Workshop 08W5091 "Quantum chaos: routes to RMT and beyond"

Gregory Berkolaiko (Texas A&M University, TX 77845, USA)
Robert S. Whitney (Institut Laue-Langevin, 38042 Grenoble, France)
Uzy Smilansky (Weizmann Institute of Science, Rehovot 76100, Israel)

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1 Summary

The objective of this workshop was to bring together applied mathematicians and theoretical physicists in order to address the following question: *what happens to chaos theory when it meets quantum mechanics?* We worked to understand why the statistical properties of quantum systems whose classical analogues are chaotic are often well-described by random matrix theory (RMT). We then looked beyond RMT, to systems which are now known to have non-RMT properties.

These issues are at the centre of the field known as *quantum chaos*, and large steps have recently been taken towards addressing them in theoretical physics. This in turn is leading to a burst of activity in applied mathematics, both in terms of trying to put the physicists' theories on a mathematically rigorous footing, and in terms of addressing a whole family of interesting combinatorial problems that arise naturally in the context of those theories. Thus the objective of the workshop was to review the state of this rapidly changing field and to stimulate future progress.

2 Overview of quantum chaos

Chaos theory and Newton's laws. Chaos theory has revolutionized our view of how complexity emerges from very simple rules. Simple rules can generate fractals, such as the Mandelbrot set, which are sets that have detailed and complex structure at whatever scale one looks at them. One aspect of the fractals' complexity is their non-integer dimension.

A marble rolling around in a bath-tub obeys Newton's laws of motion ($F = ma$, etc). It has been shown that these simple rules can cause the marble to follow a chaotic path. Chaotic paths are exponentially sensitive to the initial position and velocity of the marble. Suppose we record the path followed by a marble in the bath with a given initial position and velocity. We can then take the marble and try to make it repeat the same path. If we get the initial position and velocity *exactly* the same as before, then the marble will follow the same path. However tiny differences in the initial position or velocity will result in the marble diverging *exponentially* from the original path. This exponential sensitivity to initial conditions (which we never known exactly) makes specific predictions impossible. It means, that even the smallest of variations in the initial data makes the outcome random. Thus much of chaos theory is dedicated to predicting the probabilities of an event occurring; much like weather forecasts which say there will be a 10% probability of rain next Tuesday (the weather is a good example of a chaotic system).

Whether the paths of a marble in a bath tub are chaotic or not depends on the shape of the bath. For example, in a circular bath-tub, the motion is regular. However, for many other simple shapes, the paths turn out to be chaotic. In fact, the chaoticity is *generic* in the sense that small perturbations to the shape of a chaotic domain do not make paths regular, while arbitrarily small perturbations to a regular domain can make the paths chaotic. We stress that when talking about "chaotic domain" we do not mean that the shape of the domain is irregular. Rather, "chaotic" refers to the properties of the resulting motion.

Fractals can also arise in the context of marble's motion in a bath-tub. A simple example is the following. Imagine having two plug-holes; one at each end of the bath. We mark a line in the bottom of the bath, place the marble somewhere on that line and flick it in a certain direction. The marble will roll around in the bath and eventually fall into one of the plug-holes. Suppose you plot a graph, putting the initial position on the line on one axis and the direction the marble was flicked on the other axis. We then color *red* all points on the graph for which the marble ends up falling in one plug-hole, and color *blue* all points for which the marble falls into the other plug-hole. The plot we end up with will be a fractal, it will have structure on all scales. The large scales are for initial conditions which correspond to the marble falling quickly into a plug-hole. The small scale structures are for initial conditions which correspond to the marble rolling around for a long time without falling into a plug-hole.

Wave-particle duality. Quantum mechanics is the theory of wave-particle duality; it tells us that all objects actually move as waves. The wave spreads out and goes to many places at once. If part of that wave crosses another part (perhaps after reflecting off something) then the two parts of the wave interfere (like light interferes in the well-known Young's double-slit experiment). The probability to find the particle in the places where destructive interference (cancellations) occurs is then greatly reduced compared to the places of no interference and espesially places where two waves enhance each other. Waves can also go around and through small obstacles because they rarely have a well-defined position or energy. The position-momentum uncertainty principle means waves can diffract around obstacles, while the energy-time uncertainty principle means they can tunnel through obstacles.

Remarkably, waves often look like particles if one looks at them on a scale much greater than their wavelength. Visible light (a wave with a wavelength of a few hundred nanometres) reflects off a surface in the same way as a marble would bounce off that surface (i.e. the angle of reflection equals the angle of incidence). This is why Newton's laws work well for relatively heavy objects, whose wavelength is too tiny to ever be observed. Even a marble is heavy enough so that when moving with a velocity of just a few mm per second, it has a wavelength of about 10^{-37}m, which is much too small to be ever observed. However it does not work for lightweight (or massless) objects such as electrons, photons, etc, whose wavelength is often observable. Photons have wavelengths from hundreds of metres (radiowaves) to a few hundred nanometres (visible light) and smaller (x-rays, gamma-rays, etc); electrons in semiconductors can have wavelengths of a few tens of nanometres. Quantum mechanics tells us that we will clearly see all of the wave properties (interference, tunneling, etc) of any object, if we are able to zoom in to the scale of that object's de Broglie wavelength.

Chaos theory meeting quantum mechanics. Suppose the marble in the bath-tub is replaced by an electron or photon in a bath-shaped container. What happens when we start studying the behavior of the particle at the scale of its wavelength? At this scale the particles move much like ripples do (imagine filling the bath-tub with water and making small ripples on the surface with your finger). So one *cannot* use Newton's laws of motion to predict the particle's behavior.

