Banff International Research Station

for Mathematical Innovation and Discovery

"Special Structures in Riemannian Geometry"
Sunday, 17-02-2008 to Friday, 22-02-2008

MEALS

◦ Breakfast (Buffet): 07:00–09:00, Sally Borden Building, Monday–Friday
◦ Lunch (Buffet): 11:30–13:30, Sally Borden Building, Monday–Friday
◦ Dinner (Buffet): 17:30–19:30, Sally Borden Building, Sunday–Thursday
◦ Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

∗ *Please remember to scan your meal card at the host/hostess station in the dining room for each meal.* ∗

MEETING ROOMS

All the prescheduled lectures will be held in Max Bell 159. The Max Bell Building is accessible by bridge on the 2nd floor of Corbett Hall. An LCD projector, overhead projectors and blackboards are available for presentations.
Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any food and beverages in those areas.

SCHEDULE

Sunday, 17-02-2008

16:00	Check-in begins (Front Desk – Professional Development Centre – open 24 hours) Lecture rooms are available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour-system.

Monday, 18-02-2008

07:00–08:45	Breakfast
08:45–09:00	Introduction and Welcome to BIRS by the BIRS Station Manager, Max Bell 159
09:00–10:00	**Mark Haskins** (Imperial College London) *"Gluing constructions of special Lagrangians"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Marianty Ionel** (University of Toledo) *"Austere submanifolds of dimension* 4*"*
11:30–13:00	Lunch
13:00–14:00	Guided Tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–15:00	**Sergei Gukov** (UC Santa Barbara) *"Deformations of Hyper-Kähler Metrics and Affine Hecke Algebras"*
15:00–15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:30	**Andrew Swann** (University of Southern Denmark) *"Intrinsic Torsion and Curvature"*
16:30–17:30	Informal discussions
17:30–19:30	Dinner
19:30–20:30	**Michael Anderson** (Stony Brook University) *"Spaces of Einstein metrics on bounded domains"*

Tuesday, 19-02-2008

07:00–09:00	Breakfast
09:00–10:00	**Guofang Wei** (UC Santa Barbara) *"Comparison Geometry for the Smooth Metric Measure Spaces"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Adrian Butscher** (Stanford University) *"Gluing Constructions for Constant Mean Curvature Surfaces"*
11:30–13:30	Lunch
13:30–17:30	Informal discussions / Improptu talks / Recreation
17:30–19:30	Dinner
19:30–20:30	**Marco Gualtieri** (M.I.T.) *"Constructions of generalized Kähler structures"*

Wednesday, 20-02-2008

07:00–09:00	Breakfast
09:00–10:00	**Robin Graham** (University of Washington) *"The ambient metric beyond the obstruction in even dimensions"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Martin Reiris** (M.I.T.) *"The Einstein flow and the Yamabe invariant of three-manifolds"*
11:30–13:15	Lunch
13:15–13:30	Group Photo; meet on the front steps of Corbett Hall
13:30–14:30	**Daniel Pollack** (University of Washington) *"Singular Yamabe metrics and Space-times with Positive Cosmological Constant"*
14:30–15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:00–16:00	**Jim Isenberg** (University of Oregon) *"Constructing solutions of the Einstein constraint equations"*
16:00–17:30	Informal discussions
17:30–19:30	Dinner
19:30–20:30	**Informal Problem Session**

Thursday, 21-02-2008

07:00–09:00	Breakfast
09:00–10:00	**Andrew Dancer** (University of Oxford) *"Symplectic versus hyperKähler geometry"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**John Loftin** (Rutgers University Newark) *"Affine Hermitian-Einstein Metrics"*
11:30–13:30	Lunch
13:30–17:30	Informal discussions / Improptu talks / Recreation
17:30–19:30	Dinner

Friday, 22-02-2008

07:00–09:00	Breakfast
09:00–10:00	**Christina Tonnesen-Friedman** (Union College) *"Hamiltonian 2-forms in Kähler geometry"*
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	**Benoit Charbonneau** (Duke University) *"Existence of periodic instantons"*
11:30–13:30	Lunch
13:30–15:00	Informal discussions / Departures

Please remember to checkout by 12:00 noon.

You are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 15:00 on Friday, although participants are still required to checkout of the guest rooms by 12:00 noon.

"Special Structures in Riemannian Geometry"
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ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Anderson, Michael** (SUNY Stony Brook)
Title: "Spaces of Einstein metrics on bounded domains"

Abstract: There are two natural classes of Einstein metrics on bounded domains. (I) metrics which extend smoothly to the boundary, and (II) complete metrics which conformally extend to the boundary, (conformally compact metrics). We will discuss similarities and differences on the structure of these spaces of Einstein metrics, in particular in regard to the "natural" boundary value problems.

