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# The pdf of Sea Surface Winds: Effects of Variable Stratification and Boundary Layer Height

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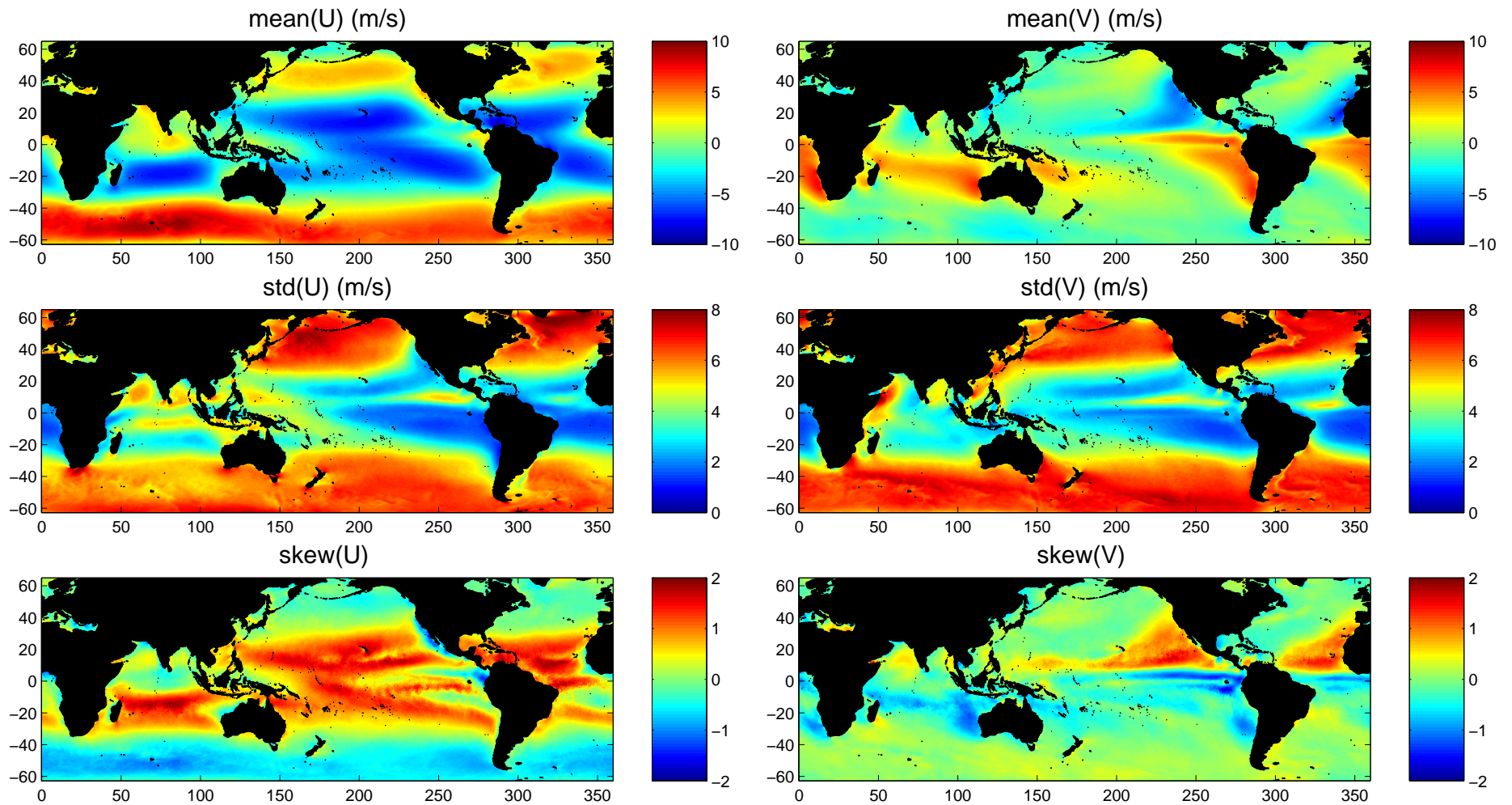