The first thing one notices is that the ripples are smooth at scales much smaller than their wavelength. This means that in quantum mechanics there are no fractals. One can have a plot that looks like a fractal at large scales, but once zoomed in to less than a wavelength the plot becomes smooth and featureless. The next thing one notices is that the structure at the scale of a wavelength is very complicated and unlike that predicted by applying chaos theory to Newton's laws. The central topic of our workshop was the study of this complicated structure at the scale of a wavelength. We are particularly interested in the statistical properties of these complicated structures (much like the example of the weather forecast we gave above).

Level-statistics in quantum chaos. In Newtonian mechanics steady-states, when the system is at rest, and periodic motion are of particular importance. In quantum mechanics, the concept of periodicity gives rise to the discrete energy levels (*eigenvalues*) of a quantum system. Indeed the name "quantum mechanics" itself is coming from the observation that atoms absorb and emit energy (typically light) in discrete portions, "quanta", and, therefore, only certain values of energy are allowed. The set of allowed energies is called the *spectrum* of a system. The *eigenfunctions* (the steady-states of the quantum system) describe the quantum system when it is in a state with a particular (allowed) energy. A quantum system is fully specified by its spectrum and the set of all eigenfunctions. Thus the *quantum chaos* can be broadly described as a study of generic properties of the eigenvalues and eigenfunctions of a quantum system which, in the limit of short wavelength, exhibits many features associated with chaotic motion.

One of the central themes of quantum chaos research has been to explain the so-called Bohigas-Giannoni-Schmit conjecture [8], made in 1984. The conjecture (based on numerical simulations of simple quantum chaotic systems) was that the statistical distribution of energy-levels in simple chaotic systems is the same as that of a suitably defined random matrix. Random matrix theory (RMT) was developed in the 1960s and has been well-studied and well-used since then; it is used to find the statistical properties of the eigenvalues of matrices whose elements are randomly distributed. The question is why a wave in a chaotically-shaped container should follow random matrix predictions.

In 1971, Gutzwiller [11] had pointed out that the Green's function of a wave in a chaotically-shaped container could be neatly expressed in terms of the stationary-phase points of a Feynman path integral. These stationary-phase points correspond to periodic classical orbits, and so the Green function's phase is given by the action of the periodic orbit. Berry ([6], 1985) then pointed out that some information about the statistical distribution of energy-levels could be obtained by considering only the diagonal terms in a double sum over such periodic orbits, and then assuming ergodicity of the classical orbits. The result of this procedure coincided with the RMT prediction for correlations between distantly spaced energy-levels (i.e. correlations between level n and level $(n+m)$ when $n, m \gg 1$), but not otherwise. Berry argued that for nearby levels (such as nearest neighbors) one could not neglect off-diagonal terms in the double sum. However despite a lot of effort, almost no progress was made in dealing with these off-diagonal terms for the next 15 years.

In 2002, Sieber and Richter [22, 21] managed to perform the first semiclassical calculation of a likely candidate for the next-order term (in an expansion in powers of the inverse distance between the distantly spaced levels). They considered the contributions from pairs of periodic orbits that consisted of two loops and that would nearly cross themselves in the phase space. If viewed from afar, the two orbits would look identical. However, they would traverse one of the loops in the same direction and the other in the opposite directions. Sieber and Richter showed that the contribution of such orbits coincided with the next term in the expansion of the RMT prediction. After this crucial step had been achieved, many researchers worked on extending these ideas to include higher orders in the expansion. Pairs of orbits constructed out of growning numbers of joined loops were considered. Finally, Müller, Heusler, Braun, Haake and Altland [17, 18] generalized the procedure to arbitrary numbers of loops, and managed to reproduce RMT to all orders in the expansion. This required the solution of a difficult problem in combinatorics since the number of ways one could connect loops to form a pair of orbits grows rapidly with the number of loops. Resummation tricks were then used to address statistics beyond the regime of convergence of the expansion, again finding RMT results. Thus in 2007 [19] they showed that this semiclassical procedure explained the Bohigas-Giannoni-Schmit conjecture.

One of the central themes of this workshop was to understand this result. It is important to appreciate that, while reproducing the conjectured result, the method is not mathematically rigorous yet. First, as for any term-by-term expansion, convergence question must be addressed. Gutzwiller trace formula is convergent only in a weak sense, so changing the order of terms can lead to differing results. A related question is whether all pairs of orbits have been accounted for (the answer is negative) and whether it is possible to extend the scheme to account for the missing terms (this question is largely open). Finally, as mentioned by N.H. Abel,[1] resummation techniques should be treated with utmost care. Answering the above questions would enable one to clearly define the regimes of validity of the method, and thereby find systems which do not have behavior which coincides with RMT.

An alternative approach to the question of eigenvalue statistics has been the super-symmetry method. In this context, it was initially use to deal with difficult averaging procedures in diordered systems. More recently it has been of great help in dealing with averages over random matrices and averages over energy in quantum graphs. In super-symmetry one makes two copies of the system, describing one with usual (commuting) variables and the other with Grassman (anti-commuting) variables. Averaging in the "doubled-system" is much easier than averaging in each one individually (due to cancellation of a "difficult" denominator).

[1] who wrote, rather strongly, that "the divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever".

However one is then forced to work with commuting and anti-commuting variables at the same time. Much progress [18] has been achieved by comparing supersymmetric expansions with periodic-orbit ones.

Quantum ergodicity and scarring. As has been mentioned above, the quantum eigenfunctions at the scales smaller than the wavelength look smooth a featureless. Zooming out to the scales comparable to the wavelength one will see complicated structure of the eigenstate. What happens when you zoom out even more? In most cases you will see a picture which is almost uniformly gray.

Coming back to the marble analogy, imagine that the marble is covered in ink and is leaving a trace on the bath-tub. If it is left to roll around for a long time (with no loss of energy due friction), it will cover the accessible areas of the bath-tub with ink. If we smudge the ink a little bit, the lines will disappear and the bath-tub will be painted with ink in a smooth way. Some areas will be darker than the others, indicating that the marble is more likely to be found there at any given time.

