20:20-21:05	Markus Kunze, TBA

Thursday

7:00–9:00	Breakfast
9:00–9:45	Wilhelm Schlag, *On inverse square potentials and applications*
9:50-10:35	Jeremy Marzuola, *Long time dynamics near the symmetry breaking bifurcation for Nonlinear Schrödinger/Gross-Pitaevskii Equations*
10:35-10:55	Coffee Break, 2nd floor lounge, Corbett Hall
10:55-11:40	Boaz Ilan, *Band-edge solitons of Nonlinear Schrodinger equations with periodic potentials*
11:40–13:30	Lunch
13:30–14:15	Thomas Chen, *Some recent developments on the well-posedness of the Cauchy problem for focusing and defocusing GP hierarchies*
14:15–14:35	Coffee Break, 2nd floor lounge, Corbett Hall
14:35–15:20	Young-Ran Lee, *Exponential decay of dispersion managed solitons with vanishing average dispersion*
15:25–16:10	Daniel Tataru, *Large data wave maps*
16:15–17:00	TBA
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:45	Discussions
9:50-10:35	Discussions
10:35-10:55	Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **Gadi Fibich** (University of Tel Aviv)
Title: *Singular standing-ring solutions of nonlinear partial differential equations*
Abstract: In this talk I will present a general framework for constructing singular solutions of nonlinear evolution equations that become singular on a d-dimensional sphere, where $d > 1$. The asymptotic profile and blowup rate of these solutions are the same as those of solutions of the corresponding one-dimensional equation that become singular at a point. The results will be demonstrated for the nonlinear Schrodinger equation, the biharmonic nonlinear Schrodinger equation, the nonlinear heat equation, and the nonlinear biharmonic heat equation.

Speaker: **Jared Bronski** (University of Illinois at Urbana-Champaign)
Title: *An index theorem for the stability of periodic waves of Korteweg-deVries type*
Abstract: We consider periodic solutions to equations of Korteweg-Devries type. While the stability theory for periodic waves has received much some attention the theory is much less developed than the analogous theory for solitary wave stability, and admits some interesting new phenomena.

We prove an index theorem giving an exact count of the number of unstable eigenvalues of the linearized operator in terms of the number of zeros of the derivative of the traveling wave profile together with geometric information about a certain map between the constants of integration of the ordinary differential equation and the conserved quantities of the partial differential equation. This index can be regarded as a generalization of both the Sturm oscillation theorem and the classical stability theory for solitary wave solutions for equations of Korteweg-de Vries type(Bona-Souganidis-Strauss, Pego-Weinstein, etc.)

In the case of a polynomial nonlinearity this index, together with a related one introduced earlier by Bronski and Johnson, can be expressed in terms of derivatives of hyperelliptic integrals on a finite genus Riemann surface. Since these hyperelliptic integrals satisfy a Picard-Fuchs relation these derivatives can be expressed in terms of the integrals themselves, leading to a closed-form expression in terms of a finite number of moments of the solution. We conclude with some illustrative examples.

This is joint work with Mathew Johnson (Indiana University) and Todd Kapitula (Calvin College).

Speaker: **Boaz Ilan** (University of California at Merced)
Title: *Band-edge solitons of Nonlinear Schrodinger equations with periodic potentials*
Abstract: Nonlinear Schrodinger (NLS) / Gross-Pitaveskii equations with periodic potentials admit positive bound states (solitons) in the semi-infinite gap (first Brillouin zone). For focusing nonlinearities these solitons bifurcate from the zero state at the band edge into the semi-infinite gap. Using the method of Multiple Scales leads to a homogenized NLS equation that depends on the band-edge Bloch wave through an effective-mass tensor and coupling constant. To leading order the soliton is constructed from the Bloch wave that is slowly modulated by a ground state of the homogenized equation. In the L2-critical case, for any non-constant periodic potential the power (L2 norm) of the soliton is strictly lower than the power of the Townes mode, which has the critical power for collapse. The implications to collapse dynamics and self-focusing instability are elucidated using computations of bound states and direct computations of critical NLS equations in 1D and 2D.