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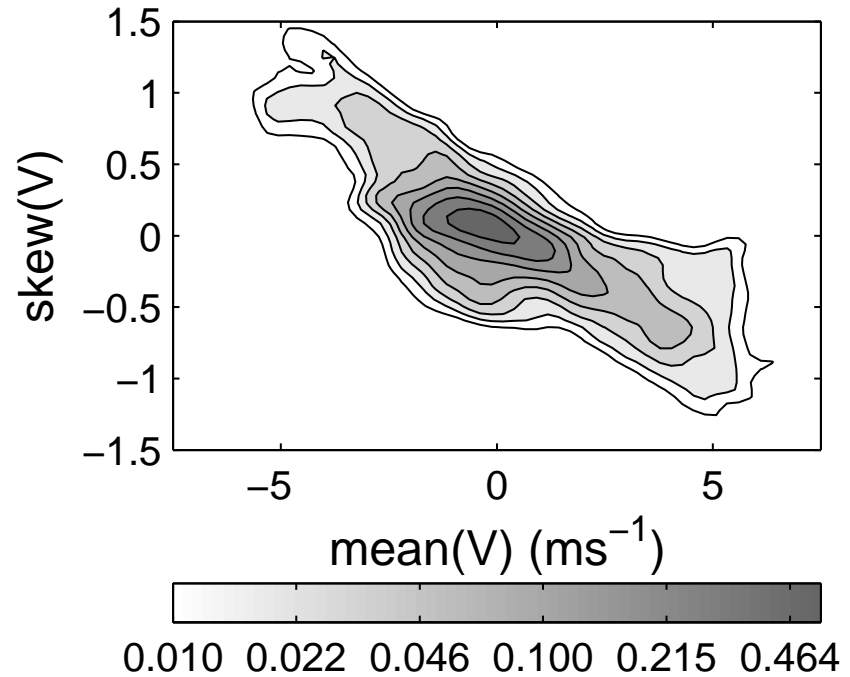
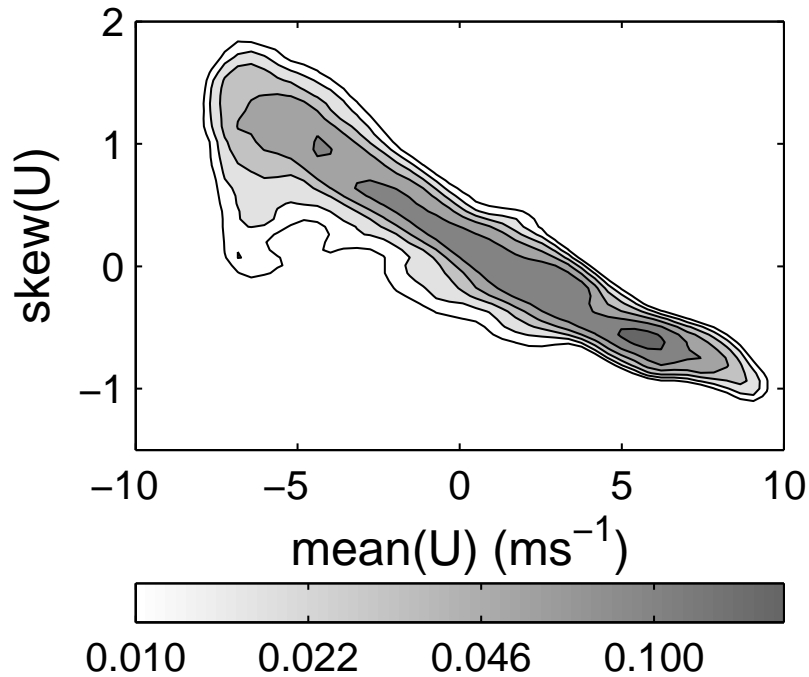
- extreme high winds have important social, economic impacts
- This talk will consider influence of variable surface stratification and boundary layer thickness on the wind speed pdf