It is quite remarkable that if we compute the high energy eigenfunctions of the (quantized) bath-tub, and smudge the intensities of the eigenfunctions a little bit, the result will look remarkably like the classical ink picture described above. This is actually a mathematical result known as Quantum Ergodicity Theorem. It is easy to understand: when marble is quantized, it is still following pretty much the same probability laws as the Newtonian marble.

However, some exceptions to the above picture have been observed by Heller [13]. Every now and then we will come across an eigenfunction which is not uniformly gray but which has a few lines of darkness standing out of the sea of light-gray. These few lines are much like the traces of the marble ball, but a one which is going on a periodic trajectory, repeatedly painting with ink the same path. Such special eigenstates were called *scars*, since they are assumed to be traces of classical periodic orbits in the quantized picture. There are many interesting questions associated with scars, for example, whether scarred eigenstates happen infinitely often, or do they stop when the energy gets higher. To put it more bluntly, do the scars really exist? Are there any systems where there are no scars (this is called Quantum Unique Ergodicity)? Can the intensity of the scar can be arbitrarily high, compared with the intensity of the background light gray?

Recently many of these questions have been resolved mathematically, at least in toy model settings (see *Quantum Maps* section below). The workshop provided a forum for mathematicians to explain the powerful new tools involved in answering these questions and for physicists to propose more difficult questions to tackle next.

Quantum chaos in open systems. Particles can escape from an open quantum system (similarly to the marble rolling around in the bath-tub and falling into a plug-hole). Quantum mechanically such systems are not described by eigenfunctions. Instead one must think in terms of the scattering matrix. If a particle is injected with given initial conditions, the scattering matrix tells us how it will escape the system.

For many years it was thought that such systems were also well-described by random-matrix theory. However it has now become clear that as one makes the wavelength smaller, the behavior changes. Many interference effects become exponentially suppressed and as a result the scattering from such a system becomes increasingly classical. The parameter which controls this cross-over from RMT behavior to "almost classical" behavior is the Ehrenfest time. Crucially the "almost classical" behavior is *universal*, by which we mean that it is not sensitive to the details of the system that one studies (unlike other previously observed deviations from RMT).

A second theme of the workshop was to discuss these results, and try to understand the "almost classical" behavior. In particular we asked which properties would be "classical" and which would not. It is hoped that a better understanding of this non-RMT behavior could lead to the discovery of other universal but non-RMT behaviors.

3 More rigorous approaches to quantum chaos

Mathematically rigorous progress on the questions of quantum chaos has been hampered by the difficulty of the questions themselves. While a lot is known about lower-lying eignevalues, quantum chaos focuses more on the small but generic variations of the high-lying eigenvalues from the well-known Weyl asymptotics. The prime tool with which one can study high eigenvalues is the trace formula, but it is only weakly convergent, involves an exponentially growing number of periodic orbits and only approximate for most systems. Due to these difficulties, two toy models have arisen: quantum maps and quantum graph.

Quantum Maps. Exactly as classical maps supply minimal models of chaotic dynamics with only one degree of freedom, quantum maps provide the simplest models for quantal manifestation of classical chaos and for the quantum-classical correspondence in this context. Simplified trace formulae help analyze the divergencies occuring in the standard trace formulae. One can probe the long time versus the short wave-length limits of the time propagators. Numerical computation is also much facilitated by the fact that, the quantum evolution is realized as the iteration of a $N \times N$ unitary matrix with N being inversely proportional to the Planck's constant. Thus the short wave-length limit corresponds to the large sizes of the propagator matrix.

The procedures associating a unitary operator to a discrete map were initially developed in [7, 12, 2]. Among the quantized maps are the Arnold "cat" maps (linear maps of the torus) and the baker's map both of which has been favourite quantized models for the mathematical studies of quantum chaos. Among notable recent successes were explanation of the violation of the Bohigas-Giannoni-Schmit conjecture on cat maps by the presense of quantum symmetries without classical analogue (Hecke symmetries) [14], quantum unique ergodicity of the Hecke eigenstates [16], construction of scarred eigenstates of weight $1/2$ [9] and use of enthropy bounds to prove impossibility of strong scars [1]. The workshop gave an opportunity to learn more about these recent breakthroughs and discuss open problems (for example, whether generic perturbation of a cat map is quantum uniquely ergodic) and possible approaches to their solution.

Quantum graphs. The introduction of quantum graphs as models for quantum chaos in 1997 [15] made it possible to study a broad range of quantum chaotic phenomena in a mathematically rigorous setting. Among the advantages of graphs is the exact trace formula and ease of numerical computations. Furthermore, while quantized maps often have non-universal statistics (successfully explained by the presence of symmetries), the graphs with rationally independent edge lengths typically follow the random matrix predictions.

Among other things, the use of quantum graphs as models have been successful in explaining the cancellation of the off-diagonal contributions in the presence of the magnetic field [5], construction of strong scars [20], application of super-symmetric methods to BGS conjecture without a disorder averaging [10], exploration [4] of the potential mathematical problems with the Müller et al enumeration scheme [17] and studies of the nodal domain count [3]. More recently, a framework for connecting the trace formula on quantum graphs with the similar results on discrete graphs (Ihara zeta function) was proposed in [23]. During workshop we had an opportunity to interact with the specialists in the study of Ihara zeta function as well as hear an update on its connection to the study of quantum graphs.

4 Conclusions

The workshops was successfull not only in letting researchers report on their latest progress, but also in bringing together people of differing traditions (mathematicians vs physicists). The underlying motive for many discussions during the workshop was to highlight the assumptions made in physical derivations and discuss prospects for their mathematical justifications. The workshop also provided a forum for the young researchers to introduce themselves to the quantum chaos community.

References

[1] N. Anantharaman and S. Nonnenmacher. Half-delocalization of eigenfunctions for the Laplacian on an Anosov manifold. *Ann. Inst. Fourier (Grenoble)*, 57(7):2465–2523, 2007. Festival Yves Colin de Verdière.