Joint work with M. I. Weinstein and Y. Sivan.

Speaker: **Milena Stanislavova** (University of Kansas)
Title: *Conditional stability theorems for Klein-Gordon type equations*

Abstract: We consider positive, radial and exponentially decaying steady state solutions of the Klein-Gordon equation with various power nonlinearities. The main result is a fairly precise construction of infinite-dimensional invariant manifolds in the vicinity of these solutions. The precise center-stable manifold theorem for the Klein-Gordon equation includes the co-dimension of the manifold, a formula for the asymptotic phase and the decay rates for even perturbations. Several outstanding open questions will be discussed.

Speaker: **Yoshio Tsutsumi** (Kyoto University)
Title: *Stability of cavity soliton for the Lugiato-Lefever equation with additive noise*
Abstract: We consider the stability of stationary solution for the Lugiato-Lefever equation with periodic boundary condition under perturbation of additive noise, to which is referred as (LL). The (LL) equation is a nonlinear Schrodinger equation with damping and spatially homogeneous forcing terms, which describes a physical model of a unidirectional ring or Fabry-Perot cavity with plane mirrors containing a Kerr medium driven by a coherent plane-wave field. The stationary solution of (LL) is called a "Cavity Sliton". We show the stability of certain stationary solutions under the perturbation of additive noise from a viewpoint of the Freidlin-Wentzell type large deviation principle. This is a joint work with T. Miyaji and I. Ohnishi, Hiroshima University.

Speaker: **Roy Goodman** (New Jersey Institute of Technology)
Title: *Bifurcations of Nonlinear Defect Modes*
Abstract: The nonlinear coupled mode equations describe the evolution of light in Bragg grating optical fibers. Defects (localized potentials) can be added to the fiber in order to trap light at a specialized location as a nonlinear defect mode. In numerical simulations these defect modes are seen to lose (linear) stability through several types of bifurcations. Inverse scattering is used to design defects in which the bifurcations can be easily observed and studied via the derivation of finite-dimensional reduced equations. Conditions are given for the existence Hamiltonian pitchfork and Hamiltonian Hopf bifurcations.

Speaker: **Mathieu LEWIN** (CNRS)
Title: *Variational models for infinite quantum systems: example of the crystal with defects*
Abstract: Describing quantum particles in a quantum medium often leads to strongly indefinite (sometimes unbounded from below) theories, for which it is usually quite hard to establish the existence and the stability of bound states. Two famous examples are relativistic electrons described by the Dirac operator and electrons close to a defect in a quantum crystal.

In this talk I will present a new method for constructing and studying a variational model for such systems. I will concentrate on the Hartree model for the crystal with a defect.

The main idea is to describe at the same time the electrons bound by the defect and the (nonlinear) behavior of the infinite crystal. This leads to a (rather peculiar) bounded-below nonlinear functional whose variable is however an operator of infinite-rank.

I will provide the correct functional setting for this functional, state the existence of global-in-time solutions to the associated time-dependent Schrodinger equation, and discuss the existence, the properties and the stability of bound states.

Speaker: **Walid K. Abou Salem** (University of Toronto)
Title: *Microscopic derivation of the magnetic Hartree equation* Abstract: I discuss the rigorous derivation of the time-dependent Hartree equation in the presence of magnetic potentials. I also remark on how to extend the analysis to the Gross-Pitaevskii equation.

Speaker: **Bjorn Sandstede** (Brown University)
Title: *Pointwise estimates and nonlinear stability of waves*
Abstract: Over the past decade, pointwise Green's function estimates have proved very useful in establishing nonlinear stability of viscous shock profiles under the assumption of spectral stability. I will report

here on recent work with Beck and Zumbrun on extending this approach to the nonlinear stability of time-periodic viscous shocks. Key to the derivation of the required pointwise bounds in the time-periodic setting are meromorphic extensions of exponential dichotomies of appropriate time-periodic eigenvalue problems. I will also show how spectral stability of weakly time-periodic shocks can be established near Hopf bifurcation using a spatial-dynamics approach. The motivation for this work comes from sources in reaction-diffusion systems, and I will outline the challenges and hopes for nonlinear stability proofs in this context.