# Vector Wind Moments

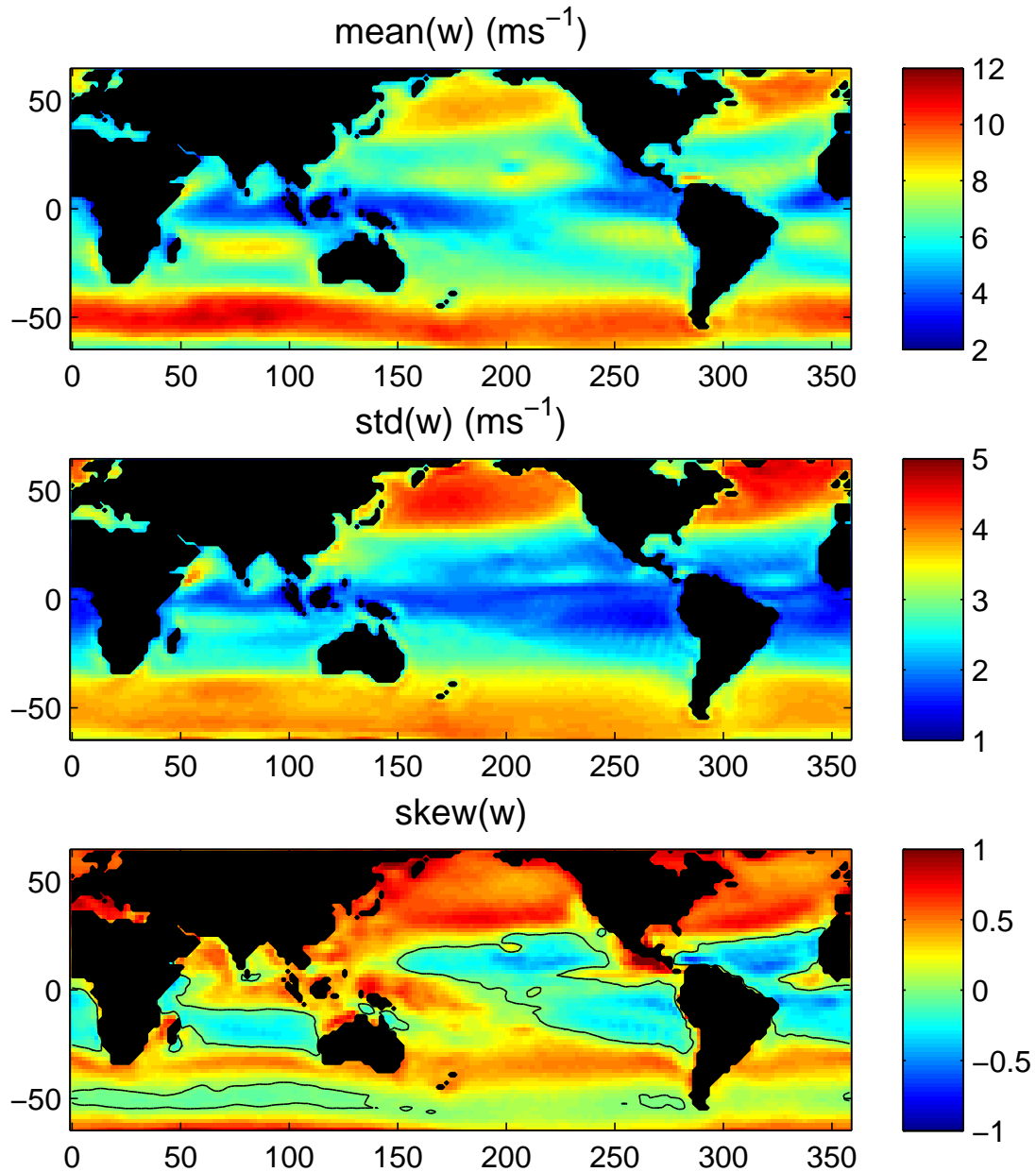


# Mean and Skewness of Vector Wind

- Joint pdfs of mean and skew for zonal and meridional winds

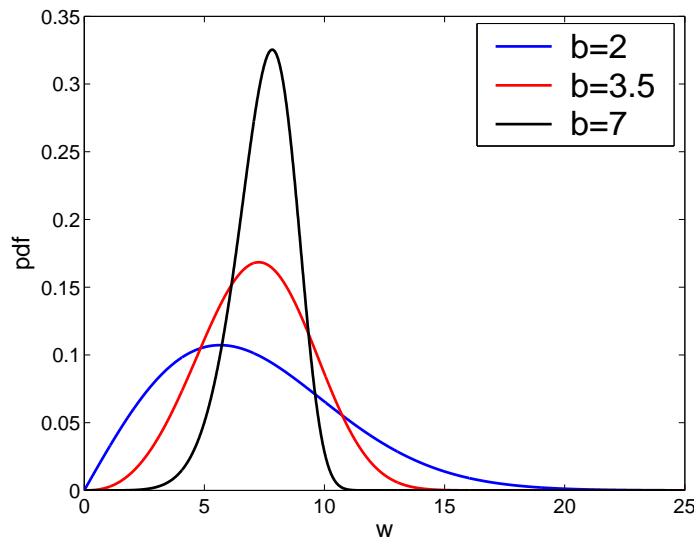


# Wind Speed Moments



# Wind Speed pdf: Weibull distribution

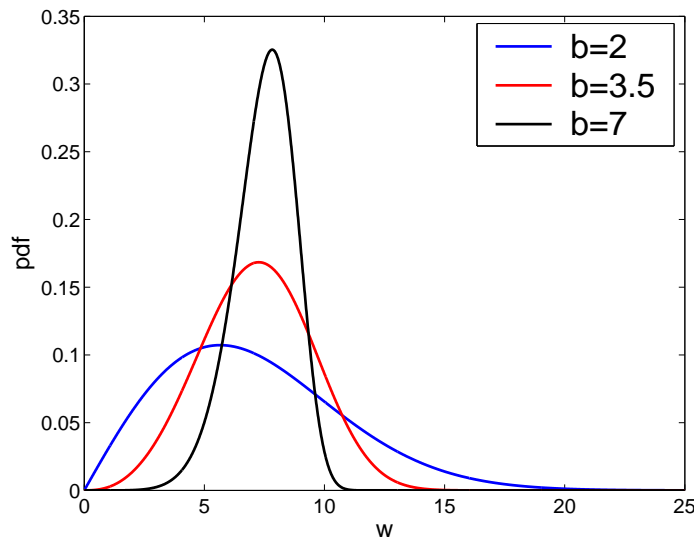
- The pdf of wind speed  $w$  has traditionally (and empirically) been represented by 2-parameter Weibull distribution:



$$p(w) = \frac{b}{a} \left(\frac{w}{a}\right)^{b-1} \exp \left[ - \left(\frac{w}{a}\right)^b \right]$$
$$\text{skew}(w) = S(b)$$
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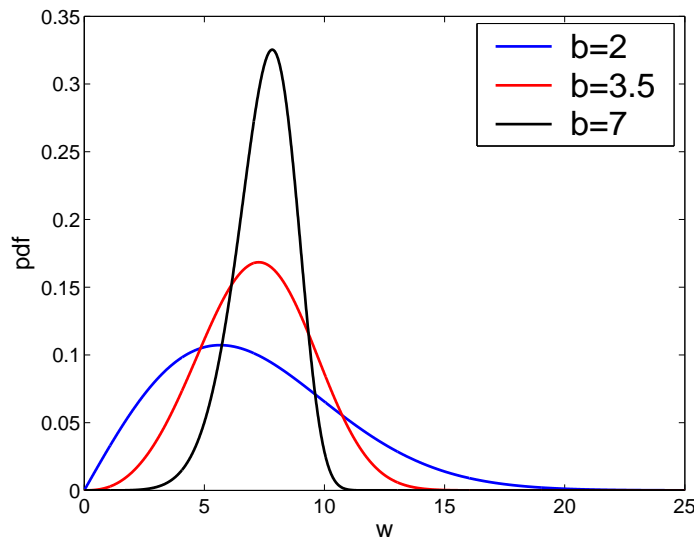
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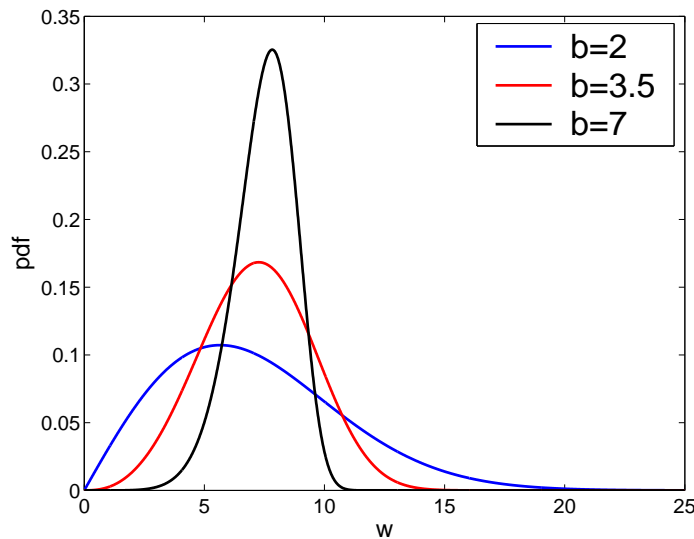
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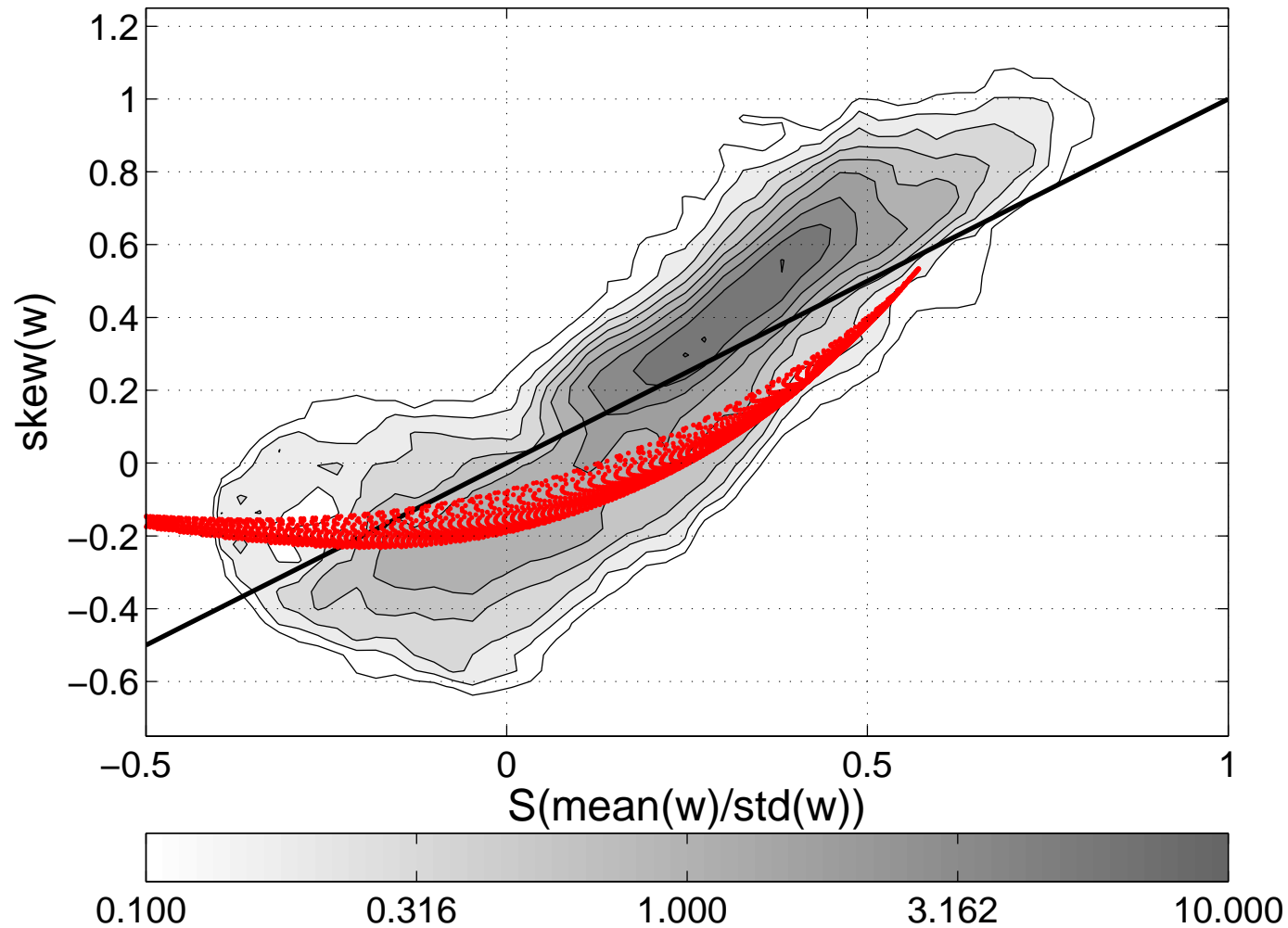
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- $a$  is the scale parameter (pdf centre)
- $b$  is the shape parameter (pdf tilt)
- $p_w(w)$  is unimodal

# Wind speed pdfs: observed and modelled



Black line: Weibull , Red dots: idealised BL momentum budget model

# Notation

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$u$  = along-mean wind component