[2] N. L. Balazs and A. Voros. The quantized baker's transformation. *Annals of Physics*, 190(1):1–31, 1989.

[3] R. Band, I. Oren, and U. Smilansky. Nodal domains on graphs — how to count them and why? In *Analysis on Graphs and Its Applications*, volume 77 of *Proceedings of Symposia in Pure Mathematics*, pages 5–27. AMS, 2008.

[4] G. Berkolaiko. Correlations within the spectrum of a large quantum graph: a diagrammatic approach. In G. Berkolaiko, R. Carlson, S. Fulling, and P. Kuchment, editors, *Proceedings of Joint Summer Research Conference on Quantum Graphs and Their Applications, 2005*. AMS, 2006.

[5] G. Berkolaiko, H. Schanz, and R. S. Whitney. Form factor for a family of quantum graphs: an expansion to third order. *J. Phys. A*, 36(31):8373–8392, 2003.

[6] M. V. Berry. Semiclassical theory of spectral rigidity. *Proc. R. Soc. Lond. A*, 400:229–251, 1985.

[7] M. V. Berry, N. L. Balazs, M. Tabor, and A. Voros. Quantum maps. *Annals of Physics*, 122(1):26–63, 1979.

[8] O. Bohigas, M.-J. Giannoni, and C. Schmit. Characterization of chaotic quantum spectra and universality of level fluctuation laws. *Phys. Rev. Lett.*, 52(1):1–4, 1984.

[9] F. Faure, S. Nonnenmacher, and S. De Bièvre. Scarred eigenstates for quantum cat maps of minimal periods. *Comm. Math. Phys.*, 239(3):449–492, 2003.

[10] S. Gnutzmann and A. Altland. Universal spectral statistics in quantum graphs. *Phys. Rev. Lett.*, 93(19):194101, 2004.

[11] M. C. Gutzwiller. Periodic orbits and classical quantization conditions. *Journal of Mathematical Physics*, 12(3):343–358, 1971.

[12] J. H. Hannay and M. V. Berry. Quantization of linear maps on a torus-fresnel diffraction by a periodic grating. *Physica D*, 1(3):267–290, 1980.

[13] E. J. Heller. Bound-state eigenfunctions of classically chaotic hamiltonian systems: Scars of periodic orbits. *Phys. Rev. Lett.*, 53(16):1515–1518, 1984.

[14] J. P. Keating and F. Mezzadri. Pseudo-symmetries of Anosov maps and spectral statistics. *Nonlinearity*, 13(3):747–775, 2000.

[15] T. Kottos and U. Smilansky. Quantum chaos on graphs. *Phys. Rev. Lett.*, 79:4794–4797, 1997.

[16] P. Kurlberg and Z. Rudnick. Hecke theory and equidistribution for the quantization of linear maps of the torus. *Duke Math. J.*, 103(1):47–77, 2000.

[17] S. Müller, S. Heusler, P. Braun, F. Haake, and A. Altland. Semiclassical foundation of universality in quantum chaos. *Phys. Rev. Lett.*, 93(1):014103, 2004.

[18] S. Müller, S. Heusler, P. Braun, F. Haake, and A. Altland. Periodic-orbit theory of universality in quantum chaos. *Phys. Rev. E*, 72(4):046207, 2005.

[19] S. Heusler, S. Müller, A. Altland, P. Braun, and F. Haake. Periodic-Orbit Theory of Level Correlations. *Phys. Rev. Lett.*, 98:044103, 2007.

[20] H. Schanz and T. Kottos. Scars on quantum networks ignore the Lyapunov exponent. *Phys. Rev. Lett.*, 90, 2003.

[21] M. Sieber. Leading off-diagonal approximation for the spectral form factor for uniformly hyperbolic systems. *J. Phys. A*, 35(42):L613–L619, 2002.

[22] M. Sieber and K. Richter. Correlations between periodic orbits and their role in spectral statistics. *Physica Scripta*, T90:128–133, 2001.

[23] U. Smilansky. Quantum chaos on discrete graphs. *J. Phys. A: Math. Theor.*, 40(27):F621–F630, 2007.

5 Participants contributions

- Michael Aizenman (Princeton, USA)
  On dynamical localization in the linear and non-linear setup

  Preprint– arXiv:0809.3436
  We consider the spectral and dynamical properties of quantum systems of n particles on the lattice Z^d, of arbitrary dimension, with a Hamiltonian which in addition to the kinetic term includes a random potential with iid values at the lattice sites and a finite-range interaction. Two basic parameters of the model are the strength of the disorder and the strength of the interparticle interaction. It is established here that for all n there are regimes of high disorder, and/or weak enough interactions, for which the system exhibits spectral and dynamical localization. The localization is expressed through bounds on the transition amplitudes, which are uniform in time and decay exponentially in the Hausdorff distance in the configuration space. The results are derived through the analysis of fractional moments of the n-particle Green function, and related bounds on the eigenfunction correlators.

- Ram Band (Weizmann, Israel)
  Spectral quantum graphs and beyond: a construction method

  Preprint– arXiv:0711.3416
  The purpose of the present manuscript is to collect known results and present some new ones relating to nodal domains on graphs, with special emphasize on nodal counts. Several methods for counting nodal domains will be presented, and their relevance as a tool in spectral analysis will be discussed.

- Harold Baranger (Duke, USA)
  Interactions in Quantum Dots: Does the RMT/Random-Wave Model Work?

  Preprint– arXiv:0707.1620
  We obtain analytic formulae for the spacing between conductance peaks in the Coulomb blockade regime, based on the universal Hamiltonian model of quantum dots. New random matrix theory results are developed in order to treat correlations between two and three consecutive spacings in the energy level spectrum. These are generalizations of the Wigner surmise for the probability distribution of single level spacing. The analytic formulae are shown to be in good agreement with numerical evaluation.