Speaker: **Justin Holmer** (Brown University)
Title: *Dynamics of KdV solitons in the presence of a slowly varying potential*
Abstract: We study the dynamics of solitons as solutions to the perturbed KdV (pKdV) equation $\partial_t u = -\partial_x(\partial_x^2 u + 3u^2 - bu)$, where $b(x,t) = b_0(hx, ht)$, $h \ll 1$ is a slowly varying potential. We are able to refine earlier work of Dejak-Sigal and obtain an estimate on the trajectory of the soliton parameters of scale and position. In addition to the Lyapunov analysis commonly applied to these problems, we use a local virial estimate due to Martel-Merle. The proof does not rely on the inverse scattering machinery and is expected to carry through for the L^2 subcritical gKdV-p equation, $1 < p < 5$. The case of $p = 3$, the modified Korteweg-de Vries (mKdV) equation, is structurally simpler and more precise results can be obtained by the method of Holmer-Zworski. This is joint work with Galina Perelman.

Speaker: **Natasa Pavlovic** (University of Texas at Austin)
Title: *The quintic NLS as the mean field limit of a Boson gas with three-body interactions*
Abstract: In this talk we will discuss joint work with Thomas Chen on the dynamics of a boson gas with three-body interactions in dimensions d=1,2. We prove that in the limit as the particle number N tends to infinity, the BBGKY hierarchy of k-particle marginals converges to a limiting Gross-Pitaevskii (GP) hierarchy for which we prove existence and uniqueness of solutions. For factorized initial data, the solutions of the GP hierarchy are shown to be determined by solutions of a quintic nonlinear Schrödinger equation.

Time permitting, we will briefly describe our new approach for studying well-posedness of the Cauchy problem for focusing and defocusing GP hierarchy. This approach and its applications will be explained in more details in the talk of Thomas Chen.

Speaker: **Eduard Kirr** (University of Illinois at Urbana-Champaign)
Title: *Asymptotic stability and resonances in subcritical NLS*
Abstract: The talk will focus on a recent method to obtain dispersive estimates for time dependent perturbations of linear Schrödinger operators. The method currently works in dimensions two and higher. The obstacles in extending it to one dimension will be discussed. Also some applications to nonlinear equations, in particular to asymptotic stability and radiative damping of ground states in NLS will be presented.

Speaker: **Stephen Gustafson** (University of British Columbia)
Title: *Schroedinger and Landau-Lifshitz maps of low degree*
Abstract: The Schroedinger (and Landau-Lifshitz) map equations are a basic model in ferromagnetism, and a natural geometric (hence nonlinear) version of the Schroedinger (and Schroedinger-heat) equation. While there has been recent progress on the question of singularity formation for the wave and heat analogues (wave map and harmonic map heat-flow), the Schroedinger case seems more elusive. We present results on global regularity and long-time dynamics for equivariant maps with near-minimal energy. We emphasize lower degree (2 and 3) maps, for which the analysis is trickier, and the dynamics more complex, phenomena related to slower spatial decay of certain eigenfunctions. This is joint work with K. Nakanishi, and T.-P. Tsai.

Speaker: **Wilhelm Schlag** (University of Chicago)
Title: *On inverse square potentials and applications*

Abstract: We discuss some recent work on dispersive estimates on a curved background. These problems arise in geometry and physics, and are reduced for fixed angular momentum to a one-dimensional problem with an inverse square potential. For the Schwarzschild case, one obtains local pointwise decay rates which increase with the angular momentum. This is joint work with R. Donninger, A. Soffer, and W. Staubach.