$v$  = cross-mean wind component

$\mathbf{u}$  =  $(u, v)$

$w$  = wind speed

$h$  = boundary layer depth

$T$  = SAT-SST

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  - surface drag (nonlinear)
  - downwards mixing of momentum from aloft (constant)
  - “ageostrophic forcing” (mean and fluctuating)
- Momentum budget averaged over BL (neglecting horiz. advection)

$$\begin{aligned}
 \frac{du}{dt} &= \underbrace{\frac{\overline{U}_s}{\tau_s}}_A + \underbrace{\frac{\eta_u}{\tau_s}}_B - \underbrace{\frac{c_d(w, T)}{h} wu}_C + \underbrace{\frac{w_e}{h} (U(h + \delta) - u)}_D + \underbrace{\sigma_u \dot{W}_1}_E \\
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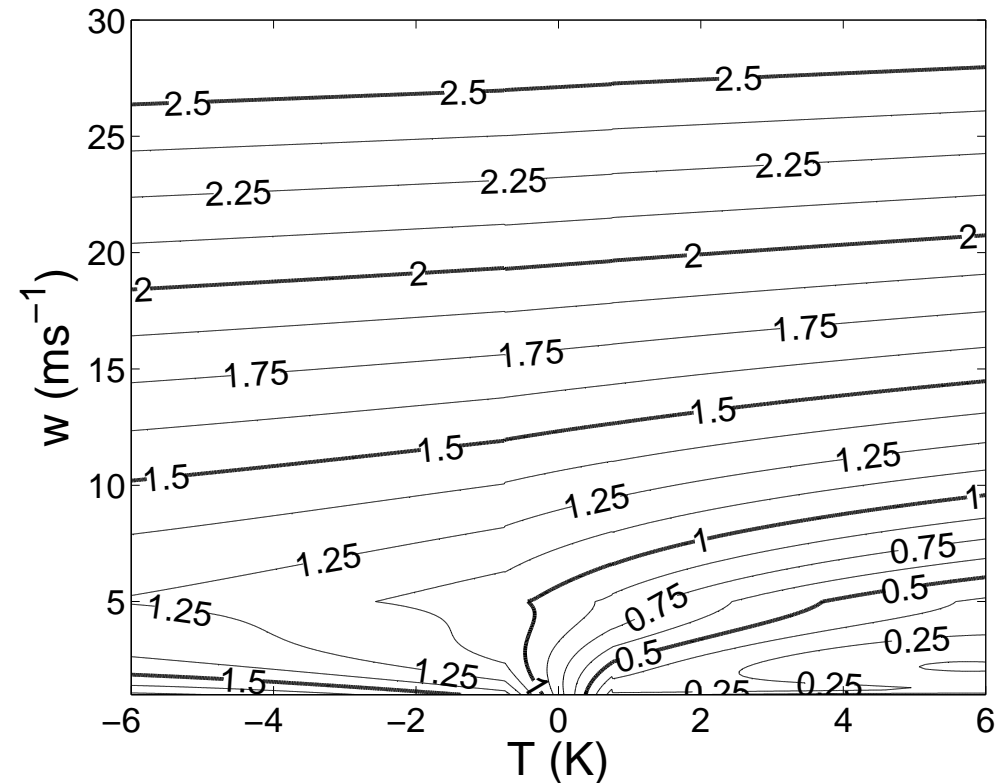
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COARE Parameterisation



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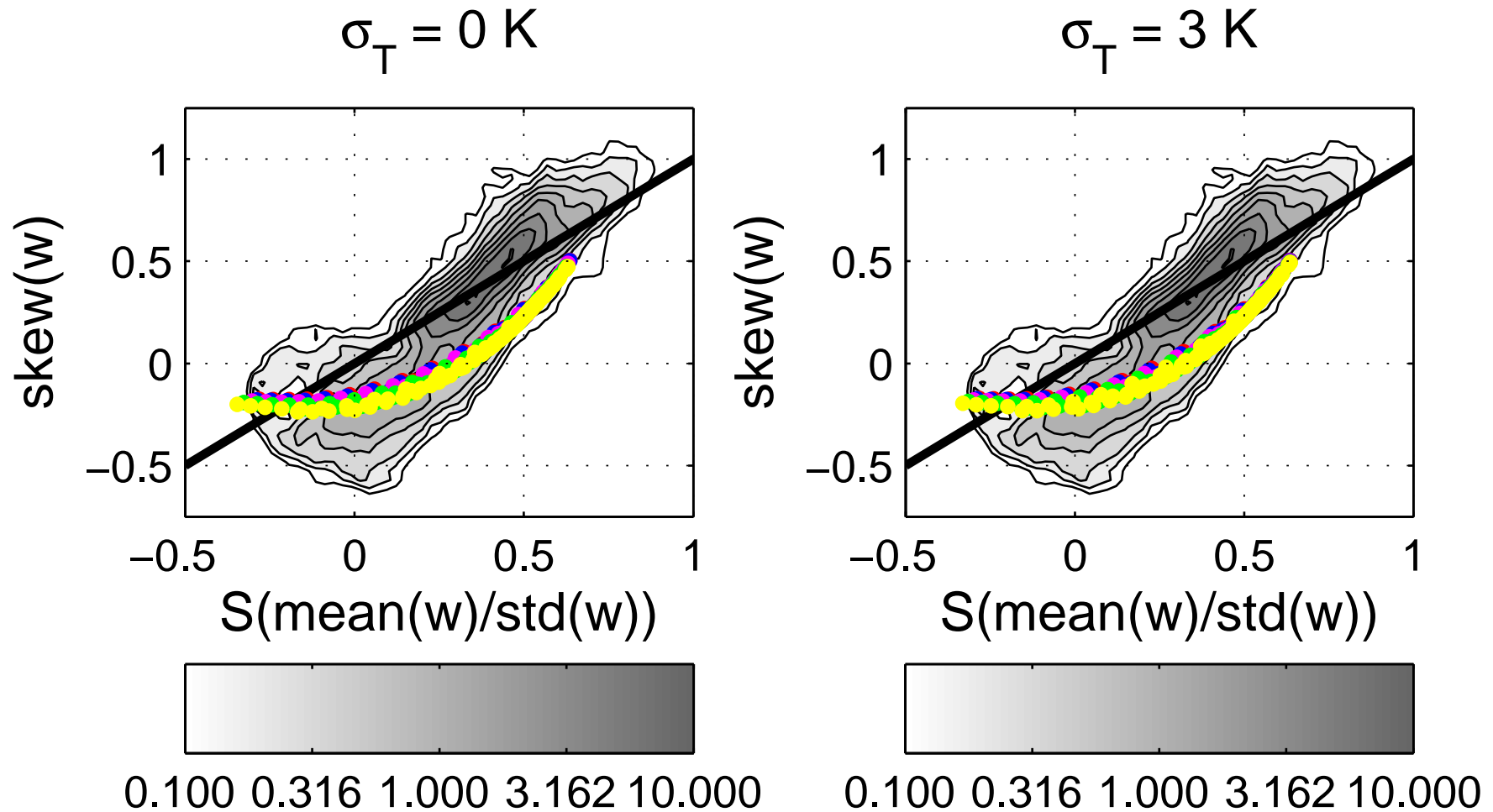
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# Wind stress - SST coupling: observations