- Gregory Berkolaiko (Texas A&M University, USA)
  Form-factor expansion: some forgotten orbits

  Preprint– arXiv:nlin/0604025
  The form factor of a quantum graph is a function measuring correlations within the spectrum of the graph. It can be expressed as a double sum over the periodic orbits on the graph. We propose a scheme which allows one to evaluate the periodic orbit sum for a special family of graphs and thus to recover the expression for the form factor predicted by the Random Matrix Theory. The scheme, although producing the expected answer, undercounts orbits of a certain structure, raising doubts about an analogous summation recently proposed for quantum billiards.

- Eugene Bolomolny (Paris-Sud, France)
  Spectral statistics of a pseudo-integrable map: the general case

- Oriol Bohigas (Paris-Sud, France)
  Some extreme value statistics problems in RMT

  Preprint– arXiv:0808.2434
  In random matrix theory (RMT), the Tracy-Widom (TW) distribution describes the behavior of the largest eigenvalue. We consider here two models in which TW undergoes transformations. In the first one disorder is introduced in the Gaussian ensembles by superimposing an external source of randomness. A competition between TW and a normal (Gaussian) distribution results, depending on the spreading of the disorder. The second model consists in removing at random a fraction of (correlated) eigenvalues of a random matrix. The usual formalism of Fredholm determinants extends

naturally. A continuous transition from TW to the Weilbull distribution, characteristc of extreme values of an uncorrelated sequence, is obtained.

- Jens Bolte (Royal Holloway, UK)
  Semiclassical theory of mesoscopic transport with spin-orbit interactions

  Preprint– arXiv:0704.2702
  We investigate the influence of spin-orbit interaction on ballistic transport through chaotic cavities by using semiclassical methods. Our approach is based on the Landauer formalism and the Fisher-Lee relations, appropriately generalized to spin-orbit interaction, and a semiclassical representation of Green functions. We calculate conductance coefficients by exploiting ergodicity and mixing of suitably combined classical spin-orbit dynamics, and making use of the Sieber-Richter method and its most recent extensions. That way we obtain weak anti-localization and confirm previous results obtained in the symplectic ensemble of Random Matrix Theory.

- Petr Braun (Essen, Germany)
  Transport through chaotic cavities: RMT reproduced from semiclassics

  Preprint– arXiv:cond-mat/0610560
  We describe a semiclassical method to calculate universal transport properties of chaotic cavities. While the energy-averaged conductance turns out governed by pairs of entrance-to-exit trajectories, the conductance variance, shot noise and other related quantities require trajectory quadruplets; simple diagrammatic rules allow to find the contributions of these pairs and quadruplets. Both pure symmetry classes and the crossover due to an external magnetic field are considered.

- Piet Brouwer (Cornell, USA)
  Anderson localization from classical trajectories

  Preprint– arXiv:0802.0976
  We show that Anderson localization in quasi-one dimensional conductors with ballistic electron dynamics, such as an array of ballistic chaotic cavities connected via ballistic contacts, can be understood in terms of classical electron trajectories only. At large length scales, an exponential proliferation of trajectories of nearly identical classical action generates an abundance of interference terms, which eventually leads to a suppression of transport coefficients. We quantitatively describe this mechanism in two different ways: the explicit description of transition probabilities in terms of interfering trajectories, and an hierarchical integration over fluctuations in the classical phase space of the array cavities.

- John Chalker (Oxford, UK)
  Network models for the quantum Hall effect and its generalisations

  Preprint– arXiv:cond-mat/0201080
  We consider network models for localisation problems belonging to symmetry class C. This symmetry class arises in a description of the dynamics of quasiparticles for disordered spin-singlet superconductors which have a Bogoliubov - de Gennes Hamiltonian that is invariant under spin rotations but not under time-reversal. Our models include but also generalise the one studied previously in the context of the spin quantum Hall effect. For these systems we express the disorder-averaged conductance and density of states in terms of sums over certain classical random walks, which are self-avoiding and have attractive interactions. A transition between localised and extended phases of the quantum system maps in this way to a similar transition for the classical walks. In the case of the spin quantum Hall effect, the classical walks are the hulls of percolation clusters, and our approach provides an alternative derivation of a mapping first established by Gruzberg, Read and Ludwig, ! Phys. Rev. Lett. 82, 4254 (1999).

- Doron Cohen (Ben-Gurion, Israel)
  The conductance of small mesoscopic disordered rings: resistor network analysis of novel sparse and textured matrices

  Preprint– arXiv:0712.0439
  The calculation of the conductance of disordered rings requires a theory that goes beyond the Kubo-

Drude formulation. Assuming "mesoscopic" circumstances the analysis of the electro-driven transitions show similarities with a percolation problem in energy space. We argue that the texture and the sparsity of the perturbation matrix dictate the value of the conductance, and study its dependence on the disorder strength, ranging from the ballistic to the Anderson localization regime. An improved sparse random matrix model is introduced to captures the essential ingredients of the problem, and leads to a generalized variable range hopping picture.

- Sven Gnutzman (Nottingham, UK)
  Quantum Graphs: From Periodic Orbits to Phase Disorder
  Preprint– arXiv:nlin/0508009
  We investigate the spectral properties of chaotic quantum graphs. We demonstrate that the 'energy'–average over the spectrum of individual graphs can be traded for the functional average over a supersymmetric non–linear σ–model action. This proves that spectral correlations of individual quantum graphs behave according to the predictions of Wigner–Dyson random matrix theory. We explore the stability of the universal random matrix behavior with regard to perturbations, and discuss the crossover between different types of symmetries.

- Fritz Haake (Essen, Germany)
  Generating function for level correlations in chaotic systems, semiclassical evaluation
  Preprint– arXiv:nlin/0610053
  We present a semiclassical explanation of the so-called Bohigas-Giannoni-Schmit conjecture which asserts universality of spectral fluctuations in chaotic dynamics. We work with a generating function whose semiclassical limit is determined by quadruplets of sets of periodic orbits. The asymptotic expansions of both the non-oscillatory and the oscillatory part of the universal spectral correlator are obtained. Borel summation of the series reproduces the exact correlator of random-matrix theory.