Speaker: **Butscher, Adrian** (Stanford University)
Title: "Gluing Constructions for Constant Mean Curvature Surfaces"

Abstract: I will review the now classical Kapouleas gluing construction for CMC surfaces in Euclidean space and present some results and work in progress concerning the extensions of this theory to general ambient manifolds. An important feature which emerges is that the ambient Riemannian curvature seems to play a significant role in the existence of such surfaces; and exploiting this, it seems possible to construct examples of CMC surfaces having properties very different from their Euclidean analogues.

Speaker: **Charbonneau, Benoit** (Duke University)
Title: "Existence of periodic instantons"

Abstract: Yang–Mills instantons on $S^1 \times \mathbb{R}^3$ (often called calorons) are in correspondence, via the Nahm transform, to solutions to Nahm's equations on the circle. In joint work with Jacques Hurtubise, we completed Nye and Singer's proof of this Nahm transform correspondence.

We also proved that the solutions on the circle are in correspondence, by a twistor transform, to certain classes of vector bundles on an associated twistor space. Those correspondence allow us to compute the moduli space of these objects, settling some very natural existence questions.

Speaker: **Dancer, Andrew** (Jesus College, University of Oxford)
Title: "Symplectic versus hyperKähler geometry"

Abstract: We look at symplectic constructions such as cutting and implosion and investigate their analogues in hyperKähler geometry.

Speaker: **Graham, Robin** (University of Washington)
Title: "The ambient metric beyond the obstruction in even dimensions"

Abstract: The ambient metric construction in conformal geometry will be reviewed, and also the equivalent formal construction of asymptotically hyperbolic Poincaré-Einstein metrics. A modification of the construction in even dimensions will be described which results in a family of smooth infinite order generalized ambient and Poincaré metrics. The generalized ambient metrics can be used to extend conformal invariant theory to all orders in even dimensions. This is joint work with Kengo Hirachi.

Speaker: **Gualtieri, Marco** (Massachusetts Institute of Technology)
Title: "Constructions of generalized Kähler structures"

Abstract: I will describe a construction of generalized Kähler structures on holomorphic Poisson manifolds, which uses the concept of a holomorphic Poisson module. I will also describe some properties of the resulting generalized Kähler metric, which is a Riemannian metric admitting two different Hermitian complex structures.

Speaker: **Gukov, Sergei** (University of California, Santa Barbara)
Title: "Deformations of Hyper-Kähler Metrics and Affine Hecke Algebras"

Abstract: I will explain how studying deformations of hyper-Kähler metrics on complex coadjoint orbits can provide a simple geometric explanation of certain deep results in representation theory, including categorification of the affine Hecke algebra. This talk is based on a joint work with Edward Witten.

Speaker: **Haskins, Mark** (Imperial College London)
Title: "Gluing constructions of special Lagrangians"

Abstract: We describe joint work with Nicos Kapouleas that constructs infinitely many special Lagrangian cones whose link is an orientable surface of genus 4 or of any odd genus. These are the first special Lagrangian cones with links that are surfaces of genus greater than one. We use a geometric PDE 'gluing' method. Time permitting, we will sketch higher dimensional generalisations of these gluing constructions.

Speaker: **Ionel, Marianty** (University of Toledo)
Title: "Austere submanifolds of dimension 4"

Abstract: An austere submanifold has the property that its second fundamental form in any normal direction has its eigenvalues symmetrically arranged around zero. The class of austere submanifolds was first introduced by Harvey and Lawson in 1982. The main motivation was their result showing that the conormal bundle of an austere submanifold in $\mathbb{R}^n$ is a special Lagrangian submanifold of $\mathbb{R}^{2n}$. The austere submanifolds of dimension 3 in Euclidean space were classified by R. Bryant. In this talk I will present some results towards a classification of austere submanifolds of dimension 4 in Euclidean space. Depending on the type of the second fundamental form, we get both non-existence results as well as new examples of austere submanifolds. This is joint work with Thomas Ivey.

Speaker: **Isenberg, Jim** (University of Oregon)
Title: "Constructing solutions of the Einstein constraint equations"

Abstract: The first step in finding a spacetime solution to the Einstein gravitational field equations via the inital value formulation is to construct initial data which satisfy the Einstein constraint equations. There are three ways of carrying out this construction which have been found to be useful: the conformal and conformal thin sandwich methods, the gluing techniques, and the quasi-spherical approaches. We describe each of these, we discuss their advantages and disadvantages, we outline some of their recent successful applications, and we present some of the outstanding questions remaining to be solved from each of these perspectives.