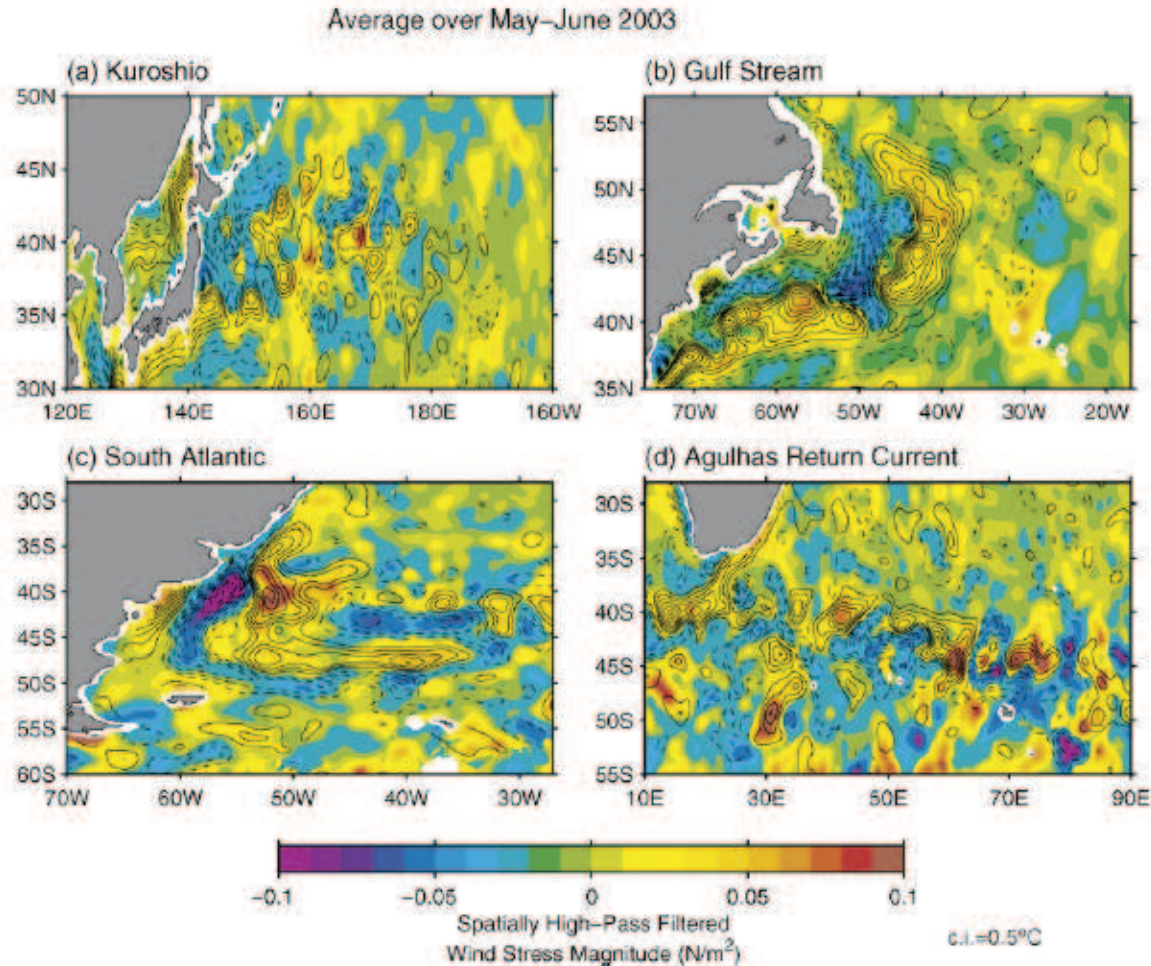


Fig. 2. Maps of spatially high-pass filtered 2 months (May–June 2003) average wind stress magnitude ( $\text{Nm}^{-2}$ , color) and SST ( $^{\circ}\text{C}$ , contours, interval  $0.5^{\circ}\text{C}$ , zero contour omitted). Data from QuikSCAT scatterometer and AMSR-E. (a) North-west Pacific, Kuroshio region, (b) North-west Atlantic, Gulf Stream and North Atlantic Current region, (c) South-west Atlantic, Brazil-Malvinas confluence, and (d) Southern Indian Ocean, Agulhas Return Current.