- Jon Harrison (Baylor, USA)
  The effect of spin in the spectral statistics of quantum graphs
  Preprint– arXiv:0712.0869
  The article surveys quantization schemes for metric graphs with spin. Typically quantum graphs are defined with the Laplace or Schrodinger operator which describe particles whose intrinsic angular momentum (spin) is zero. However, in many applications, for example modeling an electron (which has spin-1/2) on a network of thin wires, it is necessary to consider operators which allow spin-orbit interaction. The article presents a review of quantization schemes for graphs with three such Hamiltonian operators, the Dirac, Pauli and Rashba Hamiltonians. Comparing results for the trace formula, spectral statistics and spin-orbit localization on quantum graphs with spin Hamiltonians.

- Philippe Jacquod (Arizona, USA)
  Quantum chaos in mesoscopic superconductivity
  Preprint– arXiv:0712.2252
  We investigate the conductance through and the spectrum of ballistic chaotic quantum dots attached to two s-wave superconductors, as a function of the phase difference ϕ between the two order parameters. A combination of analytical techniques – random matrix theory, Nazarov's circuit theory and the trajectory-based semiclassical theory – allows us to explore the quantum-to-classical crossover in detail. When the superconductors are not phase-biased, $\phi = 0$, we recover known results that the spectrum of the quantum dot exhibits an excitation gap, while the conductance across two normal leads carrying $N_{\rm N}$ channels and connected to the dot via tunnel contacts of transparency $\Gamma_{\rm N}$ is $\propto \Gamma_{\rm N}^2 N_{\rm N}$. In contrast, when $\phi = \pi$, the excitation gap closes and the conductance becomes $G \propto \Gamma_{\rm N} N_{\rm N}$ in the universal regime. For $\Gamma_{\rm N} \ll 1$, we observe an order-of-magnitude enhancement of the conduct! ance towards $G \propto N_{\rm N}$ in the short-wavelength limit. We relate this enhancement to resonant tunneling through a macroscopic number of levels close to the Fermi energy. Our predictions are corroborated by numerical simulations.

- Dubi Kelmer (IAS Princeton, USA)
  Scarring on invariant manifolds for quantum maps on the torus

Preprint– arXiv:0801.2493
We previously introduced a family of symplectic maps of the torus whose quantization exhibits scarring on invariant co-isotropic submanifolds. The purpose of this note is to show that in contrast to other examples, where failure of Quantum Unique Ergodicity is attributed to high multiplicities in the spectrum, for these examples the spectrum is (generically) simple.

- Massimo Macucci (Pisa, Italy)
  Shot Noise Suppression in Single and Multiple Chaotic Cavities: the Role of Diffraction, Disorder and Symmetries

  Preprint– arXiv:0802.4329
  We report the results of an analysis, based on a simple quantum-mechanical model, of shot noise suppression in a structure containing cascaded tunneling barriers. Our results exhibit a behavior that is in sharp contrast with existing semiclassical models predicting a limit of 1/3 for the Fano factor as the number of barriers is increased. The origin of this discrepancy is investigated and attributed to the presence of localization on the length scale of the mean free path, as a consequence of 1-dimensional disorder, while no localization appears in common semiclassical models. The results of the quantum model seem to be compatible with the experimentally observed behavior.

- Sebastian Müller (Cambridge, UK)
  Constructing a sigma model from semiclassics

  Preprint– arXiv:nlin/0610053
  We present a semiclassical explanation of the so-called Bohigas-Giannoni-Schmit conjecture which asserts universality of spectral fluctuations in chaotic dynamics. We work with a generating function whose semiclassical limit is determined by quadruplets of sets of periodic orbits. The asymptotic expansions of both the non-oscillatory and the oscillatory part of the universal spectral correlator are obtained. Borel summation of the series reproduces the exact correlator of random-matrix theory.

- Taro Nagao (Nagoya, Japan)
  Parametric Spectral Correlation with Spin 1/2

  Preprint– arXiv:0707.2276
  The spectral correlation of a chaotic system with spin 1/2 is universally described by the GSE (Gaussian Symplectic Ensemble) of random matrices in the semiclassical limit. In semiclassical theory, the spectral form factor is expressed in terms of the periodic orbits and the spin state is simulated by the uniform distribution on a sphere. In this paper, instead of the uniform distribution, we introduce Brownian motion on a sphere to yield the parametric motion of the energy levels. As a result, the small time expansion of the form factor is obtained and found to be in agreement with the prediction of parametric random matrices in the transition within the GSE universality class. Moreover, by starting the Brownian motion from a point distribution on the sphere, we gradually increase the effect of the spin and calculate the form factor describing the transition from the GOE (Gaussian Orthogonal Ensemble) class to the GSE class.

- Shinsuke Nishigaki (Shimane, Japan)
  Critical level statistics and QCD phase transition

- Stephane Nonnenmacher (Saclay, France)
  Quantum symbolic dynamics

  Preprint– arXiv:0805.4137
  The subject area referred to as ”wave chaos”, ”quantum chaos” or ”quantum chaology” has been investigated mostly by the theoretical physics community in the last 30 years. The questions it raises have more recently also attracted the attention of mathematicians and mathematical physicists, due to connections with number theory, graph theory, Riemannian, hyperbolic or complex geometry, classical dynamical systems, probability etc. After giving a rough account on ”what is quantum chaos?”, I intend to list some pending questions, some of them having been raised a long time ago, some others more recent.