Speaker: **Loftin, John** (Rutgers University Newark)
Title: "Affine Hermitian-Einstein Metrics"

Abstract: A special affine manifold is a manifold with an atlas whose gluing maps are all constant affine maps in $\mathbb{R}^n$ preserving the standard volume form. The tangent bundle to a special affine manifold has the structure of a complex manifold with holomorphic volume form. We develop a theory of stable bundles and affine Hermitian-Einstein metrics for flat vector bundles over a special affine manifold. The proof involves adapting the proof of Uhlenbeck-Yau on the existence of Hermitian-Einstein metrics on Kähler manifolds, and the extension of this theorem by Li-Yau to the non-Kähler complex case of Gauduchon metrics. Our definition of stability involves only flat vector subbundles (and not singular subsheaves), and so is simpler than the complex case in some places.

Speaker: **Pollack, Dan** (University of Washington)
Title: "Singular Yamabe metrics and Space-times with Positive Cosmological Constant"

Abstract: The Delaunay (aka Fowler) metrics form the asymptotic models for isolated singularities of conformally flat metrics of constant positive scalar curvature metrics. The Kottler-Schwarzschild-de Sitter space-times form the model family for black hole solutions of the Einstein field equations with a positive cosmological constant. We will show why the former coincides with the time-symmetric initial data sets for that latter. We will then demonstrate how to construct large families of initial data sets for the vacuum Einstein equations with positive cosmological constant which contain exactly Delaunay ends; these are non-trivial initial data sets whose ends coincide with those for the Kottler-Schwarzschild-de Sitter metrics. From the purely Riemannian geometric point of view, this produces complete, constant positive scalar curvature metrics with exact Delaunay ends which are not globally Delaunay. The construction provided applies to more general situations where the asymptotic geometry may have non-spherical cross-sections consisting of Einstein metrics with positive scalar curvature. This is joint work with Piotr Chrusciel.

Speaker: **Reiris, Martin** (Massachusetts Institute of Technology)
Title: "The Einstein flow and the Yamabe invariant of three-manifolds"

Abstract: We will explain how the long time evolution of the Einstein flow is related with the Yamabe invariant of three-manifolds. We will set the main conjectures and elaborate on partial results. The discussion will be based on the importance of volume in General Relativity.

Speaker: **Swann, Andrew** (University of Southern Denmark)
Title: "Intrinsic Torsion and Curvature"

Abstract: For a Riemannian G-structure a large part of the Riemannian curvature is determined by the intrinsic torsion. Representation theoretic techniques lead to a number of constraints and relations and in many situations much can be gleaned from the exterior algebra. This talk will discuss recent work in this area, including particular results for almost Hermitian and almost quaternion-Hermitian structures.

Speaker: **Tønnesen-Friedman, Christina** (Union College)
Title: "Hamiltonian 2-forms in Kähler geometry"

Abstract: Hamiltonian 2-forms, introduced in [1], induce isometric Hamiltonian torus actions and underpin many explicit constructions in Kähler geometry. This talk will take off as a survey and discussion of the techniques developed in [1] and [2]. Some of these techniques have already been applied in subsequent works with my co-authors, but we believe that there are still exciting avenues to take – in particular in the case of higher order Hamiltonian 2-forms and, in general, higher order toric bundles.
[1] "Hamiltonian 2-forms in Kähler geometry, I General Theory"; V. Apostolov, D.M.J. Calderbank, and P. Gauduchon; *J. Diff. Geom.*, **73** (2006), 359–412.
[2] "Hamiltonian 2-forms in Kähler geometry, II Global Classifications"; V. Apostolov, D.M.J. Calderbank, P. Gauduchon , and C. Tønnesen-Friedman; *J. Diff. Geom.*, **68** (2004), 277–345.

Speaker: **Wei, Guofang** (University of California at Santa Barbara)
Title: "Comparison Geometry for the Smooth Metric Measure Spaces"

Abstract: For a smooth metric measure space $(M, g, e^{-f}dvol_g)$ the Bakry-Emery Ricci tensor is a natural generalization of the classical Ricci tensor. It occurs naturally in the study of diffusion processes, Ricci flow, the Sobolev inequality, warped products, and conformal geometry. We prove mean curvature and volume comparison results when the ∞-Bakry-Emery Ricci tensor is bounded from below and f is bounded or $\partial_r f$ is bounded from below, generalizing the classical ones (i.e. when f is constant.) This leads to extensions of many theorems for Ricci curvature bounded below to the Bakry-Emery Ricci tensor. In particular, we give extensions of all of the major comparison theorems when f is bounded. Simple examples show the bound on f is necessary for these results. This is a joint work with W. Wylie.