UVic

Warm SST  $\Rightarrow$  faster winds (e.g. Small et al., Dyn. Atmos. Oceans, 2008)

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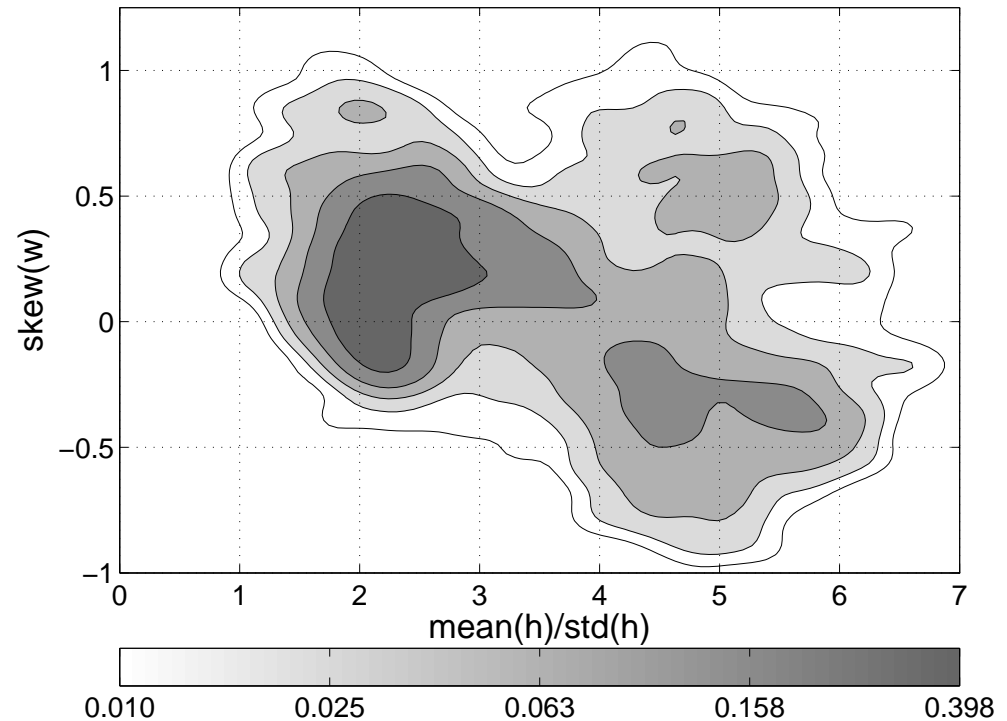
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From ERA-40 Reanalysis

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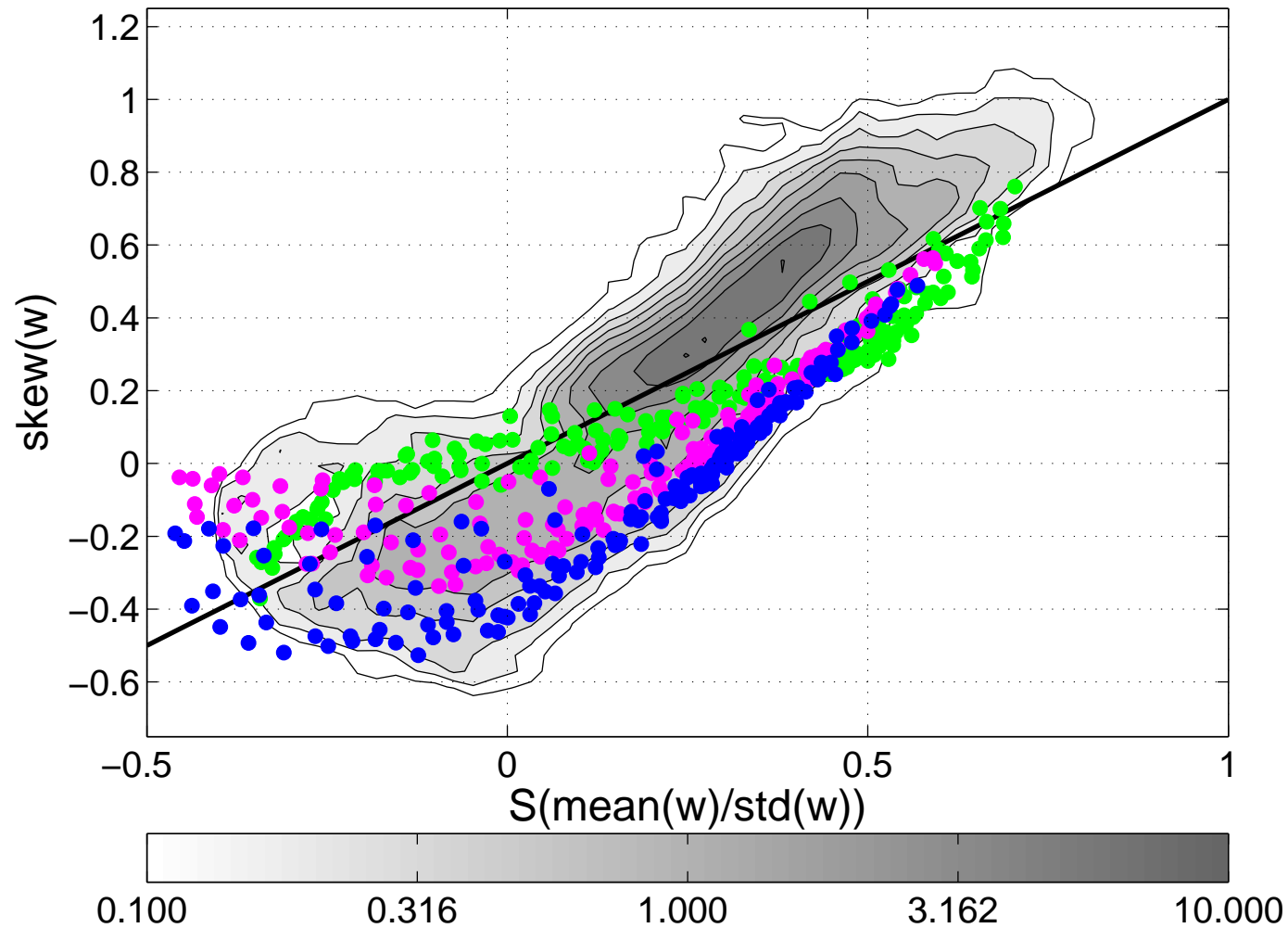
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$$\mathbf{U}(z, t) = \mathbf{u}(t) \quad z < h$$