- Marcel Novaes (Bristol, UK)
  Counting statistics of quantum chaotic cavities from classical action correlations

  Preprint– arXiv:cond-mat/0703803
  We present a trajectory-based semiclassical calculation of the full counting statistics of quantum transport through chaotic cavities, in the regime of many open channels. Our method to obtain the mth moment of the density of transmission eigenvalues requires two correlated sets of m classical trajectories, therefore generalizing previous works on conductance and shot noise. The semiclassical results agree, for all values of m, with the corresponding predictions from random matrix theory.

- Cyril Petitjean (Regensburg, Germany)
  Dephasing in quantum chaotic transport (a semiclassical approach)

  Preprint– arXiv:0710.5137
  We investigate the effect of dephasing/decoherence on quantum transport through open chaotic ballistic conductors in the semiclassical limit of small Fermi wavelength to system size ratio, $\lambda_F/L << 1$. We use the trajectory-based semiclassical theory to study a two-terminal chaotic dot with decoherence originating from: (i) an external closed quantum chaotic environment, (ii) a classical source of noise, (iii) a voltage probe, i.e. an additional current-conserving terminal. We focus on the pure dephasing regime, where the coupling to the external source of dephasing is so weak that it does not induce energy relaxation. In addition to the universal algebraic suppression of weak localization, we find an exponential suppression of weak-localization $\propto \exp[-\tilde{\tau}/\tau_\phi]$, with the dephasing rate τ_ϕ^{-1}. The parameter $\tilde{\tau}$ depends strongly on the source of dephasing. For a voltage probe, $\tilde{\tau}$ is of order the Ehrenfest time $\propto \ln[L/\lambda_F]$. In contrast, for a chaotic environment or a classical source of noise, it has the correlation length ξ of the coupling/noise potential replacing the Fermi wavelength λ_F. We explicitly show that the Fano factor for shot noise is unaffected by decoherence. We connect these results to earlier works on dephasing due to electron-electron interactions, and numerically confirm our findings.

- Saar Rahav (Maryland, USA)
  The classical limit of quantum transport

  Preprint– arXiv:0705.2337
  Quantum corrections to transport through a chaotic ballistic cavity are known to be universal. The universality not only applies to the magnitude of quantum corrections, but also to their dependence on external parameters, such as the Fermi energy or an applied magnetic field. Here we consider such parameter dependence of quantum transport in a ballistic chaotic cavity in the semiclassical limit obtained by sending Planck's constant to zero without changing the classical dynamics of the open cavity. In this limit quantum corrections are shown to have a universal parametric dependence which is not described by random matrix theory.

- Stefan Rotter (Yale, USA)
  Diffractive paths for weak localization in quantum billiards

  Preprint– arXiv:0709.3210
  We study the weak localization effect in quantum transport through a clean ballistic cavity with regular classical dynamics. We address the question which paths account for the suppression of conductance through a system where disorder and chaos are absent. By exploiting both quantum and semiclassical methods, we unambiguously identify paths that are diffractively backscattered into the cavity (when approaching the lead mouths from the cavity interior) to play a key role. Diffractive scattering couples transmitted and reflected paths and is thus essential to reproduce the weak-localization peak in reflection and the corresponding anti-peak in transmission. A comparison of semiclassical calculations featuring these diffractive paths yields good agreement with full quantum calculations and experimental data. Our theory provides system-specific predictions for the quantum regime of few open lead modes and can be expected to be relevant also for mixed as well as chaotic sys! tems.

- Henning Schomerus (Lancaster, UK)
  Staggered level repulsion for lead-symmetric transport

Preprint– arXiv:0708.0690

Quantum systems with discrete symmetries can usually be desymmetrized, but this strategy fails when considering transport in open systems with a symmetry that maps different openings onto each other. We investigate the joint probability density of transmission eigenvalues for such systems in random-matrix theory. In the orthogonal symmetry class we show that the eigenvalue statistics manifests level repulsion between only every second transmission eigenvalue. This finds its natural statistical interpretation as a staggeredsuperposition of two eigenvalue sequences. For a large number of channels, the statistics for a system with a lead-transposing symmetry approaches that of a superposition of two uncorrelated sets of eigenvalues as in systems with a lead-preserving symmetry (which can be desymmetrized). These predictions are confirmed by numerical computations of the transmission-eigenvalue spacing distribution for quantum billiards and for the open kicked rotator.

- Martin Sieber (Bristol, UK)
  Periodic orbit encounters: a mechanism for trajectory correlations

  Preprint– arXiv:0711.4537

  The Wigner time delay of a classically chaotic quantum system can be expressed semiclassically either in terms of pairs of scattering trajectories that enter and leave the system or in terms of the periodic orbits trapped inside the system. We show how these two pictures are related on the semiclassical level. We start from the semiclassical formula with the scattering trajectories and derive from it all terms in the periodic orbit formula for the time delay. The main ingredient in this calculation is a new type of correlation between scattering trajectories which is due to trajectories that approach the trapped periodic orbits closely. The equivalence between the two pictures is also demonstrated by considering correlation functions of the time delay. A corresponding calculation for the conductance gives no periodic orbit contributions in leading order.

- Harold Stark (UC San Diego, USA)
  Poles of Zeta Functions of Graphs and their Covers

- Audrey Terras (UC San Diego, USA)
  What is the Riemann Hypothesis for Zeta Functions of Irregular Graphs?

- Steven Tomsovic (Washington State, USA)
  Extreme statistics of random and quantum chaotic states

  Preprint– arXiv:0708.0176

  An exact analytical description of extreme intensity statistics in complex random states is derived. These states have the statistical properties of the Gaussian and Circular Unitary Ensemble eigenstates of random matrix theory. Although the components are correlated by the normalization constraint, it is still possible to derive compact formulae for all values of the dimensionality N. The maximum intensity result slowly approaches the Gumbel distribution even though the variables are bounded, whereas the minimum intensity result rapidly approaches the Weibull distribution. Since random matrix theory is conjectured to be applicable to chaotic quantum systems, we calculate the extreme eigenfunction statistics for the standard map with parameters at which its classical map is fully chaotic. The statistical behaviors are consistent with the finite-N formulae.