Speaker: **Benjamin Schlein** (Cambridge University)
Title: *Tutorial: Derivation of effective evolution equations from first principle quantum dynamics*
Abstract: In this talk, I am going to discuss the derivation, from microscopic many-body quantum mechanics, of effective evolution equations. In particular, I am going to show that the nonlinear Hartree equation can be used to describe the macroscopic properties of the evolution of a many body system in the so called mean field limit. Moreover, I am going to explain that the Gross-Pitaevskii equation, a cubic nonlinear Schroedinger equation, can be used to describe the dynamics of Bose-Einstein condensates.

Speaker: **Thomas Chen** (University of Texas at Austin)
Title: *Some recent developments on the well-posedness of the Cauchy problem for focusing and defocusing GP hierarchies*
Abstract: This talk surveys some recent results, all from joint works with Natasa Pavlovic, related to the Cauchy problem for the Gross-Pitaevskii (GP) hierarchy. First, we address the local well-posedness theory, in various dimensions, for the cubic and quintic case. We then introduce new conserved energy functionals which we use in the following contexts: (1) In a joint work with N. Tzirakis, we prove, on the L2 critical and supercritical level, that solutions of focusing GP hierarchies with a negative average energy per particle blow up in finite time. (2) We prove the global well-posedness of the Cauchy problem for energy subcritical, defocusing GP hierarchies, based on the conservation of higher order energy functionals. (3) We prove global well-posedness of focusing and defocusing GP hierarchies on the L2 subcritical level, based on a generalization of the Gagliardo-Nirenberg inequalities which we establish for density matrices.

Speaker: **Young-Ran Lee** (Korean Institute of Technology)
Title: *Exponential decay of dispersion managed solitons with vanishing average dispersion*
Abstract: We show that any L2 solution of the Gabitov-Turitsyn equation describing dispersion managed solitons with zero average dispersion decays exponentially in space and frequency domains. This confirms in the affirmative Lushnikovs conjecture of exponential decay of dispersion managed solitons. This is a joint work with M. Burak Erdogan and Dirk Hundertmark.

Speaker: **Gideon Simpson** (University of Toronto)
Title: *Numerical simulations of the energy-supercritical Nonlinear Schrodinger equation* Abstract: We present numerical simulations of the defocusing nonlinear Schrodinger (NLS) equation with an energy supercritical nonlinearity. These computations were motivated by recent works of Kenig-Merle and Kilip-Visan who considered some energy supercritical wave equations and proved that if the solution is a priori bounded in the critical Sobolev space (i.e. the space whose homogeneous norm is invariant under the scaling leaving the equation invariant), then it exists for all time and scatters.

We numerically investigate the boundedness of the H^2-critical Sobolev norm for solutions of the NLS equation in dimension five with quintic nonlinearity. We find that for a class of initial conditions, this norm remains bounded, the solution exists for long time, and it scatters. Authors: J. Colliander, G. Simpson, and C. Sulem

Speaker: **Keith Promislow** (Michigan State University)
Title: *Pore Formation in Polymer Electrolytes* Abstract: The efficient conversion of energy from chemical and photonic forms to useful electric voltage requires the development of nanostructured materials with interpercolating structure. In practical applications this is achieved by functionalizing polymers, attaching acid groups to short side chains which extend from long, hydrophobic polymer backbones. When placed in solvent, these functionalized polymers form nanoscale solvent-filled pores lined with the tips of the acid groups, and ideal environment for the selective conduction of properly charged ions.

We present a family of models, which we call functionalized Lagrangians, which mimic the energy landscape of the functionalized polymer/solvent mixtures. The functionalized energies are higher order, and strongly nonlinear, but with special structure which renders them amenable to analysis. We outline the properties of the functionalized Lagrangians, and the multi-stage structure of the associated gradient flows.

Speaker: **Daniel Tataru** (University of California at Berkeley)
Title: *Large data wave maps*
Abstract: For large data wave maps from $\mathbb{R}^{2+1}$ into a compact Riemmanian manifold we establish the following dichotomy: either a solution is global and dispersive, or a soliton like concentration must occur. This is joint work with Jacob Sterbenz.