and relax toward specified shear in free atmosphere

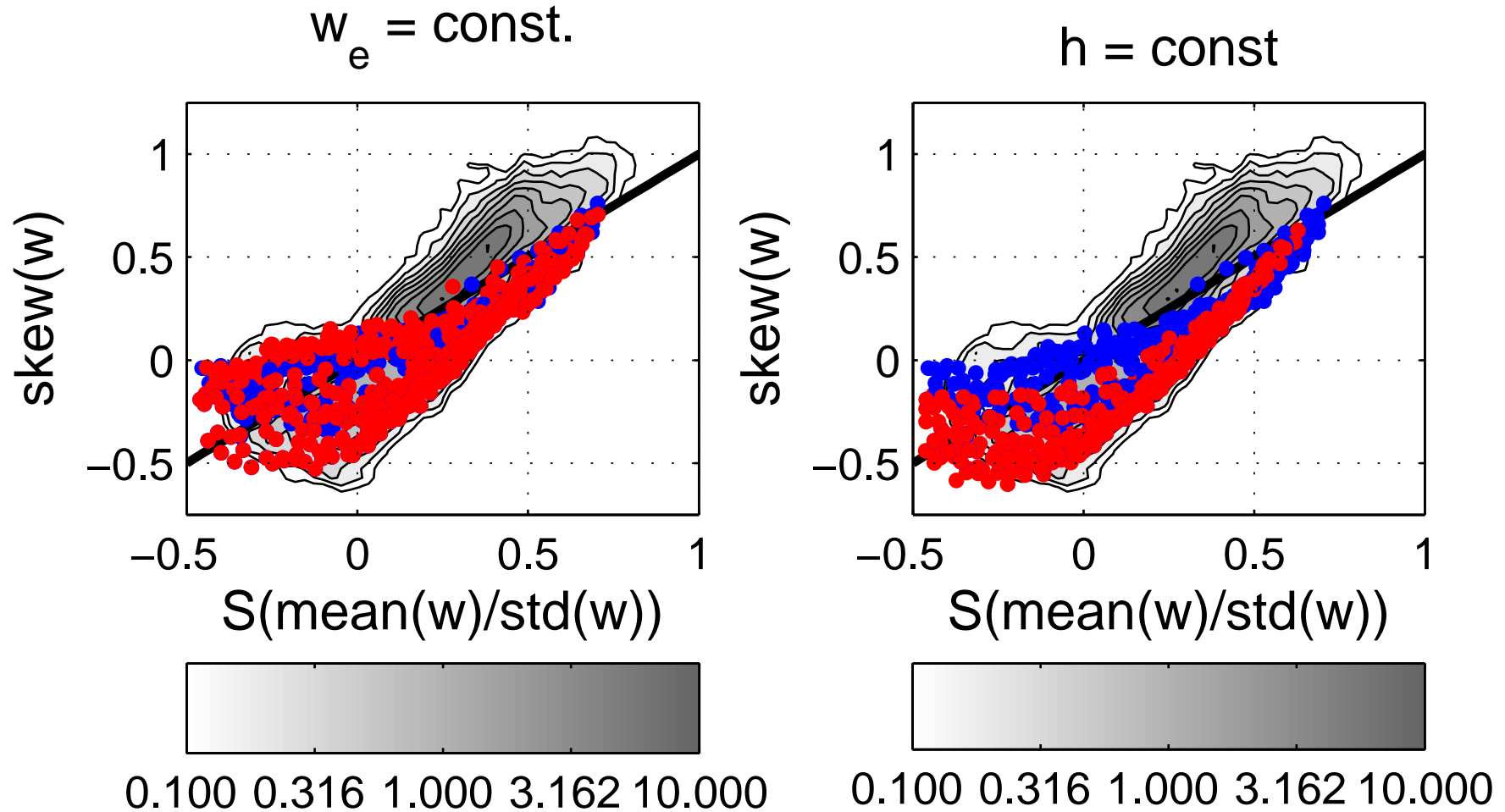
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# Wind speed pdf: variable BL effects



$\text{std}(h) = 0 \text{ m}$  (blue), 200 m (magenta), 400m (green)

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- Need for more complex BL model; goal of future research