- Denis Ullmo (Paris-Sud, France)
  Residual Coulomb interaction fluctuations in chaotic systems: the boundary, random plane waves, and semiclassical theory.

  Preprint– arXiv:0712.1154

  Experimental progresses in the miniaturisation of electronic devices have made routinely available in the laboratory small electronic systems, on the micron or sub-micron scale, which at low temperature are sufficiently well isolated from their environment to be considered as fully coherent. Some of their most important properties are dominated by the interaction between electrons. Understanding their behaviour therefore requires a description of the interplay between interference effects and interactions. The goal of this review is to address this relatively broad issue, and more specifically to address it from the perspective of the quantum chaos community. I will therefore present some of the concepts developed in the field of quantum chaos which have some application to study many-body effects in mesoscopic and nanoscopic systems. Their implementation is illustrated on a few examples of experimental relevance such as persistent currents, mesoscopic fluctuations of Kondo properties or Coulomb blockade. I will furthermore try to bring out, from the various physical illustrations, some of the specific advantages on more general grounds of the quantum chaos based approach.

- Jiri Vanicek (EPFL, Switzerland)
  Dephasing representation of quantum fidelity

  Preprint– arXiv:quant-ph/0506142
  General semiclassical expression for quantum fidelity (Loschmidt echo) of arbitrary pure and mixed states is derived. It expresses fidelity as an interference sum of dephasing trajectories weighed by the Wigner function of the initial state, and does not require that the initial state be localized in position or momentum. This general dephasing representation is special in that, counterintuitively, all of fidelity decay is due to dephasing and none due to the decay of classical overlaps. Surprising accuracy of the approximation is justified by invoking the shadowing theorem: twice–both for physical perturbations and for numerical errors. It is shown how the general expression reduces to the special forms for position and momentum states and for wave packets localized in position or momentum. The superiority of the general over the specialized forms is explained and supported by numerical tests for wave packets, non-local pure states, and for simple and random mixed states. ! The tests are done in non-universal regimes in mixed phase space where detailed features of fidelity are important. Although semiclassically motivated, present approach is valid for abstract systems with a finite Hilbert basis provided that the discrete Wigner transform is used. This makes the method applicable, via a phase space approach, e. g., to problems of quantum computation.

- Daniel Waltner (Regensburg, Germany)
  Semiclassical approach to quantum decay of open chaotic systems

  Preprint– arXiv:0805.3585
  We address the decay in open chaotic quantum systems and calculate semiclassical corrections to the classical exponential decay. We confirm random matrix predictions and, going beyond, calculate Ehrenfest time effects. To support our results we perform extensive numerical simulations. Within our approach we show that certain (previously unnoticed) pairs of interfering, correlated classical trajectories are of vital importance. They also provide the dynamical mechanism for related phenomena such as photo-ionization and -dissociation, for which we compute cross section correlations. Moreover, these orbits allow us to establish a semiclassical version of the continuity equation.

- Simone Warzel (Princeton, USA)
  On the joint distribution of energy levels for random Schroedinger operators

  Preprint– arXiv:0804.4231
  We consider operators with random potentials on graphs, such as the lattice version of the random Schroedinger operator. The main result is a general bound on the probabilities of simultaneous occurrence of eigenvalues in specified distinct intervals, with the corresponding eigenfunctions being separately localized within prescribed regions. The bound generalizes the Wegner estimate on the density of states. The analysis proceeds through a new multiparameter spectral averaging principle.

- Robert Whitney (Institut Laue-Langevin, Grenoble, France)
  Introduction - Ehrenfest time in scattering problems

  Preprint– arXiv:cond-mat/0512662
  Abstract: We investigate transport properties of quantized chaotic systems in the short wavelength limit. We focus on non-coherent quantities such as the Drude conductance, its sample-to-sample fluctuations, shot-noise and the transmission spectrum, as well as coherent effects such as weak localization. We show how these properties are influenced by the emergence of the Ehrenfest time scale τ_{E}. Expressed in an optimal phase-space basis, the scattering matrix acquires a block-diagonal form as τ_{E} increases, reflecting the splitting of the system into two cavities in parallel, a classical deterministic cavity (with all transmission eigenvalues either 0 or 1) and a quantum mechanical stochastic cavity. This results in

the suppression of the Fano factor for shot-noise and the deviation of sample-to-sample conductance fluctuations from their universal value. We further present a semiclassical theory for weak localization which captures non-ergodic phase! -space structures and preserves the unitarity of the theory. Contrarily to our previous claim [Phys. Rev. Lett. 94, 116801 (2005)], we find that the leading off-diagonal contribution to the conductance leads to the exponential suppression of the coherent backscattering peak and of weak localization at finite $\tau_{\rm E}$. This latter finding is substantiated by numerical magnetoconductance calculations.

- Brian Winn (Loughborough, UK)
  Quantum graphs where back-scattering is prohibited

  Preprint– arXiv:0708.0839
  We describe a new class of scattering matrices for quantum graphs in which back-scattering is prohibited. We discuss some properties of quantum graphs with these scattering matrices and explain the advantages and interest in their study. We also provide two methods to build the vertex scattering matrices needed for their construction.

- Martin Zirnbauer (Köln, Germany)
  On the Hubbard-Stratonovich transformation for interacting bosons

  Preprint– arXiv:0801.4960
  We revisit a long standing issue in the theory of disordered electron systems and their effective description by a non-linear sigma model: the hyperbolic Hubbard-Stratonovich (HS) transformation in the bosonic sector. For time-reversal invariant systems without spin this sector is known to have a non-compact orthogonal symmetry O(p,q). There exists an old proposal by Pruisken and Schaefer how to do the HS transformation in an O(p,q) invariant way. Giving a precise formulation of this proposal, we show that the HS integral is a sign-alternating sum of integrals over disjoint domains.